

On the Meaning of Pulsar Velocities

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Abstract. From the observed pulsar proper-motions we conclude that most pulsars were born in and released from binary systems. A minority of pulsars belong to a class characterized by low velocities, low magnetic fields and characteristic ages in excess of 10^7 yr. These were significantly spun-up during accretion from their companions. The rest belong to the high velocity, high magnetic field class. Using a remarkable relationship between the speed at present and that in the binary orbit prior to the release of the pulsar, we deduce that such binaries must have undergone a common envelope phase of stellar evolution. Limits on masses and separations of these binary systems are derived using the proper-motion observations.

Key words: Pulsars, velocities—pulsars, magnetic fields—pulsars, progenitors—stars, binary evolution.

1. Introduction

In the literature on pulsars, much attention has been paid to their angular velocities, which provide the most stable part of their signatures. It is an interesting fact that a large number, perhaps, the majority, of theoretical studies of pulsars are based on only the angular velocities and their first derivatives, ignoring all other details of the individual pulse characteristics. In contrast, the linear velocities of pulsars have received less attention, perhaps because they are not among the characteristics which can be immediately determined. Several years of observations are necessary to measure the velocities for single pulsars, and for them it is only the proper-motions which are observed. On the other hand, it is only the *orbital* velocities which can be determined when the pulsars occur in binary systems. Thus one does not know pulsar velocities completely. Even so we wish to point out here that the linear velocities of pulsars are like signatures of their progenitors, and that much can be learnt from them. Also that the best data available at present show that pulsars were born in binary systems, and enables one to derive constraints on such binary systems indicating that they must have undergone a common envelope phase of stellar evolution. The close binary systems which we believe lead to the observed velocities are described at the end.

I

Due to the special location of the Crab pulsar, its proper motion was estimated soon after its discovery. The transverse velocity inferred by optical means was $\sim 100 \text{ km s}^{-1}$ (Trimble 1968). On the basis of this value, as well as the derived heights of pulsars from the galactic plane, Gunn & Ostriker (1970) concluded that all pulsars must have high

velocities $\sim 100 \text{ km s}^{-1}$. They also suggested that these may be the runaway velocities from binary systems which were disrupted by the *first* supernova explosions in them. The proposal was similar to that advanced for runaway OB stars by Zwicky (1957) and Blaauw (1961).

Subsequent determinations of pulsar proper motions came from radio observations. Drifts of the pulsars scintillation patterns and analysis of pulse arrival times over several years (see references in Lyne, Anderson & Salter 1982) confirmed that pulsars indeed had proper motions of $100\text{--}200 \text{ km s}^{-1}$. The best measurements available today are, however, due to radio interferometric observations and give a range of 10 to 370 km s^{-1} (Lyne, Anderson & Salter 1982).

On the theoretical front two other mechanisms have been proposed to explain the pulsar velocities. They are, the asymmetric supernova explosion (Shklovskii 1970) and 'rocket' mechanisms (Le Blanc & Wilson 1970; Harrison & Tademaru 1975).

Asymmetric supernova explosions invoke a non-spherically-symmetric ejection of the supernova shell, which to conserve momentum must give a 'kick' to the new-born pulsar; but no detailed physical model has yet been proposed. In the 'rocket' mechanism of Harrison & Tademaru (1975), the oblique pulsar magnetic dipole is displaced from the centre of the neutron star and is skew to the rotation axis. The consequent asymmetry in the multipole radiation, propels the pulsar along its rotation axis soon after it is born.

II

Observationally, the 'rocket' mechanism can be tested by checking if the directions of motion and of the rotation axis do indeed coincide. The intrinsic linear polarization at the centre of the pulse is the projection of the rotation axis (and also of the rotating magnetic dipole axis at that instant) on the plane of the sky, according to the polar cap model of pulsar radio emission (Radhakrishnan 1969). The proper motion is also a projection of the space velocity on the plane of the sky. If two vectors coincide so must their projections. This test comparing proper motion directions to position angles of the polarization vectors at the centre of the pulses, was suggested and carried out by Morris, Radhakrishnan & Shukre (1976). These authors argued that the observations did not favour the 'rocket' mechanism, but the issue however remained clouded by two factors (Tademaru 1977). The pulse polarization in many pulsars jumped by 90° within the pulse, thus making the polarization direction uncertain. Secondly, the proper motion data itself was meagre and not of very high quality. Subsequently, Backer and coworkers demonstrated on the basis of refined polarization measurements that when allowance is made for polarization flips, all the observations fully support the polar cap model (Backer 1982). Using the excellent proper motion observations of Lyne, Anderson & Salter (1982), this test was repeated by Anderson & Lyne (1983). They have reinforced the conclusion of Morris, Radhakrishnan & Shukre (1976) that the 'rocket' mechanism is not operative in accelerating pulsars.

