

Coherent microwave-to-optical conversion with dilute gaseous atoms

by

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for the degree of

Doctor of Philosophy



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DECLARATION

I, **Adwaith K V** (Enrolment No.: RRI/2015/002), hereby declare that the work presented in this thesis titled ‘*Coherent microwave-to-optical conversion with dilute gaseous atoms*’ is completely original. This thesis is composed independently by me at **Raman Research Institute**, Bengaluru under the supervision of **Prof. Andal Narayanan** and is the result of my own work unless otherwise stated. I further declare that the subject matter presented in this thesis has not formed earlier the basis for the award of any degree, diploma, membership, associateship, fellowship or any other similar title of any university or institution. I also declare, this thesis has been checked through the plagiarism software **Turnitin**.

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CERTIFICATE

This is to certify that the work contained in the thesis titled ‘*Coherent microwave-to-optical conversion with dilute gaseous atoms*’, submitted by *Adwaith K V* (Enrolment No.: RRI/2015/002) to the Jawaharlal Nehru University for the award of the degree of *Doctor of Philosophy (Ph.D.)* in Physical Sciences, is the bonafide record of original research work carried out by the candidate from August 2015-August 2021, under my guidance and supervision at Raman Research Institute (RRI), Bengaluru, India. The results embodied in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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Synopsis

This thesis deals with investigations on the coherent microwave-to-optical conversion with dilute gaseous atoms. In particular, our study deals with novel effects in a three-level atomic system of room temperature Rubidium atoms that coherently interacts with one microwave and two optical, electromagnetic fields. A three-level atomic system coupled to two laser fields exhibits interference effects that can result in the cancellation of absorption at a resonance transition frequency and thereby modify the optical response. Analogous to the interference of classical waves resulting in constructive and destructive interference, it is the field amplitudes that interfere in the quantum case. If there is more than one pathway available for an atom to have a transition from the ground to the excited state, then the probability of the transition is the sum of the probability amplitude square of each possible pathway. The interference of different transition probability amplitudes can be constructive or destructive depending upon the intensity of laser fields and the choice of atomic levels.

In the case of an atomic system as shown in Fig. 1.3, if the probe field with Rabi frequency Ω_p and a coupling field with Rabi frequency Ω_c driving transitions from $|1\rangle$ to $|3\rangle$ and $|2\rangle$ to $|3\rangle$ respectively, also if the transition from $|1\rangle$ to $|2\rangle$ is electric dipole forbidden, then the atomic system is called a Λ system. In such a system, an electromagnetically induced transparency (EIT) effect is possible [1, 2]. When $\Omega_p \ll \Omega_c$, then an atom in the ground state have two possible pathways for a transition from $|1\rangle$ to $|3\rangle$, which are via a direct transition from $|1\rangle$ to $|3\rangle$, or an indirect pathway $|1\rangle \rightarrow |3\rangle \rightarrow |2\rangle \rightarrow |3\rangle$. In this particular case, the transition probability of an atom to have transition $|1\rangle$ to $|3\rangle$ is zero due to a destructive quantum interference of the two different pathways leads to an EIT effect to the probe field. Once a medium renders EIT effect, it transforms an initially highly opaque medium to almost transparent to a resonant laser field field. Also the

refractive properties of the medium will be greatly modified [3].

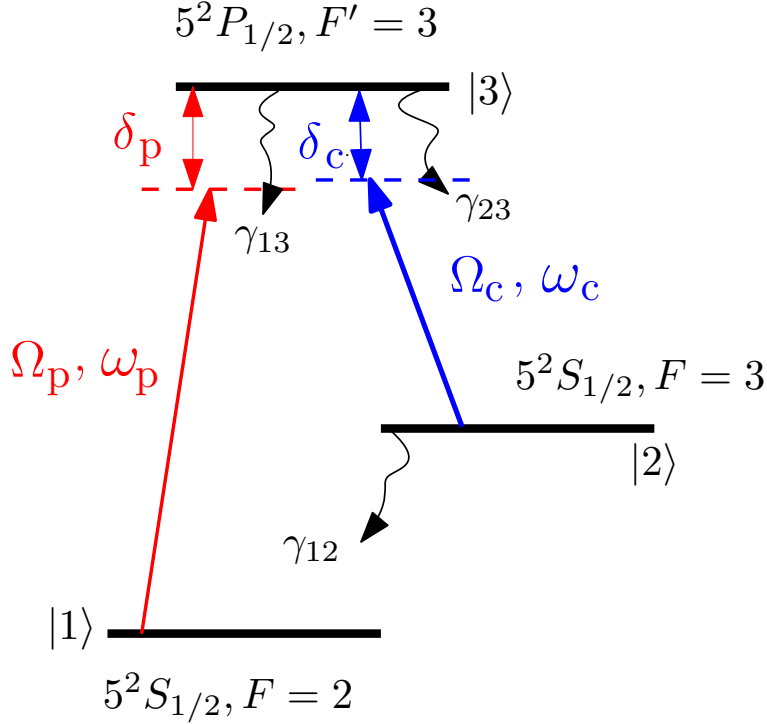


Figure 1: Λ System formation using hyper-fine levels of ^{85}Rb in its D1 manifold.

Many experiments exploit coherent interactions between the atom and electromagnetic waves in a regime where interference effects dominate transition amplitudes governing transitions between a pair of levels. These coherent interactions between atoms and electromagnetic (EM) fields can dramatically modify the optical properties of a medium, leading to electromagnetically induced transparency (EIT) and related effects [4]. EIT and the associated dark state formation are exploited in many systems to achieve lasing without inversion [5], slow and stopped light [6], quantum memory devices [7], and nonlinear wave mixing with ultra-low intensities [8, 9].

Recent studies have focused on the interaction of electromagnetic fields with atoms in a closed level configuration which shows EIT. In particular, studies have focused on the effect of connecting the two ground states of a regular Λ system by a third field, whose frequency for alkali metal atoms is in the microwave regime. These studies show that the coherence and population in a closed system depend on the phase of all the interacting fields [10]. Thus it opens up a plethora of novel effects in coherent atom EM field interaction, such as enhanced nonlinearity,

phase-dependent EIT [11] and phase-dependent optical switches [12].

This thesis deals with investigations on the coherent microwave-to-optical conversion with dilute gaseous atoms. In particular, our study deals with novel effects in a three-level atomic system that undergoes an EIT effect and having a hybrid interaction between optical electric dipole and microwave magnetic dipole transitions. The hybrid interaction breaks the centrosymmetry present in the atomic system. We have used room temperature Rubidium (^{85}Rb) atoms as the medium. The three hyperfine energy levels of ^{85}Rb atom interacting with two optical and one microwave field forms a Δ system as shown in Fig. 2, thereby making the system phase-dependent. We also present a novel three-wave mixing process enabling a coherent microwave-to-optical conversion, which leads to a phase-dependent amplification (PDA).

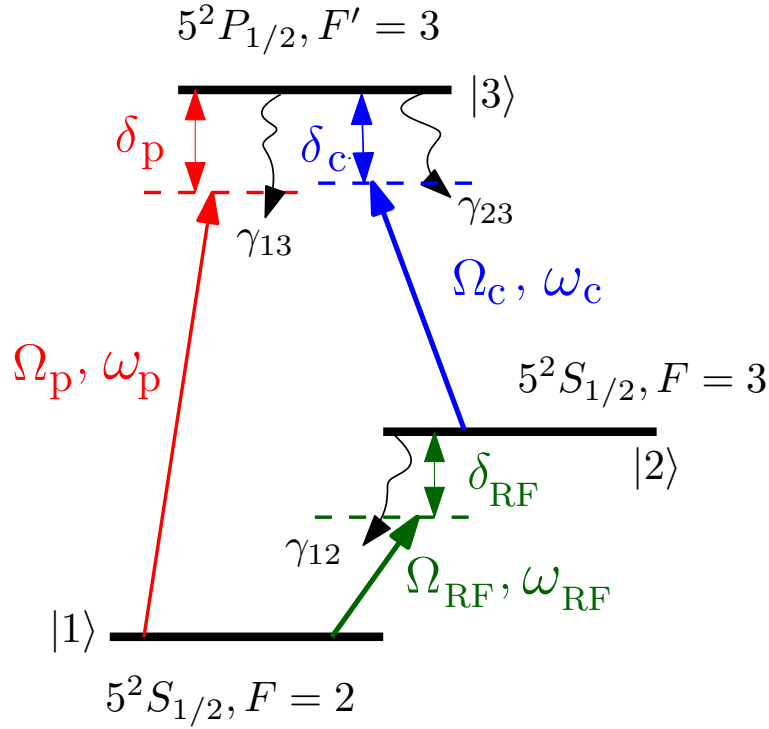


Figure 2: Δ System formation using hyper-fine levels of ^{85}Rb in its D1 manifold. Introducing an additional microwave field with Rabi frequency Ω_{RF} driving the transition between ground and metastable levels, changes the Λ system to a closed level scheme.

This thesis consists of 5 chapters. Chapter 1 starts by giving an overview and motivation to the research work carried out. A brief introduction to coherent processes in light-matter interaction, in particular, the nonlinear processes and discussion on nonlinearity with low light intensity, are included. Later a detailed

discussion on electromagnetically induced transparency (EIT) is given. Chapter 1 gives an overview of the systems which we are working on, as well as of the various experimental techniques involved.

Chapter 2 describes a coherent generation of a near infrared optical field through a three-wave mixing phenomenon in an atomic energy level scheme of ^{85}Rb atoms. In systems with centrosymmetry, second-order nonlinear processes are absent [13]. A nonlinear generation process in a centrosymmetric thermally broadened atomic system is made possible through a novel interaction between induced electric and magnetic dipoles [14]. The spectral profile of the generated probe field is studied. We have shown that the generation processes will follow a three-photon resonance condition between all the three electromagnetic fields, which is a direct consequence of energy conservation in this sum-frequency process. The bandwidth of the generation process is decided by the Raman coherence present in the system, and this is evident from the generated probe power vs two-photon resonance graph as shown in Fig. 3. When a two-photon resonance between the coupling field and the generated probe field is satisfied, the probe field also undergoes an EIT effect [4]. The EIT phenomenon is suppressing all linear absorption process and the dark state associated with EIT is responsible for the suppression of all the decoherence effect such as spontaneous emission from the excited state. The bandwidth for coherent conversion from microwave-to-optical frequencies with this scheme is shown to be about 825 kHz. The efficiency of our conversion process is about $2.6 \times 10^{-4}\%$. The chapter concludes by summarizing the results.

Chapter 3 covers the phase-dependent amplification of an optical field using microwaves. We have done a theoretical analysis of the three-wave mixing present in our Δ system. We have obtained an analytical expression for the probe field using density matrix analysis. Our analysis shows that, in addition to a generation of probe field which is discussed in earlier chapter, there will be an amplification also possible for the probe field in our Δ system. The physical mechanism of the amplification process reveals that the amplification is due to an interference of the input probe field and the generated probe field. This interference effect can give rise to an amplification or a de-amplification depends on the phase of

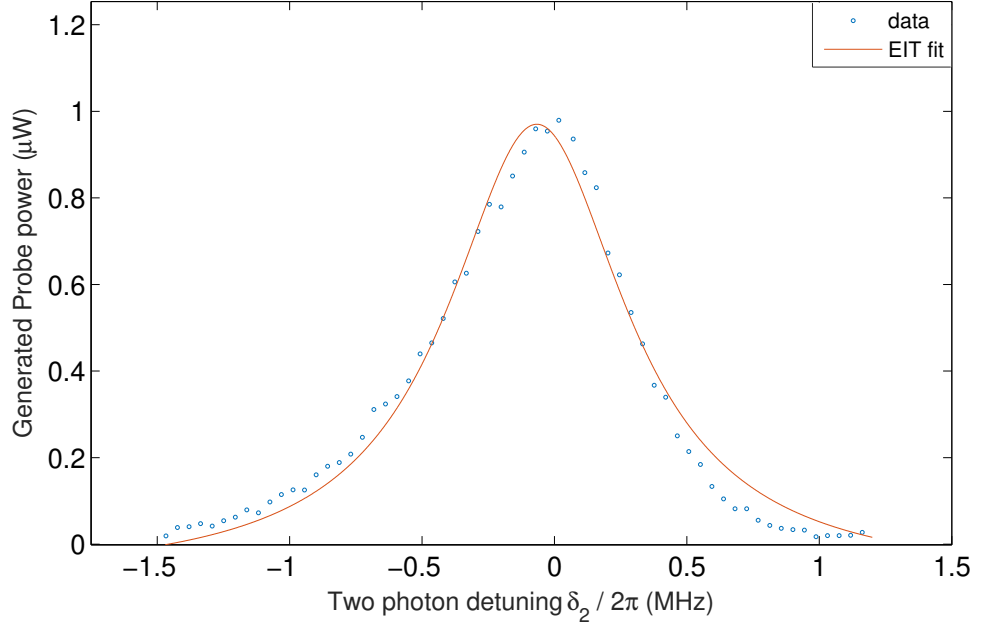


Figure 3: Generated probe power as a function of probe field detuning δ_p when $\delta_c = 0$. The continuous line is the EIT fit to the experimental data.

the generated probe field. This probe phase can be controlled by the phase of the microwave field as described in previous chapter. Thus making the system as a phase depended amplifier (PDA). The dependence of amplification and de-amplification of probe field is studied with various microwave field intensities, and with various two-photon detuning. The chapter ends with concluding the results.

Chapter 4 is an experimental demonstration of the results presented in chapter 3. We observe amplification and de-amplification of the input probe beam as a result of nonlinear three-wave mixing interaction between the microwave and the two optical fields mediated by the ^{85}Rb atoms. The amplification is dependent on the relative phase between all the three fields [10]. In the current experiment, we investigate the gain experienced by the probe field both in the presence and absence of two-photon induced ground state coherence. By carefully choosing an experimental arrangement, we could control the two-photon detuning by changing the microwave detuning which is connecting the ground and the excited states. In Fig. 4 we have shown a PDA of the probe field, when the relative phase difference between all the three electromagnetic field ($\Delta\Phi$) changes the probe gain also changes. We show that a simultaneous fulfilment of a two-photon resonance con-

dition that creates ground state coherence and a three-photon resonance condition leads to a significantly higher amplification of 7.5 dB of the optical probe field with a visibility of 98.8 %. By selectively breaking the ground state coherence using microwaves, we show that the amplification reduces with a bandwidth of 5 MHz. Nevertheless, the system shows non-zero PDA for large two-photon detunings of 15 MHz with high visibility of 66.8 %. This novel, controllable hybrid-PDA can be potentially used to trade-off amplification for bandwidth during the transmission of phase coherent classical and quantum information.

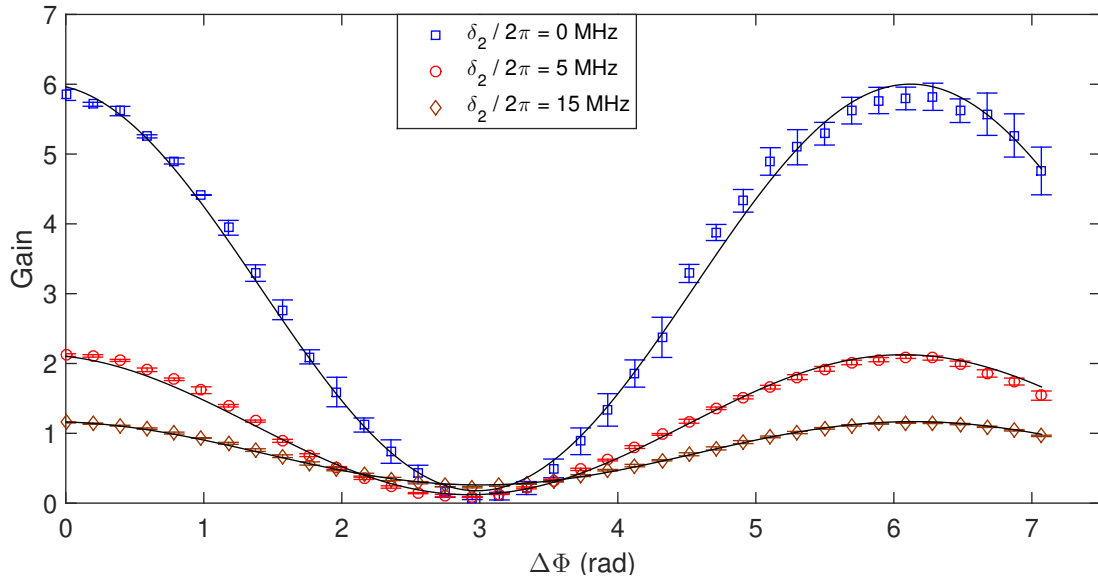


Figure 4: Probe gain vs $\Delta\Phi$ for different values of two photon detuning (δ_2) at microwave intensity 1.03 mW/cm^2 . The continuous line is the theoretical fit.

Chapter 5 summarizes the important observations and conclusions drawn from the studies presented in this thesis. In addition, we also discuss possible directions for future work on the systems described here. In summary, a unique hybrid atomic scheme is presented which can coherently interact with both microwaves and optical frequencies. Generation of a new optical field is demonstrated using this hybrid room temperature atomic system. This three-wave mixing is characterised with parameters like three-photon and two-photon detuning. A theoretical analysis of this hybrid Δ system results in a PDA of the optical probe field, which can be controlled by the phase change in the microwave field. A seed probe field and the generated probe field interfere inside the medium resulted in a PDA, which

changes from amplification region to a de-amplification decided by the phase of the microwave field. A significantly higher amplification is possible in the hybrid Δ system when the two optical field is following a two-photon resonance condition, while amplification reduces with increasing two-photon resonance. This microwave controlled hybrid system can be a potential candidate for coherent conversion and amplification of classical and quantum information signals from microwave-to-optical frequencies.

Bibliography

- [1] K.-J. Boller, A. Imamoglu, and S. E. Harris. Observation of electromagnetically induced transparency. *Phys. Rev. Lett.*, 66:2593–2596, May 1991.
- [2] J. E. Field, K. H. Hahn, and S. E. Harris. Observation of electromagnetically induced transparency in collisionally broadened lead vapor. *Phys. Rev. Lett.*, 67:3062–3065, Nov 1991.
- [3] J. P. Marangos. Electromagnetically induced transparency. *Journal of Modern Optics*, 45(3):471–503, 1998.
- [4] Michael Fleischhauer, Atac Imamoglu, and Jonathan P. Marangos. Electromagnetically induced transparency: Optics in coherent media. *Rev. Mod. Phys.*, 77:633–673, Jul 2005.
- [5] S. E. Harris. Lasers without inversion: Interference of lifetime-broadened resonances. *Phys. Rev. Lett.*, 62:1033–1036, Feb 1989.
- [6] Chien Liu, Zachary Dutton, Cyrus H. Behroozi, and Lene Vestergaard Hau. Observation of coherent optical information storage in an atomic medium using halted light pulses. *Nature*, 409:490–493, 2001.
- [7] T. Chanelière, D. N. Matsukevich, S. D. Jenkins, S.-Y. Lan, T. A. B. Kennedy, and A. Kuzmich. Storage and retrieval of single photons transmitted between remote quantum memories. *Nature*, 438:833—836, 2005.

- [8] S. E. Harris, J. E. Field, and A. Imamoglu. Nonlinear optical processes using electromagnetically induced transparency. *Phys. Rev. Lett.*, 64:1107–1110, Mar 1990.
- [9] M. D. Lukin, P. R. Hemmer, M. Löffler, and M. O. Scully. Resonant enhancement of parametric processes via radiative interference and induced coherence. *Phys. Rev. Lett.*, 81:2675–2678, Sep 1998.
- [10] Manukumara Manjappa, Satya Sainadh Undurti, Asha Karigowda, Andal Narayanan, and Barry C. Sanders. Effects of temperature and ground-state coherence decay on enhancement and amplification in a Δ atomic system. *Phys. Rev. A*, 90:043859, Oct 2014.
- [11] Hebin Li, Vladimir A. Sautenkov, Yuri V. Rostovtsev, George R. Welch, Philip R. Hemmer, and Marlan O. Scully. Electromagnetically induced transparency controlled by a microwave field. *Phys. Rev. A*, 80:023820, Aug 2009.
- [12] Megha Ghosh, Asha Karigowda, Ayyappan Jayaraman, Fabien Bretenaker, Barry C Sanders, and Andal Narayanan. Demonstration of a high-contrast optical switching in an atomic delta system. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 50(16):165502, jul 2017.
- [13] Robert W. Boyd. *Nonlinear Optics*. Academic Press, Inc., Orlando, FL, USA, 3rd edition, 2008.
- [14] P. S. Pershan. Nonlinear optical properties of solids: Energy considerations. *Phys. Rev.*, 130:919–929, May 1963.

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Chapter 1

Introduction

1.1 Overview

The coherent interaction of electromagnetic waves with atoms has attracted a lot of attention in the last few decades. Advances in precise manipulation and control of atoms using electromagnetic waves aided much development in information processing and communication systems. Of electromagnetic waves, optical and microwave frequencies are the most widely used tools in classical and quantum information processing, signal transmission and communication. In particular, numerous attempts have been made to build quantum processors and quantum simulators, which exploit excitations in the optical and microwave domain. These studies can fundamentally transform how we receive, process and transmit classical and quantum information. A range of quantum bit (Qubit) systems has been realized in the microwave and optical frequency regimes in recent years. Realization of qubits using defects in solids [1, 2], an ensemble of trapped ions [3], neutral atomic systems [4], and quantum dots [5, 6] have been employed in the optical frequency regime. While microwaves have been used in superconducting qubits of diverse flavours [7], charge qubits in various material systems [8], electron spin systems [9], and nuclear spins [10] to realize qubits. Out of these systems, superconducting qubits operating at several gigahertz (microwave frequency) have recently emerged as the most favourable quantum computing architecture due to their ability to perform quantum operations with high fidelity [11, 12, 13].

Nevertheless, it is inconvenient to transmit quantum information encoded in

microwave frequency over a long distance [14]. The microwave signal decays rapidly during propagation and is easily disturbed by thermal noise at room temperature. On the other hand, optical photons show complimentary features. There have been demonstrations of optical quantum communication over hundreds of kilometres [15]. In particular, the transmission of information in the optical domain over large scale distances is envisaged using fibre-based quantum networks [16]. Thus it creates a strong interest in developing interfaces between microwave and optical frequencies. Apart from the efficient long-distance transmission, the optical domain has already been exploited to store and retrieve quantum memory using solid state systems [17] ions [18], spin ensembles [19], and nitrogen-vacancy centres [20]. Thus coherent conversion of microwave-to-optical frequency would offer tremendous advancement in developing next-generation hybrid classical and quantum technologies [21, 22, 23].

Microwaves and optical waves are separated by several orders of magnitude in frequency, making it challenging to find a suitable platform that couples these two widely separated frequency bands. Several promising approaches have been taken to implement coherent conversion from microwave-to-optical frequency. There have been microwave-to-optical conversions using carefully engineered whispering-gallery-mode resonators [24, 25]. Several authors exploited micro and nano-sized hybrid electro-optomechanical devices which operate at a few Kelvin [26, 27]. There were demonstrations of efficient microwave frequency conversion mediated by a photonics compatible silicon nitride nano beam oscillator [28]. They also require a liquid helium temperature environment. Recent investigation on rare-earth-doped crystals [29] and gaseous atomic systems with induced coherence [30] found to be a viable architecture for effecting non-linear frequency generation and signal transfer.

There have been several theoretical proposals of microwave-to-optical conversion in atomic systems [31, 32, 33, 34, 35], but very few experimental demonstrations. The only atom-based platform that successfully demonstrated microwave-to-optical conversion is Rydberg atom's cold atomic cloud, which employs a six-wave mixing process [36, 37]. The challenge in microwave-to-optical conversion using

atom-based schemes is to devise an energy level scheme that interacts strongly with both fields and leads to a nonlinear interaction in the system. Effecting a nonlinear resonant interaction in an atomic system is not feasible in traditional nonlinear optics due to the linear and nonlinear absorption near-atomic resonance [38].

Many experiments exploit coherent interactions between the atom and electromagnetic waves in a regime where interference effects dominate transition amplitudes governing transitions between a pair of levels. These coherent interactions between atoms and electromagnetic (EM) fields can dramatically modify the optical properties of a medium, leading to electromagnetically induced transparency (EIT) and related effects [39]. EIT and the associated dark state formation are exploited in many systems to achieve lasing without inversion [40], slow and stopped light [41], quantum memory devices [42], and nonlinear wave mixing with ultra-low intensities [43, 44, 45].

Recent studies have focused on the interaction of electromagnetic fields with atoms in a closed level configuration which shows EIT. In particular, studies have focused on the effect of connecting the two ground states of a regular Λ system by a third field, whose frequency for Rubidium atom is in the microwave regime. These studies show that the coherence and population in a closed system depend on the phase of all the interacting fields [46, 47]. Coherent processes like EIT and the novel effects in closed level systems opens up a plethora of novel effects in coherent atom EM field interaction, such as enhanced nonlinearity [48], phase-dependent EIT [49], subluminal and superluminal propagation of light [50], efficient light storage [51]. A cyclic atomic level scheme interacting with an optical and a microwave field is used for microwave generation [52], efficient Raman sub-harmonics generation [53], arbitrary optical-pulse generation in a thermal vapor [54] and phase-dependent optical switches [55].

This thesis deals with investigations on the coherent microwave-to-optical conversion with dilute gaseous atoms. In particular, our study deals with novel effects in a three-level atomic system that undergoes an EIT effect and having a hybrid interaction between optical electric dipole and microwave magnetic dipole transi-

tions. The hybrid interaction breaks the centrosymmetry present in the atomic system. We have used room temperature Rubidium (^{85}Rb) atoms as the medium. The three hyperfine energy levels of ^{85}Rb atom interacting with two optical and one microwave field forms a Δ system, thereby making the system phase-dependent. We also present a novel three-wave mixing process enabling a coherent microwave-to-optical conversion, which leads to a phase-dependent amplification (PDA).

1.2 Motivation and objectives

The motivation for the problems addressed in this thesis is to develop a suitable platform that strongly couples microwave and optical frequencies that are separated by five orders of magnitude in frequency. A three-level system that can employ a three-wave mixing process between optical and microwave frequencies will be the simplest scheme one could imagine. One of the possible schemes is the three hyperfine energy levels of a ^{85}Rb atom. However, in neutral atomic systems, three-wave mixing nonlinear-optical processes are forbidden by symmetry. It is known that the utilisation of a magnetic-dipole interaction breaks the symmetry requirement and enables a three-wave mixing response to emerge in the centrosymmetric atomic system. This allows us to demonstrate a coherent microwave-to-optical conversion by three-wave mixing using ^{85}Rb atoms. Another major objective of the present study is to develop an atom-based amplifier that could be used as an interface for coherent conversion and amplification of quantum information generated in the microwave domain and converted to optical frequencies. To summarize the objectives of this thesis are,

- Perform an experiment to demonstrate coherent microwave-to-optical conversion by three-wave mixing in a room temperature atomic vapour of ^{85}Rb atoms.
- Develop a theoretical model to study the coherent conversion and amplification process of an optical field using microwaves in a closed three-level scheme (Δ system).
- Perform an experiment to investigate the role of ground state coherence in

an atom-based optical amplifier.

- Explore the possibility of extending our amplification scheme for digital communication and optical switches that a microwave field can control.

1.3 Overview of experimental concepts and techniques

1.3.1 Nonlinear wave mixing-three wave mixing

In Fig. 1.1 shows a three-wave mixing process. We have two input fields with frequency ω_a and ω_b , which interacts with the atoms in the medium. Due to non-linearity in atomic response, atoms develops an oscillating dipole moment which gives rise to a new field with frequency $\omega_c = \omega_a + \omega_b$. In this section we are trying to understand the theory of three wave mixing process. This section is based on the calculations done in "Nonlinear Optics" book by Boyd [38]. Let us assume a nonlinear medium, with no free charge and no free current. Also assume that the medium is non magnetic. Then we can write the Maxwell equations in such a media as,

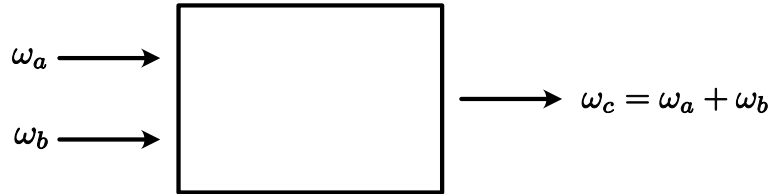


Figure 1.1: Three-wave mixing process.

$$\nabla \cdot \mathbf{D} = 0 \quad (1.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (1.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1.3)$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial \mathbf{D}}{\partial t} \quad (1.4)$$

where $\mathbf{B}$, $\mathbf{E}$ are the magnetic and electric fields respectively. Here $\mathbf{D}$ is the displacement operator and it is defined as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (1.5)$$

where ϵ_0 is the permittivity of free space and $\mathbf{P}$ is the polarization vector which depends nonlinearly on $\mathbf{E}$. Now we take the curl of third equation in Eq. (1.4),

$$\begin{aligned} \nabla \times \nabla \times \mathbf{E} &= -\nabla \times \frac{\partial \mathbf{B}}{\partial t} \\ &= -\frac{\partial(\nabla \times \mathbf{B})}{\partial t} \end{aligned} \quad (1.6)$$

Substituting for $\nabla \times \mathbf{B}$ from fourth equation in Eq. (1.4), we obtain

$$\nabla \times \nabla \times \mathbf{E} + \mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2} = 0 \quad (1.7)$$

By applying Eq. (1.5) and using the identity $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$, we get

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} + \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} = 0. \quad (1.8)$$

Using Eq. (1.4), we can simplify the wave equation as

$$-\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad (1.9)$$

where c is the speed of light in vacuum, $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$. The wave equation can also be written in terms of displacement field as

$$-\nabla^2 \mathbf{E} - \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{D}}{\partial t^2} = 0. \quad (1.10)$$

The total polarization can be understood as the sum of linear ($\mathbf{P}^{(1)}$) and nonlinear ($\mathbf{P}^{(NL)}$) polarization

$$\mathbf{P} = \mathbf{P}^{(1)} + \mathbf{P}^{(NL)} \quad (1.11)$$

similarly the displacement field $\mathbf{D}$ can also be split into linear and nonlinear part

$$\mathbf{D} = \mathbf{D}^{(1)} + \mathbf{P}^{(NL)} \quad (1.12)$$

where

$$\begin{aligned} \mathbf{D}^{(1)} &= \epsilon_0 \mathbf{E} + \mathbf{P}^{(1)} \\ &= \epsilon_0 \mathbf{E} + \epsilon_0 \chi^{(1)} \mathbf{E} \\ &= \epsilon_0 \epsilon_r \mathbf{E}. \end{aligned} \quad (1.13)$$

Using Eq. (1.12) one can write the wave equation Eq. (1.10) as

$$-\nabla^2 \mathbf{E} - \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{D}^{(1)}}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}^{(NL)}}{\partial t^2}. \quad (1.14)$$

In a dispersive medium each frequency component of the medium has to be considered separately. Then the wave equation can be written as

$$-\nabla^2 \mathbf{E}_i - \frac{\epsilon_r(\omega_i)}{c^2} \frac{\partial^2 \mathbf{E}_i}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}_i^{(NL)}}{\partial t^2}. \quad (1.15)$$

This wave equation has a form of an inhomogeneous wave equation. The term on the right hand side acts as a source term. If the source term is absent, then Eq. (1.15) will be representing a free wave propagation equation with a velocity of c/n , where n is the refractive index of the medium.

