

Lecture Notes in Physics

Sumati Surya

The Causal Set Approach to Quantum Gravity

An Introduction

 Springer

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The Causal Set Approach to Quantum Gravity

An Introduction

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*To Amma and Appa who gave me the freedom
to question.*

Preface

This book is intended for current practitioners in quantum gravity, as well beginners to research in the field. It aims to give a broad overview of the contours of the causal set approach, starting from the main hypothesis, through some well-established results, arriving all the way to the frontiers of our understanding. It presents some of the key results in causal set theory, with enough detail to make it possible for the interested reader to begin working in the subject. Scattered throughout the book are exercises that will give the reader simple practice with causal sets.

Chapter 1 offers a non-technical overview of Causal Set Theory (CST) within the context of quantum gravity, while Chap. 2 introduces the causal set hypothesis, foundational theorems in Lorentzian geometry, and the discrete-continuum approximation that supports CST's core ideas. Chapter 3 explores how continuum geometry and topology are approximated by random causal sets, showcasing CST's ability to encode continuum spacetime structures in a discrete substratum. Chapter 4 examines quantum scalar fields on causal sets, discussing the potential effects of discreteness on quantum fields and introducing a causal set based calculation of entanglement entropy. CST dynamics, covered in Chap. 5, leverages nonlocality in the non-perturbative CST path sum, to prove that the most dominant non-spacetimelike contributions are suppressed. This chapter also elaborates on the Lorentzian state-sum models and the more fundamental sequential growth models. The final Chap. 6 is a brief discussion of CST's phenomenological implications, starting with Sorkin's surprisingly accurate 1987 prediction for the cosmological constant, and the discreteness induced particle "swerves", ending with a short list of open questions.

Bengaluru, India
August 2025

Sumati Surya

*No one can teach you aught but that which lies half asleep in the
dawning of your knowledge.*

—Khalil Gibran

Acknowledgements The quote above captures how the relativist inside us might respond to the ideas behind the causal set approach to quantising gravity.

This book is a result of the work and dedication of those who have contributed to causal set theory. Rafael Sorkin's deep insight into Lorentzian geometry and quantum foundations has helped forge the two into a coherent theory of quantum gravity. I am forever indebted to him for opening the doors to a whole new way of thinking of physics, and for his generosity in sharing his ideas. Fay Dowker has been a long term collaborator and friend without whose encouragement I may never have begun the daring foray into causal set waters. Her ability to concretise vague thoughts into actionable mathematical ideas has been invaluable, and my understanding has been enriched by countless discussions with her. I am also very lucky to have worked with many others on causal set theory who I list here in alphabetical order: Anish Bhattacharya, Graham Brightwell, Steve Carlip, Will Cunningham, Astrid Eichhorn, L. Glaser, Joe Henson, Steven Johnston, Ian Jubb, Seth Major, Abhishek Mathur, Denjoe O'Connor, Abhishek Ravishankar, Kasia Rejzner, David Rideout, Alan Daniel Santhosh, Anup Anand Singh, Ritesh Srivastava, Fleur Versteegen, Nomaan X., Yasaman Yazdi and Stav Zalel. Together, we have scratched our heads and argued our way through thick forests of ideas, making progress when possible, leaving some ideas to slowly simmer. This community has given each of us enough room to be creative, while at the same time providing ample stimulation, thanks in large measure to the energies and insights of Rafael Sorkin and Fay Dowker.

I am deeply grateful to the Raman Research Institute for providing a calm, supportive environment over the years, conducive to work in quantum gravity. Work like ours would not be possible without several kinds of help. I thank the administration at RRI, especially Manju, Gayathri and Mahadeva for their kind support over the years.

A special thanks to Nomaan X for regenerating plots from unused data and for Christoph Minz for his causal set graphics package. I also thank Abhishek Mathur for his patient reading of parts of the text. During the last stages of writing this book I was also partly supported by the Blaumann Foundation Grant and the SERB MATRICS Grant Number MTR/2023/000831.

I thank my editors for their patience (and final impatience) at my continual missing of deadlines. I would have liked to write this book over a decade, slowly and with care, but as they say, time waits for no one.

Finally, I am delighted to thank my family: Sachin, Mythili, Yasmin, Amma, Appa and Arjun, for their love and patience as I slogged through the writing of

this book. Thanks in particular to Yasmin who helped me type in changes to the manuscript when I was nursing an injury. It is my hope that our daughters will continue to carry with them a love of mathematics and the natural world as they make their way in this increasing complex world.

