

RESEARCH NEWS

Materials with negative Poisson's ratio

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It is a matter of common experience that solids when stretched in a particular direction suffer a contraction in the lateral direction. The ratio of lateral strain of contraction to longitudinal strain of extension is the Poisson's ratio, σ . It is always a constant for a given material. Most solids have a positive Poisson's ratio. Interestingly, the physics of elastic bodies does not impose any restrictions on the sign of σ . When a solid is stretched in one direction it can also expand in the perpendicular direction (see Figure 1). For isotropic materials it

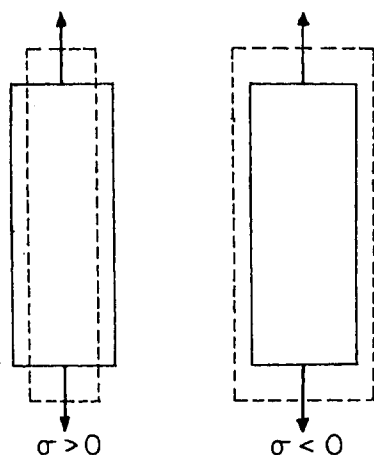


Figure 1. Effect of uniaxial tension on a strip of an isotropic material. The deformed structure is indicated by the dotted line.

has a lower bound of -1 and an upper bound of $+1/2$. At $\sigma = -1$ the material has infinite rigidity modulus and at $\sigma = +1/2$ the material has infinite bulk modulus. It therefore intrigued many, including Poisson, that a large class of materials should have only a positive Poisson's ratio. Poisson himself came up with a molecular model that lead to $\sigma = +1/4$. In this model an assembly of spheres interact only along the line joining the centres with no force required for tangential displacements. This triggered an extensive search for materials with negative σ . The lowest value of σ was reported by Poisson himself in the case of cork ($\sigma \approx 0$). Poisson argued that bottle stoppers made of cork can be cylindrical in shape due to σ of cork being nearly zero. They will not laterally expand when axially compressed. On the other hand, it is nearly impossible to push in, through a bottleneck, a cylindrical stopper made of rubber since its σ is close to 0.5 . Hence the stopper has to be of a tapered conical form. In the beginning, workers in this field confined themselves to looking at isotropic solids and without a single exception they all had positive σ . It was in the world of crystals that they had some success in finding negative Poisson's ratio. Here again a majority of crystals had a positive σ but

a limited number of them exhibited negative σ . Single crystals of zinc and ammonium dihydrogen phosphate are examples of this rare class¹. It must be remarked that both these are anisotropic crystals and negative σ is observed only for certain directions of axial strain. However, Love² mentions a single example of a cubic crystal of pyrite with $\sigma = -0.14$. He suggested that this negative value may have resulted from crystal twinning.

It is against such a background that the recent paper in *Nature*³ on 'Microstructure of isotropic materials with negative Poisson's ratio' by Rothenburg *et al.* assumes some significance. The authors have undertaken an *engineering* analysis of the problem. They were inspired in this exercise by the work of Lakes⁴ on foams. Lakes produced, by a special process, foams of negative Poisson's ratio from conventional low-density open-cell polymer foams, which in their natural state have positive σ like any other isotropic material. Lakes subjected this normal foam to a triaxial compression, i.e. equal compression in three perpendicular directions and heated the foam to a temperature slightly above its softening temperature. The mold was then cooled to room temperature. This foam had undergone a compression by a factor of 1.4 to 4 . Lakes found such compressed foams to exhibit negative Poisson's ratio. He got consistent results with foams made of different polymers and having different cell sizes. Interestingly reticulated metal

foams transformed by plastic deformation at room temperature also exhibited negative σ . This unusual behaviour of compressed foams was traced to their unusual cell structure—described as *reentrant structure* by Lakes. An example of such a reentrant cell is shown in Figure 2. This was obtained by a symmetrical collapse of a 24-sided polyhedron of cubic symmetry. A tension applied to the vertical links of such a structure will lead to cell unfolding and lateral expansion, i.e. negative Poisson's ratio. Study of the compressed foams under an optical microscope indicated the existence of such cells with ribs protruding into the cell.