III

Ironically enough, while Anderson & Lyne failed to find any evidence for the directions of velocities to be as predicted by the 'rocket' mechanism which in essence invokes magnetic *multipole* moments for the purpose, they found the magnitudes of the velocities to be correlated with the derived magnetic *dipole* moments (Fig. 1). Though

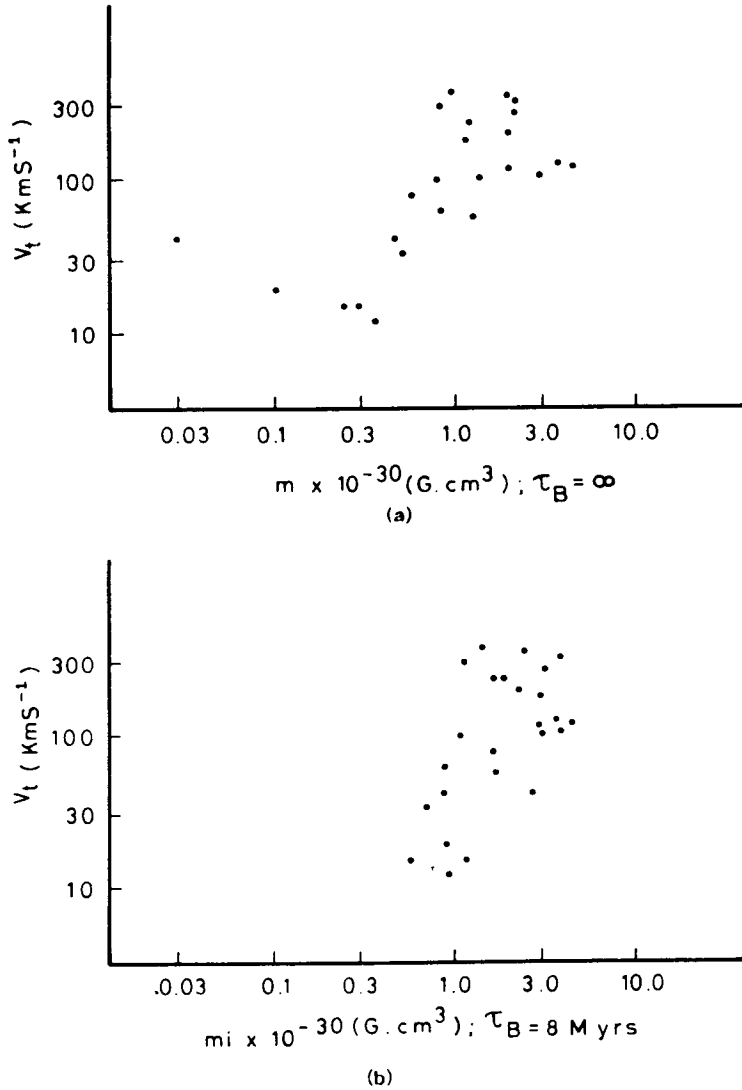


Figure 1. Transverse velocities of 25 pulsars plotted against initial magnetic dipole moments, (a) assuming no magnetic field decay, and (b) assuming exponential decay with characteristic time of 8 Myr. (From Anderson & Lyne 1983.)

they did not come to any firm conclusions on the origin of pulsar velocities, these authors suggested that the velocities are probably due to asymmetric supernova explosions, and that the asymmetry could be influenced by the magnetic dipole moment.

The possibility that the velocities could result from an asymmetric explosion governed by the magnetic field is most unlikely because the kinetic energy of a pulsar moving with 250 km s^{-1} velocity is $\sim 10^{48} \text{ erg}$ while its magnetic energy content is $\sim 10^{43} \text{ erg}$. Thus the energy stored in the dipole fields is many orders of magnitude lower than the kinetic energy of the pulsar. (This point was noted in passing by

Anderson & Lyne). Another good reason for doubting asymmetric explosions as a mechanism to produce pulsar velocities in general is the fine tuning required. The amount of energy in a supernova explosion ($\sim 10^{51}$ erg) is about two orders of magnitude higher than the kinetic energy of the pulsar. Thus, while it is true that a minute asymmetry *could* provide the energy, it would be extraordinary if the spread in asymmetry were so small as to produce the relatively narrow distribution of velocities observed.

IV

Nevertheless, the correlation found by Anderson & Lyne is there and a satisfactory explanation is required for it. Such an explanation was provided on the basis that the sample of pulsars is a mixture of two classes, one with 'high' velocities and the other with 'low' velocities (Radhakrishnan 1984; Radhakrishnan & Shukre 1985). It should be mentioned that Lyne, Anderson & Salter (1982) clearly noted that the distribution of velocities did not seem to be Maxwellian, there being excesses of both high and low values. Fig. 2 shows a plot of velocity *vs.* characteristic age which is very suggestive of a division into two groups. Those with low fields and $P/2\dot{P} \geq 10^7$ yr have low velocities and the high field ones have high velocities.