Qn. Do you think you can travel back in time?

Ans. Not in Reality. Only in your imagination.

Conversation with Yasmin, aged 12

Bengaluru, India
August 2025

Sumati Surya

Competing Interests The author has no competing interests to declare that are relevant to the content of this manuscript.

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Acronyms

BDG	Benincasa-Dowker-Glaser
CHK	Caratheodory-Hahn-Kluvanek
CNN	Convex Normal Neighbourhood
CSG	Classical Sequential Growth
CSG	Complex Sequential Growth
CST	Causal Set Theory
EE	Entanglement Entropy
GNN	Gaussian Normal Neighbourhood
GPPP	General Poisson Point Process
HH	Hartle-Hawking
HKMMKP	Hawking-King-McCarthy-Malament-Kronheimer-Penrose
KCH	Kolmogorov-Caratheodary-Hahn
KR	Kleitmann-Rothschild
MCMC	Markov Chain Monte Carlo
PJ	Pauli-Jordan
QSG	Quantum Sequential Growth
RNN	Riemann Normal Neighbourhood
SJ	Sorkin-Johnston
SOH	Sum-Over-Histories
SSEE	Sorkin's Spacetime Entanglement Entropy

Notations

General Notation

$\mathbb{N}$	Natural Numbers
$\mathbb{R}$	Real Line
$\mathbb{C}$	Complex Plane
$ A $	Cardinality of the set A
A^c	Complement of a subset $A \subset B$ in the set B
$\bar{A}$	Closure of a set A

Continuum Notation

$\prec$	Irreflexive causality relation $x \prec y$, $x \not\prec x$
$\preceq$	Reflexive causality relation, with $x \preceq y \preceq x \Rightarrow x = y$
$\ll$	Irreflexive chronology relation $x \ll y$
$\rightarrow$	Horismotic or light-like relation $x \rightarrow y$, reflexive
$\parallel$	Spacelike relation
$I^\pm(p)$	Chronological future and past, respectively, of an event p
$J^\pm(p)$	Causal future and past, respectively, of an event p
$I(p, q)$	$\equiv \{r p \ll r \ll q\}$ is an Alexandrov interval
$\text{vol}_g(X)$	Spacetime volume of the region X in a spacetime (M, g)
$\text{vol}(p, q)$	$\equiv \text{vol}(I(p, q))$, spacetime volume of $I(p, q)$
$\mathbb{M}^d$	d -dimensional Minkowski spacetime
η_{ab}	Minkowski metric
$\mathbb{D}^d$	$\equiv I(p, q) \subset \mathbb{M}^d$ for some $p, q \in \mathbb{M}^d$
$\tau(p, q)$	Proper time from p to q
$\text{vol}(\mathbb{D}^d)$	$\equiv \zeta_d \tau(p, q)^d$
ζ_d	$\equiv S_{d-2} \frac{2^{1-d}}{d(d-1)}$

$$S_{d-2} \equiv \frac{2\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)}, \text{ the volume of the round } (d-2)\text{-dimensional unit sphere}$$