In essence, the work of Rothenburg *et al.* is an elaboration of this interesting idea of *reentrant* cells. They have suggested other possible types of structural units that lead to a negative Poisson's ratio. The common feature of all these structural elements is that they have a very low *axial stiffness* (akin to Young's modulus y) compared to *tangential stiffness* (rigidity modulus n). This is rather understandable because the continuum theory of elasticity of isotropic bodies leads to the relation $\sigma = (y - 2n)/(y + 2n)$. This implies that y should be less than $2n$ for σ to be negative. In a simpler language this can be interpreted to mean that it is far easier to stretch or compress the structural element than shear it. Often this is stated differently to mean that if the elements of a structure are rigid as regards their shape but flexible as regards their size, then an isotropic network of such elements usually leads to negative values for σ . In fact in two dimensions one can show that $\sigma = (1 - \lambda)/(3 + \lambda)$, with λ as the ratio of the tangential stiffness to the axial stiffness. Whenever $\lambda > 1$, we end up with negative

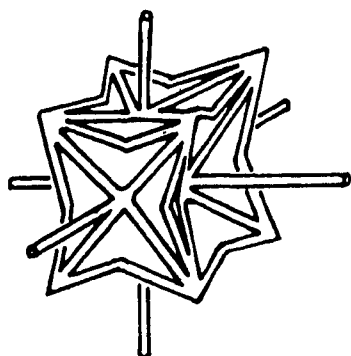


Figure 2. A possible reentrant structure (after Lakes*).

σ . This paper presents results on isotropic networks made up of identical structural units. Three different types of structural units have been discussed in detail to bring home this point that negative σ is a consequence of axial stiffness being much smaller than tangential stiffness.

Shock absorber-like units

Let us say the structural element is very much like a shock absorber, i.e. a spring cushioned piston moving inside a socket (Figure 3a). This object can be easily

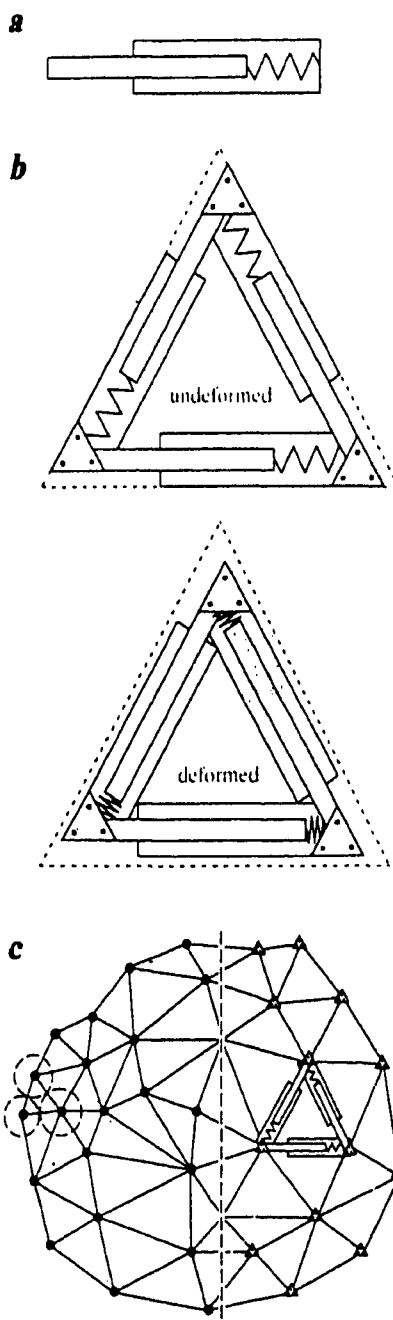


Figure 3. a, A shock-absorber unit; b, a rigid equilateral triangle of shock absorbers; c, an isotropic network of shock absorbers. (After Rothenburg *et al.*.)

stretched or compressed but it is nearly impossible to shear it. We consider, as an example, an assembly of such units all rigidly fixed to be in the vertical direction. Then under a tensile (or compressive) stress applied along the vertical, we find only axial extension (or compression). There will be no accompanying lateral contraction (or extension). In other words such a network of shock absorbers has $\sigma = 0$. Rothenburg *et al.* considered an equilateral triangular element (Figure 3b) as a structural unit. Under an axial compression, the unit shrinks overall to an equilateral triangular element of smaller size, since the shock absorbers are rigidly fixed at the vertices. This means that we have a lateral contraction under longitudinal compression. Hence the triangular structure has a negative σ . One can build an isotropic network (Figure 3c) with such triangles allowing for all possible orientations. These authors showed that such a network will also exhibit a negative σ . The whole network will shrink under an axial compression or swell under an axial extension.