It was proposed by Radhakrishnan & Srinivasan (1981) and later independently by Backus, Taylor & Damashek (1982) that there must exist a class of 'recycled' pulsars which were spun-up in binary systems through accretion, and later released as single pulsars on the disruption of the binary by the second explosion. Such pulsars would have a low field due to its decay during the evolution time of the companion star, but the neutron star would still be observable as a radio pulsar due to its anomalously high spin (for the field). A necessary but not sufficient condition, which is an excellent diagnostic for such recycled pulsars, is that they must have characteristic ages ($P/2\dot{P} \geq 10^7$ yr. The reason for this can be seen from Fig. 3 which is a Log B –Log P plot showing the field-period equilibrium condition or 'spin-up' line (taken from Srinivasan & van den

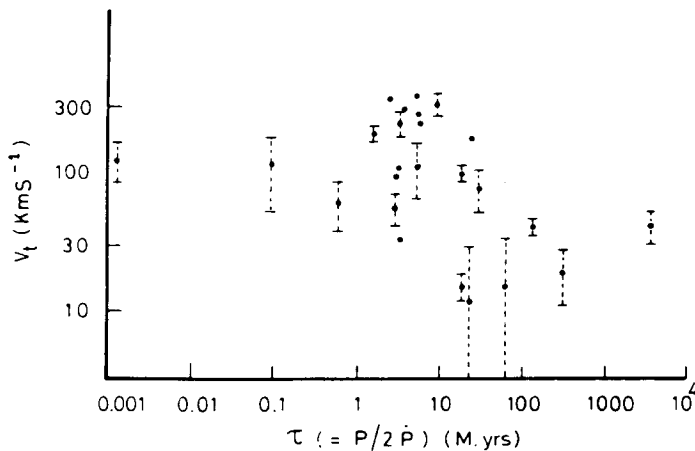


Figure 2. Transverse velocities from Lyne, Anderson & Salter (1982) plotted against characteristic age $P/2\dot{P}$. Small error bars for some of the pulsars have been omitted from the figure for clarity. (From Radhakrishnan 1984.)