We are going to use the nonlinear wave equation to study the three-wave mixing process. Let us consider a sum-frequency generation in a nonlinear optical medium as shown in Fig. 1.2. We have input electromagnetic fields with frequency ω_a and ω_b which are collimated, continuous and propagating along $+z$ direction. The input fields can be written as,

$$\begin{aligned} \mathbf{E}_a(z, t) &= \mathcal{E}_a e^{-i(\omega_a t - k_a z)} + \text{C.C} \\ \mathbf{E}_b(z, t) &= \mathcal{E}_b e^{-i(\omega_b t - k_b z)} + \text{C.C} \end{aligned} \quad (1.16)$$

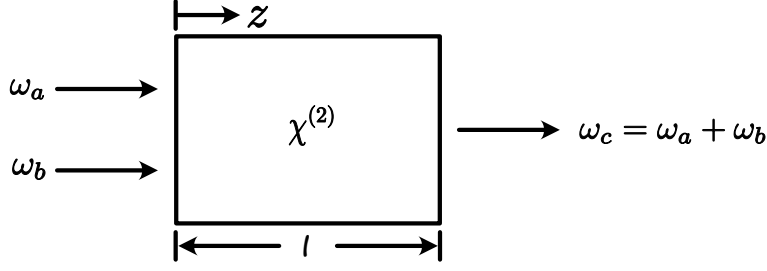


Figure 1.2: Three-wave mixing process. A $\chi^{(2)}$ nonlinear medium with a length of l is interacting with two input fields with frequency ω_a and ω_b , and a new field is generated with a frequency of $\omega_c = \omega_a + \omega_b$.

where C.C. is the complex conjugate, $k_i = n_i \omega_i / c$ and $n_i = \sqrt{\epsilon_r(\omega_i)}$ ($i = a, b$), are the wave vector and the refractive index of the medium respectively. Now let us assume a plane wave propagating in $+z$ direction,

$$\mathbf{E}_c(z, t) = \mathcal{E}_c e^{-i(\omega_c t - k_c z)}. \quad (1.17)$$

The amplitude $\mathcal{E}_c$ will be a constant for a plane wave. Now to solve the wave equation Eq. (1.15) for the sum-frequency generation process, where two input fields with frequency ω_a and ω_b interacting with the nonlinear medium gives rise to a sum-frequency generation of a new field having frequency of $\omega_c = \omega_a + \omega_b$. In this case we assume the solution will be of the form as in Eq. (1.17), except that the amplitude $\mathcal{E}_c$ will be a function of z . The nonlinear source term relevant in this case is

$$\mathbf{P}_c(z, t) = \mathcal{P}_c(z) e^{-i\omega_c t} + \text{C.C.} \quad (1.18)$$

where

$$\mathcal{P}_c(z) = 2\epsilon_0 \chi^{(2)} \mathcal{E}_a \mathcal{E}_b e^{i(k_a + k_b)z} \quad (1.19)$$

with this information one can write the wave equation for the sum-frequency generation as,

$$\begin{aligned} \left[\frac{d^2 \mathcal{E}_c}{dz^2} + 2ik_c \frac{d\mathcal{E}_c}{dz} - k_c^2 \mathcal{E}_c + \frac{\epsilon_r(\omega_c)\omega_c^2 \mathcal{E}_c}{c^2} \right] e^{-i(\omega_c t - k_c z)} + \text{C.C.} \\ = -\frac{2\chi^{(2)}\omega_c^2}{c^2} \mathcal{E}_a \mathcal{E}_b e^{-i(\omega_c t - (k_a + k_b)z)} + \text{C.C.} \end{aligned} \quad (1.20)$$

since $k_c^2 = \epsilon_r(\omega_c)\omega_c^2/c^2$ we can simplify Eq. (1.20) as

$$\frac{d^2 \mathcal{E}_c}{dz^2} + 2ik_c \frac{d\mathcal{E}_c}{dz} = -\frac{2\chi^{(2)}\omega_c^2}{c^2} \mathcal{E}_a \mathcal{E}_b e^{i(k_a + k_b - k_c)z}. \quad (1.21)$$

Here we assume the slowly varying amplitude approximation, and it is valid whenever $\left| \frac{d^2 \mathcal{E}_c}{dz^2} \right| \ll \left| k_c \frac{d\mathcal{E}_c}{dz} \right|$, then Eq. (1.21) becomes

$$\frac{d\mathcal{E}_c}{dz} = \frac{i\chi^{(2)}\omega_c^2}{k_c c^2} \mathcal{E}_a \mathcal{E}_b e^{i\Delta k z} \quad (1.22)$$

where $\Delta k = k_a + k_b - k_c$, is the wavevector mismatch. The total amplitude of the general field power can be obtained by integrating Eq. (1.22) from $z = 0$ to $z = l$,

$$\begin{aligned} \mathcal{E}_c(l) &= \frac{i\chi^{(2)}\omega_c^2}{k_c c^2} \mathcal{E}_a \mathcal{E}_b \int_0^l e^{i\Delta k z} dz \\ &= \frac{i\chi^{(2)}\omega_c^2}{k_c c^2} \mathcal{E}_a \mathcal{E}_b \left(\frac{e^{i\Delta k l} - 1}{\Delta k} \right) \end{aligned} \quad (1.23)$$

The intensity of the generated field can be written as,

$$\begin{aligned} I_c &= 2n_c \epsilon_0 c |\mathcal{E}_c(l)|^2 \\ &= \frac{2n_c \epsilon_0 (\chi^{(2)})^2 \omega_c^4 |\mathcal{E}_a|^2 |\mathcal{E}_b|^2}{k_c^2 c^3} \left| \frac{e^{i\Delta k l} - 1}{\Delta k} \right|^2. \end{aligned} \quad (1.24)$$

When the input waves are in copropagating configuration, for $\Delta k \approx 0$, which is the phase matching condition, then the generated field intensity can be written as

$$I_c = \frac{2n_c \epsilon_0 (\chi^{(2)})^2 \omega_c^4 |\mathcal{E}_a|^2 |\mathcal{E}_b|^2 l^2}{k_c^2 c^3}. \quad (1.25)$$

The generated field intensity given in Eq. (1.25) is valid in the limit in which the two input fields are undepleted by the nonlinear interaction.

A gaseous atomic system possess an inversion symmetry, also known as the center of symmetry. This centrosymmetry plays a crucial role in the nonlinear response of gaseous atoms. To understand this, we first consider the polarization responsible for the second order process,

$$\mathbf{P}(t) = \epsilon_0 \chi^{(2)} \mathbf{E}^2(t) \quad (1.26)$$

Due to the centrosymmetry, if we change the sign of the electric field, then the sign of the polarization also changes,

$$\begin{aligned} -\mathbf{P}(t) &= \epsilon_0 \chi^{(2)} [-\mathbf{E}(t)]^2 \\ &= \mathbf{P}(t) \end{aligned} \quad (1.27)$$

This can occur only if $\chi^{(2)} = 0$. Thus for a centrosymmetric nonlinear medium the $\chi^{(2)}$ nonlinear susceptibility is absent.

1.3.2 Electromagnetically induced transparency

Coherent interactions of electromagnetic waves with a three-level atom can result in diverse physical phenomena. Using one or many electromagnetic fields in such a system, one can control the medium's optical properties. A three-level atomic system coupled to two laser fields exhibits interference effects that can cancel absorption at a resonance transition frequency and thereby modify the optical response of the medium. Analogous to the interference of classical waves resulting in constructive and destructive interference, the probability amplitudes of various pathways interfere in the quantum case. Suppose there is more than one pathway available for an atom to have a transition from the ground to the excited state; in that case, the transition probability is the squared sum of the probability amplitude of each possible pathway. The interference of various transition probability amplitudes can be constructive or destructive depending upon the intensity of laser fields and the choice of atomic levels.

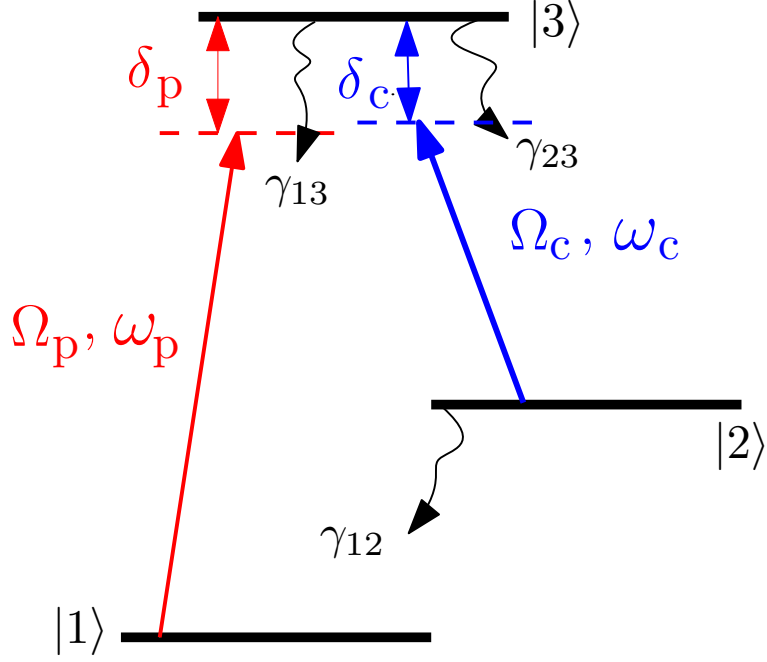


Figure 1.3: Λ System formation using a three level system.

In the case of an atomic system as shown in Fig. 1.3, if the probe field with Rabi frequency Ω_p and a coupling field with Rabi frequency Ω_c driving transitions from $|1\rangle$ to $|3\rangle$ and $|2\rangle$ to $|3\rangle$ respectively, also if the transition from $|1\rangle$ to $|2\rangle$ is electric dipole forbidden, then the atomic system is called a Λ system. In such a system, an electromagnetically induced transparency (EIT) effect is possible [56, 57]. When $\Omega_p \ll \Omega_c$, then an atom in the ground state have two possible pathways for a transition from $|1\rangle$ to $|3\rangle$, which are via a direct transition from $|1\rangle$ to $|3\rangle$, or an indirect pathway $|1\rangle \rightarrow |3\rangle \rightarrow |2\rangle \rightarrow |3\rangle$. In this particular case, the transition probability of an atom to have transition $|1\rangle$ to $|3\rangle$ is zero due to a destructive quantum interference of the two different pathways leads to an EIT effect to the probe field. Once a medium renders EIT effect, it transforms an initially highly opaque medium to almost transparent to a resonant laser field. Also the refractive properties of the medium will be greatly modified [58].

The first observation of EIT is carried out by Harris et al in 1991 [56, 57]. They have used initially a strontium vapour and later lead vapour to demonstrate the EIT effect. After that, a series of investigations on EIT has been done by several groups of researchers. Apart from causing a highly opaque medium to being rendered highly transparent, the EIT effect will modify the refractive properties

of the medium as well. This leads to the development of a medium with very unusual optical properties. EIT and the associated dark state formation are exploited in many systems to achieve lasing without inversion [40], slow and stopped light [41], quantum memory devices [42], and nonlinear wave mixing with ultra-low intensities [43, 44].

1.3.2.1 Theoretical calculation

In this section we are going to derive the EIT Hamiltonian in a Λ system. Further, we will calculate the linear susceptibility of the three-level atom interacting with two electromagnetic fields using density matrix calculation. In Fig. 1.3 shows a three-level atom interacting with two electromagnetic fields, which forms a Λ configuration. The ground state, metastable state and the excited state energy levels of the atom is denoted as $|1\rangle$, $|2\rangle$ and $|3\rangle$ respectively. The electric dipole transition between $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ transitions are driven by the probe field (ω_p) with Rabi frequency Ω_p and coupling field (ω_c) with Rabi frequency Ω_c respectively. The transition between $|1\rangle$ and $|2\rangle$ is electric dipole forbidden. The detuning of the coupling field from the $|2\rangle \rightarrow |3\rangle$ transition is denoted by δ_c and δ_p is the detuning of the probe field from the $|1\rangle \rightarrow |3\rangle$ transition. Here γ_{12} and $\gamma_{23} = \gamma_{13}$ representing natural linewidth of levels $|2\rangle$ and $|3\rangle$ respectively. The total electric field $\mathbf{E}$ in the system can be written under dipole approximation ($|k|R \ll 1$, where $\vec{k}$ is the wave vector in the medium, and R is the radius of the atom) as

$$\mathbf{E}(t) = \vec{e}_c \mathcal{E}_c \cos(\omega_c t) + \vec{e}_p \mathcal{E}_p \cos(\omega_p t) \quad (1.28)$$

where $\mathcal{E}_c$ and $\mathcal{E}_p$ are the electric field amplitudes of the coupling and probe field respectively. ω_c and ω_p are the angular frequencies of coupling and probe fields. Here $\vec{e}_c$ and $\vec{e}_p$ being the polarization of electromagnetic fields. The total Hamiltonian of the system can be written as,

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t) \quad (1.29)$$

where H_0 is the bare state Hamiltonian,

$$\hat{H}_0 = \hbar \begin{pmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{pmatrix} \quad (1.30)$$

with $E_j = \hbar\omega_j$ being the energy of level $j = 1, 2, 3$. The perturbed Hamiltonian H_1 can be written as

$$\hat{H}_1(t) = -\hat{Q} \cdot \mathbf{E}(t) \quad (1.31)$$

where $\hat{Q}$ is the dipole moment operator defined as $\hat{Q} = q\hat{r}$. The dipole matrix elements are represented as $Q_{ij} = \langle i|\hat{Q}|j\rangle$. Due to the spherical symmetry of the atoms, the diagonal elements Q_{ii} will be zero. Also Q_{12} and Q_{21} will be zero due to which the electric dipole forbidden transition between the levels $|1\rangle$ and $|2\rangle$.

Now we can write the perturbed Hamiltonian $\hat{H}_1$

$$\hat{H}_1(t) = -(\mathbf{E}(t)) \cdot (Q_{21}|2\rangle\langle 1| + Q_{23}|2\rangle\langle 3| + \text{H.C.}) \quad (1.32)$$

where H.C is the Hermitian conjugate. With expanding the electric field $\mathbf{E}$ using Eq. (3.1), $\hat{H}_1$ becomes,

$$\begin{aligned} \hat{H}_1(t) = & -\left[\frac{\vec{e}_c \mathcal{E}_c}{2}(e^{i\omega_c t} + e^{-i\omega_c t}) + \frac{\vec{e}_p \mathcal{E}_p}{2}(e^{i\omega_p t} + e^{-i\omega_p t})\right] \\ & \cdot (Q_{21}|2\rangle\langle 1| + Q_{23}|2\rangle\langle 3| + \text{H.C.}). \end{aligned} \quad (1.33)$$

We can simplify Eq. (1.33) by applying the rotating wave approximation. To do that we first move to the interaction picture by using the unitary transformation $\hat{U} = e^{i\hat{H}_0 t/\hbar}$. The Hamiltonian under this transformation can be written as $\hat{H}_I = \hat{U}\hat{H}_1\hat{U}^\dagger$

$$\begin{aligned} \hat{H}_I(t) = & -\frac{Q_{13}}{2}[\mathcal{E}_c(e^{-i(\omega_3-\omega_1-\omega_c)t} + e^{-i(\omega_3-\omega_1+\omega_c)t}) + \\ & \mathcal{E}_p(e^{-i(\omega_3-\omega_1-\omega_p)t} + e^{-i(\omega_3-\omega_1+\omega_p)t})]|1\rangle\langle 3| \\ & -\frac{Q_{23}}{2}[\mathcal{E}_c(e^{-i(\omega_3-\omega_2-\omega_c)t} + e^{-i(\omega_3-\omega_2+\omega_c)t}) + \\ & \mathcal{E}_p(e^{-i(\omega_3-\omega_2-\omega_p)t} + e^{-i(\omega_3-\omega_2+\omega_p)t})]|2\rangle\langle 3| + \text{H.C.} \end{aligned} \quad (1.34)$$

We can now assume that the rapidly oscillating terms will be averaged out to zero on time scale of observation. Since we have already assumed that the probe and coupling fields are near resonant with the transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ respectively, the term $e^{\pm i(\omega_3 - \omega_1 - \omega_p)t}$ and $e^{\pm i(\omega_3 - \omega_2 - \omega_c)t}$ will be slowly oscillating.

$$\begin{aligned}\hat{H}_1(t) = & -\frac{Q_{13}}{2}[\mathcal{E}_p e^{-i(\omega_3 - \omega_1 - \omega_p)t}]|1\rangle\langle 3| \\ & -\frac{Q_{23}}{2}[\mathcal{E}_c e^{-i(\omega_3 - \omega_2 - \omega_c)t}]|2\rangle\langle 3| + \text{H.C.}\end{aligned}\quad (1.35)$$

To move back to Schrödinger picture, we reverse the transformation and obtain $\hat{H}_1$,

$$\begin{aligned}\hat{H}_1(t) = & \hat{U}^\dagger \hat{H}_1 \hat{U} \\ = & -\frac{Q_{13}}{2}[\mathcal{E}_p e^{i\omega_p t}]|1\rangle\langle 3| - \frac{Q_{23}}{2}[\mathcal{E}_c e^{i\omega_c t}]|2\rangle\langle 3| + \text{H.C.}\end{aligned}\quad (1.36)$$

The dipole elements can be written in polar form as

$$\begin{aligned}Q_{13} = Q_{31}^* &= |Q_{13}|e^{i\phi_p} \\ Q_{23} = Q_{32}^* &= |Q_{23}|e^{i\phi_c}\end{aligned}\quad (1.37)$$

here ϕ_i with $i = (p, c)$ is the respective angle between the wave polarization and dipole moment. using this, we define the Rabi frequencies of the probe and coupling fields as

$$\begin{aligned}\Omega_p &= \frac{|Q_{13}|\mathcal{E}_p}{\hbar} \\ \Omega_c &= \frac{|Q_{23}|\mathcal{E}_c}{\hbar}.\end{aligned}\quad (1.38)$$

Using Eq. (1.29, 1.30, 1.36, 1.37, 1.38), the total Hamiltonian of the system can be written as

$$\begin{aligned}\hat{H}(t) = & \hbar\omega_1|1\rangle\langle 1| + \hbar\omega_2|2\rangle\langle 2| + \hbar\omega_3|3\rangle\langle 3| - \frac{\hbar}{2}\Omega_p e^{i(\omega_p t + \phi_p)}|1\rangle\langle 3| \\ & - \frac{\hbar}{2}\Omega_c e^{i(\omega_c t + \phi_c)}|2\rangle\langle 3| + \text{H.C.}\end{aligned}\quad (1.39)$$

The total EIT Hamiltonian is given in Eq. (1.39). since it is often inconvenient to work with time-dependent Hamiltonian, we transform the basis states to a co-rotating frame; by doing this transformation, we can avoid the explicit time dependence of the Hamiltonian. The unitary transformation used here is

$$\hat{U}'(t) = |1\rangle\langle 1| + e^{i[(\omega_p - \omega_c)t + \phi_p - \phi_c]}|2\rangle\langle 2| + e^{i(\omega_p t + \phi_p)}|3\rangle\langle 3|. \quad (1.40)$$

The Schrödinger equation is applicable in the new basis as well, we can write

$$\begin{aligned} i\hbar \frac{\partial \psi'}{\partial t} &= i\hbar \frac{\partial \hat{U}'(t)\psi}{\partial t} \\ &= i\hbar \left(\frac{\partial \hat{U}'(t)}{\partial t} \psi + \hat{U}'(t) \frac{\partial \psi}{\partial t} \right) \\ &= \left(i\hbar \frac{\partial \hat{U}'(t)}{\partial t} \hat{U}'^\dagger(t) + \hat{U}'(t) \hat{H}(t) \hat{U}'^\dagger(t) \right) \psi' \\ &= \hat{H}' \psi' \end{aligned} \quad (1.41)$$

where ψ' is the new basis state, it is related to old basis state via the relation $\psi' = \hat{U}'(t)\psi$. From the Eq. (1.41), we can deduce that the time dependent Hamiltonian in the co-rotating frame are related by the following relation,

$$\hat{H}' = i\hbar \frac{\partial \hat{U}'(t)}{\partial t} \hat{U}'^\dagger(t) + \hat{U}'(t) \hat{H}(t) \hat{U}'^\dagger(t). \quad (1.42)$$

Substituting for $\hat{H}(t)$, $\hat{U}'(t)$ and $\hat{U}'^\dagger(t)$ from Eq. (1.39, 1.40) in Eq. (1.42), we obtain,

$$\begin{aligned} \hat{H}' &= -\frac{\hbar}{2} [2\omega_1 |1\rangle\langle 1| + 2(\omega_2 - \omega_p + \omega_c) |2\rangle\langle 2| + 2(\omega_3 - \omega_p) |3\rangle\langle 3| + \Omega_p |1\rangle\langle 3| \\ &\quad + \Omega_c |2\rangle\langle 3| + \text{H.C.}]. \end{aligned} \quad (1.43)$$

Now we have obtained the time independent Hamiltonian given by Eq. (1.43). To write the Hamiltonian in terms of detuning of probe and coupling fields, we have taken energy level $|1\rangle$ as the reference level, we have incorporated this by adding $-\hbar\omega_1 I$ with the time independent Hamiltonian. In Eq. (1.44) given the total time independent EIT Hamiltonian

$$\begin{aligned}\hat{H}' = & -\hbar(\delta_p - \delta_c)|2\rangle\langle 2| - \hbar\delta_p|3\rangle\langle 3| - \frac{\hbar}{2}\Omega_p|1\rangle\langle 3| \\ & - \frac{\hbar}{2}\Omega_c|2\rangle\langle 3| + \text{H.C.}.\end{aligned}\tag{1.44}$$

The detunings are defined as

$$\begin{aligned}\delta_p &= \omega_p - (\omega_3 - \omega_1) \\ \delta_c &= \omega_c - (\omega_3 - \omega_2).\end{aligned}\tag{1.45}$$

The dynamical evolution of the rotated density matrix $\hat{\sigma}(t) = \hat{U}'(t)\hat{\rho}(t)\hat{U}'^\dagger(t)$, is given by the master equation [\[47\]](#)

$$\frac{\partial \hat{\sigma}(t)}{\partial t} = -i[\hat{H}', \hat{\sigma}(t)] + \sum_{k=1}^3 \mathcal{L}(\hat{a}_k)\hat{\sigma}(t)\tag{1.46}$$

with $\mathcal{L}(\hat{a}_k)$ the Lindblad superoperator

$$\mathcal{L}(\hat{a})\hat{\sigma}(t) = \hat{a}\hat{\sigma}(t)\hat{a}^\dagger - \frac{1}{2}\{\hat{\sigma}(t), \hat{a}^\dagger\hat{a}\}\tag{1.47}$$

acting on operators

$$\begin{aligned}\hat{a}_1 &= \sqrt{\gamma_{12}}|1\rangle\langle 2| \\ \hat{a}_2 &= \sqrt{\gamma_{13}}|1\rangle\langle 3| \\ \hat{a}_3 &= \sqrt{\gamma_{23}}|2\rangle\langle 3|\end{aligned}\tag{1.48}$$

with γ_{12} and $\gamma_{23} = \gamma_{13}$ representing natural linewidth of levels $|2\rangle$ and $|3\rangle$ respectively. The density matrix equations will be,

$$\begin{aligned}
\frac{\partial \sigma_{33}(z, t)}{\partial t} &= \frac{i\Omega_c}{2} \sigma_{23}(z, t) + \frac{i\Omega_p}{2} \sigma_{13}(z, t) - \frac{i\Omega_p}{2} \sigma_{31}(z, t) - \frac{i\Omega_c}{2} \sigma_{32}(z, t) \\
&\quad - \gamma_{13} \sigma_{33}(z, t) - \gamma_{23} \sigma_{33}(z, t) \\
\frac{\partial \sigma_{22}(z, t)}{\partial t} &= \frac{i\Omega_c}{2} \sigma_{32}(z, t) - \frac{i\Omega_c}{2} \sigma_{23}(z, t) + \gamma_{23} \sigma_{33}(z, t) \\
&\quad - \gamma_{12} \sigma_{22}(z, t) \\
\frac{\partial \sigma_{32}(z, t)}{\partial t} &= \frac{i\Omega_p}{2} \sigma_{12}(z, t) + \frac{i\Omega_c}{2} [\sigma_{22}(z, t) - \sigma_{33}(z, t)] \\
&\quad + i\delta_c \sigma_{32}(z, t) - \frac{\gamma_{13}}{2} \sigma_{32}(z, t) - \frac{\gamma_{23}}{2} \sigma_{32}(z, t) \\
&\quad - \frac{\gamma_{12}}{2} \sigma_{32}(z, t) \\
\frac{\partial \sigma_{31}(z, t)}{\partial t} &= \frac{i\Omega_p}{2} [\sigma_{11}(z, t) - \sigma_{33}(z, t)] + \frac{i\Omega_c}{2} \sigma_{21}(z, t) \\
&\quad + i\delta_p \sigma_{31}(z, t) - \frac{\gamma_{13}}{2} \sigma_{31}(z, t) - \frac{\gamma_{23}}{2} \sigma_{31}(z, t) \\
\frac{\partial \sigma_{21}(z, t)}{\partial t} &= \frac{i\Omega_c}{2} \sigma_{31}(z, t) - \frac{i\Omega_p}{2} \sigma_{23}(z, t) \\
&\quad + i[\delta_p - \delta_c] \sigma_{21}(z, t) - \frac{\gamma_{12}}{2} \sigma_{21}(z, t) \\
\sigma_{11}(z, t) &= 1 - \sigma_{22}(z, t) - \sigma_{33}(z, t)
\end{aligned} \tag{1.49}$$

In the limit of weak probe field, $\Omega_p \ll \Omega_c$, we can assume that all the atoms are in the ground state. Solving Eq. (1.49) for σ_{31} at steady state (i.e., $\sigma_{ij} = 0$) by assuming $\sigma_{11} = 1$, we obtain,

$$\sigma_{31}(z) = \frac{2i\Gamma_2\Omega_p(\mathbf{z})}{4\Gamma_1\Gamma_2 + |\Omega_c(\mathbf{z})|^2}. \tag{1.50}$$

Here

$$\begin{aligned}
\Gamma_1 &= \frac{\gamma_{13}}{2} + \frac{\gamma_{23}}{2} - i\delta_p \\
\Gamma_2 &= \frac{\gamma_{12}}{2} - i\delta_2
\end{aligned} \tag{1.51}$$

where $\delta_2 = \delta_p - \delta_c$ is the two photon detuning.