$$\mathbb{S}^{d-2}$$

$$\mathbf{s}_d \equiv S_{d-2} \frac{1}{d(d-1)}$$

Causal Set Notation

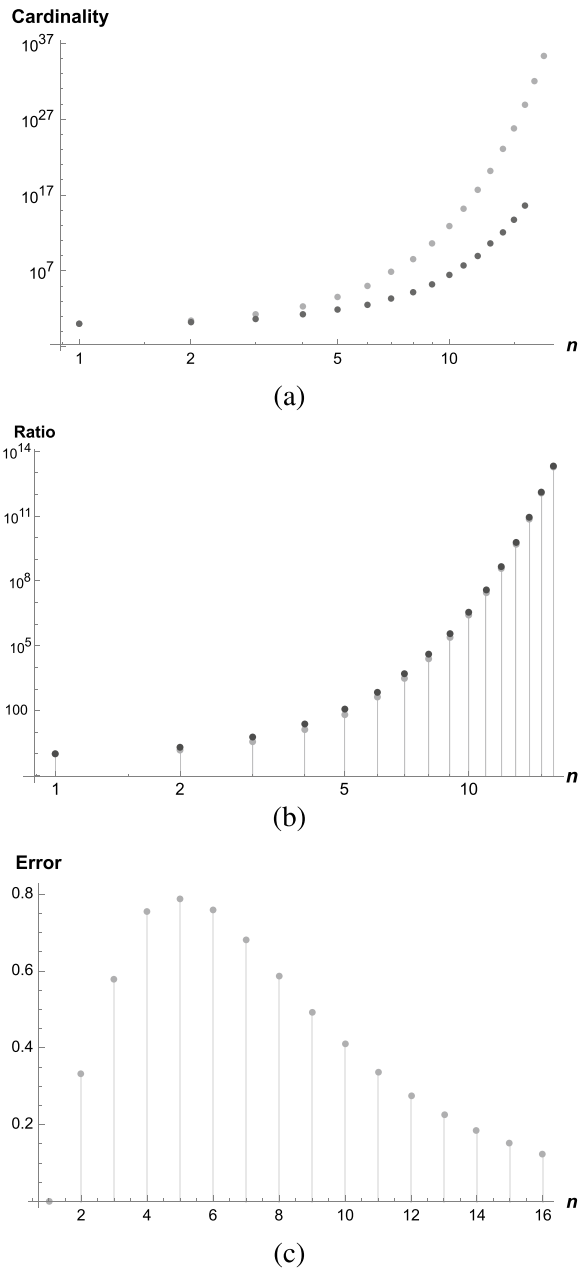
$<$	Irreflexive order relation
$\parallel$	Incomparable or spacelike: $e \parallel e' \Rightarrow e \not\prec e' \ \& \ e' \not\prec e$
C	Causal set with an irreflexive order relation $<$
$\text{Fut}(e)$	$\equiv \{e' \in C \mid e < e'\}$, Future of e
$\text{Past}(e)$	$\equiv \{e' \in C \mid e' < e\}$, Past of e
$\text{IFut}(e)$	$\equiv \{e' \in C \mid e \preceq e'\}$, Inclusive future of e
$\text{IPast}(e)$	$\equiv \{e' \in C \mid e' \preceq e\}$, Inclusive past of e
(e, e')	$\equiv \{e'' \in C \mid e < e'' < e'\} = \text{Fut}(e) \cap \text{Past}(e')$, Order interval
$[e, e']$	$\equiv \{e'' \in C \mid e \preceq e'' \preceq e'\} = \text{IFut}(e) \cap \text{IPast}(e')$, Closed order interval
$M_C(e, e')$	Causal Matrix element
$\mathfrak{C}_\rho(M, g)$	Ensemble of causal sets generated from a Poisson sprinkling into (M, g) at density ρ
A^{cc}	Causal complement of the subcausal set $A \subset C$ in C

In this appendix we provide a list of different types of causal sets, some of which can be used as alternatives for the exercises.

An explicit enumeration of the set Ω_n of unlabelled posets is known upto $n = 16$ [238] and the values of $|\Omega_n|$ are available as Sequence A000112 of “The On-Line Encyclopedia of Integer Sequences” [239]. The enumeration of the set $\tilde{\Omega}_n$ of the set of labelled posets is known upto $n = 18$ and the values of $|\tilde{\Omega}_n|$ are available as the Sequence A001035 in the same reference [240]. Figure A.1a shows $|\Omega_n|$ and $|\tilde{\Omega}_n|$ as a function of n .

As we have discussed earlier in the text, the ratio $R_n = |\tilde{\Omega}_n|/|\Omega_n|$ can be compared to $n!$, the number of relabellings on n elements, as shown in Fig. A.1b. As n becomes larger, the two approach each other with the “error” $(n! - R_n)/R_n$ in Fig. A.1c dipping to 0.123611 at $n = 16$. The difference is caused by the possibility of non-trivial **automorphisms** in a labelled causal set. Let $\tilde{C} \equiv l(C)$ be a labelling of C and consider a relabelling $L : \tilde{C} \rightarrow \tilde{C}'$ of C . If $\tilde{C}' = \tilde{C}$, then L is said to be an automorphism of $\tilde{C}$. Such automorphisms exist whenever there are **indistinguishable pairs** $e, e' \in C$, i.e., for which $Past(e) = Past(e')$ and $Fut(e) = Fut(e')$. Starting with any labeling $\tilde{C} \equiv l(C)$ the switching of labels of indistinguishable pairs, $l(e) \leftrightarrow l(e')$ gives back the same labelled causal set $\tilde{C}$. Hence the number of labelings is less than $n!$ —one must instead divide $n!$ by the number of automorphisms.