Kink-like units

The previous example is more in the nature of an engineering structure and is thus not relevant to natural physical systems. A structural unit that is rather realistic in the world of polymers was considered next. It is very easy to see that a rod is difficult to compress or extend axially. But the same rod bent into a kink (as shown in Figure 4a) is easy to compress or extend axially along the arms. It has a much reduced axial stiffness. The shear stiffness for both the rod and the kink forms are nearly the same. Hence a network (shown in Figure 4b) built out of such kinked elements is likely to have negative σ . Calculations by Rothenburg *et al.* indicate this to be indeed the case. But interestingly this happens only when the compressional ratio (i.e. ratio of the length of the rod to that of the kink) is greater than 1.5. Only then $\lambda > 1$ (Figure 4a). For smaller values of the compressional ratio we have positive Poisson's ratio. Hence a network of kinked elements can exhibit negative σ . This example can be looked upon as the two-dimensional analogue of Lakes' compressed foam with reentrant structural units.

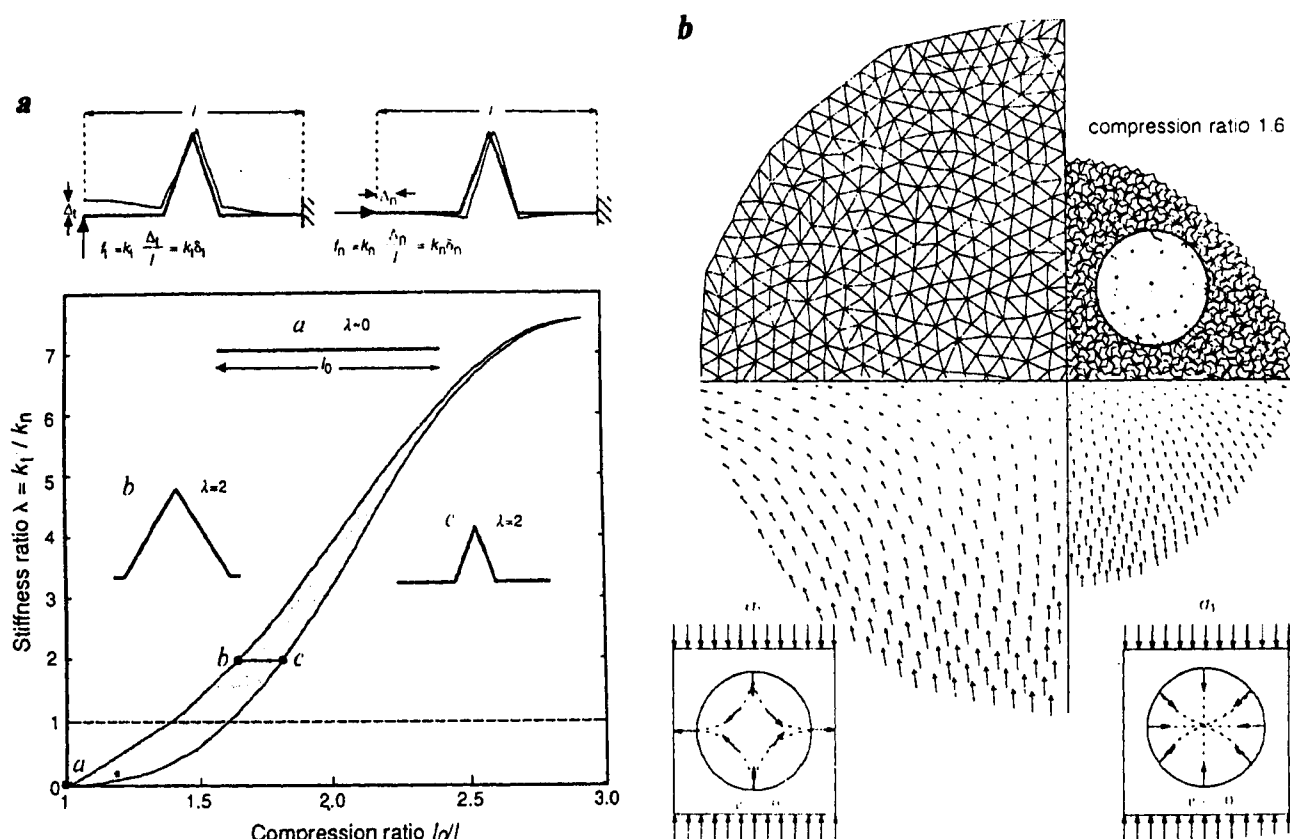


Figure 4. *a*, Compressed bar elements. Insets *a*, *b* and *c* show bar shape corresponding to points *a*, *b* and *c* on the curve. (After Rothenburg *et al.*³) *b*, Behaviour of an isotropic network of bars (left) and of kinks (right) under a uniaxial compression. Arrows indicate direction of movement of junction points.