To understand the optical response of the EIT medium, as measured by the probe field, we need to determine the linear susceptibility. The linear susceptibility and polarization of the medium are related by $\mathbf{P} = \epsilon_0 \chi^{(1)} \mathbf{E}$. In our case, we can write the polarization as

$$P = \frac{\epsilon_0}{2} [\chi^{(1)}(\omega_p) \mathcal{E}_p e^{-i\omega_p t} + \chi^{(1)}(-\omega_p) \mathcal{E}_p e^{i\omega_p t} + \chi^{(1)}(\omega_c) \mathcal{E}_c e^{-i\omega_c t} + \chi^{(1)}(-\omega_c) \mathcal{E}_c e^{i\omega_c t}] \quad (1.52)$$

The polarization of the medium is defined as the dipole moment per unit volume, that is,

$$\begin{aligned} P &= N \langle \hat{Q} \rangle \\ &= N \text{Tr}(\hat{Q} \hat{\rho}) \\ &= N [Q_{13} \rho_{31} + Q_{31} \rho_{13} + Q_{23} \rho_{32} + Q_{32} \rho_{23}] \end{aligned} \quad (1.53)$$

where N is the number density of atoms and $\langle \hat{Q} \rangle$ is the expectation value of dipole operator. Here $\hat{\rho}$ is the non-rotating density matrix, and it is related to the rotated density matrix $\hat{\sigma}$ by the relation

$$\begin{aligned} \hat{\rho}(t) &= \hat{U}'^\dagger(t) \hat{\sigma}(t) \hat{U}'(t) \\ &= \begin{bmatrix} \sigma_{11} & \sigma_{12} e^{i((\omega_p - \omega_c)t + \phi_p - \phi_c)} & \sigma_{13} e^{i(\omega_p t + \phi_p)} \\ \sigma_{21} e^{-i((\omega_p - \omega_c)t + \phi_p - \phi_c)} & \sigma_{22} & \sigma_{23} e^{i(\omega_c t + \phi_c)} \\ \sigma_{31} e^{-i(\omega_p t + \phi_p)} & \sigma_{32} e^{-i(\omega_c t + \phi_c)} & \sigma_{33} \end{bmatrix} \end{aligned} \quad (1.54)$$

Substituting for $\hat{\rho}(t)$ from Eq. (1.54) in Eq. (1.53), we obtain

$$P = N [Q_{13} \sigma_{31} e^{-i(\omega_p t + \phi_p)} + Q_{31} \sigma_{13} e^{i(\omega_p t + \phi_p)} + Q_{23} \sigma_{32} e^{-i(\omega_c t + \phi_c)} + Q_{32} \sigma_{23} e^{i(\omega_c t + \phi_c)}]. \quad (1.55)$$

Using the polar form of dipole matrix elements from Eq. (1.37), we can remove the phase terms in Eq. (1.55)

$$P = N [Q_{13} \sigma_{31} e^{-i\omega_p t} + Q_{31} \sigma_{13} e^{i\omega_p t} + Q_{23} \sigma_{32} e^{-i\omega_c t} + Q_{32} \sigma_{23} e^{i\omega_c t}]. \quad (1.56)$$

In order to understand the absorption and dispersion of the probe field, we need to evaluate only the linear susceptibility related to the probe field $\chi^{(1)}(\omega_p)$. Comparing the frequency component of Eq. (1.52) and Eq. (1.56), we obtain

$$N|Q_{13}|\sigma_{31} = \frac{\epsilon_0\chi^{(1)}(\omega_p)}{2}\mathcal{E}_p \quad (1.57)$$

Now the probe susceptibility can be written as

$$\chi^{(1)}(\omega_p) = \frac{2N|Q_{13}|}{\epsilon_0\mathcal{E}_p}\sigma_{31} \quad (1.58)$$

By substituting for σ_{31} from Eq. (1.50), we can express the probe susceptibility

$$\chi^{(1)}(\omega_p) = \frac{2N|Q_{13}|}{\epsilon_0\mathcal{E}_p} \frac{2i\Gamma_2\Omega_p}{4\Gamma_1\Gamma_2 + |\Omega_c|^2} \quad (1.59)$$

The probe field parameters Ω_p and $\mathcal{E}_p$ can be removed by substituting for Ω_p from Eq. (1.38). Then Eq. (1.59) becomes

$$\chi^{(1)}(\omega_p) = \frac{4iN|Q_{13}|^2}{\hbar\epsilon_0} \frac{\Gamma_2}{4\Gamma_1\Gamma_2 + |\Omega_c|^2} \quad (1.60)$$

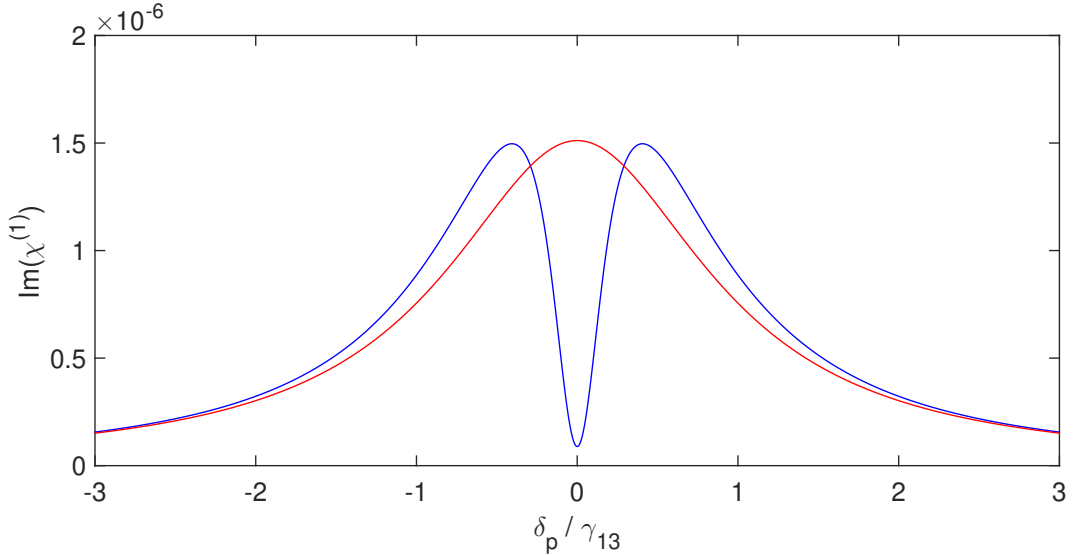


Figure 1.4: Imaginary part of $\chi^{(1)}$ vs δ_p/γ_{13}

Using Eq. (1.51) one can write the linear susceptibility in terms of the natural linewidth of the energy levels, also taking $\gamma_{13} = \gamma_{23}$, which is the natural line width of the level $|3\rangle$

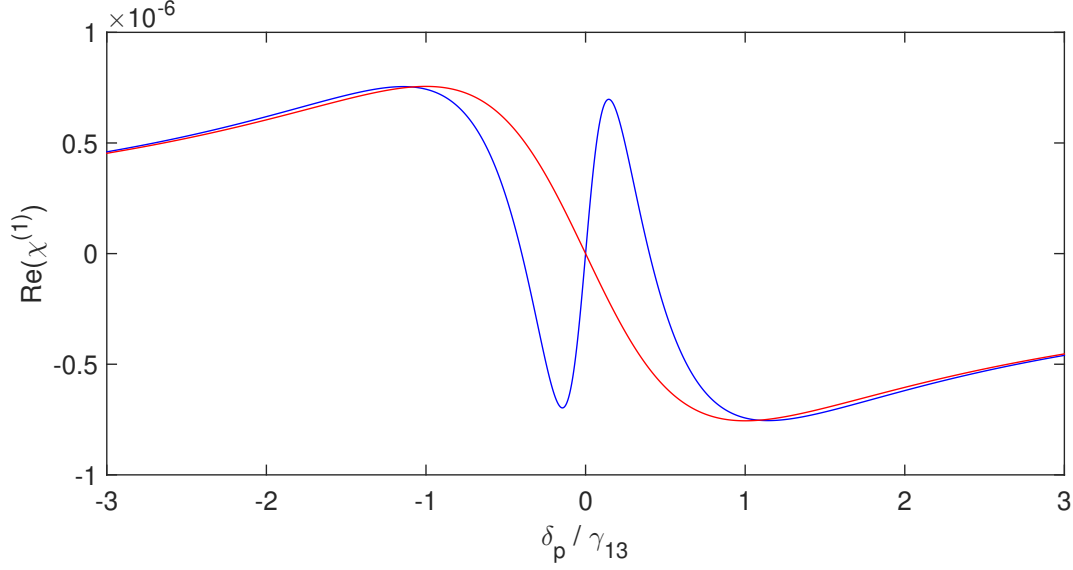


Figure 1.5: Real part of $\chi^{(1)}$ vs δ_p/γ_{13}

$$\chi^{(1)}(\omega_p) = \frac{N|Q_{13}|^2}{\hbar\epsilon_0} \left(\frac{\frac{\gamma_{12}}{2} - i\delta_2}{(\gamma_{13} - i\delta_p)(\frac{\gamma_{12}}{2} - i\delta_2) + (\frac{|\Omega_c|}{2})^2} \right) \quad (1.61)$$

The real and imaginary part of linear susceptibility is given as

$$Im(\chi^{(1)}(\omega_p)) = \frac{2N|Q_{13}|^2[2\gamma_{12}^2\gamma_{13} + 8\gamma_{13}\delta_2^2 + \gamma_{12}|\Omega_c|^2]}{\hbar\epsilon_0[16\gamma_{13}^2\delta_2^2 + 4\gamma_{12}^2(\gamma_{13}^2 + \delta_p^2) + 4\gamma_{12}\gamma_{13}|\Omega_c|^2 + (4\delta_p\delta_c - 4\delta_p^2 + |\Omega_c|^2)]} \quad (1.62)$$

$$Re(\chi^{(1)}(\omega_p)) = \frac{-4N|Q_{13}|^2[\gamma_{12}^2\delta_p + 4\delta_c^2\delta_p + 4\delta_p^3 - \delta_p|\Omega_c|^2 + \delta_c(-8\delta_p^2 + |\Omega_c|^2)]}{\hbar\epsilon_0[16\gamma_{13}^2\delta_2^2 + 4\gamma_{12}^2(\gamma_{13}^2 + \delta_p^2) + 4\gamma_{12}\gamma_{13}|\Omega_c|^2 + (4\delta_p\delta_c - 4\delta_p^2 + |\Omega_c|^2)]} \quad (1.63)$$

The imaginary part of the linear susceptibility ($Im(\chi^{(1)})$) represents the absorption profile of the probe field, while the real part ($Re(\chi^{(1)})$) shows the dispersive nature of the probe field in the EIT medium. In Fig. 1.4 we have plotted the imaginary part of the linear susceptibility ($Im(\chi^{(1)})$) against detuning of the probe field δ_p . Here the continuous red line represents the absorptive profile of the probe field in the absence of the coupling field, which has Lorentzian profile. In the presence of the coupling field, the absorptive nature of the probe field will get drastically modified, going from absorption to near-transparent in the medium, which is known

as the electromagnetically induced transparency (EIT) effect [56, 57], shown as a continuous blue line. The dependence of real part of the linear susceptibility ($\text{Re}(\chi^{(1)})$) on detuning of the probe field δ_p is plotted in Fig. 1.5. The dispersive nature of the probe field in the medium in the absence of the coupling field is shown as a continuous red line. The slope of the $\text{Re}(\chi^{(1)})$ is negative around the transparency window. The continuous blue line shows the $\text{Re}(\chi^{(1)})$ in the presence of the coupling field. It is evident from the Fig. 1.5 that the slope of the curve is positive, and it can lead to the reduction in group velocity of the probe field in the medium, this feature is exploited in employing slow light and stopped light experiments in atomic media [39, 41]. The EIT and associated Raman coherence are well studied in the past decade [59]. The dark state polariton [60] presence in an EIT medium is well studied and is used in understanding the quantum memory and its physical realization [61, 62, 42]. The nonlinear optical response of a undergoes an EIT effect is exploited in employing nonlinear wave mixing with ultra-low intensities [43, 44, 45] and Giant Kerr nonlinearity [63]

1.3.3 Saturated absorption spectroscopy

Saturated absorption spectroscopy (SAS) is a spectroscopic techniques. It helps us lock the external cavity diode laser (ECDL) to a particular transition with greater accuracy and precision. One can resolve all the relevant transition lines of an atomic transition with the help of the saturated absorption spectroscopic technique, which otherwise appears as a single broad line due to the Doppler broadening [64, 65].

The velocity of room temperature atoms is dictated by the Maxwell-Boltzmann velocity distribution. Therefore the absorption profile of a resonant field will have a Gaussian profile due to the Doppler shift. Hence the laser field cannot be precisely locked to a particular transition.

We have used Rubidium (^{85}Rb) atoms in our experiments. Rb belongs to the group I elements in the periodic table, and they have a single electron in the valence shell. There are two isotopes of Rb found in nature with a natural abundance

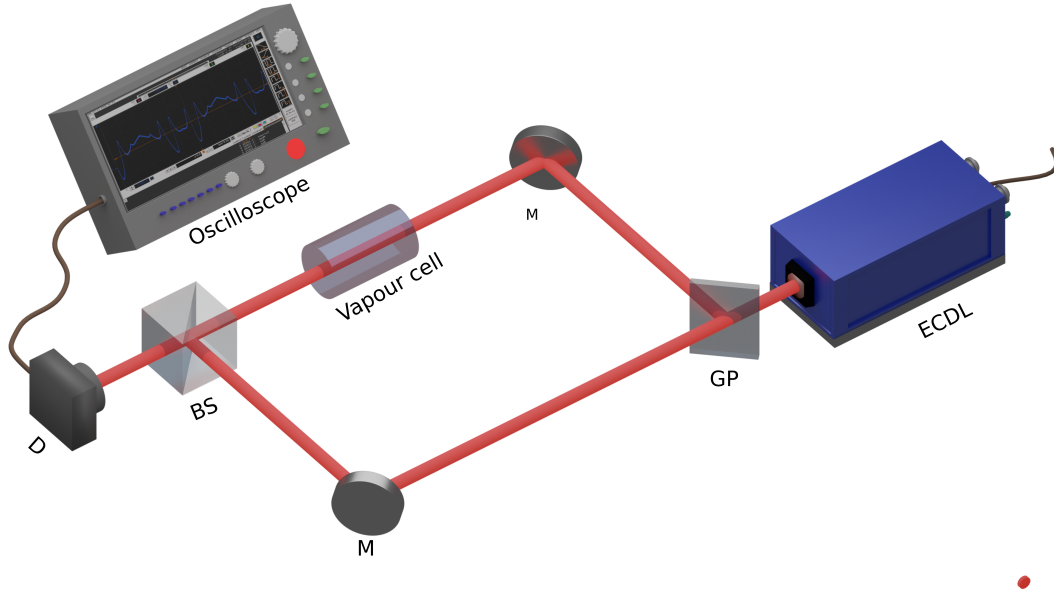


Figure 1.6: Schematic of experimental set up of the saturated absorption spectroscopy. Here ECDL —External Cavity Diode Laser, M —Mirror, D —Detector, BS —Beam splitter, GP —Glass plate.

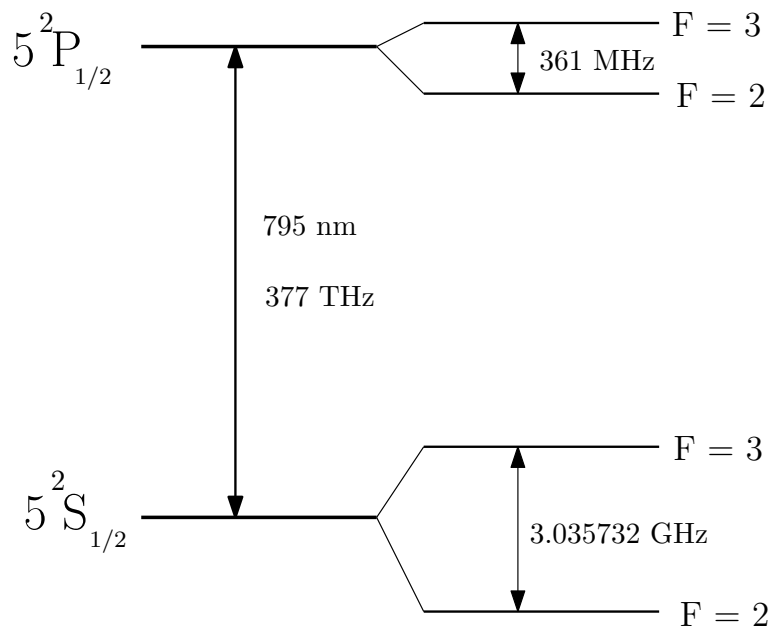


Figure 1.7: Hyperfine energy levels of ^{85}Rb atoms in its D1 manifold.

of 72.2 % of ^{85}Rb isotope and 27.8 % of ^{87}Rb isotope. A vapour cell containing enriched ^{85}Rb atoms have been used throughout our experiments. The energy level structure of ^{85}Rb atom is given in Fig. 1.7.

To observe Doppler free spectrum of ^{85}Rb atoms we have used a saturated absorption spectroscopic arrangement as shown in Fig. 1.6. A strong pump beam and a weak probe beam derived from the same external cavity diode laser (ECDL) will propagate through a vapour cell containing ^{85}Rb atoms in a counter propagating configuration. The counter-propagating configuration of the pump and probe beam make sure that a particular velocity class of atoms will be seeing both the beams on resonant. Since a single ECDL is used to derive both pump and probe beam, the frequency, and the detuning of both the fields are same. Due to the higher power of the pump beam, most of the atoms will get excited and the transition gets saturated. Hence the probe beam won't get absorbed much and transmitted through the vapour cell. The transmitted probe beam will further detect using a photo detector and analyzed in an oscilloscope. The spectral profile of the probe beam will be Doppler free spectrum of the corresponding transition.

In Fig. 1.8 shown the saturated absorption spectrum of D1 transition line of ^{85}Rb atom.

1.4 Organization of the thesis

Chapter 2 describes a coherent generation of a near infrared optical field through a three-wave mixing phenomenon in an atomic energy level scheme of ^{85}Rb atoms. In systems with centrosymmetry, second-order nonlinear processes are absent [38]. We have shown that the generation processes will follow a three-photon resonance condition between all the three electromagnetic fields, which is a direct consequence of energy conservation in this sum-frequency process. The bandwidth of the generation process is decided by the Raman coherence present in the system. When a two-photon resonance between the coupling field and the generated probe field is satisfied, the probe field also undergoes an EIT effect [39]. The EIT phe-

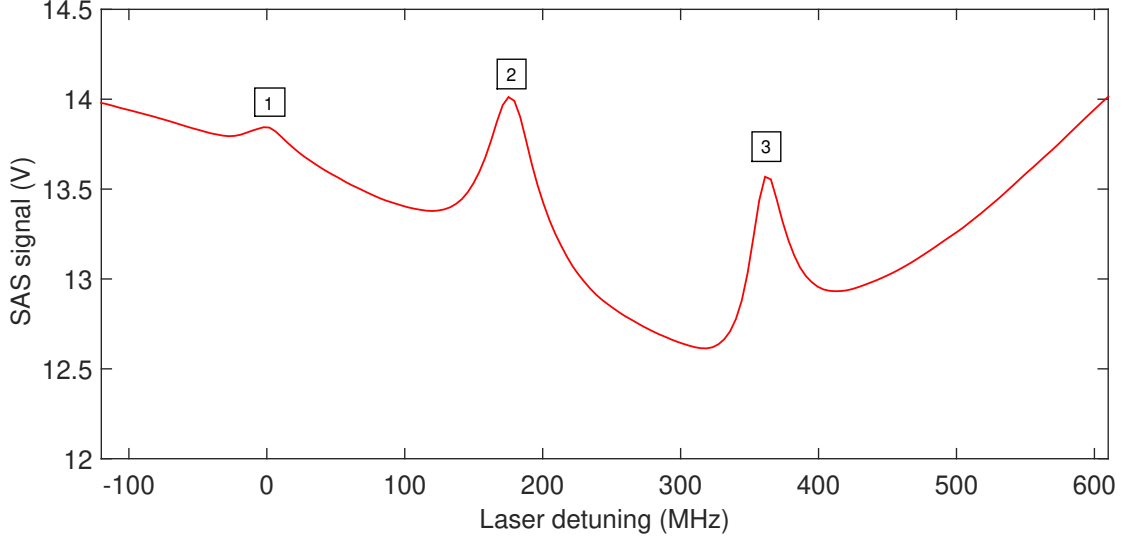


Figure 1.8: Saturated absorption spectrum of $F = 3 \rightarrow F' = 2$ and 3 transition of ^{85}Rb atom in its D1 manifold. Here 1, 2 and 3 represents the saturation absorption spectrum peak for the $F = 3 \rightarrow F' = 2$ transition, cross over resonance peak and $F = 3 \rightarrow F' = 3$ transition respectively.

nomenon is suppressing all linear absorption process and the dark state associated with EIT is responsible for the suppression of all the decoherence effect such as spontaneous emission from the excited state. The chapter concludes by comparing our scheme with other physical realization for microwave-to-optical frequency conversion.

Chapter 3 covers the phase-dependent amplification of an optical field using microwaves. We have done a theoretical analysis of the three-wave mixing present in our Δ system. We have obtained an analytical expression for the probe field using density matrix analysis. Our analysis shows that, in addition to a generation of probe field which is discussed in earlier chapter, there will be an amplification also possible for the probe field in our Δ system. The physical mechanism of the amplification process reveals that the amplification is due to an interference of the input probe field and the generated probe field. This interference effect can give rise to an amplification or a de-amplification depends on the phase of the generated probe field. This probe phase can be controlled by the phase of the microwave field as described in previous chapter. Thus making the system as a phase depended amplifier (PDA). The dependence of amplification and de-

amplification of probe field is studied with various microwave field intensities, and with various two-photon detuning. The chapter ends with concluding the results.

Chapter 4 is an experimental demonstration of the results presented in chapter 3. We observe amplification and de-amplification of the input probe beam. In this experiment, we investigate the gain experienced by the probe field both in the presence and absence of two-photon induced ground state coherence. By carefully choosing an experimental arrangement, we could control the two-photon detuning by changing the microwave detuning which is connecting the ground and the excited states. We show that a simultaneous fulfilment of a two-photon resonance condition that creates ground state coherence and a three-photon resonance condition leads to a significantly higher amplification of the optical probe field. By selectively breaking the ground state coherence using microwaves, we show that the amplification reduces. Nevertheless, the system shows non-zero PDA for large two-photon detunings with high visibility. This novel, controllable hybrid-PDA can be potentially used to trade-off amplification for bandwidth during the transmission of phase coherent classical and quantum information.

In the last chapter, we conclude the thesis by giving a summary. Also, a brief overview of future works is given.

Bibliography

- [1] Emre Togan, Yiwen Chu, Alexei S Trifonov, Liang Jiang, Jeronimo Maze, Lilian Childress, MV Gurudev Dutt, Anders Søndberg Sørensen, Phillip R Hemmer, Alexander S Zibrov, et al. Quantum entanglement between an optical photon and a solid-state spin qubit. *Nature*, 466(7307):730–734, 2010.
- [2] William F Koehl, Bob B Buckley, F Joseph Heremans, Greg Calusine, and David D Awschalom. Room temperature coherent control of defect spin qubits in silicon carbide. *Nature*, 479(7371):84–87, 2011.
- [3] Boris B Blinov, David L Moehring, L-M Duan, and Chris Monroe. Observation of entanglement between a single trapped atom and a single photon. *Nature*, 428(6979):153–157, 2004.

- [4] DN Matsukevich and A Kuzmich. Quantum state transfer between matter and light. *Science*, 306(5696):663–666, 2004.
- [5] WB Gao, Parisa Fallahi, Emre Togan, Javier Miguel-Sánchez, and Atac Imamoglu. Observation of entanglement between a quantum dot spin and a single photon. *Nature*, 491(7424):426–430, 2012.
- [6] Kristiaan De Greve, Leo Yu, Peter L McMahon, Jason S Pelc, Chandra M Natarajan, Na Young Kim, Eisuke Abe, Sebastian Maier, Christian Schneider, Martin Kamp, et al. Quantum-dot spin–photon entanglement via frequency downconversion to telecom wavelength. *Nature*, 491(7424):421–425, 2012.
- [7] M. H. Devoret and R. J. Schoelkopf. Superconducting circuits for quantum information: An outlook. *Science*, 339(6124):1169–1174, 2013.
- [8] Yu. A. Pashkin, O. Astafiev, T. Yamamoto, Y. Nakamura, and J. S. Tsai. Josephson charge qubits: a brief review. *Quantum Inf. Process.*, 8:55–80, 2009.
- [9] Ronald Hanson and David D Awschalom. Coherent manipulation of single spins in semiconductors. *Nature*, 453(7198):1043–1049, 2008.
- [10] L. M. K. Vandersypen and I. L. Chuang. Nmr techniques for quantum control and computation. *Rev. Mod. Phys.*, 76:1037–1069, Jan 2005.
- [11] Göran Wendin. Quantum information processing with superconducting circuits: a review. *Rep. Prog. Phys.*, 80(10):106001, 2017.
- [12] Nissim Ofek, Andrei Petrenko, Reinier Heeres, Philip Reinhold, Zaki Leghtas, Brian Vlastakis, Yehan Liu, Luigi Frunzio, SM Girvin, Liang Jiang, et al. Extending the lifetime of a quantum bit with error correction in superconducting circuits. *Nature*, 536(7617):441–445, 2016.
- [13] Yuwei Ma, Yuan Xu, Xianghao Mu, Weizhou Cai, Ling Hu, Weiting Wang, Xiaoxuan Pan, Haiyan Wang, YP Song, C-L Zou, et al. Error-transparent operations on a logical qubit protected by quantum error correction. *Nat. Phys.*, 16(8):827–831, 2020.

- [14] Philipp Kurpiers, Paul Magnard, Theo Walter, Baptiste Royer, Marek Pechal, Johannes Heinsoo, Yves Salathé, Abdulkadir Akin, Simon Storz, J-C Besse, et al. Deterministic quantum state transfer and remote entanglement using microwave photons. *Nature*, 558(7709):264–267, 2018.
- [15] Sheng-Kai Liao, Wen-Qi Cai, Wei-Yue Liu, Liang Zhang, Yang Li, Ji-Gang Ren, Juan Yin, Qi Shen, Yuan Cao, Zheng-Ping Li, et al. Satellite-to-ground quantum key distribution. *Nature*, 549(7670):43–47, 2017.
- [16] Alberto Boaron, Gianluca Boso, Davide Rusca, Cédric Vulliez, Claire Autebert, Misael Caloz, Matthieu Perrenoud, Gaëtan Gras, Félix Bussièrès, Ming-Jun Li, et al. Secure quantum key distribution over 421 km of optical fiber. *Phys. Rev. Lett.*, 121(19):190502, 2018.
- [17] J. J. Longdell, E. Fraval, M. J. Sellars, and N. B. Manson. Stopped light with storage times greater than one second using electromagnetically induced transparency in a solid. *Phys. Rev. Lett.*, 95:063601, Aug 2005.
- [18] Ye Wang, Mark Um, Junhua Zhang, Shuoming An, Ming Lyu, Jing-Ning Zhang, L-M Duan, Dahyun Yum, and Kihwan Kim. Single-qubit quantum memory exceeding ten-minute coherence time. *Nat. Photonics*, 11(10):646–650, 2017.
- [19] Manjin Zhong, Morgan P Hedges, Rose L Ahlefeldt, John G Bartholomew, Sarah E Beavan, Sven M Wittig, Jevon J Longdell, and Matthew J Sellars. Optically addressable nuclear spins in a solid with a six-hour coherence time. *Nature*, 517(7533):177–180, 2015.
- [20] Thomas Astner, Johannes Gugler, Andreas Angerer, Sebastian Wald, Stefan Putz, Norbert J Mauser, Michael Trupke, Hitoshi Sumiya, Shinobu Onoda, Junichi Isoya, et al. Solid-state electron spin lifetime limited by phononic vacuum modes. *Nat. Mater.*, 17(4):313–317, 2018.
- [21] Nicholas J. Lambert, Alfredo Rueda, Florian Sedlmeir, and Harald G. L. Schwefel. Coherent conversion between microwave and optical photons—an overview of physical implementations. *Adv. Quantum Technol.*, 3(1):1900077, 2020.

- [22] Nikolai Lauk, Neil Sinclair, Shabir Barzanjeh, Jacob P Covey, Mark Saffman, Maria Spiropulu, and Christoph Simon. Perspectives on quantum transduction. *Quantum Sci. Technol.*, 5(2):020501, 2020.
- [23] Xu Han, Wei Fu, Chang-Ling Zou, Liang Jiang, and Hong X Tang. Microwave-optical quantum frequency conversion. *Optica*, 8(8):1050–1064, 2021.
- [24] D. V. Strekalov, H. G. L. Schwefel, A. A. Savchenkov, A. B. Matsko, L. J. Wang, and N. Yu. Microwave whispering-gallery resonator for efficient optical up-conversion. *Phys. Rev. A*, 80:033810, Sep 2009.
- [25] Alfredo Rueda, Florian Sedlmeir, Michele C. Collodo, Ulrich Vogl, Birgit Stiller, Gerhard Schunk, Dmitry V. Strekalov, Christoph Marquardt, Johannes M. Fink, Oskar Painter, Gerd Leuchs, and Harald G. L. Schwefel. Efficient microwave to optical photon conversion: an electro-optical realization. *Optica*, 3(6):597–604, Jun 2016.
- [26] Reed W Andrews, Robert W Peterson, Tom P Purdy, Katarina Cicak, Raymond W Simmonds, Cindy A Regal, and Konrad W Lehnert. Bidirectional and efficient conversion between microwave and optical light. *Nat. Phys.*, 10(4):321, 2014.
- [27] Joerg Bochmann, Amit Vainsencher, David D Awschalom, and Andrew N Cleland. Nanomechanical coupling between microwave and optical photons. *Nat. Phys.*, 9(11):712, 2013.
- [28] J M Fink, M Kalaei, R Norte, A Pitanti, and O Painter. Efficient microwave frequency conversion mediated by a photonics compatible silicon nitride nanobeam oscillator. *Quantum Sci. Technol.*, 5(3):034011, jun 2020.
- [29] Christopher O’Brien, Nikolai Lauk, Susanne Blum, Giovanna Morigi, and Michael Fleischhauer. Interfacing superconducting qubits and telecom photons via a rare-earth-doped crystal. *Phys. Rev. Lett.*, 113(6):063603, 2014.
- [30] Susanne Blum, Christopher O’Brien, Nikolai Lauk, Pavel Bushev, Michael Fleischhauer, and Giovanna Morigi. Interfacing microwave qubits and optical photons via spin ensembles. *Phys. Rev. A*, 91:033834, Mar 2015.

- [31] Mohammad Hafezi, Zaeill Kim, Steven L Rolston, Luis A Orozco, BL Lev, and Jacob M Taylor. Atomic interface between microwave and optical photons. *Phys. Rev. A*, 85(2):020302, 2012.
- [32] Martin Kiffner, Amir Feizpour, Krzysztof T Kaczmarek, Dieter Jaksch, and Joshua Nunn. Two-way interconversion of millimeter-wave and optical fields in rydberg gases. *New J. Phys.*, 18(9):093030, 2016.
- [33] Bryan T. Gard, Kurt Jacobs, R. McDermott, and M. Saffman. Microwave-to-optical frequency conversion using a cesium atom coupled to a superconducting resonator. *Phys. Rev. A*, 96:013833, Jul 2017.
- [34] Lewis A Williamson, Yu-Hui Chen, and Jevon J Longdell. Magneto-optic modulator with unit quantum efficiency. *Phys. Rev. Lett.*, 113(20):203601, 2014.
- [35] Yu-Guo Liu, Keyu Xia, and Shi-Liang Zhu. Efficient microwave-to-optical single-photon conversion with a single flying circular rydberg atom. *Opt. Express*, 29(7):9942–9959, Mar 2021.
- [36] Jingshan Han, Thibault Vogt, Christian Gross, Dieter Jaksch, Martin Kiffner, and Wenhui Li. Coherent microwave-to-optical conversion via six-wave mixing in rydberg atoms. *Phys. Rev. Lett.*, 120:093201, Mar 2018.
- [37] Thibault Vogt, Christian Gross, Jingshan Han, Sambit B. Pal, Mark Lam, Martin Kiffner, and Wenhui Li. Efficient microwave-to-optical conversion using rydberg atoms. *Phys. Rev. A*, 99:023832, Feb 2019.
- [38] Robert W. Boyd. *Nonlinear Optics, Third Edition*. Academic Press, Inc., Orlando, FL, USA, 3rd edition, 2008.
- [39] Michael Fleischhauer, Atac Imamoglu, and Jonathan P. Marangos. Electromagnetically induced transparency: Optics in coherent media. *Rev. Mod. Phys.*, 77:633–673, Jul 2005.
- [40] S. E. Harris. Lasers without inversion: Interference of lifetime-broadened resonances. *Phys. Rev. Lett.*, 62:1033–1036, Feb 1989.

- [41] Chien Liu, Zachary Dutton, Cyrus H. Behroozi, and Lene Vestergaard Hau. Observation of coherent optical information storage in an atomic medium using halted light pulses. *Nature*, 409:490–493, 2001.
- [42] T. Chanelière, D. N. Matsukevich, S. D. Jenkins, S.-Y. Lan, T. A. B. Kennedy, and A. Kuzmich. Storage and retrieval of single photons transmitted between remote quantum memories. *Nature*, 438:833—836, 2005.
- [43] S. E. Harris, J. E. Field, and A. Imamoglu. Nonlinear optical processes using electromagnetically induced transparency. *Phys. Rev. Lett.*, 64:1107–1110, Mar 1990.
- [44] M. D. Lukin, P. R. Hemmer, M. Löffler, and M. O. Scully. Resonant enhancement of parametric processes via radiative interference and induced coherence. *Phys. Rev. Lett.*, 81:2675–2678, Sep 1998.
- [45] Maneesh Jain, Hui Xia, G. Y. Yin, A. J. Merriam, and S. E. Harris. Efficient nonlinear frequency conversion with maximal atomic coherence. *Phys. Rev. Lett.*, 77:4326–4329, Nov 1996.
- [46] D V Kosachiov, B G Matisov, and Y V Rozhdestvensky. Coherent phenomena in multilevel systems with closed interaction contour. *J. Phys. B: At. Mol. Opt. Phys.*, 25(11):2473–2488, jun 1992.
- [47] Manukumara Manjappa, Satya Sainadh Undurti, Asha Karigowda, Andal Narayanan, and Barry C. Sanders. Effects of temperature and ground-state coherence decay on enhancement and amplification in a Δ atomic system. *Phys. Rev. A*, 90:043859, Oct 2014.
- [48] H. R. Hamed and G. Juzeliūnas. Phase-sensitive kerr nonlinearity for closed-loop quantum systems. *Phys. Rev. A*, 91:053823, May 2015.
- [49] Hebin Li, Vladimir A. Sautenkov, Yuri V. Rostovtsev, George R. Welch, Philip R. Hemmer, and Marlan O. Scully. Electromagnetically induced transparency controlled by a microwave field. *Phys. Rev. A*, 80:023820, Aug 2009.