Fig. A.1 **a** A log-log plot of the cardinality of the set of causal sets as a function of the size n of the causal set, for $n = 1$ to $n = 16$. The points in dark grey are the unlabelled causal sets, i.e., $|\Omega_n|$, while those in light grey are the labelled causal sets $|\tilde{\Omega}_n|$. **b** A log-log plot of the ratio of the number of labelled to unlabelled posets (dark grey) compared to $n!$ (light grey). As n becomes larger, the two graphs merge. **c** The “error” as a function of n



In [191] it was shown that

Theorem A.1 (Promel) *There exists a constant s such that for all n ,*

$$|\Omega_n| \leq \frac{|\tilde{\Omega}_n|}{n!} \left(1 + \frac{s}{2^{n/4}} \right)$$

This implies that

Corollary A.1 (Promel) *Almost all partial orders are **rigid**, i.e., have no non-trivial automorphism.*

The following figures are examples of unlabelled causal sets that can be used to supplement the exercises in the book (Figs. A.2, A.3, A.4, A.5, A.6, A.7, A.8, A.9).

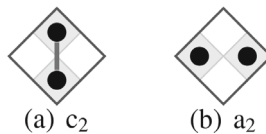


Fig. A.2 The set Ω_2 of $n = 2$ unlabelled posets, $|\Omega_2| = 2$ consists of **a** the chain c_2 and **b** the antichain a_2 . The number of naturally labelled $n = 2$ posets $|\Omega_2^{(nl)}| = 2$ as well, since there is only one natural labelling each for both the c_2 and the a_2 . The number of labelled posets on the other hand is $|\tilde{\Omega}_2| = 3$: c_2 allows $2!$ relabellings but a_2 , whose automorphism group is also of cardinality 2, does not admit any relabellings

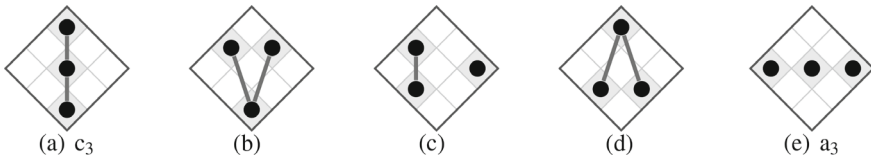


Fig. A.3 The set Ω_3 of $n = 3$ unlabelled posets, has cardinality $|\Omega_3| = 5$. The posets **a**, **b**, **d** and **e** allow only one natural labelling each, while **c** admits three. Hence, $|\Omega_3^{(nl)}| = 7$. The number of all possible labellings is distributed as follows: **a** has $3! = 6$, **b** has $3!/2! = 3$ since the number of automorphism is 2, **c** has $3! = 6$, **d** has again $3!/2!$ since the number of automorphisms is 2 and finally **e** has $3!/3! = 1$ since the number of automorphisms is $3!$. Thus, $|\tilde{\Omega}_3| = 19$