Interpenetrating hexagons

The interesting fact that a very low axial stiffness leads to negative σ helps us to appreciate an interesting result of Wojciechowski⁵ obtained around the same time. A computer simulation showed that an isotropic 2D solid built out of

hexamer molecular clusters (Figure 5) leads to negative σ provided they can penetrate each other. On the other hand, non-penetrating hexamers always leads to positive σ . One can interpret this result in terms of axial stiffness. It is not difficult to convince oneself that in the presence of interpenetration axial

stiffness is very much reduced, compared to tangential stiffness which will be nearly the same whether interpenetration exists or not. An extension of this work can easily be imagined in 3D. Interpenetrating spherical molecular clusters can, in principle, lead to negative σ . One is thus at the threshold of

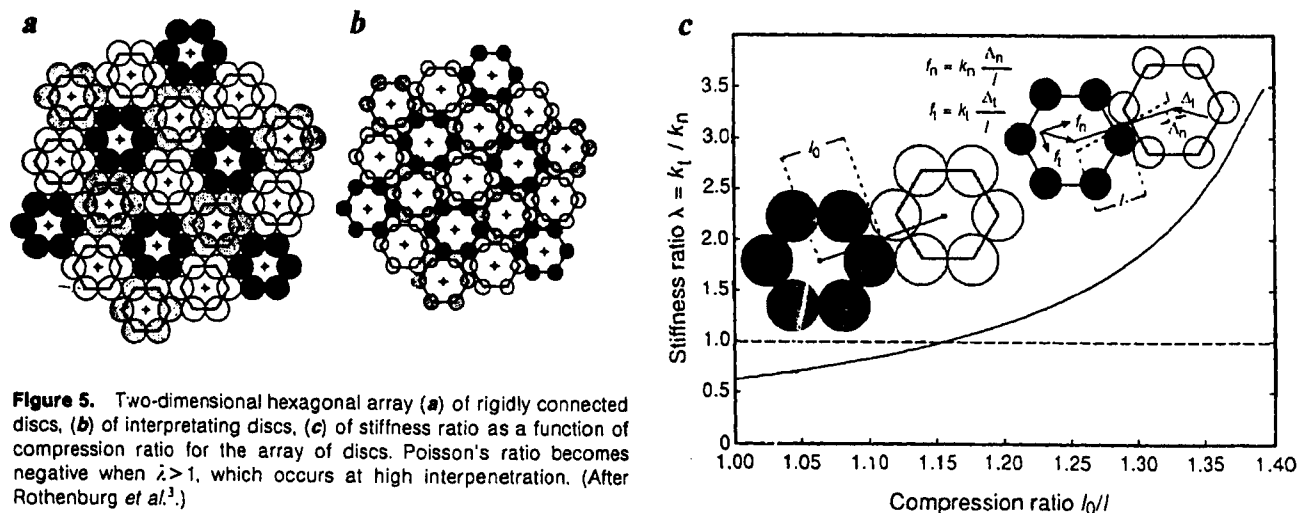


Figure 5. Two-dimensional hexagonal array (*a*) of rigidly connected discs, (*b*) of interpenetrating discs, (*c*) of stiffness ratio as a function of compression ratio for the array of discs. Poisson's ratio becomes negative when $\lambda > 1$, which occurs at high interpenetration. (After Rothenburg *et al.*³)

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almost synthesizing an isotropic solid with negative Poisson's ratio.

Materials with negative σ are expected to have many interesting elastic properties. Since real isotropic materials with negative σ are yet to be synthesized, one can only extrapolate from the results obtained with foams of negative σ . Such materials can be expected to have a large elastic resilience. Normal materials are linearly elastic only up to 5% strain. But foams with reentrant

units are linear up to 40% strain. These materials are also expected to have large energy absorption and fracture resistance⁴. In the light of these facts we can look forward to a fertile field of study on the elastic properties of solids with negative Poisson's ratio.

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