- [50] G. S. Agarwal, Tarak Nath Dey, and Sunish Menon. Knob for changing light propagation from subluminal to superluminal. *Phys. Rev. A*, 64:053809, Oct 2001.
- [51] A. Eilam, A. D. Wilson-Gordon, and H. Friedmann. Efficient light storage in a λ system due to coupling between lower levels. *Opt. Lett.*, 34(12):1834–1836, Jun 2009.
- [52] Sankar Davuluri and Yuri Rostovtsev. Optical control of backward and forward microwave generation. *Phys. Rev. A*, 88:053847, Nov 2013.
- [53] Rajitha K. V., Tarak N. Dey, Sumanta Das, and Pankaj K. Jha. Microwave-controlled efficient raman sub-harmonic generation. *Opt. Lett.*, 40(10):2229–2232, May 2015.
- [54] Rajitha K. V. and Tarak N. Dey. Microwave-assisted arbitrary optical-pulse generation in a thermal vapor. *Phys. Rev. A*, 94:053851, Nov 2016.
- [55] Megha Ghosh, Asha Karigowda, Ayyappan Jayaraman, Fabien Bretenaker, Barry C Sanders, and Andal Narayanan. Demonstration of a high-contrast optical switching in an atomic delta system. *J. Phys. B: At. Mol. Opt. Phys.*, 50(16):165502, jul 2017.
- [56] K.-J. Boller, A. Imamoglu, and S. E. Harris. Observation of electromagnetically induced transparency. *Phys. Rev. Lett.*, 66:2593–2596, May 1991.
- [57] J. E. Field, K. H. Hahn, and S. E. Harris. Observation of electromagnetically induced transparency in collisionally broadened lead vapor. *Phys. Rev. Lett.*, 67:3062–3065, Nov 1991.
- [58] J. P. Marangos. Electromagnetically induced transparency. *J. Mod. Opt.*, 45(3):471–503, 1998.
- [59] Sergey Novikov, T Sweeney, JE Robinson, SP Premaratne, B Suri, FC Wellstood, and BS Palmer. Raman coherence in a circuit quantum electrodynamics lambda system. *Nat. Phys.*, 12(1):75–79, 2016.

- [60] M. Fleischhauer and M. D. Lukin. Dark-state polaritons in electromagnetically induced transparency. *Phys. Rev. Lett.*, 84:5094–5097, May 2000.
- [61] M. Fleischhauer and M. D. Lukin. Quantum memory for photons: Dark-state polaritons. *Phys. Rev. A*, 65:022314, Jan 2002.
- [62] M. D. Lukin, S. F. Yelin, and M. Fleischhauer. Entanglement of atomic ensembles by trapping correlated photon states. *Phys. Rev. Lett.*, 84:4232–4235, May 2000.
- [63] H. Schmidt and A. Imamoglu. Giant kerr nonlinearities obtained by electromagnetically induced transparency. *Opt. Lett.*, 21(23):1936–1938, Dec 1996.
- [64] T. W. Hänsch, I. S. Shahin, and A. L. Schawlow. High-resolution saturation spectroscopy of the sodium *d* lines with a pulsed tunable dye laser. *Phys. Rev. Lett.*, 27:707–710, Sep 1971.
- [65] R C Thompson. High resolution laser spectroscopy of atomic systems. *Rep. Prog. Phys.*, 48(4):531–578, apr 1985.

Chapter 2

Coherent microwave-to-optical conversion by three-wave mixing in a room temperature atomic system.

2.1 Introduction

In the past decade, using ultra-cold solid-state devices quantum bits (qubits) have been realized in the microwave frequency regime. These include superconducting qubits of diverse flavours [1], charge qubits in various material systems [2], spin qubits [3], electron spin systems [4], and nuclear spins [5]. However, it is inconvenient to transmit quantum information encoded in microwave frequency over a long distance. The microwave signal decays rapidly during propagation and is easily disturbed by thermal noise at room temperature. In stark contrast, the optical photons are robust to thermal noise, and optical fibers can efficiently transmit quantum information over long distances. Thus it creates a strong interest in developing interfaces between microwave and optical frequencies [6]. Single-photon detectors and quantum memories [7] use optical, electromagnetic fields, suggesting the need for the bi-directional transfer of quantum information between microwave and optical photons. Thus coherent conversion of microwave-to-optical frequency has tremendous potential for developing next-generation classical and quantum technologies.

Microwaves and optical fields have a five-order difference in frequency, making

it difficult to find a suitable platform for interaction and signal transfer between these two widely separated fields. In recent times nonlinear frequency conversion from microwave to optical frequencies has garnered a lot of attention [8, 9, 10]. This is owing to the ease of transport and detection of gigahertz signals through optical channels and the essentially noise-free nature of frequency conversion process. The latter attribute has been central to proposing high-fidelity classical and quantum conversion of signals between microwave and optical frequencies [11, 12]. A range of devices have been proposed to demonstrate this frequency conversion phenomena.

Starting from the commonly used electro-optic-modulator, several authors have investigated carefully engineered whispering-gallery-mode resonators [13, 14], micro and nano sized hybrid electro-opto-mechanical devices operating at a few kelvin [15, 16], efficient microwave frequency conversion mediated by a photonics compatible silicon nitride nano beam oscillator [17], rare-earth doped crystals [18] and gaseous atomic systems with induced coherence [19] as viable architectures for effecting non-linear frequency generation and signal transfer. In atomic systems, there have been several theoretical proposals [20, 21, 22, 23, 24], but very few experimental demonstrations [25, 26]. These studies highlight various aspects of the non-linear generation phenomenon such as efficiency, bi-directionality and non-reciprocity.

Four-wave mixing phenomenon has been a widely utilized non-linear frequency generation process in atomic systems [27, 28]. Beginning with lasing without inversion [29], experiments showing new frequency generation have been performed, which combine atomic coherence effects with four-wave-mixing process [30, 31, 32, 33]. To increase the efficiency of nonlinear process the more efficient three-wave mixing could be used. However in neutral atomic systems three-wave mixing nonlinear-optical processes are forbidden by symmetry [34]. In order to observe these nonlinear optical processes in an atomic vapour, some method must be used to break the symmetry of the medium. The utilisation of a magnetic-dipole interaction breaks the symmetry requirement and enables a three-wave mixing response to emerge in the centro-symmetric atomic system [35]. To achieve this, we have chosen a three hyperfine atomic level scheme in room temperature Rubidium atoms

which is coupled with microwave and optical fields.

In this chapter, we demonstrate a novel three-wave mixing process in a room temperature gaseous atomic vapour of ^{85}Rb atoms, which utilizes induced atomic coherence to generate a near infrared optical field. This three-wave mixing process combines an electric dipole and a magnetic dipole transition, in optical and microwave frequency domains respectively, leading to generation of a new infra-red frequency. A unique feature of our system is that despite thermal motion of our atoms which are at room temperature and despite resonant non-linear interaction, the generation process is coherent. It is neither affected by the Doppler width nor by decay processes from excited states. This is due to the fulfillment of both two-photon and three-photon resonance conditions for our system by all the three fields. During the generation process of the probe field, the probe and the coupling satisfy a two-photon resonance that shelves the atoms in a dark state owing to the electromagnetically induced transparency effect [36], making the process immune to noise effects due to spontaneous decay from excited states. As is well known the two-photon resonance condition is Doppler free. This Doppler-free condition remains valid even during the fulfillment of the three-photon resonance condition between all the three electromagnetic fields due to the very narrow Doppler width seen by the atoms for microwaves.

Thus due to a non-linear interaction between induced electric and magnetic dipoles and due to formation of atomic ground state coherence we have demonstrated for the first time the lowest order coherent non-linear frequency conversion process from an atomic ensemble. The band width for coherent conversion from microwave to optical frequencies with this scheme is shown to be about 825 kHz. The efficiency of our conversion process is about 2.6×10^{-6} . The efficiency is mainly dictated by the weakness of magnetic-dipole interaction induced by the microwave field. Nevertheless the fidelity that could be achieved by our scheme for signal transfer will be very high due to absence of major sources of decoherence in our conversion mechanism. This aspect can be exploited especially for transfer of quantum signals as previously shown [37].

2.2 Experimental details

We have chosen a three-level system around the D1 manifold of hyperfine levels of ^{85}Rb atoms. The three atomic energy levels chosen for our experiment are the ground state $5S_{1/2}, F = 2$, the metastable state $5S_{1/2}, F = 3$ and the excited state $5P_{1/2}, F = 3'$ labeled as $|1\rangle$, $|2\rangle$ and $|3\rangle$ respectively as shown in Fig. 2.1. The metastable ground state $5S_{1/2}, F = 3$ state ($|2\rangle$) to the excited state $5P_{1/2}, F = 3'$ ($|3\rangle$) transition is driven by an optical coupling laser having an angular frequency of ω_c . While the ground state $5S_{1/2}, F = 2$ state ($|1\rangle$) is connected to the metastable state $5S_{1/2}, F = 3$ state ($|2\rangle$) with a microwave drive field with an angular frequency of ω_{RF} . In this energy level structure, due to selection rules, the transition $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ are electric dipole allowed transitions, while $|1\rangle \rightarrow |2\rangle$ transition is electric dipole forbidden transition. Thus the magnetic dipole transition $|1\rangle \rightarrow |2\rangle$ is driven by a microwave field (ω_{RF}).

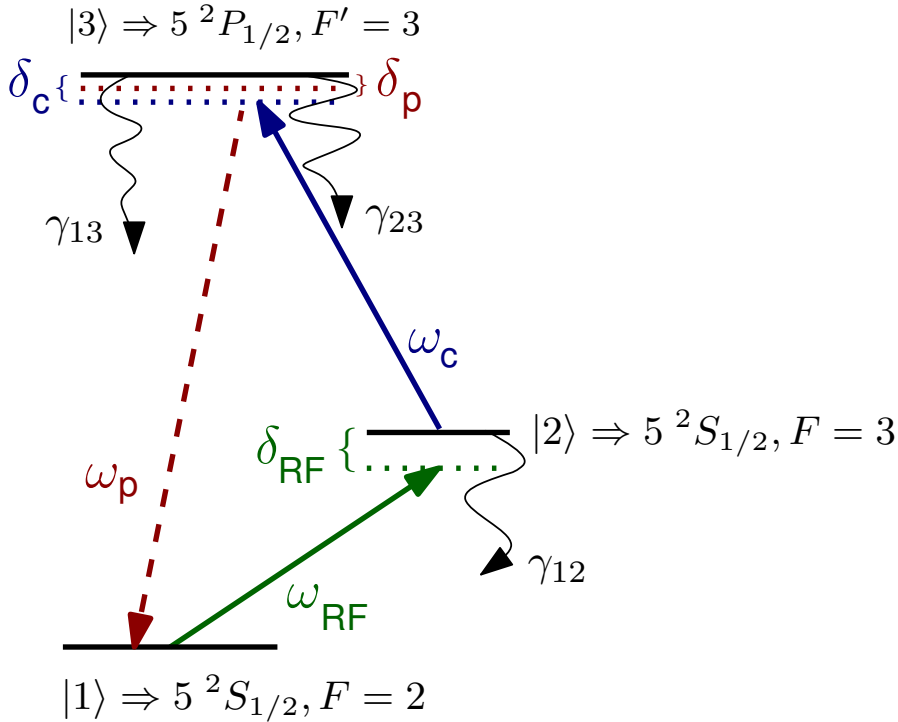


Figure 2.1: A Δ system formation using hyperfine levels of ^{85}Rb in its D1 manifold.

In our experiment the optical coupling beam is derived from an external cavity diode laser (ECDL) with a line-width of a few hundred kHz. The frequency of

the coupling laser ω_c is controlled by a saturated absorption spectroscopy (SAS) setup. The detailed experimental setup is shown in Fig. 2.2. The detuning of the coupling field from the $|2\rangle \rightarrow |3\rangle$ transition is denoted by δ_c . The microwave drive field is generated from a microwave source, the detuning of the microwave drive field is denoted as δ_{RF} . A vapour cell of length 5 cm containing enriched ^{85}Rb atoms at a temperature of 300 K, is used in our experiment. To overcome the weakness of the magnetic-dipole transition driven by the microwave field the vapour cell is placed inside a cylindrical microwave cavity. This cavity supports a TE_{011} mode of the microwave field with a high quality factor of 9000 ± 1000 at 3.0357 GHz, which is the separation between the levels $|1\rangle$ and $|2\rangle$. This is shown in the schematic of the experimental setup, also given in Fig. 2.2.

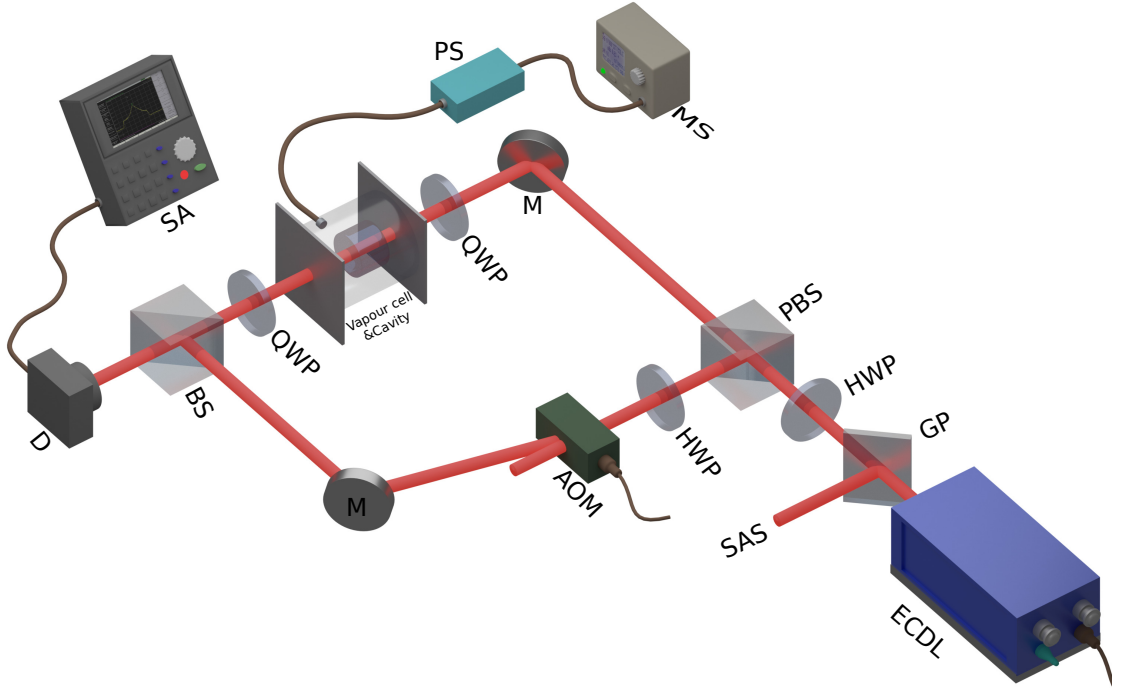


Figure 2.2: Schematic of experimental set up. Here ECDL —External Cavity Diode Laser, GP —Glass plate, HWP —half-wave plate, QWP —quarter wave-plate, PBS —polarizing beam-splitter, AOM —Acousto-optic modulator, M —Mirror, MS —Microwave source, PS —Phase shifter, D —Detector, SA —Spectrum analyzer.

A typical experimental run consists of locking the coupling laser at resonance using the saturation spectroscopy signal, $\delta_c = 0$. The microwave drive field is then

scanned around its resonance with δ_{RF} spanning through ± 2 MHz. The value of the Doppler width of the microwave transition is around 4 kHz and it is less than the scanning range. During this scan, the coupling optical field and the microwave drive field interact non-linearly with the atoms. A sum-frequency generation takes place, giving rise to a probe field ω_p with a frequency $\omega_p = \omega_c + \omega_{\text{RF}}$. In our experiment, the phase-matching requirement $\vec{k}_p = \vec{k}_c + \vec{k}_{\text{RF}}$ for efficient generation and the fact that $\vec{k}_{\text{RF}}$ is small ensures the generated beam copropagate with the coupling beam.

A heterodyne detection is employed to separate probe field from other fields. An acousto-optic modulator (AOM) is used to generate the local oscillator beam by sending a part of the coupling beam. The AOM is driven by a radio frequency source at 70 MHz. After the AOM double pass, we chose the local oscillator beam using an iris having a frequency of $\omega_c + 140$ MHz. If we were using the coupling laser (ω_c) as a local oscillator for the heterodyne detection scheme the beatnote would have appeared exactly at the microwave frequency. The high bandwidth detector (25 GHz) used for the heterodyne detection was also picking up the frequency of the microwave source due to leakage. This was a dominant contribution towards the signal appear at the microwave frequency (3.0357 GHz) in the spectrum analyzer. To avoid the beatnote due to the generated probe field being affected by the leakage from the microwave source, we have used a local oscillator with frequency $\omega_c + 140$ MHz for the heterodyne detection, so that the probe field detection can be done unambiguously. The heterodyne detection of the probe field is done using a spectrum analyzer (Fig. 2.2).

2.3 Results and discussion

Now we proceed to show the experimental results. We have studied the characteristics of the generated probe field by measuring the beat-note between the generated probe beam and the local oscillator beam. The spectral profile of the generated probe field is shown in Fig. 2.3. This measurement is taken when the detunings of the optical coupling and microwave drive field are at resonance with

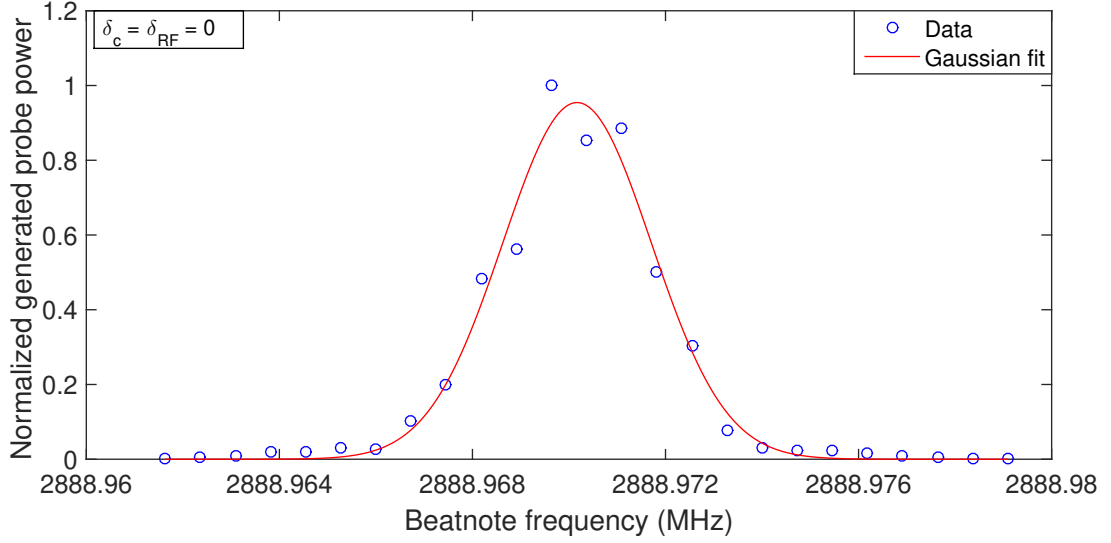


Figure 2.3: The spectral width of the generated probe field is shown when both the coupling and the microwave fields are at resonance with their respective transitions. The experimental points are shown as circles with a Gaussian fit shown as a continuous line. The resolution bandwidth (RBW) of the spectrum analyzer was 100 Hz for this measurement.

their respective transitions. The x axis in Fig. 2.3 refers to this beat-note frequency.

The width of the generated probe field is calculated from the Gaussian fit to the experimental data. The width of the generated probe field is found to be about 3 kHz at a resolution bandwidth of 100 Hz. Such a narrow bandwidth for the generated probe field is due to the parametric nature of the process. The width of the generated probe field is decided by the noise of the microwave source which induces the three-wave mixing process.

We now demonstrate that the sum-frequency generation satisfies a three-photon resonance condition, a direct consequence of energy conservation when one coupling optical photon combines with a microwave drive photon to generate an optical probe photon. The three-photon resonance condition is defined between the two optical and one microwave field as

$$\delta_p - \delta_c - \delta_{RF} = 0. \quad (2.1)$$

Detuning δ_p at which maximum probe light generation is observed as a function

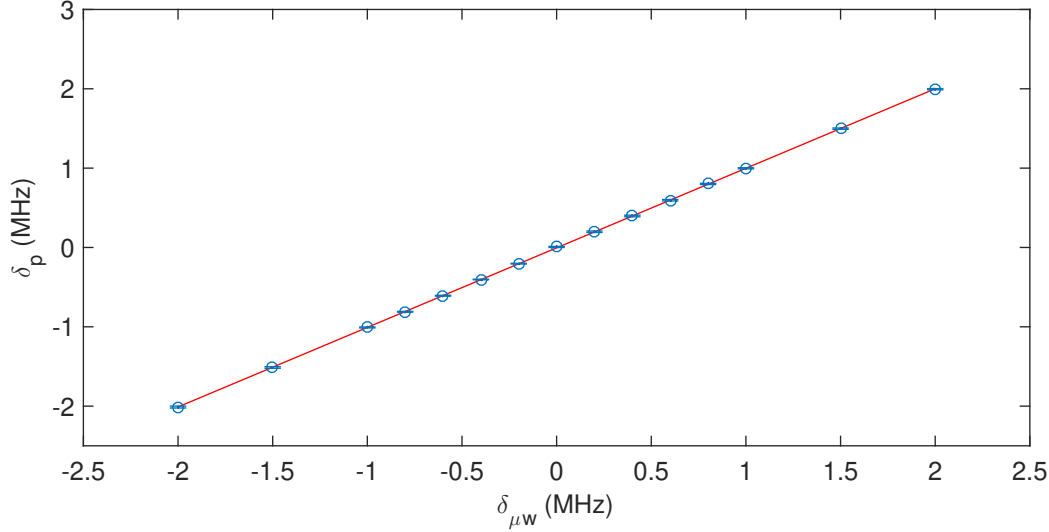


Figure 2.4: Detuning δ_p at which maximum probe light generation is seen as a function of detuning δ_{RF} of the microwave drive field with the coupling field on resonance with $\delta_c = 0$. We have measured a maximum standard deviation of 0.0241 MHz. The experimental points are shown as circles with a linear fit shown as a continuous line.

of detuning δ_{RF} of the microwave drive field, is plotted in Fig. 2.4. This plot is obtained by varying the detuning of the microwave field δ_{RF} while keeping the coupling field resonant with the $|2\rangle \rightarrow |3\rangle$ transition (i.e. $\delta_c = 0$). It is evident from Fig. 2.4 that the detuning at which maximum probe field generation is observed denoted by δ_p has a one-to-one scaling with the detuning of the microwave drive field δ_{RF} . This is expected from the three-photon resonance condition as given in Eq.(2.1) with $\delta_c = 0$. The straight-line fit to the data unambiguously confirms that the sum-frequency generation process follows a three-photon resonance condition. The error bars in this plot denote the narrow range over which this condition is satisfied. Thus this plot not only confirms the non-linear frequency conversion but also shows that the generation process occurs even when the microwave drive field is non-resonant with its transition.

In order to confirm the suppression of one-photon optical processes in our generation scheme, we examined the possibility that our undepleted coupling beam and the generated probe beam are shelving the atom in a dark state due to the fulfillment of a two-photon Raman resonance condition.

To substantiate this, we measured the power of the generated probe field with

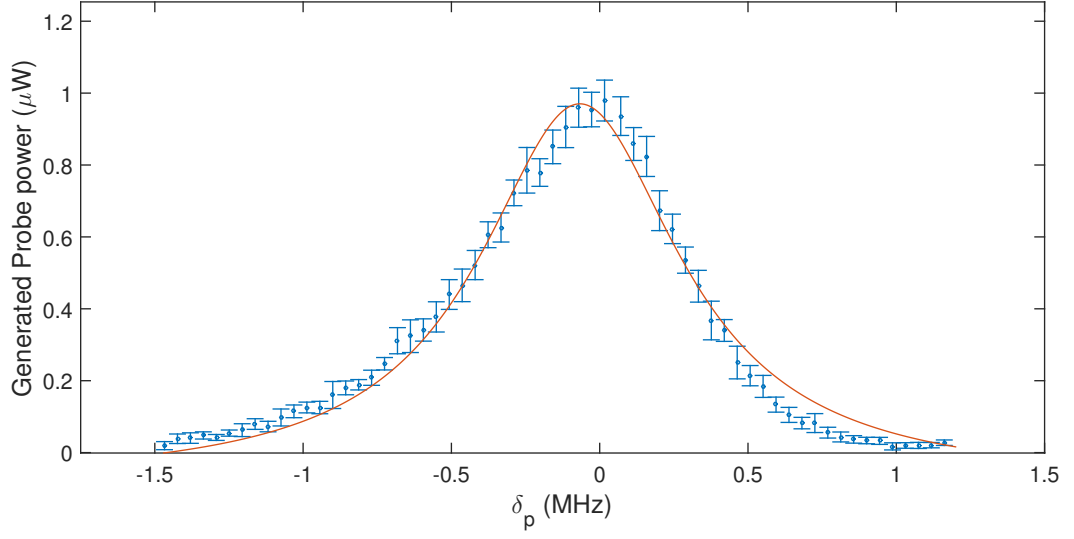


Figure 2.5: Generated probe power as a function of probe field detuning δ_p when $\delta_c = 0$. Values of δ_p are extracted from Fig 2.4 for corresponding values of microwave detuning δ_{RF} . We have measured a maximum standard deviation of $0.43 \mu\text{W}$. The experimental points are shown as circles with an EIT fit shown as a continuous line.

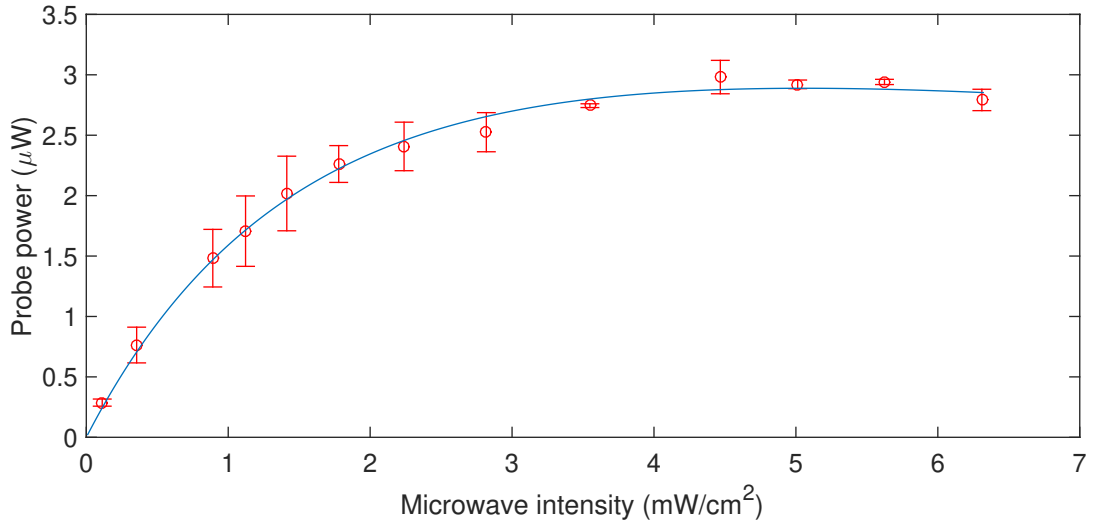


Figure 2.6: Generated optical probe power as a function of input microwave intensity at constant coupling field intensity of $8.9 \text{ mW}/\text{cm}^2$. The experimental data points are shown as red circles with a theoretical fit shown as a continuous blue line.

varied values of microwave detuning δ_{RF} while keeping $\delta_c = 0$. The corresponding probe field detunings δ_p for generation are acquired from Fig. 2.4. The result of these measurements is plotted in Fig. 2.5. Every point along this curve satisfies the three-photon resonance condition which is a necessary and sufficient condition for probe generation. But as can be seen in Fig. 2.5 the generated probe power does not remain constant and varies along this curve. We find that the probe beam has maximum power only at $\delta_p = \delta_c$. This establishes that the sum-frequency generation has maximum efficiency when a two-photon resonance condition is also satisfied. Under two-photon resonance condition, the atoms are shelves in a dark state, giving rise to an induced transparency effect [36] for the generated probe field. Therefore uniquely for our system, the non-linear generation of the probe light also minimizes its one-photon optical processes and the consequent losses.

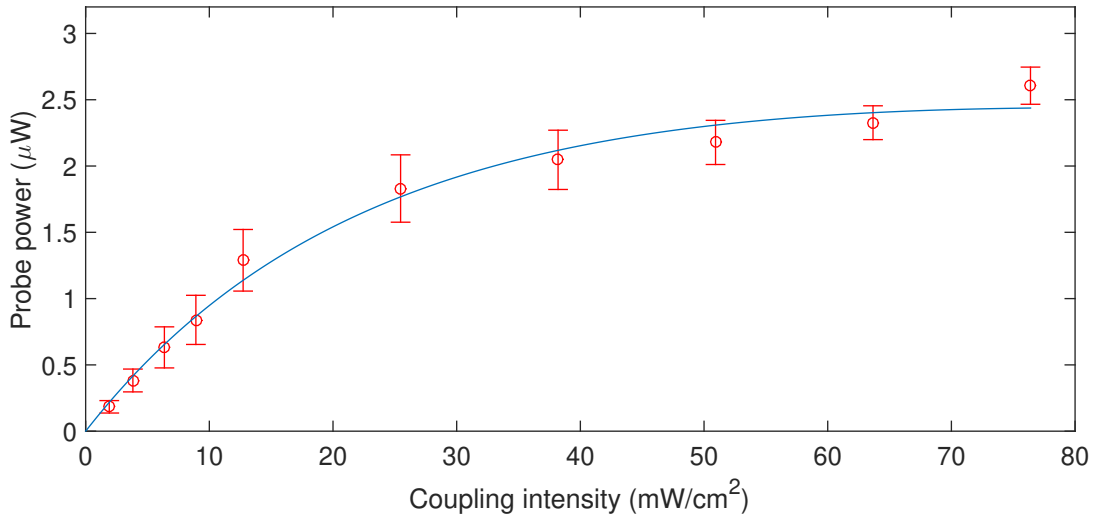


Figure 2.7: Generated optical probe power as a function of coupling field intensity at a constant microwave field intensity of 1.12 mW/cm^2 . The experimental data points are shown as red circles with a theoretical fit shown as a continuous blue line.

The solid line in Fig. 2.5 is a fit to our experimental data fitted with the formula [38]

$$P_p = C - \left(\frac{C_1^2}{\Gamma_1^2 + \delta^2} - \frac{C_2^2}{\Gamma_2^2 + \delta^2} \right). \quad (2.2)$$

This fit represents the fact that the generation process undergoes an electromagnetically induced transparency effect [39]. This formula decomposes the power

of the generated probe P_p as a difference of two Lorentzians centered around the same value of probe detuning $\delta_p = 0$. That we are measuring a transmitted beam is taken care by subtracting the sum from an overall constant C . From the fitted values, we find the width $2\Gamma_2$ which arises from Raman coherence to be about 825 kHz. In Fig. 2.5, the asymmetric nature of the emitted radiation is not known to us.

We have measured the dependence of the generated probe power as a function of both the drive microwave field intensity and the coupling optical field intensity. As shown in Fig. 2.6, the generated probe power varies linearly with input microwave intensity for small intensities and reaches saturation beyond an input intensity of about 4.4 mW/cm².