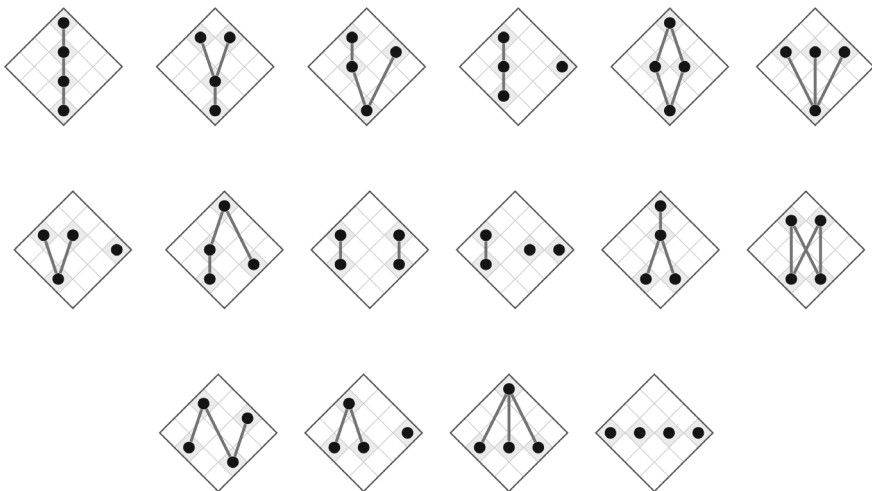


Fig. A.4 The set of $n = 4$ posets

Fig. A.5 The set of
 $n = 4, 6, 8$ crown posets

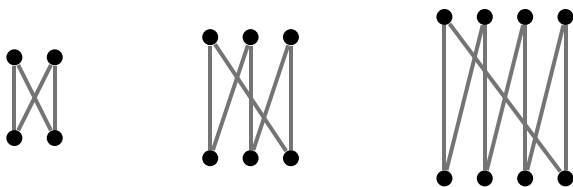


Fig. A.6 An example of a
dominant $n = 16$ bilayer
order with $dn^2 = \frac{1}{8}n^2$
relations

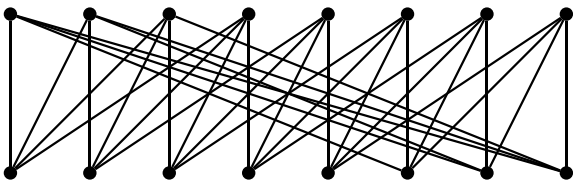
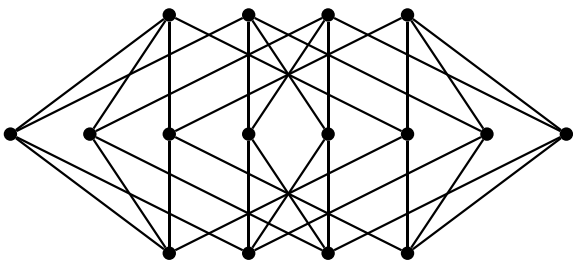


Fig. A.7 An example of an
 $n = 16$ KR order with
 $dn^2 = \frac{3}{16}n^2$ relations