A similar trend is also seen in Fig. 2.7 where the generated probe power is plotted as a function of input coupling intensity. We see that beyond a coupling intensity of about 38 mW/cm² the generated probe power remains constant. In Fig. 2.6 and Fig. 2.7 the continuous line is the theoretical fit to the corresponding experimental data. The theoretical formula used for fitting is explained in the next chapter. Also the nature of the curve in Fig. 2.6 and Fig. 2.7 are explained in the next chapter.

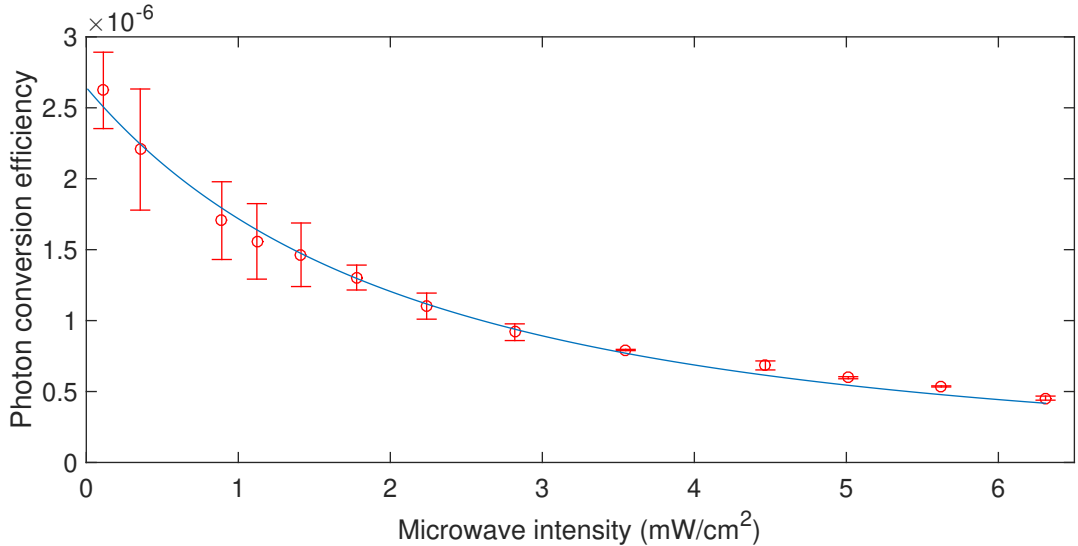


Figure 2.8: Photon conversion efficiency as a function of microwave field intensity at a constant coupling intensity of 8.9 mW/cm². Red circles are experimental data and the continuous blue line is the theoretical fit.

The efficiency η of probe generation is calculated using the definition [25],

$$\eta = \frac{\frac{P_p}{h\nu_p}}{\frac{I_{\text{RF}}A_{\text{RF}}}{h\nu_{\text{RF}}}} \quad (2.3)$$

where P_p is the generated probe power at frequency ν_p and I_{RF} is the input microwave intensity at frequency ν_{RF} . Here A_{RF} is the cross sectional area of microwave with intensity I_{RF} that interacts with the atoms. The maximum value for η that is seen in our experiment is 2.6×10^{-6} . The photon conversion efficiency dependence with microwave intensity and coupling field intensity are shown in Fig. 2.8 and Fig. 2.9, respectively. The equation used for the theoretical fitting is given in the next chapter.

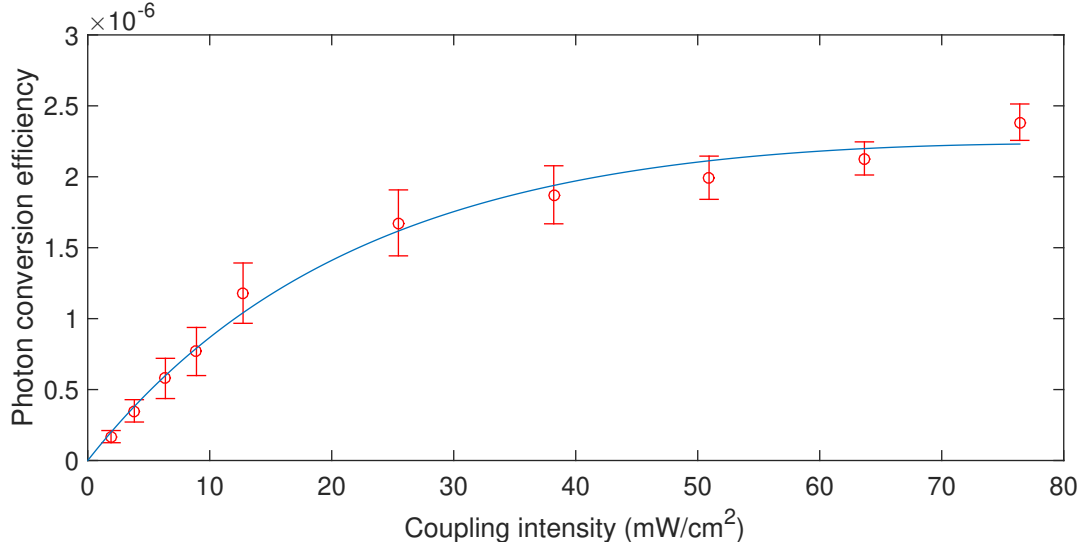


Figure 2.9: Photon conversion efficiency as a function of coupling field intensity at a constant microwave intensity of 1.12 mW/cm^2 . Red circles are experimental data and the continuous blue line is the theoretical fit.

The factors that affect the efficiency are the weakness of magnetic dipole transition induced by the microwave field and the lower number density obtainable with room temperature atomic vapours.

We have explicitly confirmed the coherent nature of our generation process by the following experiment. The Schematic of experimental set up for measuring the phase of the generated probe field is shown in Fig. 2.10. We have measured

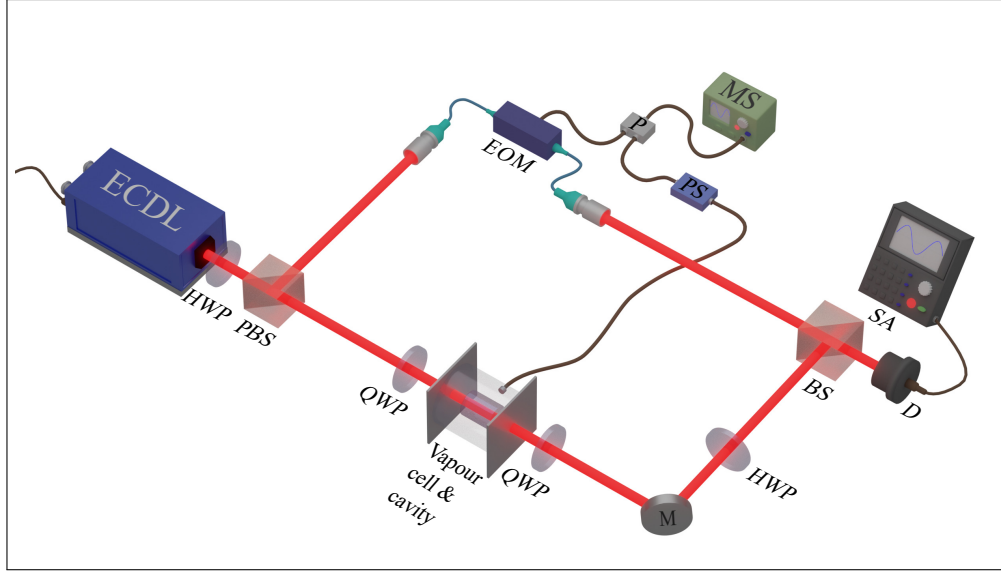


Figure 2.10: Schematic of experimental set up for measuring the phase of the generated probe field. Here ECDL —External Cavity Diode Laser, HWP —half-wave plate, QWP —quarter wave-plate, PBS —polarizing beam-splitter, BS —beam-splitter, AOM —Acousto-optic modulator, M —Mirror, EOM —Electro-optic modulator, MS —Microwave source, P —Power splitter, PS —Phase shifter, D —Detector, SA —Spectrum analyzer.

the phase of the generated probe field by the following experimental arrangement. We derive a local oscillator beam by sending a part of the coupling beam to an electro-optic modulator (EOM). The EOM is driven by the same microwave source at 3.0357 GHz which is used to drive our microwave cavity. The EOM generates two side-bands at $\omega_c \pm 3.0357$ GHz frequencies. We now mix the generated probe frequency ω_p with the frequencies generated by the EOM $\omega_c \pm 3.0357$ GHz. Since the probe frequency is $\omega_p = \omega_c + 3.0357$, the beat note of ω_p with one of the sideband of EOM $\omega_c - 3.0357$ will appear at 6.0714 GHz. We detect this beat note using a spectrum analyzer. We now introduce known phase shifts to the microwave which is feeding our cavity keeping its power remaining constant through an external microwave phase shifter. We then measure the changing power at our heterodyne measurement frequency 6.0714 GHz, since the local oscillator phase remains constant the change in power results from changes in the generated optical phase. We then extract the optical phase from this data. This is given in Fig. 2.11. From this figure it is clear that the probe phase changes commensurately

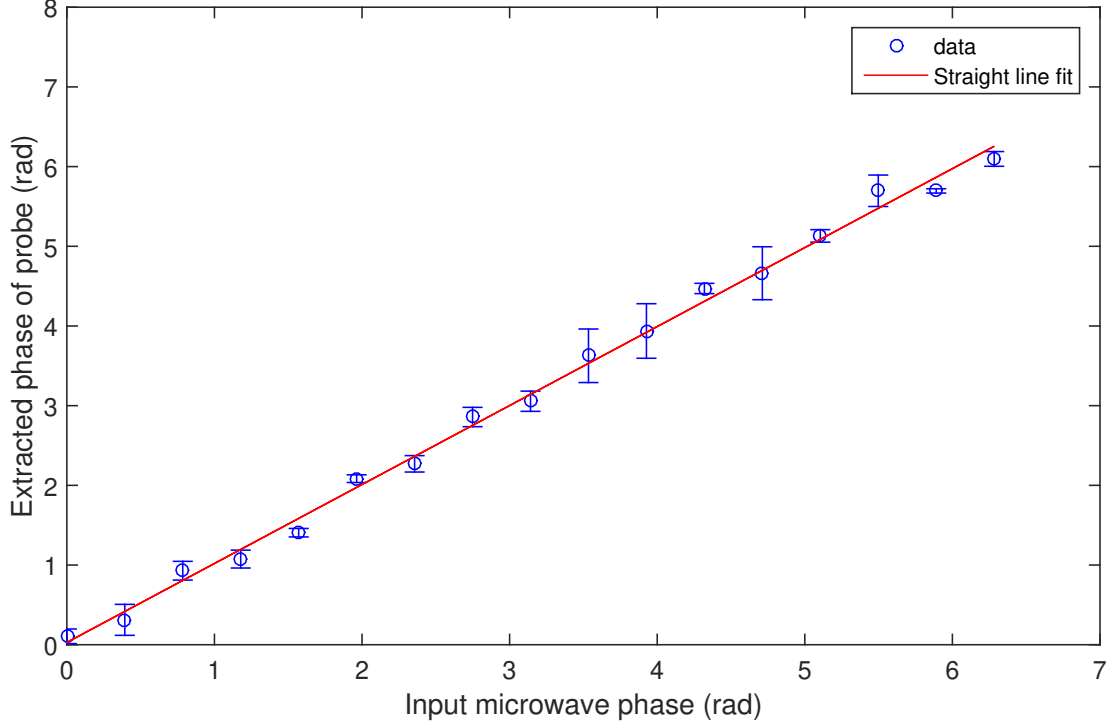


Figure 2.11: Extracted optical phase of the generated probe field as a function of input phase imposed on the microwave field. The circles are experimental points with a linear fit shown as a continuous line.

with a change in the microwave phase proving the coherent nature of probe field generation.

2.4 Conclusions

Many physical systems have been successfully used to demonstrate coherent microwave-to-optical transduction. A comparison of these physical system based on their photon conversion efficiency and conversion bandwidth will be useful. Fig. 2.12 shows the different transduction approaches having their highest achieved photon conversion efficiency. The optomechanical system has achieved the highest efficiency of 0.47, amongst other physical realization, but a relatively low bandwidth of 12 kHz [40]. Optomechanical systems also require various techniques to suppress thermal noise. With the help of double cavities, electro-optics experiments have demonstrated a higher bandwidth of 1.38 MHz with an efficiency of

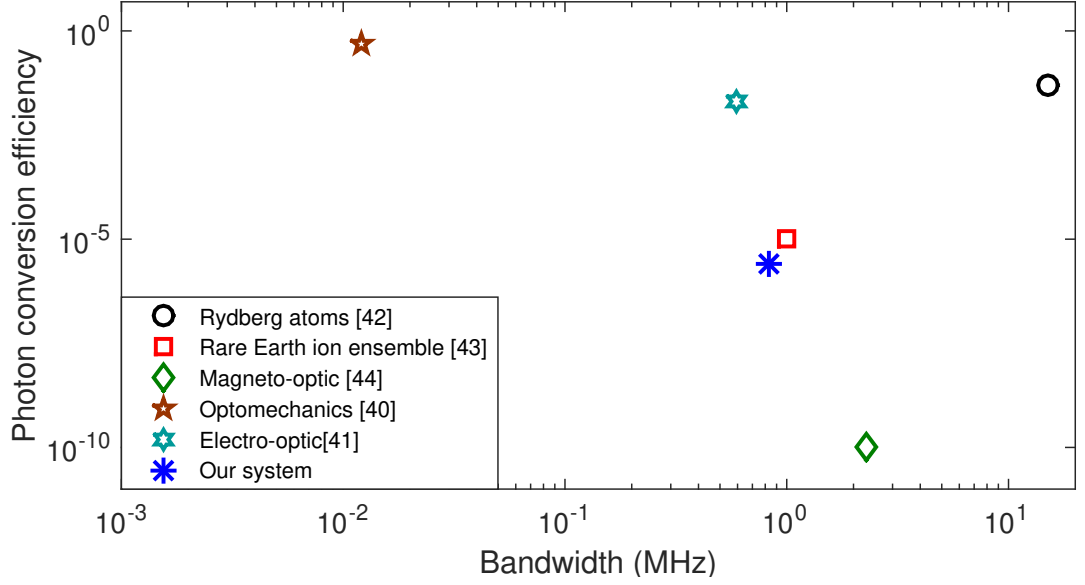


Figure 2.12: Photon conversion efficiency vs conversion bandwidth for different transduction approaches.

1.09×10^{-2} [41]. Recent investigations on Rydberg atomic ensemble has given a chance to improve the conversion bandwidth to the highest value of 15 MHz but having an efficiency of 5×10^{-2} [42]. Rare-Earth doped crystals are also an appealing candidate for the microwave-to-optical conversion. Employing Erbium-doped crystal as the conversion medium and using optical Fabry–Perot resonator and superconducting magnet at liquid Helium temperature, microwave-to-optical telecom-wavelength conversion has been demonstrated [43]. They have achieved an efficiency of 10^{-5} with a bandwidth of 1 MHz. One of the other approaches is the magnon based transducer. The fundamental idea in this method is to use collective magnetostatic spin excitations (magnons) as the intermediary mode. Using this magneto-optic medium, they have obtained a photon conversion efficiency of 10^{-10} and a conversion bandwidth of 2.28 MHz [44].

The coherent microwave-to-optical conversion described in this chapter is the first observation reported so far from atomic systems using a three-wave mixing process. Such an observation has not been possible due to the absence of any three-wave processes for electric-dipole mediated non-linear generation in centrosymmetric atomic systems. In our system, the symmetry is broken because of the interaction of induced magnetic and electric dipoles, allowing for a three-wave

mixing process. Our experiment significantly reduces the number of input beams required and enormously simplifies the experimental requirements. To put this in context the only experimental demonstration of coherent generation between microwave and optical frequency regimes from atomic systems utilized a six-wave mixing process and also required ultra-cold atomic samples [25, 42].

Despite the low efficiency of our microwave-to-optical conversion scheme due to the magnetic dipole interaction, our conversion mechanism has the potential for high fidelity signal transfer between these two frequency regimes. Fidelity is a measure of correlation between the experimental signal produced and the expected result. We expect to have a high because the two-photon resonance in our scheme prevents decoherence by spontaneous emission by shelving the atom in a dark state, which are the very states used traditionally as quantum memory states. Hence our scheme is ideally suited for transfer of quantum information where fidelity of transfer is essentially limited by optical quantum memory [19] and not by loss of signal strength [37].

Bibliography

- [1] M. H. Devoret and R. J. Schoelkopf. Superconducting circuits for quantum information: An outlook. *Science*, 339(6124):1169–1174, 2013.
- [2] Yu. A. Pashkin, O. Astafiev, T. Yamamoto, Y. Nakamura, and J. S. Tsai. Josephson charge qubits: a brief review. *Quantum Inf. Process.*, 8:55–80, 2009.
- [3] L. M. K. Vandersypen, H. Bluhm, J. S. Clarke, A. S. Dzurak, R. Ishihara, A. Morello, D. J. Reilly, L. R. Schreiber, and M. Veldhorst. Interfacing spin qubits in quantum dots and donors—hot, dense, and coherent. *Npj Quantum Inf.*, 3(34):1–10, 2017.
- [4] Ronald Hanson and David D Awschalom. Coherent manipulation of single spins in semiconductors. *Nature*, 453(7198):1043–1049, 2008.
- [5] L. M. K. Vandersypen and I. L. Chuang. Nmr techniques for quantum control and computation. *Rev. Mod. Phys.*, 76:1037–1069, Jan 2005.

- [6] Nicholas J. Lambert, Alfredo Rueda, Florian Sedlmeir, and Harald G. L. Schwefel. Coherent conversion between microwave and optical photons—an overview of physical implementations. *Adv. Quantum Technol.*, 3(1):1900077, 2020.
- [7] J. J. Longdell, E. Fraval, M. J. Sellars, and N. B. Manson. Stopped light with storage times greater than one second using electromagnetically induced transparency in a solid. *Phys. Rev. Lett.*, 95:063601, Aug 2005.
- [8] Guilherme Temporão, Sébastien Tanzilli, Hugo Zbinden, Nicolas Gisin, Thierry Aellen, Marcella Giovannini, and Jérôme Faist. Mid-infrared single-photon counting. *Opt. Lett.*, 31(8):1094–1096, Apr 2006.
- [9] M. Jalal Khan, Jerry C. Chen, and Sumanth Kaushik. Optical detection of terahertz radiation by using nonlinear parametric upconversion. *Opt. Lett.*, 32(22):3248–3250, Nov 2007.
- [10] D. V. Strekalov, A. A. Savchenkov, A. B. Matsko, and N. Yu. Efficient up-conversion of subterahertz radiation in a high-q whispering gallery resonator. *Opt. Lett.*, 34(6):713–715, Mar 2009.
- [11] Ganesh K Gopalakrishnan, William K Burns, and Catherine H Bulmer. Microwave-optical mixing in linbo/sub 3/modulators. *IEEE Trans. Microw. Theory Techn.*, 41(12):2383–2391, 1993.
- [12] Ze-Liang Xiang, Sahel Ashhab, J. Q. You, and Franco Nori. Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems. *Rev. Mod. Phys.*, 85:623–653, Apr 2013.
- [13] D. V. Strekalov, H. G. L. Schwefel, A. A. Savchenkov, A. B. Matsko, L. J. Wang, and N. Yu. Microwave whispering-gallery resonator for efficient optical up-conversion. *Phys. Rev. A*, 80:033810, Sep 2009.
- [14] Alfredo Rueda, Florian Sedlmeir, Michele C. Collodo, Ulrich Vogl, Birgit Stiller, Gerhard Schunk, Dmitry V. Strekalov, Christoph Marquardt, Johannes M. Fink, Oskar Painter, Gerd Leuchs, and Harald G. L. Schwefel.

- Efficient microwave to optical photon conversion: an electro-optical realization. *Optica*, 3(6):597–604, Jun 2016.
- [15] Joerg Bochmann, Amit Vainsencher, David D Awschalom, and Andrew N Cleland. Nanomechanical coupling between microwave and optical photons. *Nat. Phys.*, 9(11):712, 2013.
 - [16] Reed W Andrews, Robert W Peterson, Tom P Purdy, Katarina Cicak, Raymond W Simmonds, Cindy A Regal, and Konrad W Lehnert. Bidirectional and efficient conversion between microwave and optical light. *Nat. Phys.*, 10(4):321, 2014.
 - [17] J M Fink, M Kalae, R Norte, A Pitanti, and O Painter. Efficient microwave frequency conversion mediated by a photonics compatible silicon nitride nanobeam oscillator. *Quantum Sci. Technol.*, 5(3):034011, jun 2020.
 - [18] Christopher O’Brien, Nikolai Lauk, Susanne Blum, Giovanna Morigi, and Michael Fleischhauer. Interfacing superconducting qubits and telecom photons via a rare-earth-doped crystal. *Phys. Rev. Lett.*, 113(6):063603, 2014.
 - [19] Susanne Blum, Christopher O’Brien, Nikolai Lauk, Pavel Bushev, Michael Fleischhauer, and Giovanna Morigi. Interfacing microwave qubits and optical photons via spin ensembles. *Phys. Rev. A*, 91:033834, Mar 2015.
 - [20] Mohammad Hafezi, Zaeill Kim, Steven L Rolston, Luis A Orozco, BL Lev, and Jacob M Taylor. Atomic interface between microwave and optical photons. *Phys. Rev. A*, 85(2):020302, 2012.
 - [21] Martin Kiffner, Amir Feizpour, Krzysztof T Kaczmarek, Dieter Jaksch, and Joshua Nunn. Two-way interconversion of millimeter-wave and optical fields in rydberg gases. *New J. Phys.*, 18(9):093030, 2016.
 - [22] Bryan T. Gard, Kurt Jacobs, R. McDermott, and M. Saffman. Microwave-to-optical frequency conversion using a cesium atom coupled to a superconducting resonator. *Phys. Rev. A*, 96:013833, Jul 2017.

- [23] Lewis A Williamson, Yu-Hui Chen, and Jevon J Longdell. Magneto-optic modulator with unit quantum efficiency. *Phys. Rev. Lett.*, 113(20):203601, 2014.
- [24] Yu-Guo Liu, Keyu Xia, and Shi-Liang Zhu. Efficient microwave-to-optical single-photon conversion with a single flying circular rydberg atom. *Opt. Express*, 29(7):9942–9959, Mar 2021.
- [25] Jingshan Han, Thibault Vogt, Christian Gross, Dieter Jaksch, Martin Kiffner, and Wenhui Li. Coherent microwave-to-optical conversion via six-wave mixing in rydberg atoms. *Phys. Rev. Lett.*, 120:093201, Mar 2018.
- [26] Thibault Vogt, Christian Gross, Jingshan Han, Sambit B. Pal, Mark Lam, Martin Kiffner, and Wenhui Li. Efficient microwave-to-optical conversion using rydberg atoms. *Phys. Rev. A*, 99:023832, Feb 2019.
- [27] Zhongzhong Qin, Leiming Cao, Hailong Wang, A. M. Marino, Weiping Zhang, and Jietai Jing. Experimental generation of multiple quantum correlated beams from hot rubidium vapor. *Phys. Rev. Lett.*, 113:023602, Jul 2014.
- [28] V. Boyer, A. M. Marino, and P. D. Lett. Generation of spatially broadband twin beams for quantum imaging. *Phys. Rev. Lett.*, 100:143601, Apr 2008.
- [29] Marlan O Scully and Michael Fleischhauer. Lasers without inversion. *Science*, 263(5145):337–339, 1994.
- [30] S. E. Harris, J. E. Field, and A. Imamoglu. Nonlinear optical processes using electromagnetically induced transparency. *Phys. Rev. Lett.*, 64:1107–1110, Mar 1990.
- [31] T Meijer, JD White, B Smeets, M Jeppesen, and RE Scholten. Blue five-level frequency-upconversion system in rubidium. *Opt. Lett.*, 31(7):1002–1004, 2006.
- [32] Alexander M. Akulshin, Russell J. McLean, Andrei I. Sidorov, and Peter Hannaford. Coherent and collimated blue light generated by four-wave mixing in rb vapour. *Opt. Express*, 17(25):22861–22870, Dec 2009.

- [33] Alexander Akulshin, Dmitry Budker, and Russell McLean. Directional infrared emission resulting from cascade population inversion and four-wave mixing in rb vapor. *Opt. Lett.*, 39(4):845–848, Feb 2014.
- [34] Robert W. Boyd. *Nonlinear Optics, Third Edition*. Academic Press, Inc., Orlando, FL, USA, 3rd edition, 2008.
- [35] P. S. Pershan. Nonlinear optical properties of solids: Energy considerations. *Phys. Rev.*, 130:919–929, May 1963.
- [36] S. E. Harris. Lasers without inversion: Interference of lifetime-broadened resonances. *Phys. Rev. Lett.*, 62:1033–1036, Feb 1989.
- [37] Alessandro Fedrizzi, Rupert Ursin, Thomas Herbst, Matteo Nespoli, Robert Prevedel, Thomas Scheidl, F. Tiefenbacher, Thomas Jennewein, and Anton Zeilinger. High-fidelity transmission of entanglement over a high-loss freespace channel. *Nat. Phys.*, 5:389, May 2009.
- [38] P. M. Anisimov, J. P. Dowling, and B. C. Sanders. Objectively discerning autler-townes splitting from electromagnetically induced transparency. *Phys. Rev. Lett.*, 107:163604, Oct 2011.
- [39] P. Anisimov and O. Kocharovskaya. Decaying-dressed-state analysis of a coherently driven three-level λ system. *J. Mod. Opt.*, 55(19-20):3159–3171, 2008.
- [40] Andrew P Higginbotham, PS Burns, MD Urmei, RW Peterson, NS Kampel, BM Brubaker, G Smith, KW Lehnert, and CA Regal. Harnessing electro-optic correlations in an efficient mechanical converter. *Nat. Phys.*, 14(10):1038–1042, 2018.
- [41] Linran Fan, Chang-Ling Zou, Risheng Cheng, Xiang Guo, Xu Han, Zheng Gong, Sihao Wang, and Hong X. Tang. Superconducting cavity electro-optics: A platform for coherent photon conversion between superconducting and photonic circuits. *Sci. Adv.*, 4(8), 2018.
- [42] Thibault Vogt, Christian Gross, Jingshan Han, Sambit B. Pal, Mark Lam, Martin Kiffner, and Wenhui Li. Efficient microwave-to-optical conversion using rydberg atoms. *Phys. Rev. A*, 99:023832, Feb 2019.

- [43] Xavier Fernandez-Gonzalvo, Sebastian P. Horvath, Yu-Hui Chen, and Jevon J. Longdell. Cavity-enhanced raman heterodyne spectroscopy in $\text{er}^{3+} : \text{y}_2\text{siO}_5$ for microwave to optical signal conversion. *Phys. Rev. A*, 100:033807, Sep 2019.
- [44] R. Hisatomi, A. Osada, Y. Tabuchi, T. Ishikawa, A. Noguchi, R. Yamazaki, K. Usami, and Y. Nakamura. Bidirectional conversion between microwave and light via ferromagnetic magnons. *Phys. Rev. B*, 93:174427, May 2016.

Chapter 3

Phase dependent amplification of an optical field using microwaves

3.1 Introduction

Optical amplifiers are vital in various fields, such as optical communications, enhancement of optical resolution [1, 2], quantum information processing [3] and in possible quantum computing [4] devices. Due to their ability to amplify optical signals directly, they became the essential element in communication systems. Among optical amplifiers, a linear optical amplifier is well studied both theoretically and experimentally. Worth emphasizing here is the fact that the linear amplifier yields a formalism that is analytically tractable but still applicable to a broad class of amplifiers. A linear optical amplifier can amplify the input signal, and the output of the amplifier will be linearly dependent on the input signal. Linear amplifiers can be classified into two major categories based on their gain and noise features. One is a phase-insensitive amplifier (PIA), where both the quadrature of the input signal gets amplified by the same amount [5]. In contrast, the second kind is a phase-sensitive amplifier (PSA) whose gain is different in orthogonal quadratures of the input signal [6]. If one quadrature of the signal is getting amplified, then the orthogonal quadrature of the signal gets de-amplification. Apart from the gain of the amplifier, noise added in the amplification process also differ in these two kinds of amplifiers. In a PIA, the minimum added noise is half a quantum due to the commutation relation. For PSA, which has a quadrature dependent noise it is

possible to make one of the quadratures to be noiseless [6].

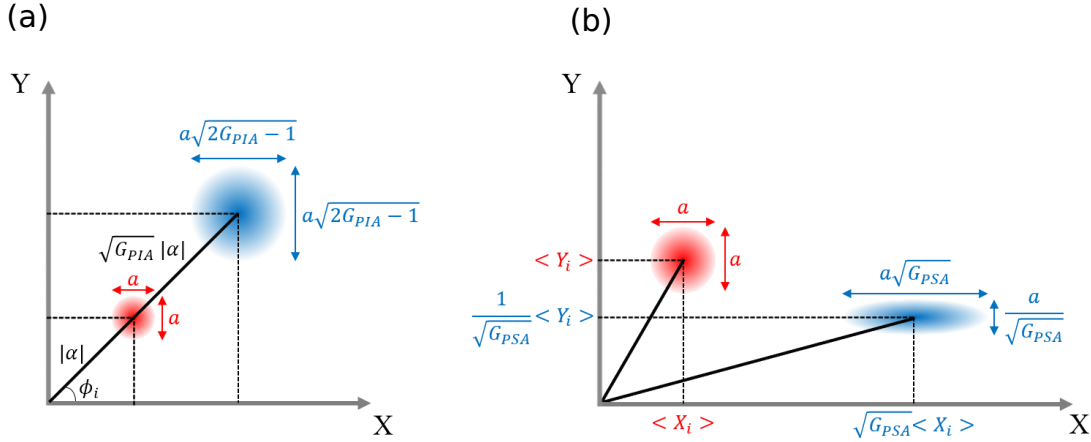


Figure 3.1: (a) Phase space representation of a PIA. (b) Phase space representation of a PSA.

The working principle of a PIA and PSA can be explained with the help of a phase space diagram, as shown in Fig. 3.1. Here X and Y are the field quadratures in units of $\sqrt{\hbar\omega/2}$. In Fig. 3.1(a), shows the phase space representation of a PIA. The input state to the amplifier has an amplitude of $|\alpha|$, and it has a noise value of a in both quadratures, which is represented as a red disk. Since both the quadratures amplify by the same amount, the output of the amplifier will have an amplitude of $\sqrt{G_{PIA}}|\alpha|$, and the noise spot remains as a disk with an added noise of $\sqrt{2G_{PIA} - 1}$ times the input noise.

In stark contrast, a PSA will amplify one quadrature and de-amplify its conjugate, resulting in distortion of the noise spot into an ellipse. However, despite the distortion, the area of the noise spot is unchanged and remains equal to that of the noise of the input signal. Thus a PSA can act as a noiseless amplifier. Hence PSAs has found extensive applications in various fields such as optical telecommunications [7, 8], metrology [9], quantum information processing [10, 11] and imaging industry [12, 1]. Experimental realisation of PSA in different physical systems like nonlinear crystals [13], nonlinear fibers [14], and in atomic systems via four-wave mixing [15] and three wave mixing [16] have been reported. The ground state coherence was explicitly exploited in achieving a PSA using atomic vapours in atleast one previous work [15, 16]. In all these demonstrations all the electromag-

netic fields where of similar wavelengths. Phase space representation of a PSA is shown in Fig: 3.1(b). Here one quadrature of the input signal amplify by $\sqrt{G_{PSA}}$, while the orthogonal quadrature gets de-amplify by the same amount (Amplify by $1/\sqrt{G_{PSA}}$).

The motivation for the problems addressed in this chapter is to develop a theoretical model to study the coherent conversion and amplification process of an optical field using microwaves in a closed three-level scheme (Δ system).

In this chapter, we have done a theoretical analysis of the three-wave mixing present in a cyclic atomic level scheme interacting with two optical and one microwave field. We have obtained an analytical expression for the probe field using density matrix analysis. Our analysis shows that, in addition to a generation of probe field, there will be an amplification also possible for the probe field in our cyclic system. The physical mechanism of the amplification process is entirely different from a phase sensitive amplification as well as from a phase insensitive amplification process. Our theoretical analysis reveals that the amplification is due to an interference of the input probe field and the generated probe field. This interference effect can give rise to amplification or a de-amplification depending on the phase of the generated probe field. This probe amplification can be controlled by the phase of the microwave field. This makes the system as a phase depended amplifier (PDA). The dependence of amplification and de-amplification of probe field is studied with various microwave field intensities, and with various two-photon detunings.

3.2 Theoretical analysis

We have chosen a three-level atomic system which can be realized in a hyperfine energy levels of ^{85}Rb atomic system in its D1 manifold. The three energy levels are interacting with two optical and one microwave electromagnetic field. This will be hereafter referred to as a Δ system due to their closed nature of interaction, is shown in Fig. 3.2. The ground state, metastable state and the excited state are represented as $|1\rangle$, $|2\rangle$ and $|3\rangle$. The electric dipole transitions $|1\rangle \rightarrow |3\rangle$

and $|2\rangle \rightarrow |3\rangle$ are driven by optical coupling and probe fields respectively. While $|1\rangle \rightarrow |2\rangle$ transition is electric dipole forbidden transition, thus a magnetic dipole transition is employed between these two states using a microwave drive field. The detunings of the probe, coupling and microwave frequency relative to the corresponding transition frequencies are δ_p , δ_c and δ_{RF} respectively. The electric

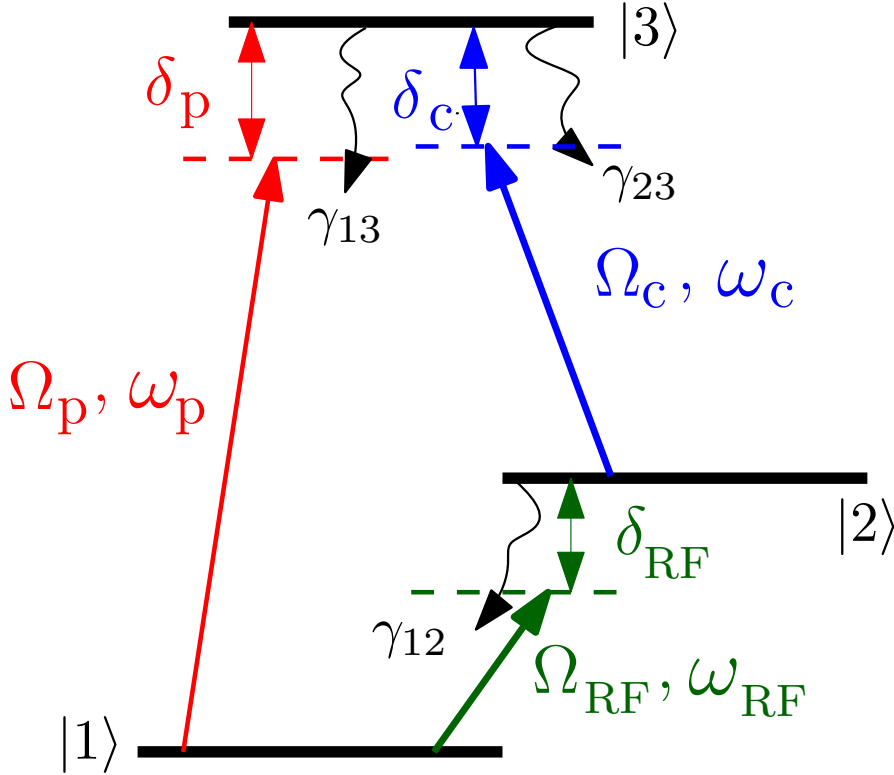


Figure 3.2: Δ System formation using a generic three-level scheme interacting with three electromagnetic fields.

fields of the optical coupling and probe fields propagating along z direction are expressed as

$$\begin{aligned}\mathbf{E}_c(\mathbf{r}) &= \vec{e}_c \mathcal{E}_c \cos(\omega_c t - \vec{k}_c \cdot \vec{r} + \phi_c) \\ \mathbf{E}_p(\mathbf{r}) &= \vec{e}_p \mathcal{E}_p \cos(\omega_p t - \vec{k}_p \cdot \vec{r} + \phi_p)\end{aligned}\quad (3.1)$$

and the microwave magnetic field can be written as

$$\mathbf{B}_{\text{RF}}(\mathbf{r}) = \vec{e}_{\text{RF}} \mathcal{B}_{\text{RF}} \cos(\omega_{\text{RF}} t - \vec{k}_{\text{RF}} \cdot \vec{r} + \phi_{\text{RF}}) \quad (3.2)$$

where $\vec{e}_i$, ω_i , $\vec{k}_i$ and ϕ_i for $i \in \{p, c, \text{RF}\}$ are the polarization, amplitude, frequency, wave vector and initial phase labels of the respective fields. The amplitudes of the

coupling, probe and microwave field are represented as $\mathcal{E}_c, \mathcal{E}_p$ and $\mathcal{B}_{\text{RF}}$ respectively. The interaction Hamiltonian of our Δ system under rotating wave approximation can be written as,

$$\begin{aligned} \hat{H}(\mathbf{r}, \mathbf{v}, t) = & \omega_1 |1\rangle\langle 1| + \omega_2 |2\rangle\langle 2| + \omega_3 |3\rangle\langle 3| - \frac{\Omega_p(\mathbf{r})}{2} e^{-i(\omega_p t - \vec{k}_p \cdot \vec{r} + \phi_p)} |3\rangle\langle 1| \\ & - \frac{\Omega_c(\mathbf{r})}{2} e^{-i(\omega_c t - \vec{k}_c \cdot \vec{r} + \phi_c)} |3\rangle\langle 2| - \frac{\Omega_{\text{RF}}(\mathbf{r})}{2} e^{-i(\omega_{\text{RF}} t - \vec{k}_{\text{RF}} \cdot \vec{r} + \phi_{\text{RF}})} |2\rangle\langle 1| + \text{H.C.} \end{aligned} \quad (3.3)$$

Here onwards we have taken all optical fields to be propagating along the z direction. We transform the Hamiltonian using unitary transformation

$$\hat{U}(t) = |1\rangle\langle 1| + e^{i((\omega_p - \omega_c)t - (k_p - k_c)z + \phi_p - \phi_c)} |2\rangle\langle 2| + e^{i(\omega_p t - k_p z + \phi_p)} |3\rangle\langle 3| \quad (3.4)$$

and obtain a time independent Hamiltonian under three-photon resonance condition defined as $\delta_3 = \delta_p - \delta_c - \delta_{\text{RF}} = 0$, with ground state $|1\rangle$ as the reference level.

$$\begin{aligned} \hat{H}(z, \mathbf{v}) = & -\Delta_p(\mathbf{v}) |3\rangle\langle 3| - (\Delta_p(\mathbf{v}) - \Delta_c(\mathbf{v})) |2\rangle\langle 2| - \frac{\Omega_p(z)}{2} |3\rangle\langle 1| \\ & - \frac{\Omega_c(z)}{2} |3\rangle\langle 2| - \frac{\Omega_{\text{RF}}(z) e^{-i\Delta\Phi}}{2} |2\rangle\langle 1| + \text{H.C.}, \end{aligned} \quad (3.5)$$

with H.C. denoting Hermitian conjugate and $\hbar$ is taken to be 1. The Hamiltonian in Eq. (3.5) is only valid under three-photon resonance condition. $\Delta\Phi$ is the relative phase between all the fields defined as

$$\Delta\Phi(z) = \phi_{\text{RF}} + (k_p - k_c)z. \quad (3.6)$$

since the microwave is a standing wave we have neglected to term $k_{\text{RF}}z$. The Doppler-shifted detunings of the probe and coupling fields are given by $\Delta_p(\mathbf{v}) := \delta_p - \mathbf{k}_p \cdot \mathbf{v}$ and $\Delta_c(\mathbf{v}) := \delta_c - \mathbf{k}_c \cdot \mathbf{v}$ where $\mathbf{v}$ is the velocity of the atom. Here

$$\begin{aligned} \Omega_p(z) &= \mathbf{d}_{13} \cdot \mathbf{E}_p(z) \\ \Omega_c(z) &= \mathbf{d}_{23} \cdot \mathbf{E}_c(z) \\ \Omega_{\text{RF}}(z) &= \boldsymbol{\mu}_{12} \cdot \mathbf{B}_{\text{RF}}(z) \end{aligned} \quad (3.7)$$

are the Rabi frequencies. $\mathbf{d}_{ij}$ and $\boldsymbol{\mu}_{ij}$ are the electric and magnetic dipole matrix elements respectively. The dynamical evolution of rotated density matrix $\hat{\sigma}(t) = \hat{U}(t) \hat{\rho}(t) \hat{U}^\dagger(t)$, is given by the master equation [17]

$$\frac{\partial \hat{\sigma}(\mathbf{v}, z, t)}{\partial t} = -i[\hat{H}(\mathbf{v}, z), \hat{\sigma}(\mathbf{v}, z, t)] + \sum_{k=1}^5 \mathcal{L}(\hat{a}_k) \hat{\sigma}(\mathbf{v}, z, t) \quad (3.8)$$

with $\mathcal{L}(\hat{a}_k)$ the Lindblad superoperator

$$\mathcal{L}(\hat{a})\hat{\sigma}(\mathbf{v}, z, t) = \hat{a}\hat{\sigma}(\mathbf{v}, z, t)\hat{a}^\dagger - \frac{1}{2}\{\hat{\sigma}(\mathbf{v}, z, t), \hat{a}^\dagger\hat{a}\} \quad (3.9)$$

acting on operators

$$\begin{aligned} \hat{a}_1 &= \sqrt{(\bar{n} + 1)\gamma_{12}}|1\rangle\langle 2|, \quad \hat{a}_2 = \sqrt{\bar{n}\gamma_{12}}|2\rangle\langle 1| \\ \hat{a}_3 &= \sqrt{\gamma_{13}}|1\rangle\langle 3|, \quad \hat{a}_4 = \sqrt{\gamma_{23}}|2\rangle\langle 3| \\ \hat{a}_5 &= \sqrt{\gamma_c}(|1\rangle\langle 1| - |2\rangle\langle 2|) \end{aligned} \quad (3.10)$$

with γ_{12} and $\gamma_{23} = \gamma_{13}$ representing natural linewidth of levels $|2\rangle$ and $|3\rangle$ respectively, $\bar{n}$ is the average number of thermal photons and γ_c being the phenomenological decay constant modeling the finite quality factor of our microwave cavity. The density matrix equations can now be written using Eq. (3.5,3.8,3.9,3.10),

$$\begin{aligned} \frac{\partial \sigma_{33}(\mathbf{v}, z, t)}{\partial t} &= \frac{i\Omega_c}{2}\sigma_{23}(\mathbf{v}, z, t) + \frac{i\Omega_p}{2}\sigma_{13}(\mathbf{v}, z, t) - \frac{i\Omega_p^*}{2}\sigma_{31}(\mathbf{v}, z, t) - \frac{i\Omega_c^*}{2}\sigma_{32}(\mathbf{v}, z, t) \\ &\quad - \gamma_{13}\sigma_{33}(\mathbf{v}, z, t) - \gamma_{23}\sigma_{33}(\mathbf{v}, z, t) \\ \frac{\partial \sigma_{22}(\mathbf{v}, z, t)}{\partial t} &= \frac{i\Omega_{\text{RF}}}{2}e^{-i\Delta\Phi}\sigma_{12}(\mathbf{v}, z, t) + \frac{i\Omega_c^*}{2}\sigma_{32}(\mathbf{v}, z, t) - \frac{i\Omega_{\text{RF}}^*}{2}e^{i\Delta\Phi}\sigma_{21}(\mathbf{v}, z, t) \\ &\quad - \frac{i\Omega_c}{2}\sigma_{23}(\mathbf{v}, z, t) + \bar{n}\gamma_{12}\sigma_{11}(\mathbf{v}, z, t) + \gamma_{23}\sigma_{33}(\mathbf{v}, z, t) \\ &\quad - (\bar{n} + 1)\gamma_{12}\sigma_{22}(\mathbf{v}, z, t) \\ \frac{\partial \sigma_{32}(\mathbf{v}, z, t)}{\partial t} &= \frac{i\Omega_p}{2}\sigma_{12}(\mathbf{v}, z, t) + \frac{i\Omega_c}{2}[\sigma_{22}(\mathbf{v}, z, t) - \sigma_{33}(\mathbf{v}, z, t)] - \frac{i\Omega_{\text{RF}}^*}{2}e^{i\Delta\Phi}\sigma_{31}(\mathbf{v}, z, t) \\ &\quad + i\Delta_c\sigma_{32}(\mathbf{v}, z, t) - \frac{\gamma_{13}}{2}\sigma_{32}(\mathbf{v}, z, t) - \frac{\gamma_{23}}{2}\sigma_{32}(\mathbf{v}, z, t) \\ &\quad - \frac{(\bar{n} + 1)}{2}\gamma_{12}\sigma_{32}(\mathbf{v}, z, t) + \frac{\gamma_c}{2}\sigma_{32}(\mathbf{v}, z, t) \\ \frac{\partial \sigma_{31}(\mathbf{v}, z, t)}{\partial t} &= \frac{i\Omega_p}{2}[\sigma_{11}(\mathbf{v}, z, t) - \sigma_{33}(\mathbf{v}, z, t)] + \frac{i\Omega_c}{2}\sigma_{21}(\mathbf{v}, z, t) - \frac{i\Omega_{\text{RF}}}{2}e^{-i\Delta\Phi}\sigma_{32}(\mathbf{v}, z, t) \\ &\quad + i\Delta_p\sigma_{31}(\mathbf{v}, z, t) - \frac{\gamma_{13}}{2}\sigma_{31}(\mathbf{v}, z, t) - \frac{\gamma_{23}}{2}\sigma_{31}(\mathbf{v}, z, t) \\ &\quad - \frac{\bar{n}\gamma_{12}}{2}\sigma_{31}(\mathbf{v}, z, t) + \frac{\gamma_c}{2}\sigma_{31}(\mathbf{v}, z, t) \\ \frac{\partial \sigma_{21}(\mathbf{v}, z, t)}{\partial t} &= \frac{i\Omega_{\text{RF}}}{2}e^{-i\Delta\Phi}[\sigma_{11}(\mathbf{v}, z, t) - \sigma_{22}(\mathbf{v}, z, t)] + \frac{i\Omega_c^*}{2}\sigma_{31}(\mathbf{v}, z, t) - \frac{i\Omega_p}{2}\sigma_{23}(\mathbf{v}, z, t) \\ &\quad + i[\Delta_p - \Delta_c]\sigma_{21}(\mathbf{v}, z, t) - \frac{(\bar{n} + 1)}{2}\gamma_{12}\sigma_{21}(\mathbf{v}, z, t) - \frac{\bar{n}\gamma_{12}}{2}\sigma_{21}(\mathbf{v}, z, t) \\ \sigma_{11}(\mathbf{v}, z, t) &= 1 - \sigma_{22}(\mathbf{v}, z, t) - \sigma_{33}(\mathbf{v}, z, t) \end{aligned} \quad (3.11)$$

At steady state, from Eq. (3.11) one can write σ_{31} , σ_{21} and σ_{32} as,

$$\begin{aligned}\sigma_{31}(\mathbf{v}, z) &= \frac{i\Omega_p[\sigma_{11}(\mathbf{v}, z) - \sigma_{33}(\mathbf{v}, z)] + i\Omega_c\sigma_{21}(\mathbf{v}, z) - i\Omega_{\text{RF}}e^{-i\Delta\Phi}\sigma_{32}(\mathbf{v}, z)}{-2i\Delta_p + \gamma_{13} + \gamma_{23} - \bar{n}\gamma_{12} + \gamma_c} \\ \sigma_{21}(\mathbf{v}, z) &= \frac{i\Omega_{\text{RF}}e^{-i\Delta\Phi}[\sigma_{11}(\mathbf{v}, z) - \sigma_{22}(\mathbf{v}, z)] + i\Omega_c^*\sigma_{31}(\mathbf{v}, z) - i\Omega_p\sigma_{23}(\mathbf{v}, z)}{-2i[\Delta_p - \Delta_c] + (\bar{n} + 1)\gamma_{12} + \bar{n}\gamma_{12}} \\ \sigma_{32}(\mathbf{v}, z) &= \frac{i\Omega_p\sigma_{12}(\mathbf{v}, z) + i\Omega_c[\sigma_{22}(\mathbf{v}, z) - \sigma_{33}(\mathbf{v}, z)] - i\Omega_{\text{RF}}^*e^{i\Delta\Phi}\sigma_{31}(\mathbf{v}, z)}{-2i\Delta_c + \gamma_{13} + \gamma_{23} + (\bar{n} + 1)\gamma_{12} - \gamma_c}. \quad (3.12)\end{aligned}$$

Now the quantity of interest is σ_{31} , we solve Eq. (3.12) for σ_{31} .

$$\begin{aligned}\sigma_{31}(\mathbf{v}, z) &= \frac{i\Omega_p[\sigma_{11}(\mathbf{v}, z) - \sigma_{33}(\mathbf{v}, z)]}{-2i\Delta_p + \gamma_{13} + \gamma_{23} - \bar{n}\gamma_{12} + \gamma_c} \\ &+ \frac{i\Omega_c(i\Omega_{\text{RF}}e^{-i\Delta\Phi}[\sigma_{11}(\mathbf{v}, z) - \sigma_{22}(\mathbf{v}, z)] + i\Omega_c^*\sigma_{31}(\mathbf{v}, z) - i\Omega_p\sigma_{23}(\mathbf{v}, z))}{-2i\Delta_p + \gamma_{13} + \gamma_{23} - \bar{n}\gamma_{12} + \gamma_c(-2i[\Delta_p - \Delta_c] + (\bar{n} + 1)\gamma_{12} + \bar{n}\gamma_{12})} \\ &- \frac{i\Omega_{\text{RF}}e^{-i\Delta\Phi}(i\Omega_p\sigma_{12}(\mathbf{v}, z) + i\Omega_c[\sigma_{22}(\mathbf{v}, z) - \sigma_{33}(\mathbf{v}, z)] - i\Omega_{\text{RF}}^*e^{i\Delta\Phi}\sigma_{31}(\mathbf{v}, z))}{-2i\Delta_p + \gamma_{13} + \gamma_{23} - \bar{n}\gamma_{12} + \gamma_c(-2i\Delta_c + \gamma_{13} + \gamma_{23} + (\bar{n} + 1)\gamma_{12} - \gamma_c)} \quad (3.13)\end{aligned}$$

In the limit of weak probe and microwave fields, $\Omega_p < \Omega_{\text{RF}} \ll \Omega_c$ we can assume $\sigma_{11} \approx 1$. Also we neglect the terms of order Ω_p^2 as our probe field is very weak compared to coupling and microwave fields. We obtain,

$$\begin{aligned}\sigma_{31}(\mathbf{v}, z) &= \frac{2i\Gamma_2\Gamma_3\Omega_p(z)}{4\Gamma_1\Gamma_2\Gamma_3 + \Gamma_3|\Omega_c(z)|^2 + \Gamma_2|\Omega_{\text{RF}}(z)|^2} \\ &- \frac{\Gamma_3\Omega_c(z)\Omega_{\text{RF}}(z)e^{-i\Delta\Phi}}{4\Gamma_1\Gamma_2\Gamma_3 + \Gamma_3|\Omega_c(z)|^2 + \Gamma_2|\Omega_{\text{RF}}(z)|^2}. \quad (3.14)\end{aligned}$$

Here

$$\begin{aligned}\Gamma_1 &= \frac{\gamma_{13}}{2} + \frac{\gamma_{23}}{2} + \frac{\bar{n}\gamma_{12}}{2} + \frac{\gamma_c}{2} - i\Delta_p, \\ \Gamma_2 &= (\bar{n} + \frac{1}{2})\gamma_{12} - i(\delta_2 - \mathbf{k}_p \cdot \mathbf{v} + \mathbf{k}_c \cdot \mathbf{v}), \\ \Gamma_3 &= \frac{\gamma_{13}}{2} + \frac{\gamma_{23}}{2} + (\frac{\bar{n} + 1}{2})\gamma_{12} + \frac{\gamma_c}{2} - i\Delta_c\end{aligned} \quad (3.15)$$

where $\delta_2 = \delta_p - \delta_c$ is the two photon detuning. In Eq. (3.14) the coefficient of Ω_p in the first term is proportional to the linear susceptibility of the probe field and the coefficient of $\Omega_c\Omega_{\text{RF}}$ in the second term is proportional to the hybrid second order susceptibility of probe field. This nonlinear susceptibility results from a combined magnetic and an electric dipole transition induced by microwave field and optical coupling field respectively resulting in a new optical probe field generation. A detailed calculation of linear susceptibility and hybrid second order susceptibility in this system can be found in [18].

Using the slowly-varying envelope approximation, we calculate the propagation equation for the probe field inside our nonlinear medium as given below.

$$\frac{\partial \Omega_p}{\partial z} = i\eta_p \sigma_{31}, \quad (3.16)$$

where $\eta_p = (\mathbf{d}_{13}^2 \omega_p N / 2\epsilon_0 c \hbar)$ is the coupling constant for the probe field and N is the number density of atoms. The propagation effect of the microwave field is not taken into account due to the fact that the ratio $\eta_{\text{RF}}/\eta_p = (\omega_{\text{RF}}/\omega_p)\alpha^2$, where $\alpha \approx 1/137$ is the fine structure constant. This results in $\eta_{\text{RF}} \ll \eta_p$ and the spatial variation of microwave intensity is thus ignored. We therefore maintain $\Omega_{\text{RF}}(z) \equiv \Omega_{\text{RF}}(0)$ in our theoretical analysis. In addition, the undepleted pump approximation is valid for the coupling field in our system. This fact is incorporated by taking $\Omega_c(z) \equiv \Omega_c(0)$. We thus solve for Eq. (3.16) in spatially uniform coupling and microwave fields, resulting in the following solution for the probe field.

$$\Omega_p(l) = \Omega_p(0)e^{-\beta l} + \frac{i\Omega_c(0)\Omega_{\text{RF}}(0)e^{-i\Delta\Phi}}{2\Gamma_2}(e^{-\beta l} - 1) \quad (3.17)$$

where

$$\beta = \frac{2\eta_p\Gamma_2\Gamma_3}{4\Gamma_1\Gamma_2\Gamma_3 + \Gamma_3|\Omega_c(0)|^2 + \Gamma_2|\Omega_{\text{RF}}(0)|^2} \quad (3.18)$$

is a complex number. The total probe amplitude at the end of the cell with length l is given by Eq. (3.17). The Doppler averaging of the Eq. (3.17) is performed for theoretical fitting to the experimental observations in Chapter 2 and Chapter 4.

3.3 Probe field generation

The second order process responsible for the three-wave mixing phenomenon can be understood from Eq. (3.17). The second term represents a three-wave mixing process present in our system. The experimental demonstration of this three-wave mixing is discussed in the previous chapter. The relation between the generated probe field with the coupling and microwave field power can be understood from our theoretical calculation. From Eq. (3.14) the coefficient of $\Omega_c\Omega_{\text{RF}}$ in the second term is proportional to the hybrid second order susceptibility of probe field. As we can see that the coefficient of $\Omega_c\Omega_{\text{RF}}$ is inversely proportional to $|\Omega_{\text{RF}}(\mathbf{z})|^2$, and $|\Omega_c(\mathbf{z})|^2$. Hence the second order susceptibility will reduce as we increase the microwave and coupling power. As the generation process also linearly dependent on $\Omega_c\Omega_{\text{RF}}$, this will lead to a saturation in the parameter regime with which we are working. This saturation effect can be found in Fig. 2.6 and Fig. 2.7.

3.3.1 Photon conversion efficiency

Photon conversion efficiency can be calculated using the formula

$$\eta = \frac{n_p}{n_{\text{RF}}} \quad (3.19)$$

where, n_p is the number of probe photon generated and n_{RF} is the microwave field photons required for the conversion process. The generated probe power can be calculated as

$$P_p = n_p \hbar \omega_p B_p = I_p A_p \quad (3.20)$$

where, $\hbar$ is reduced Plank's constant. The generated probe field has an angular frequency of ω_p and bandwidth of B_p . I_p and A_p are the intensity and area of cross section of the probe field. Microwave drive field power is given as

$$P_{\text{RF}} = n_{\text{RF}} \hbar \omega_{\text{RF}} B_{\text{RF}} = I_{\text{RF}} A_{\text{RF}} \quad (3.21)$$

The microwave drive field has an angular frequency of ω_{RF} and bandwidth of B_{RF} . I_p and A_p are the intensity and area of cross section of the microwave drive

field. The efficiency of conversion can be written using Eq. (3.19), Eq. (3.20) and Eq. (3.21) as,

$$\eta = \frac{\omega_{\text{RF}} B_{\text{RF}} I_{\text{p}} A_{\text{p}}}{\omega_{\text{p}} B_{\text{p}} I_{\text{RF}} A_{\text{RF}}} \quad (3.22)$$

The intensity of the probe and microwave field can be expressed in terms of the Rabi frequency of the corresponding electromagnetic field by,

$$I_{\text{p}} = \frac{\epsilon_0 c \hbar^2 |\Omega_{\text{p}}|^2}{d_{13}^2} I_{\text{RF}} = \frac{c \hbar^2 |\Omega_{\text{RF}}|^2}{\mu_0 \mu_{12}^2}. \quad (3.23)$$

. The photon conversion efficiency can be written as,

$$\eta = \frac{\omega_{\text{RF}} B_{\text{RF}} A_{\text{p}} \mu_{12}^2}{\omega_{\text{p}} B_{\text{p}} A_{\text{RF}} d_{31}^2} \left(\frac{|\Omega_{\text{p}}|^2}{|\Omega_{\text{RF}}|^2} \right) \quad (3.24)$$

Substituting $\Omega_{\text{p}}(l)$ from Eq. (3.17), then Eq: (3.24) becomes,

$$\eta = \alpha \frac{|\Omega_{\text{c}}(0)|^2 |e^{-\beta l} - 1|^2}{4\Gamma_2} \quad (3.25)$$

where $\alpha = \frac{\omega_{\text{RF}} B_{\text{RF}} A_{\text{p}} \mu_{12}^2}{\omega_{\text{p}} B_{\text{p}} A_{\text{RF}} d_{31}^2}$.

In terms of coupling field intensity,

$$\eta = \alpha \frac{I_{\text{c}} d_{23}^2 |e^{-\beta l} - 1|^2}{4\Gamma_2 \epsilon_0 c \hbar^2} \quad (3.26)$$

The photon conversion efficiency as a function of microwave power is given in Fig. 2.8. The nature of this curve can be explained with Eq. (3.26). When the microwave power (proportional to $|\Omega_{\text{RF}}(0)|^2$) increases, the quantity $|e^{-\beta l} - 1|^2$ decreases. Also from Eq. (3.17), it is clear that the term $(e^{-\beta l} - 1)$ is deciding the generation process. This causes the degradation in efficiency in Fig. 2.8.

3.4 Phase dependent amplification

The physical picture of the amplification process present in our system can be explained with the semi classical field calculation done in the previous section. With this calculation, we can discern a PDA from other linear amplifiers. A general PSA amplifiers output field can be explained using the semi classical field expression given below [15]

$$\Omega_{\text{p}}(l) = M\Omega_{\text{p}}(0) + L\Omega_{\text{p}}^*(0). \quad (3.27)$$

Where M and L are gain coefficients. From this expression it is expected that the periodicity of the power gain vs input probe phase curve to be π , as shown in [15]. In addition to this if M and L are following the relation $|M|^2 - |L|^2 = 1$, then the PSA will be a noiseless amplifier and it can generate squeezed states of light [6]. The amplification process presented in our scheme is distinctively different from the PSA process. As can be noted from Eq. (3.17), we can simplify it and write as,

$$\Omega_p(l) = M\Omega_p(0) + Q\Omega_c(0)\Omega_{\text{RF}}(0). \quad (3.28)$$

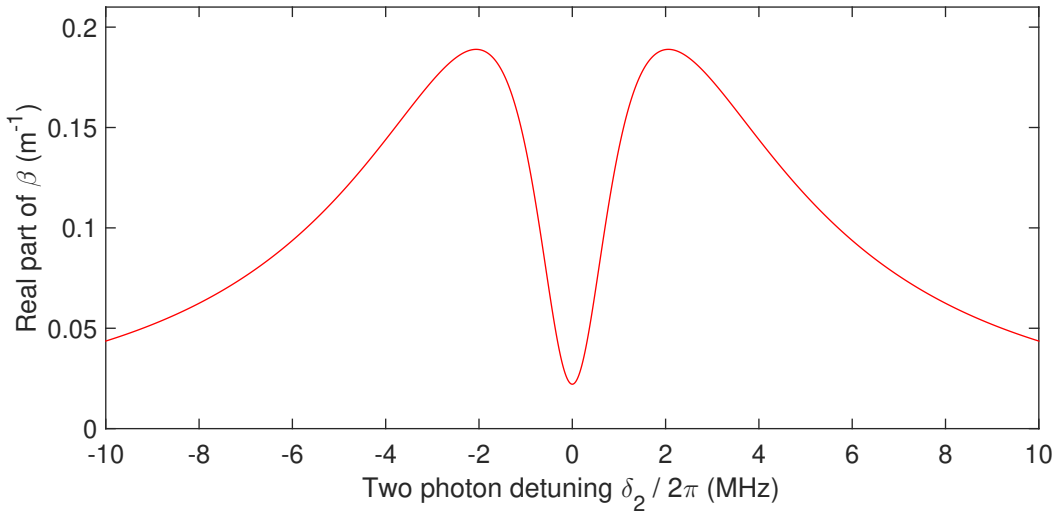


Figure 3.3: Real part of β vs two photon detuning δ_2 .

Here the conjugate of $\Omega_p(0)$ is absent. In this case $L = 0$, which is the coefficient of $\Omega_p^*(0)$. It is evident from Eq. (3.17) and from Fig. 3.4 that the power gain vs $\Delta\Phi$ curve having a periodicity of 2π . The second term in Eq. (3.28) represents the three-wave mixing process. The generated probe field and the seed probe $\Omega_p(0)$ will interfere resulting in an amplification or a de-amplification which is dictated by $\Delta\Phi$. Hence our amplifier is not a phase sensitive amplifier but it is a phase dependent amplifier, as the gain still depend on the input phase of the electromagnetic fields.