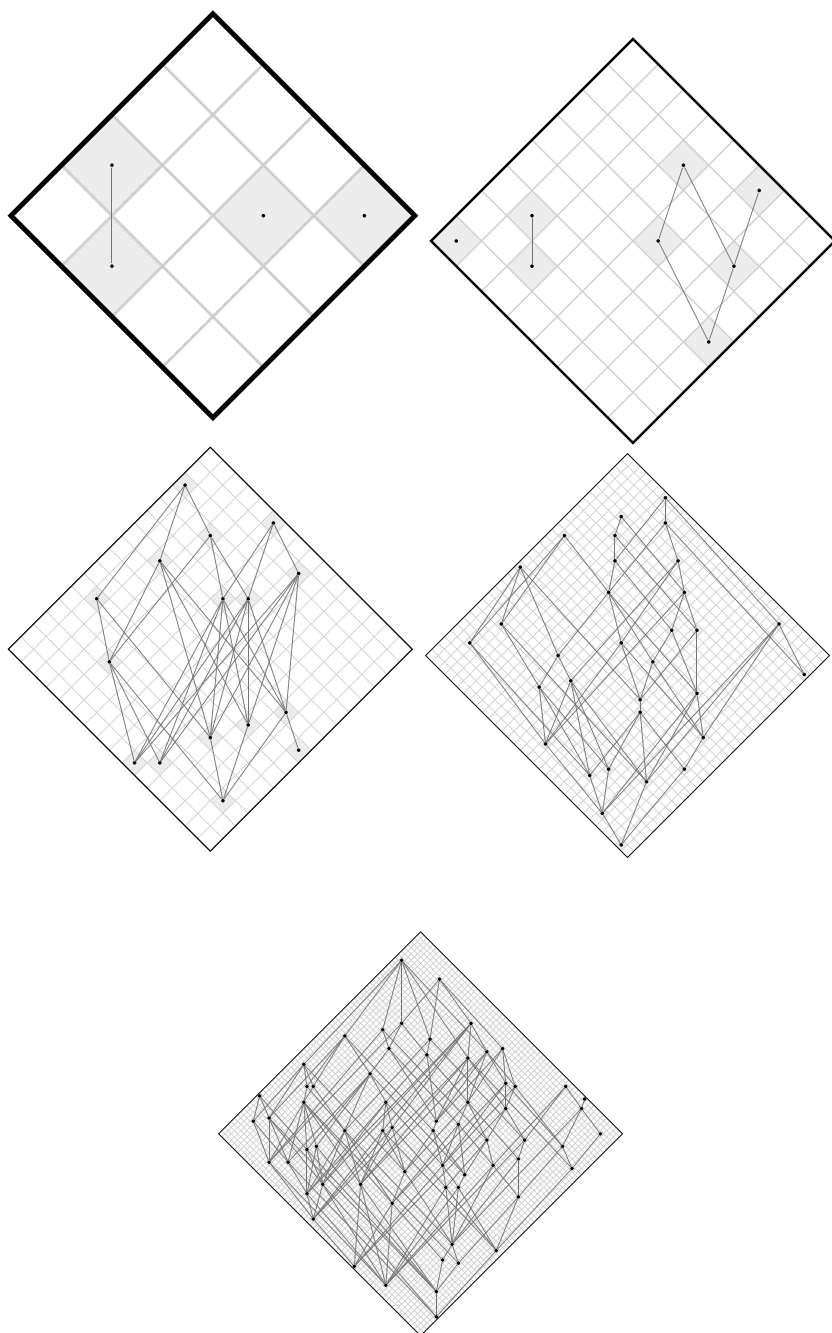


Fig. A.8 Random 2d orders of size $n = 4, 8, 16, 32, 64$. As the density of sprinkling increases, the connectivity does as well, and hence captures more of the continuum geometry

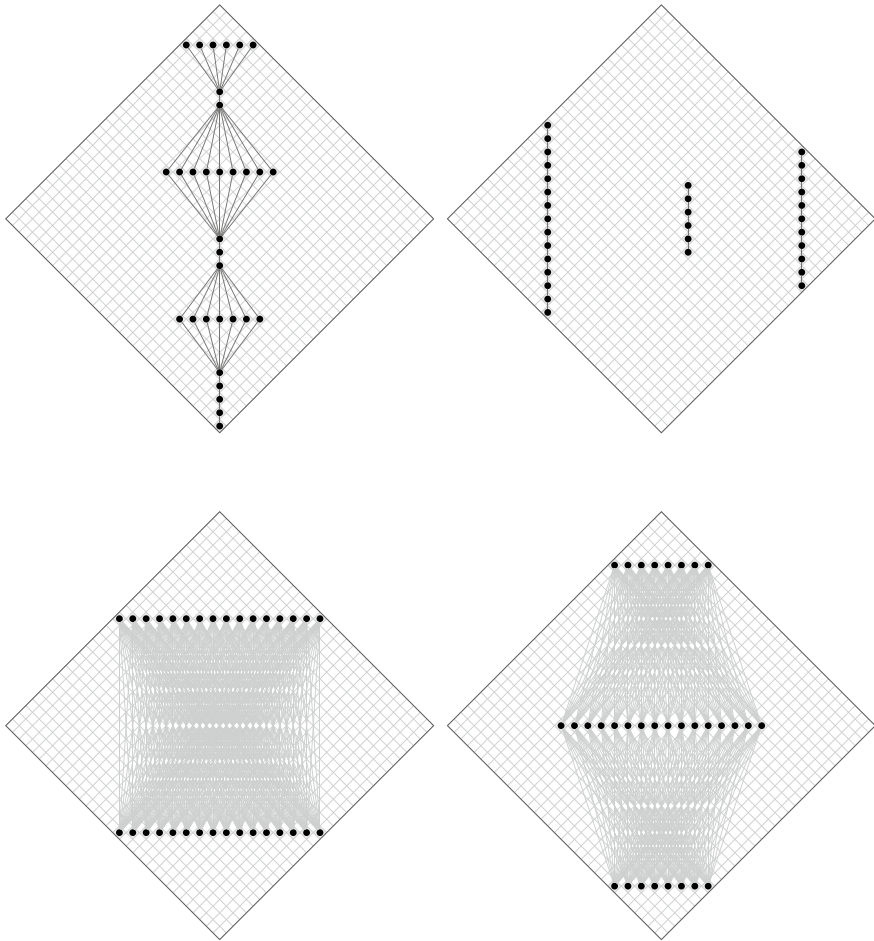


Fig. A.9 Examples of 2d orders of size $n = 32$ which embed into the causal diamond but not faithfully. They are not continuumlike. The two causal sets at the bottom are layered and maximally linked, and do not therefore represent the dominant bilayer posets or the KR posets of [20,21,50]

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