For a phase insensitive amplifier the output field can be written as [5],

$$\Omega_p(l) = M\Omega_p(0). \quad (3.29)$$

Here the output probe field only depends on the input probe field and the gain provided by the system. Here $M > 1$, where $|M|^2$ will be the power gain of the PIA amplifier. In our case the coefficient of $\Omega_p(0)$ is $M = e^{-\beta l}$. In Fig. 3.3 we have plotted the real part of β with various two photon detuning. It is evident from the Fig. 3.3 that the real part of β is always positive under three photon resonance condition. Thus the value of M will always be less than 1, and M is not a gain parameter in our amplification scheme. In contrast to the PIA, the amplification in our system occurs due to a three-wave mixing phenomenon and the gain of the system is dictated by the parameter Q from Eq. (3.28). Therefore our amplification process cannot be classified under a phase insensitive amplification process.

3.5 Results

The amplification present in our system can be explained using Eq. (3.17). The total probe amplitude at the end of the cell with length l is given by Eq. (3.17). The first term in Eq. (3.17) represents phase changes and losses suffered by the input probe field due to linear susceptibility and the second term represents similar changes experienced by the generated probe field. The input probe and the generated probe field amplitudes interfere constructively or destructively for appropriate values of $\Delta\Phi$ which results in either amplification or de-amplification of the probe field respectively. Here the gain G of the amplifier can be defined as the ratio of the transmitted probe power to the input probe power. In Fig. 3.4 shows the variation of gain with relative phase between all the three fields $\Delta\Phi$ for different values of two photon detuning δ_2 . A value $G > 1$ means an amplification of the probe field and $G < 1$ means a de-amplification of the probe field. It is evident from our result that the PDA process has a periodicity of 2π . When the relative phase $\Delta\Phi$ varies from 0 to π , then the probe gain varies from an amplification regime to a de-amplification regime. It is apparent from the Fig. 3.4 that the PDA reduces with increasing two photon detuning δ_2 .

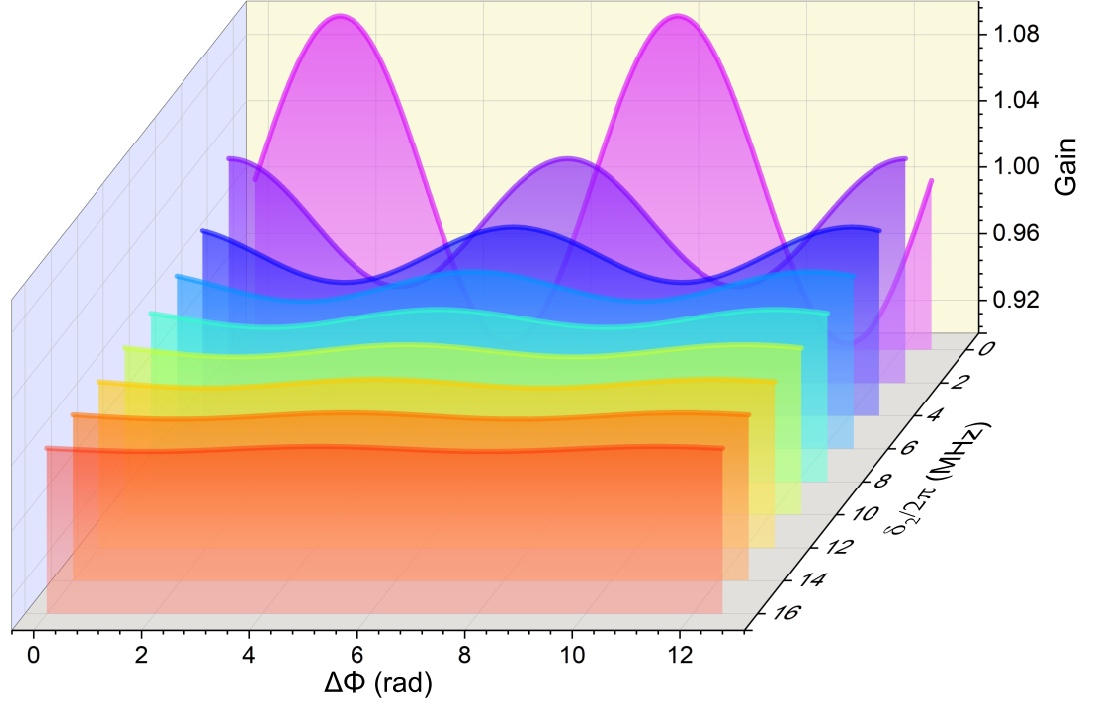


Figure 3.4: Probe gain vs relative phase between all the three fields $\Delta\Phi$ for different values of two photon detuning δ_2 at $\Omega_c = 0.4\gamma_{13}$, $\Omega_{\text{RF}} = 0.2\gamma_{13}$ and $\Omega_p = 0.02\gamma_{13}$

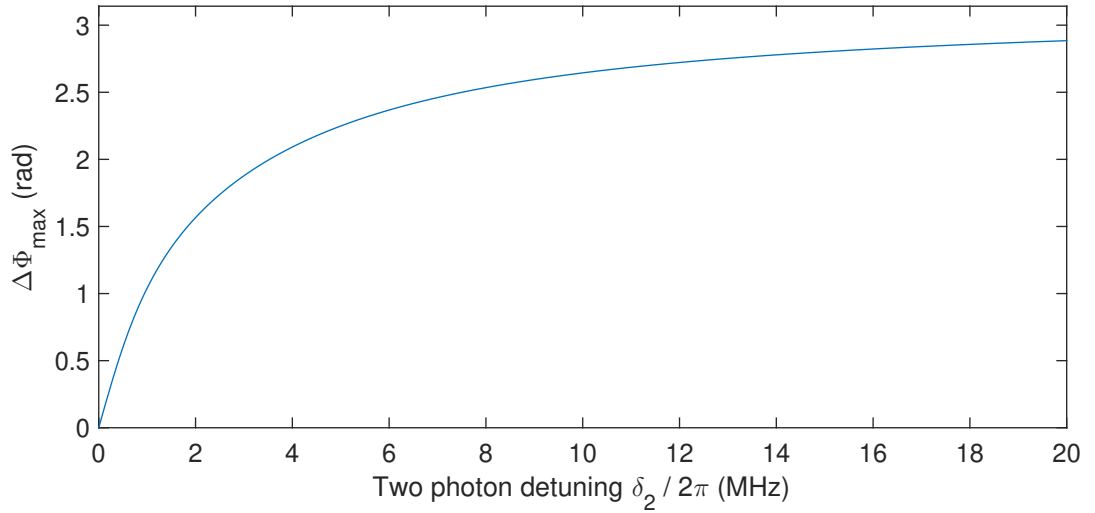


Figure 3.5: $\Delta\Phi_{\text{max}}$ with two photon detuning δ_2 at $\Omega_c = 0.4\gamma_{13}$, $\Omega_{\text{RF}} = 0.2\gamma_{13}$ and $\Omega_p = 0.02\gamma_{13}$

Apart from the reduction in gain, the phase at which maximum gain occurs denoted as $\Phi_{\max}$, also changes with two photon detuning δ_2 . The change in phase shift at which maximum gain occurs with respect to $\Phi_{\max}$ at $\delta_2 = 0$ ($\Phi_{\max}^0$) is defined as $\Delta\Phi_{\max} = \Phi_{\max}^0 - \Phi_{\max}^{\delta_2}$ for a two photon detuning of δ_2 . In Fig. 3.5 we have plotted $\Delta\Phi_{\max}$ with two photon detuning δ_2 . It is to be noted that, when the two photon detuning δ_2 changes, then in Eq.(3.17) the first and the second term changes and develop a phase shift, due to this $\Phi_{\max}$ also changes. As the two photon detuning increases the quantity $\Delta\Phi_{\max}$ saturates. This is owing to the decrease in the total gain of the amplifier with higher δ_2 , hence the phase sensitivity is also reduces.

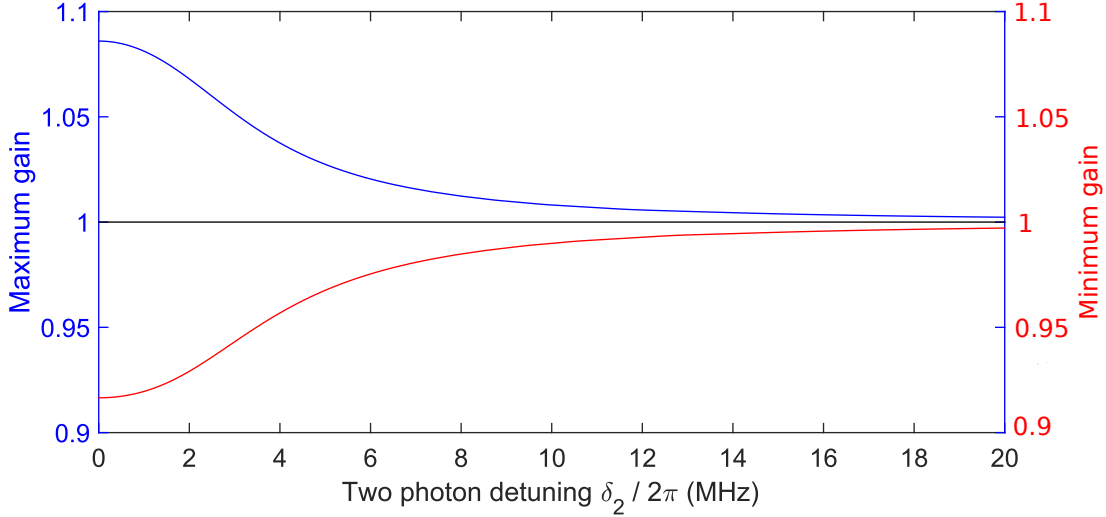


Figure 3.6: Gain vs two photon detuning at $\Omega_c = 0.625\gamma_{13}$, $\Omega_{\text{RF}} = 0.312\gamma_{13}$ and $\Omega_p = 0.02\gamma_{13}$

The variation of maximum gain and minimum gain of our PDA as a function of δ_2 is plotted in Fig. 3.6. At two-photon resonance ($\delta_2 = 0$), the input probe field and generated probe field will propagate with minimal losses due to EIT effect. In addition, probe field generation is most efficient at $\delta_2 = 0$ resulting in a maximum gain for the probe. When the two photon resonance is not satisfied ($\delta_2 \neq 0$), the input probe field experiences linear absorption due to absence of EIT and also the probe generation reduces as $1/\delta_2$ due to which the maximum gain decreases. The de-amplification effect is due to destructive interference and therefore the second

term acquires a negative sign during de-amplification resulting in an increase in the minimum gain value. As can be seen from the theoretical plot in Fig. 3.6, the maximum gain and minimum gain curves reflect this symmetry.

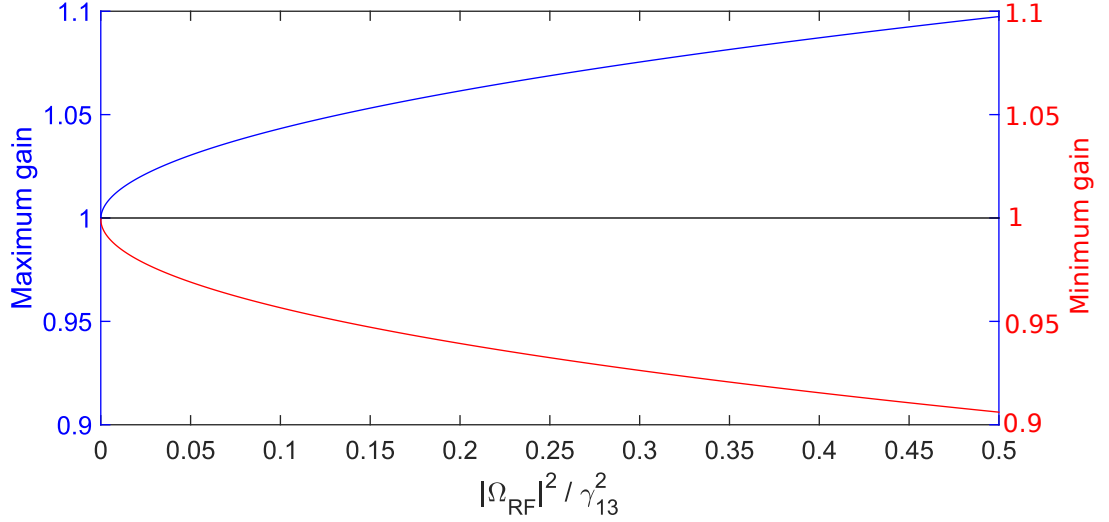


Figure 3.7: Gain vs $|\Omega_{\text{RF}}|^2 / \gamma_{13}^2$ at $\Omega_c = 0.625\gamma_{13}$ and $\Omega_p = 0.02\gamma_{13}$

Further, we have studied the dependence of our amplifier gain on the power of optical fields. Fig. 3.7 shows the variation of maximum and minimum gain of the PDA with input microwave power. The maximum and minimum gain of the PDA as a function of input coupling power is plotted in Fig. 3.8. Initially, the maximum gain increases linearly with input optical power and saturates to a maximum value. The minimum gain follows a symmetric pattern with the maximum gain curve; it reduces initially and saturates as the input optical power increases.

In Fig. 3.6, Fig. 3.7 and Fig. 3.8, the blue line is represents maximum gain, red line corresponds to minimum gain. The black line represents a unit gain, i.e. $G = 1$, above region of this line is amplification and below region corresponds to a de-amplification regime.

3.6 Conclusion

A theoretical analysis of the three-wave mixing present in a cyclic atomic level scheme interacting with two optical and a microwave field is done. A semi-classical density matrix analysis of the cyclic level scheme interacting with three electro-

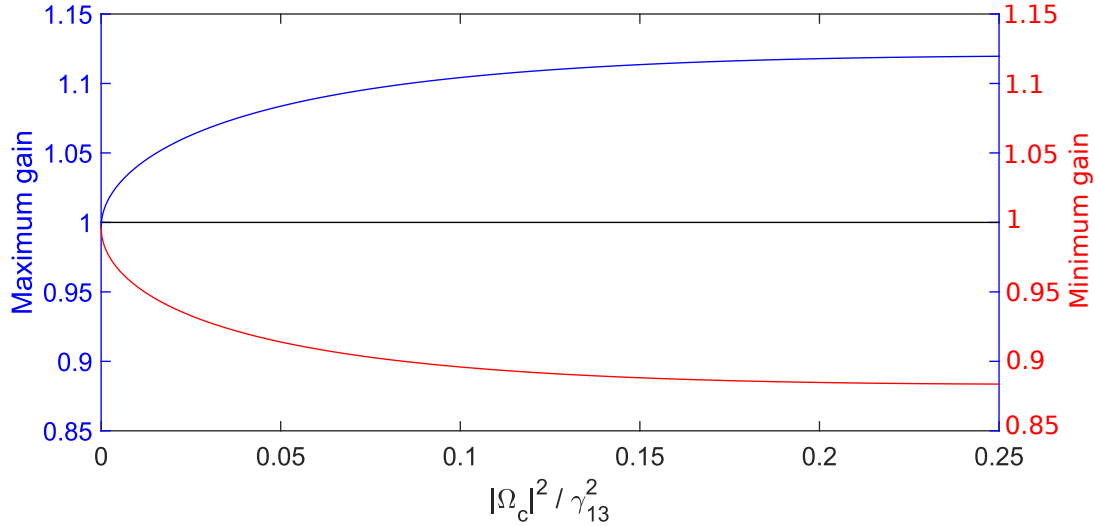


Figure 3.8: Gain vs $|\Omega_c|^2/\gamma_{13}^2$ at $\Omega_{\text{RF}} = 0.312\gamma_{13}$ and $\Omega_p = 0.02\gamma_{13}$

magnetic fields is done. Our theoretical study proposes a phase dependent amplification of an optical probe field which can be controlled by a microwave field. The amplification is due to an interference of the input probe field and the generated probe field. This interference effect can give rise to amplification or a de-amplification depending on the input microwave field phase. The distinctive difference between the physical process in our amplification process and other linear amplifiers is discussed.

Further, the power gain of the probe field is studied with various parameters such as two photon detuning, the relative phase difference between three electromagnetic fields, and field power. Noise figure calculation of our phase dependent amplifier can be pursued as a future extension of this work. This study will reveal more insight into the phase dependent amplification and difference from other linear amplification processes.

Bibliography

- [1] Mikhail I. Kolobov. *Quantum Imaging*. Springer, New York, NY, 2007.
- [2] Oo-Kaw Lim, Zachary Dutton, Gideon Alon, Chao-Hsiang Chen, Michael Vasilyev, and Prem Kumar. Enhanced optical resolution in target detection with phase-sensitive versus phase-insensitive pre-amplification. In Ronald E.

- Meyers, Yanhua Shih, and Keith S. Deacon, editors, *Quantum Communications and Quantum Imaging IX*, volume 8163, pages 45 – 50. International Society for Optics and Photonics, SPIE, 2011.
- [3] Samuel L. Braunstein and Peter van Loock. Quantum information with continuous variables. *Rev. Mod. Phys.*, 77:513–577, Jun 2005.
 - [4] S Fossier, E Diamanti, T Debuisschert, R Tualle-Brouri, and P Grangier. Improvement of continuous-variable quantum key distribution systems by using optical preamplifiers. *J. Phys. B: At. Mol. Opt. Phys.*, 42(11):114014, may 2009.
 - [5] Carlton M. Caves, Joshua Combes, Zhang Jiang, and Shashank Pandey. Quantum limits on phase-preserving linear amplifiers. *Phys. Rev. A*, 86:063802, Dec 2012.
 - [6] Carlton M. Caves. Quantum limits on noise in linear amplifiers. *Phys. Rev. D*, 26:1817–1839, Oct 1982.
 - [7] Radan Slavík, Francesca Parmigiani, Joseph Kakande, Carl Lundström, Martin Sjödin, Peter A. Andrekson, Ruwan Weerasuriya, Stylianos Sygletos, Andrew D. Ellis, Lars Grüner-Nielsen, Dan Jakobsen, Søren Herstrøm, Richard Phelan, James O’Gorman, Adonis Bogris, Dimitris Syvridis, Sonali Dasgupta, Periklis Petropoulos, and David J. Richardson. All-optical phase and amplitude regenerator for next-generation telecommunications systems. *Nat. Photonics*, 4(10):690–695, Oct 2010.
 - [8] Carl Lundström, Zhi Tong, Magnus Karlsson, and Peter A. Andrekson. Phase-to-phase and phase-to-amplitude transfer characteristics of a nondegenerate-idler phase-sensitive amplifier. *Opt. Lett.*, 36(22):4356–4358, Nov 2011.
 - [9] Kirk McKenzie, Daniel A. Shaddock, David E. McClelland, Ben C. Buchler, and Ping Koy Lam. Experimental demonstration of a squeezing-enhanced power-recycled michelson interferometer for gravitational wave detection. *Phys. Rev. Lett.*, 88:231102, May 2002.

- [10] Samuel L. Braunstein and Peter van Loock. Quantum information with continuous variables. *Rev. Mod. Phys.*, 77:513–577, Jun 2005.
- [11] S Fossier, E Diamanti, T Debuisschert, R Tualle-Brouri, and P Grangier. Improvement of continuous-variable quantum key distribution systems by using optical preamplifiers. *J. Phys. B: At. Mol. Opt. Phys.*, 42(11):114014, may 2009.
- [12] Ivan V. Sokolov and Mikhail I. Kolobov. Squeezed-light source for superresolving microscopy. *Opt. Lett.*, 29(7):703–705, Apr 2004.
- [13] J. A. Levenson, I. Abram, Th. Rivera, and Ph. Grangier. Reduction of quantum noise in optical parametric amplification. *J. Opt. Soc. Am. B*, 10(11):2233–2238, Nov 1993.
- [14] C. J. McKinstrie and S. Radic. Phase-sensitive amplification in a fiber. *Opt. Express*, 12(20):4973–4979, Oct 2004.
- [15] P Neveu, C Banerjee, J Lugani, F Bretenaker, E Brion, and F Goldfarb. Phase sensitive amplification enabled by coherent population trapping. *New J. Phys.*, 20(8):083043, aug 2018.
- [16] Asha K. Phase sensitive coherent control in atom-photon interactions and its application to quantum information processing, 2019.
- [17] Manukumara Manjappa, Satya Sainadh Undurti, Asha Karigowda, Andal Narayanan, and Barry C. Sanders. Effects of temperature and ground-state coherence decay on enhancement and amplification in a Δ atomic system. *Phys. Rev. A*, 90:043859, Oct 2014.
- [18] J. K. Saaswath, K. N. Pradosh, K. V. Adwaith, Barry C. Sanders, Fabien Bretenaker, and Andal Narayanan. Microwave-driven generation and group delay control of optical pulses from an ultra-dilute atomic ensemble. *Opt. Express*, 29(11):15940–15952, May 2021.

Chapter 4

Microwave controlled ground state coherence in an atom-based optical amplifier

4.1 Introduction

Atom-based sensors and radio-over-fibre atomic antennas are rapidly emerging as preferred architectures for sensing platforms and signal communication between microwave and optical frequencies [1, 2, 3]. Advances in quantum electrodynamical superconducting circuits operating in the microwave domain for quantum information processing, has stimulated strong interest in developing interfaces between these two frequencies [4, 5]. In this context hybrid-amplifiers which will enable coherent conversion and amplification of signals between microwave and optical frequencies play a crucial role.

In traditional nonlinear optics, amplification of a probe field saturates due to linear and nonlinear absorption, especially near atomic resonances [6]. Experimental realization of electromagnetically induced transparency(EIT) [7, 8] and coherent population trapping (CPT) [9, 10, 11] eliminates linear absorption by a large fraction, thereby enabling nonlinear amplification close to resonance. This opened the new field of low light intensity nonlinear optics. The physical effect of CPT in inducing a ground state coherence through the formation of dark state is at the heart of elimination of linear response [12] and modification of nonlinear re-

sponses in atomic media [13]. There have been demonstrations of nonlinear wave mixing and optical amplification using resonant atomic systems [14, 15, 16, 17] which exploit this ground state coherence. However, no thorough investigation of ground state coherence linked to two-photon mechanisms and their effect on three-photon processes has been experimentally performed.

In this chapter, we present experimental results of an atomic PDA whose ground state coherence is controlled using microwaves and the amplification is observed at optical frequencies. We have used a three-level atomic configuration in room temperature ^{85}Rb atoms to achieve the amplification. These three levels are connected cyclically by two optical fields and one microwave field rendering a closed, cyclic interaction of atomic dipoles with all the three electromagnetic fields. This closed system is sensitive to the relative phase of all the three fields, thus making the output probe intensity dependent on the relative phase between all the three fields.

In our earlier work [18, 19], phase dependent amplification (PDA) of the probe field was seen using this system. In the present study, we investigate the explicit role of ground state coherence in our probe transmission by selectively breaking the two-photon resonance, which creates ground state coherence. We, therefore for the first time, experimentally quantify the role of ground state coherence in a phase dependent atomic amplifier. We have achieved a maximum probe amplification of 7.5 dB with a visibility of 98.8% in the presence of ground state coherence. The amplification is shown to decrease sharply with increasing two-photon detuning. However, a nonzero amplification is present even with a large two-photon detuning of 15 MHz with reasonably high visibility of 66.8%. Our experimental results agree very well with our theoretical calculations.

Our experiment shows that for high amplification in a PDA, it is necessary to establish ground state coherence. However, for practical implementations of a PDA, we can sacrifice amplification for bandwidth. Thus one can make a judicious choice between amplification and bandwidth depending upon the application. This atom-based PDA can potentially serve as a very good interface for coherent conversion and amplification of quantum information generated in the microwave domain and converted to optical frequencies.

4.2 Experimental details

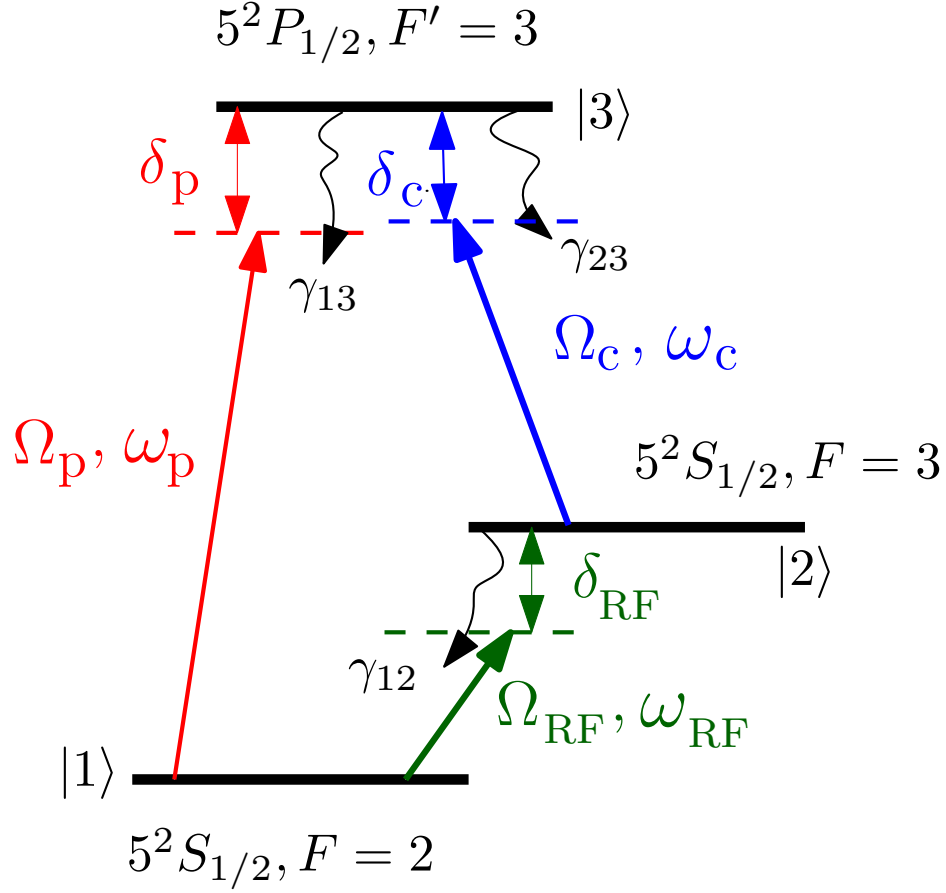


Figure 4.1: Energy level scheme.

The hyperfine energy levels used in the experiment are $5^2S_{1/2}, F=2$ ($|1\rangle$), $5^2S_{1/2}, F=3$ ($|2\rangle$) and $5^2P_{1/2}, F'=3$ ($|3\rangle$). An optical coupling field (ω_c) with Rabi frequency Ω_c connects the transition $5^2S_{1/2}, F=3$ ($|2\rangle$) $\rightarrow$ $5^2P_{1/2}, F'=3$ ($|3\rangle$). A microwave field (ω_{RF}) at 3.0357 GHz, having a Rabi frequency Ω_{RF} induces a magnetic dipole transition between the states $5^2S_{1/2}, F=2$ ($|1\rangle$) and $5^2S_{1/2}, F=3$ ($|2\rangle$). The weak magnetic dipole transition is enhanced using a microwave cavity resonating at 3.0357 GHz frequency with a quality factor of 10000 ± 1000 . A probe field (ω_p) connects the states $|1\rangle$ and $|3\rangle$, with a Rabi frequency Ω_p . The microwave and the two coupling and probe optical fields connect the hyper-fine levels of the room temperature ^{85}Rb atoms in a cyclic configuration. This will be hereafter referred to as a Δ system shown in Fig. 4.1.

A schematic representation of our experimental setup is shown in Fig. 4.2.

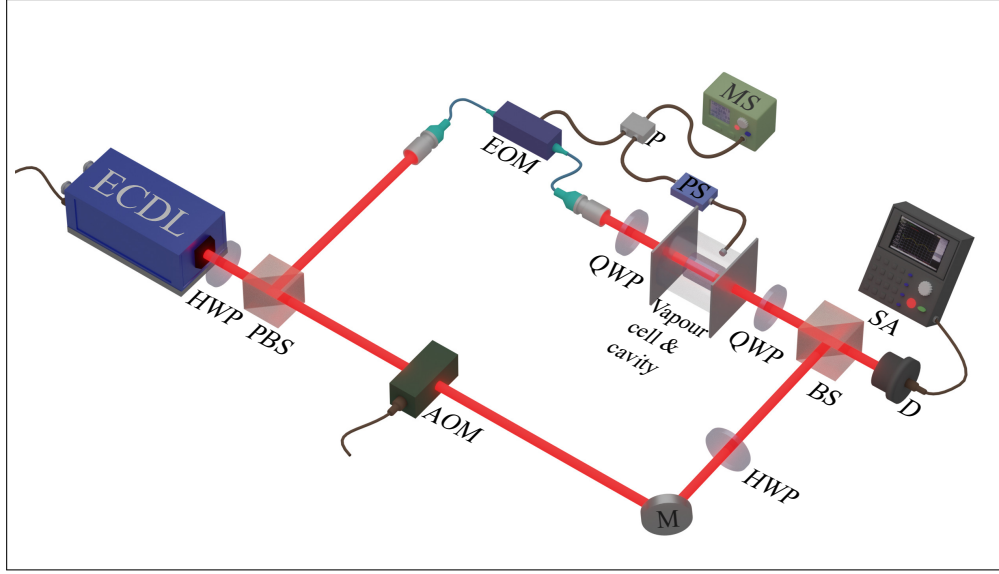


Figure 4.2: Schematic of experimental set up. Here ECDL —External Cavity Diode Laser, HWP —half-wave plate, QWP —quarter wave-plate, PBS —polarizing beam-splitter, AOM —Acousto-optic modulator, M —Mirror, EOM —Electro-optic modulator, MS —Microwave source, P —Power splitter, PS —Phase shifter, D —Detector, SA —Spectrum analyzer.

An ECDL has been used to derive the coupling beam, which enters a fiber coupled electro-optic modulator (EOM) driven by the microwave source to produce the probe field. The probe and coupling beams pass through a vapour cell kept inside the microwave cavity at room temperature. The cavity supports a standing wave at 3.0357 GHz and so the atoms inside the cavity are subjected to two optical and one microwave field. The optical fields after interaction emerge from the cavity and the probe field is separated from the coupling field by heterodyne detection. The amplitude of the probe field is analyzed using a spectrum analyzer. A digital phase shifter has been used to control the phase of the microwave field, which enters the cavity. The optical power of probe and coupling fields are kept constant at $33 \mu\text{W}$ and $166 \mu\text{W}$ respectively and the optical density of the sample is calculated to be 1.0. The power of optical fields are chosen for values where the gain is a maximum. The optical depth is fixed by the lowest detectable probe power by our high band-width detector.

The microwave cavity and the EOM are driven by the same source of microwave field, ensuring relative phase stability between all the three fields. Throughout the

experiment, a three-photon resonance condition $\delta_3 = \delta_p - \delta_c - \delta_{\text{RF}} = 0$, is satisfied. Here δ_c , δ_p and δ_{RF} refer to the detunings of the coupling, probe and microwave fields from their respective transitions respectively (Fig. 4.1). During all runs of our experiment, the value of δ_c is kept zero and the equality $\delta_p = \delta_{\text{RF}}$ is maintained. A change in δ_{RF} from its resonance thus enables selective breaking of two-photon resonance ($\delta_2 = \delta_p - \delta_c = 0$) while ensuring that the three-photon resonance condition remains satisfied.

We perform our experiment at a room temperature of 300 K. At this temperature we can still observe true amplification of the input seed probe field for appropriate values of input microwave and coupling powers.

4.3 Experimental observation

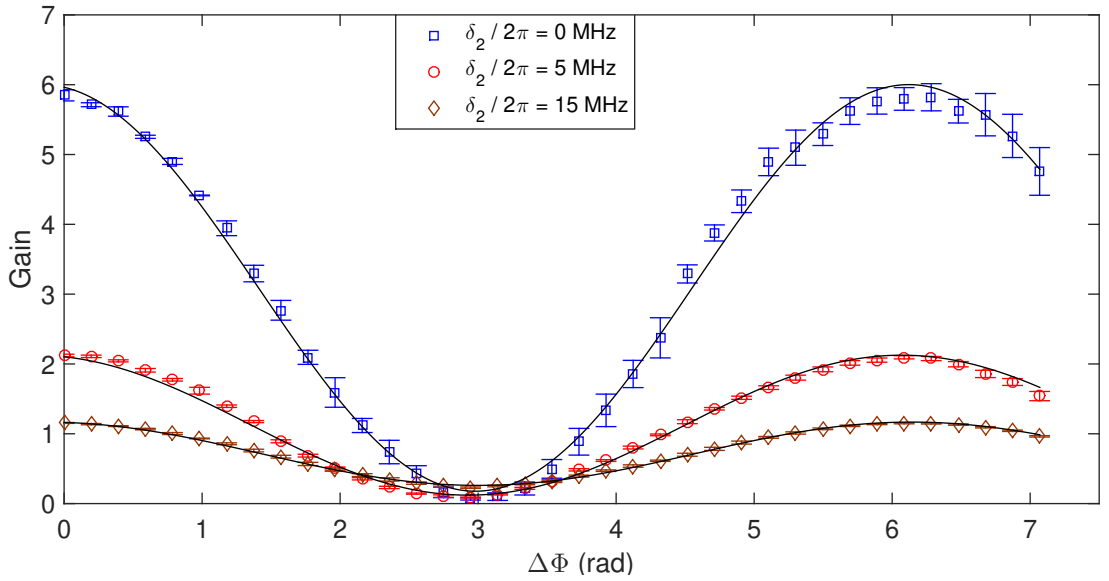


Figure 4.3: Probe gain vs $\Delta\Phi$ for different values of δ_2 at microwave intensity 1.03 mW/cm^2 . The continuous line is the theoretical fit.

We observe amplification and de-amplification of the input probe beam as a result of three-wave mixing interaction present in our Rb atomic system. This is possible with the help of a microwave magnetic dipole transition, which allows a three-wave mixing phenomenon by breaking the centro-symmetry of the atom [20]. The amplification is dependent on the relative phase between all the three fields.

This relative phase is defined as

$$\Delta\Phi(z) = \phi_{\text{RF}} + (k_p - k_c)z \quad (4.1)$$

where ϕ_{RF} is the phase of the microwave field and $k_p z$, $k_c z$ are the propagation phases of the probe and coupling field respectively [18]. Since the microwave field is inside a cavity, there are no propagation phases associated with it. The phases of probe and coupling field are taken to be the same as they are derived from the same source. When the relative phase $\Delta\Phi$ changes from 0 to π , the probe beam transits from a regime of amplification to a regime of de-amplification with a periodicity of 2π as reported in our previous study [18].

As a significant new step, in the current experiment, we investigate the gain (G) experienced by the probe field both in the presence and absence of two-photon induced ground state coherence. This is achieved by selectively detuning the probe field away from its resonance while maintaining the coupling field at its resonance. This breaks the two-photon resonance condition and renders $\delta_2 = \delta_p - \delta_c \neq 0$. Since our microwave source controls the EOM from which the probe laser is generated any microwave detuning δ_{RF} results in the same numerical value for the detuning δ_p of the probe laser. Thus a selective breaking of two-photon resonance is achieved while simultaneously maintaining the three-photon resonance necessary for efficient nonlinear wave mixing. The effect of this selective breaking of ground-state coherence on probe amplification is shown in Fig. 4.3.

In this plot, we have shown the variation of gain (G) of the probe field as a function of relative phase difference ($\Delta\Phi$) at different two-photon detunings δ_2 . Here the gain G is calculated as the ratio of the transmitted probe power at a particular δ_2 to that at far-off-resonance where both the optical fields are away from their respective resonance by more than 1 GHz.

The maximum and minimum gain are defined as the highest amplification and de-amplification respectively, which is observed in the probe field. As can be seen in this figure, for $\delta_2 \neq 0$, the maximum gain decreases. Nevertheless, since $\delta_3 = 0$ and since our system is phase dependent, we can see that even at non-zero δ_2 we achieve phase dependent amplification ($G > 1$) and de-amplification ($G < 1$).

The variation of maximum gain and minimum gain as a function of δ_2 is plotted in Fig. 4.4. As can be seen from the plot the nonlinear wave mixing is

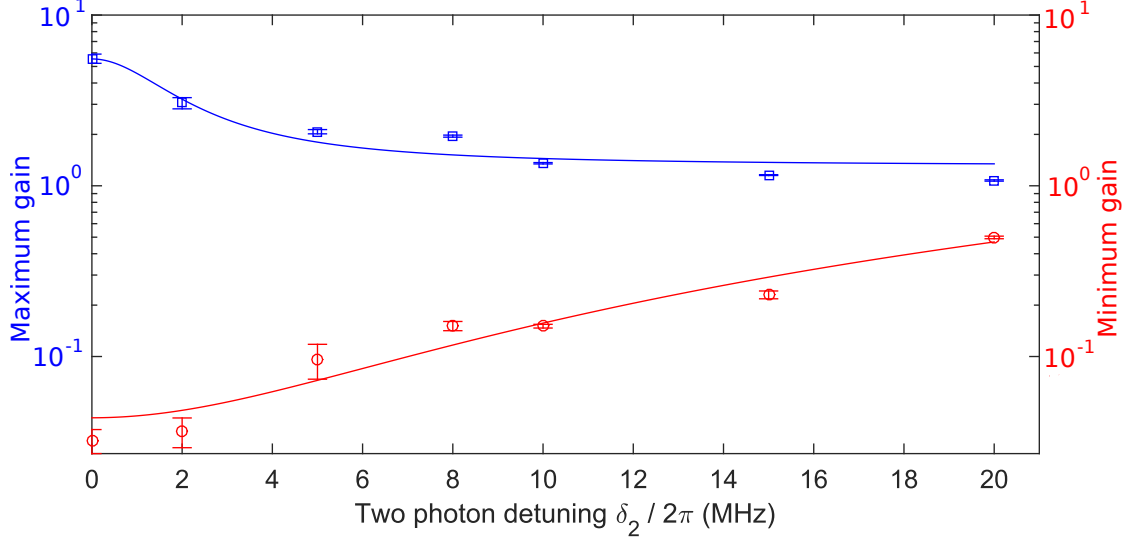


Figure 4.4: Maximum and minimum gain vs δ_2 in log scale for microwave intensity of 1.03 mW/cm^2 . Blue squares and red circles are experimental data points for maximum and minimum gain respectively. The continuous lines are theoretical fit.

highly efficient and coherently adds to the input probe resulting in maximum amplification of the probe in the presence of ground state coherence ($\delta_2 = 0$). As δ_2 increases, the maximum gain asymptotically reaches unity reflecting no gain. In a symmetric fashion, the minimum gain increases from its lowest value as δ_2 increases and also seen to tend to unity at large δ_2 values. The full width at half maximum (FWHM) of the amplification is found to be 5 MHz.

In Fig. 4.5, the maximum and the minimum gain as a function of microwave power is plotted. The reason for the saturation nature of the curve can be explained with the theoretical calculation we have done in the previous chapter. The second order process responsible for the three-wave mixing phenomenon can be understood from our Eq. (3.14), the coefficient of $\Omega_c \Omega_{\text{RF}}$ in the second term is proportional to the hybrid second order susceptibility of probe field. As we can see that the coefficient of $\Omega_c \Omega_{\text{RF}}$ is inversely proportional to $|\Omega_{\text{RF}}(\mathbf{z})|^2$. Hence the second order susceptibility will reduce as we increase the microwave and coupling power. As the generation process and the phase dependent amplification also linearly dependent on Ω_{RF} , this will lead to a saturation in the parameter regime with which we are working.

A single data point represents an average of 150 data points. The error bars denote the statistical standard deviation in our data sets. All the theoretical fit-

ting is done based on the calculations in Chapter 3.

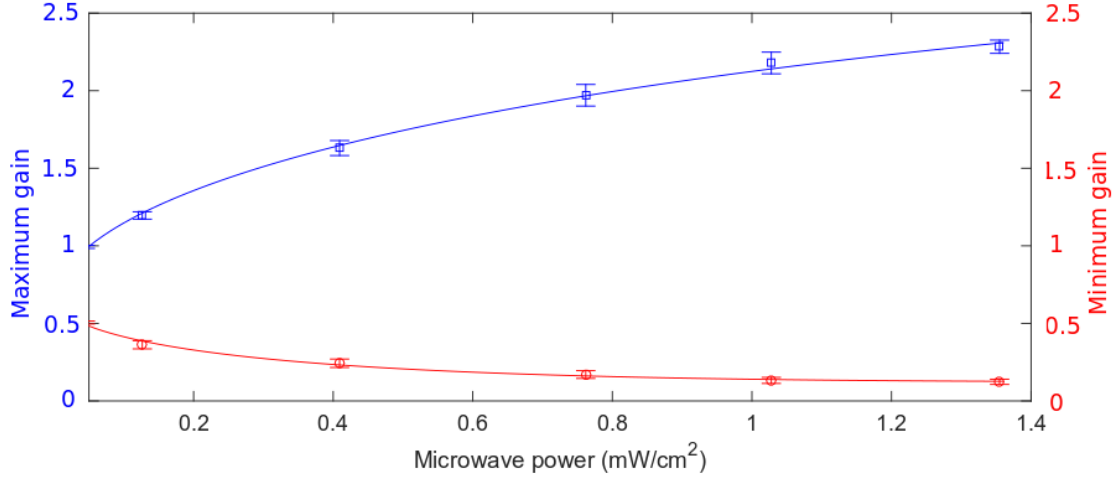


Figure 4.5: Maximum and minimum gain vs microwave intensity at $\delta_2/2\pi = 5$ MHz. Blue squares and red circles are experimental data points for maximum and minimum gain respectively. The continuous lines are theoretical fit.

Visibility is an important parameter for interference. In analogy with the interference of two fields arising from the two slits of a double-slit experiment, we identify the generated probe and the seed probe to be the two interfering fields. Extending this analogy, we can associate the maximum gain and minimum gain in our experiment to the bright and dark fringes of the interference pattern respectively. Using this comparison, the visibility at any given δ_2 is defined as the ratio between the difference of maximum gain and minimum gain to their sum. From Fig. 4.4, it is clear that the maximum gain and the minimum gain asymptotically tend to unity for high two photon detunings as explained earlier. For $\delta_2 = 0$, both the two-photon and three-photon resonance conditions are satisfied which maximises the ground state coherence assisted generation of the probe. Hence visibility is maximum for this value and under ideal conditions will be unity. On the other end, for large non-zero values of δ_2 , visibility will be at its lowest due to the negligible generation of the probe field and will tend to zero. This behaviour is experimentally observed, as seen in Fig. 4.6.

A large visibility value in our experiment indicates that the generated and in-

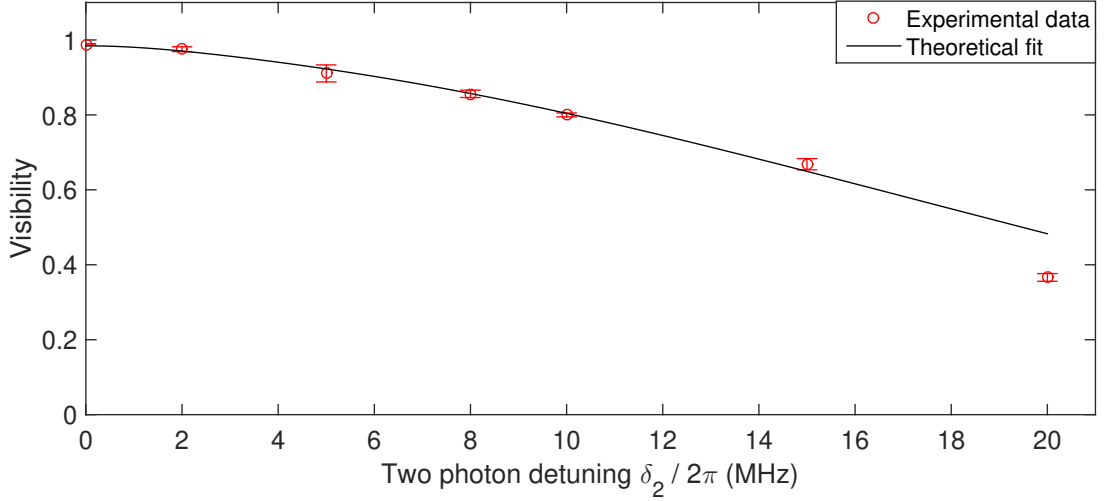


Figure 4.6: Visibility vs two photon detuning δ_2 for microwave intensity of 1.03 mW/cm^2 .

put probe fields have similar amplitudes. This is desirable in optical switches and digital communication [1, 2]. In our experiment, we obtain high visibility of 98.8% at $\delta_2/2\pi = 0$ and visibility of 66.8% at $\delta_2/2\pi = 15 \text{ MHz}$ as shown in Fig. 4.6. It is important to note from Fig. 4.6 that faithful transmission of signals is possible with very little amplification for two-photon detunings as large as 20 MHz with visibility close to 37 % which is $1/e$ value of the visibility curve. Thus our PDA gives us the freedom to choose amplification over bandwidth and vice-versa depending upon the application.

In order to understand the dependence of visibility on microwave intensity, we have plotted the variation of visibility with microwave intensity in Fig. 4.7 for $\delta_2/2\pi = 5 \text{ MHz}$. Visibility increases as microwave intensity increases and shows saturation beyond 0.4 mW/cm^2 . This is because, as microwave intensity increases, the probe generation will increase up to a point where absorption associated with nonlinearity will saturate any further increase [20]. This in-turn leads to a saturation in the maximum gain and the minimum gain and consequently the visibility.

We have measured the variation of signal to noise ratio (SNR) of the optical output probe field for different values of two photon detuning to understand the signal quality of the output signal. As expected the SNR is found to be decreasing

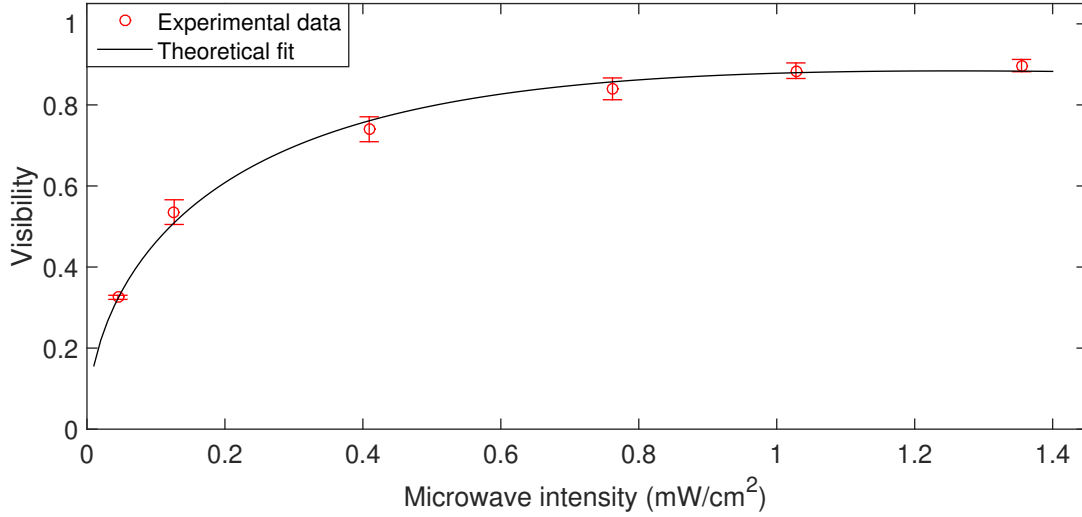


Figure 4.7: Visibility vs microwave intensity for $\delta_2/2\pi = 5$ MHz.

with increasing δ_2 as shown in Fig. 4.8. However this decrease is very less. From a large SNR of 90 dB at resonance for a $\delta_2/2\pi = 0$, the SNR decreases only to 71 dB even at a $\delta_2/2\pi = 20$ MHz. This signifies that our atomic PDA has a large data capacity for communication purposes even at a bandwidth of 20 MHz.

4.4 Significance of our experimental results

Amplification of an optical field has been experimentally demonstrated using a three wave mixing ($\chi^{(2)}$) process earlier using classical nonlinear crystals [13]. In alkali atomic vapours possessing ground state coherence, all previous demonstrations of an amplifier used a four-wave mixing ($\chi^{(3)}$) nonlinear interaction [18,25]. In most demonstrations, the interacting fields are within a narrow band-width of frequencies and amplification is seen in an all-optical domain. Our experimental demonstration of a PDA where amplification is demonstrated in an optical field by controlling parameters of a microwave field is the first of its kind across various physical systems.

Our hybrid-PDA spanning electromagnetic fields, which are several orders apart in frequency, is of immense practical interest. In particular, our PDA between microwave and optical fields can be used to obtain radio-over-fiber implementations for coherent amplification of classical signals. It is to be noted that a Rydberg atom-based optical communication protocol interfacing with a fiber-optic

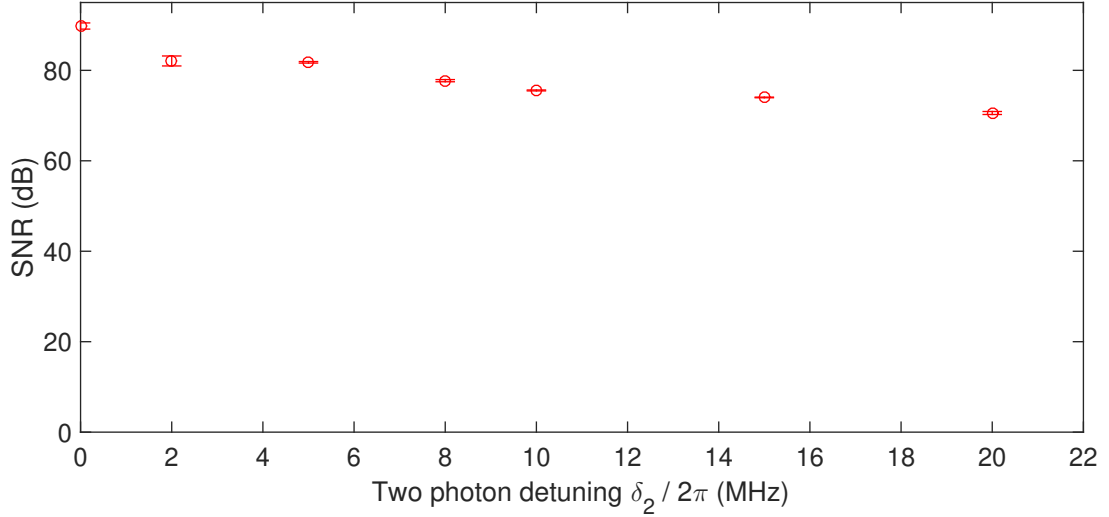


Figure 4.8: SNR of the optical output probe field vs two photon detuning δ_2 for microwave intensity of 1.03 mW/cm^2 .

link for the transmission of RF signals has already been demonstrated [21]. Such communication protocols can readily be implemented with our system with the added advantage of high amplification and a good extinction ratio. More importantly, if our scheme is implemented with cold atoms, it can readily serve as an ideal interface for coherent transfer and amplification of quantum signals created in the microwave domain [22] to the optical domain. Such ultra-cold atom-based implementation can significantly increase amplification, as was explicitly calculated in an earlier publication [23]. Low-field-few-atom nonlinearity is a desirable goal [24] for the design of quantum logic gates. In this experiment, we demonstrated nonlinear amplification of the probe with very low optical densities of our Rb vapour, achieving a high gain of 7.5 dB with an optical density of 0.45. This is one of the lowest density nonlinearities demonstrated amongst atomic vapours showing amplification [25, 26] and the first such low-density demonstration in a hybrid architecture.

4.5 Conclusion

We experimentally investigate the role of microwave controlled ground state coherence on a phase dependent optical amplifier. We show that the highest amplification is achieved only in the presence of ground state coherence. Through

our experiment and subsequent analysis, we show that amplification is due to an interference process resulting in a maximum probe amplification of 7.5 dB with visibility of 98.8%. We also obtain a non-zero amplification with a visibility of 66.8% at a two-photon detuning of 15 MHz. The SNR of the output optical probe field is found to be high with values ranging from 90 dB at two-photon resonance to 70 dB at a bandwidth of 20 MHz. This enables us to make a judicious choice between amplification and bandwidth in accordance with our end application. We envisage that our atom-based hybrid optical amplifier will serve as a good interface for coherent transfer and amplification of classical and quantum microwave signals to optical frequencies.

Bibliography

- [1] David H. Meyer, Kevin C. Cox, Fredrik K. Fatemi, and Paul D. Kunz. Digital communication with rydberg atoms and amplitude-modulated microwave fields. *Appl. Phys. Lett.*, 112(21):211108, 2018.
- [2] C. L. Holloway, M. T. Simons, J. A. Gordon, and D. Novotny. Detecting and receiving phase-modulated signals with a rydberg atom-based receiver. *IEEE Antennas and Wireless Propagation Letters*, 18(9):1853–1857, 2019.
- [3] A. Tretiakov, C. A. Potts, T. S. Lee, M. J. Thiessen, J. P. Davis, and L. J. LeBlanc. Atomic microwave-to-optical signal transduction via magnetic-field coupling in a resonant microwave cavity. *Appl. Phys. Lett.*, 116(16):164101, 2020.
- [4] Ze-Liang Xiang, Sahel Ashhab, J. Q. You, and Franco Nori. Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems. *Rev. Mod. Phys.*, 85:623–653, Apr 2013.
- [5] Nicholas J. Lambert, Alfredo Rueda, Florian Sedlmeir, and Harald G. L. Schwefel. Coherent conversion between microwave and optical photons—an overview of physical implementations. *Adv. Quantum Technol.*, 3(1):1900077, 2020.

- [6] Robert W. Boyd. *Nonlinear Optics, Third Edition*. Academic Press, Inc., Orlando, FL, USA, 3rd edition, 2008.
- [7] K.-J. Boller, A. Imamoglu, and S. E. Harris. Observation of electromagnetically induced transparency. *Phys. Rev. Lett.*, 66:2593–2596, May 1991.
- [8] J. E. Field, K. H. Hahn, and S. E. Harris. Observation of electromagnetically induced transparency in collisionally broadened lead vapor. *Phys. Rev. Lett.*, 67:3062–3065, Nov 1991.
- [9] G. Alzetta, A. Gozzini, L. Moi, and G. Orriols. An experimental method for the observation of r.f. transitions and laser beat resonances in oriented sodium vapour. *Il Nuovo Cimento B (1971-1996)*, 36(1):5–20, Nov 1976.
- [10] E. Arimondo and G. Orriols. Nonabsorbing atomic coherences by coherent two-photon transitions in a three-level optical pumping. *Lettere al Nuovo Cimento (1971-1985)*, 17(10):333–338, Nov 1976.
- [11] H. R. Gray, R. M. Whitley, and C. R. Stroud. Coherent trapping of atomic populations. *Opt. Lett.*, 3(6):218–220, Dec 1978.
- [12] Michael Fleischhauer, Atac Imamoglu, and Jonathan P. Marangos. Electromagnetically induced transparency: Optics in coherent media. *Rev. Mod. Phys.*, 77:633–673, Jul 2005.
- [13] S. E. Harris, J. E. Field, and A. Imamoglu. Nonlinear optical processes using electromagnetically induced transparency. *Phys. Rev. Lett.*, 64:1107–1110, Mar 1990.
- [14] M. D. Lukin, P. R. Hemmer, M. Löffler, and M. O. Scully. Resonant enhancement of parametric processes via radiative interference and induced coherence. *Phys. Rev. Lett.*, 81:2675–2678, Sep 1998.
- [15] Feng Wen, Huaibin Zheng, Xinxin Xue, Haixia Chen, Jianping Song, and Yanpeng Zhang. Electromagnetically induced transparency-assisted four-wave mixing process in the diamond-type four-level atomic system. *Opt. Mat.*, 37:724 – 726, 2014.

- [16] Chenguang Ye and Jing Zhang. Electromagnetically induced transparency-like effect in the degenerate triple-resonant optical parametric amplifier. *Opt. Lett.*, 33(16):1911–1913, Aug 2008.
- [17] Kangkang Li, Yin Cai, Wei Li, Huanrong Fan, Shaohuan Ning, Siqiang Zhang, and Yanpeng Zhang. All-optical wavelength division multiplexing amplifier based on vacuum induced nonreciprocal bistability in six-wave mixing process with a ring cavity. *Results Phys.*, 16:102822, 2020.
- [18] Asha Karigowda, Adwaith K V, Pradosh K. Nayak, S. Sudha, Barry C. Sanders, Fabien Bretenaker, and Andal Narayanan. Phase-sensitive amplification of an optical field using microwaves. *Opt. Express*, 27(22):32111–32121, Oct 2019.
- [19] Asha K. *Phase sensitive coherent control in atom-photon interactions and its application to quantum information processing*. 2019.
- [20] K. V. Adwaith, Asha Karigowda, Charudatta Manwatkar, Fabien Bretenaker, and Andal Narayanan. Coherent microwave-to-optical conversion by three-wave mixing in a room temperature atomic system. *Opt. Lett.*, 44(1):33–36, Jan 2019.
- [21] A. B. Deb and N. Kjærgaard. Radio-over-fiber using an optical antenna based on rydberg states of atoms. *Appl. Phys. Lett.*, 112(21):211106, 2018.
- [22] G Wendin. Quantum information processing with superconducting circuits: a review. *Rep. Prog. Phys.*, 80(10):106001, sep 2017.
- [23] Manukumara Manjappa, Satya Sainadh Undurti, Asha Karigowda, Andal Narayanan, and Barry C. Sanders. Effects of temperature and ground-state coherence decay on enhancement and amplification in a Δ atomic system. *Phys. Rev. A*, 90:043859, Oct 2014.
- [24] Alisa Javadi, I Söllner, Marta Arcari, S Lindskov Hansen, Leonardo Midolo, Sahand Mahmoodian, G Kiršanskė, Tommaso Pregnolato, EH Lee, JD Song, et al. Single-photon non-linear optics with a quantum dot in a waveguide. *Nat. Commun.*, 6(1):1–5, 2015.

- [25] N. V. Corzo, A. M. Marino, K. M. Jones, and P. D. Lett. Noiseless optical amplifier operating on hundreds of spatial modes. *Phys. Rev. Lett.*, 109:043602, Jul 2012.
- [26] P Neveu, C Banerjee, J Lugani, F Bretenaker, E Brion, and F Goldfarb. Phase sensitive amplification enabled by coherent population trapping. *New J. Phys.*, 20(8):083043, aug 2018.

Chapter 5

Summary and future works

5.1 Summary

The problems addressed in this thesis is related to the development of an atom-based platform that strongly couples microwave and optical frequencies thereby making a coherent microwave-to-optical converter. To achieve this, we have chosen a three hyperfine atomic level scheme in a room temperature ^{85}Rb atoms which is coupled with microwave and optical fields. With this scheme we have demonstrated a coherent microwave-to-optical conversion and a phase dependent amplification of an optical probe field using microwaves.

The initial work reported in the thesis is the coherent generation of a near-infrared optical field through a three-wave mixing phenomenon in an atomic energy level scheme of ^{85}Rb atoms. This nonlinear generation process in a centrosymmetric thermally broadened atomic system is made possible through a novel interaction between induced electric and magnetic dipoles. During the generation process of the probe field, the probe and the coupling field satisfy a two-photon resonance condition that shelves the atoms in a dark state due to the electromagnetically induced transparency effect, making the process immune to noise effects due to spontaneous decay from excited states. The photon conversion efficiency and bandwidth of the conversion process in our scheme are further compared with other physical systems that have been successfully used to demonstrate coherent microwave-to-optical transduction. Our scheme represents a minimal optical decoherence scheme which could be used to transfer quantum states between microwave-to-optical frequency

regimes with near-unit fidelity.

Next, we developed a theoretical model to study the coherent conversion and amplification process of an optical field using microwaves in our previous closed three-level system. We have obtained an analytical expression for the probe field using density matrix analysis. Our analysis shows that, in addition to a generation of probe field which is discussed in the earlier chapter, the optical probe field can be amplified as well. The physical mechanism of the amplification process reveals that the amplification is due to an interference of the input probe field and the generated probe field. This interference effect can give rise to an amplification or a de-amplification depends on the phase of the generated probe field. The amplification process can be controlled by the phase of the microwave field, thus making the system a phase-dependent amplifier (PDA). The phase-dependent amplification of the probe field is studied with various microwave field intensities, and with various two-photon detuning.

We then performed an experiment to investigate the role of ground state coherence in an atom-based optical amplifier which we have theoretically studied. We have investigated the Phase dependent amplification (PDA) experienced by the probe field both in the presence and absence of two-photon induced ground state coherence. We have shown that a simultaneous fulfilment of a two-photon resonance condition that creates ground state coherence and a three-photon resonance condition leads to a significantly higher amplification of the optical probe field. By selectively breaking the ground state coherence using microwaves, we show that the amplification reduces. Nevertheless, the system shows non-zero PDA for large two-photon detunings with high visibility. This novel, controllable hybrid-PDA can be potentially used to trade-off amplification for bandwidth during the transmission of phase coherent classical and quantum information.

5.2 Future works

The work presented in this thesis opens up a lot of prospects for the future. Some of them are discussed in this section.

The physical mechanism of the phase dependent amplification (PDA) process studied in the thesis is very different from other optical linear amplification processes. However further studies are required to understand the noise properties of the amplifier. This study will help us to compare and contrast our PDA with other optical linear amplifiers.

The other possible future work is to explore the possibility of exploiting the hybrid-phase dependent amplifier for radio-over-fibre communications and microwave controlled optical switches. The higher visibility and bandwidth of our amplifier will be the key features for these future applications.

The traditional Λ system is extensively used to realize dark state polariton, slow light, quantum memory, and even as a single photon source. In this thesis we have theoretically and experimentally studied a closed Λ system also known as the Δ system. The extra field we have introduced make the system phase dependent. Apart from this, the microwave controlled optical amplification also makes our Δ system a potential candidate for future applications. Our system could be used to realize and study dark state polariton, slow light, quantum memory, and single photon source.