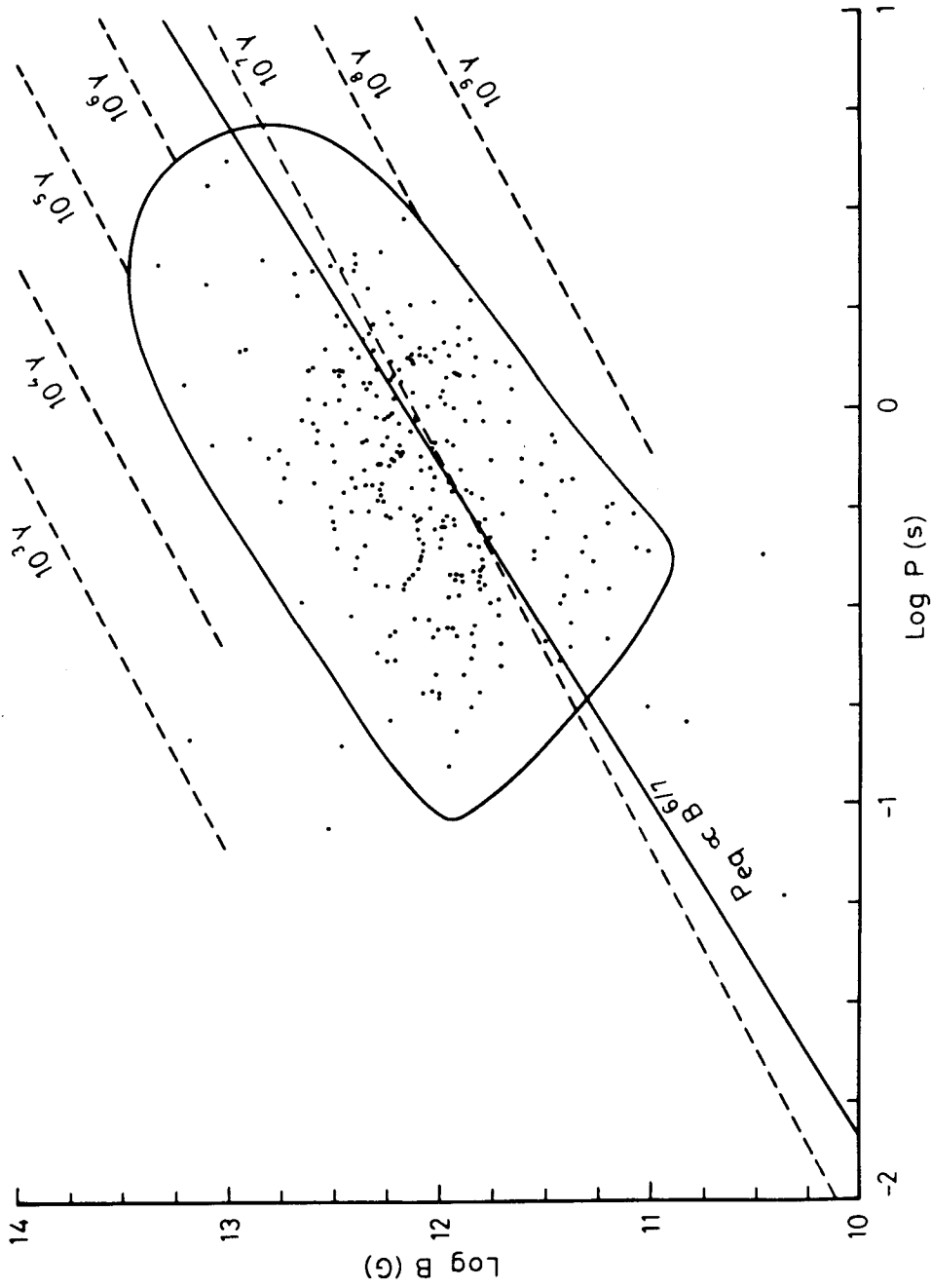


Figure 3. Plot of derived magnetic field *vs.* period for pulsars. Solid line is the field-period equilibrium condition for spin-up given by Srinivasan & van den Heuvel (1982). Broken lines are the characteristic age $P/2\dot{P} = \text{constant}$ lines. (Figure adapted from Radhakrishnan, 1982).

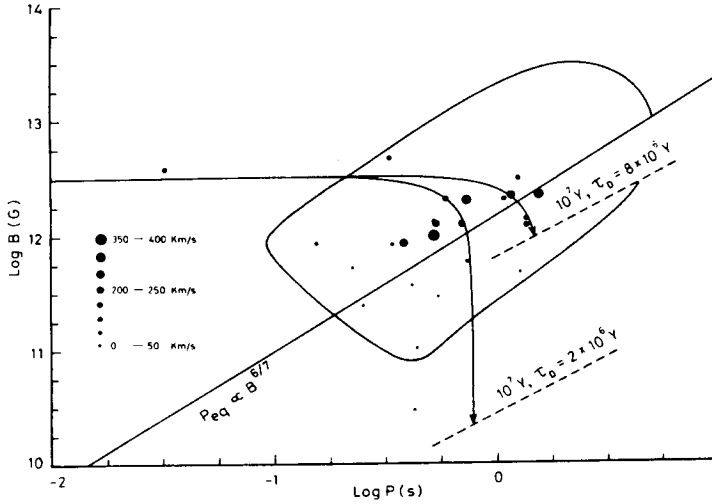


Figure 4. Plot of derived magnetic field *vs.* period for the 25 pulsars. Size of the dot reflects the magnitude of the transverse velocity of the pulsar. The straight solid line is the field-period equilibrium line. Broken lines depict age = 10^7 yr for the two indicated values of the magnetic decay timescale τ_d . Solid lines with arrows depict evolutionary tracks of a pulsar for the same two values of τ_d .

Heuvel 1982) for accretion in binaries. This line has almost the same slope as the characteristic age lines in this plot, and its position practically coincides with the age line for 10^7 yr. Therefore this provides us with the diagnostic referred to above. It is important to note that this condition is *independent* of the decay time scale for the magnetic fields.

The 26 pulsars chosen for the proper motion measurements by Lyne, Anderson & Salter (1982) were selected on the basis of positional nearness on the sky to extragalactic point sources, and not on the basis of any pulsar characteristic. For this reason, they must constitute a representative sample. It would appear therefore, that all single pulsars observed fall into two classes; a low-velocity minority, the recycled ones referred to above, and the majority with high velocities ($\sim 250 \text{ km s}^{-1}$) and high fields (as can be seen in Fig. 4).

V

It was predicted by Radhakrishnan & Srinivasan (1981) that low-field recycled pulsars will have high velocities because they were the less massive of the two stars in the pre-explosion binary system. By the same token, the fresh high-field pulsars should have low velocities. We have just seen, however, that the observations indicate the opposite. Thus, if we identify the low velocity pulsars as recycled ones, it still remains to explain the low magnitudes of their velocities. Note, however, that these velocities are too high to be attributed to random motions of single stars, and cannot be explained by assuming that pulsar progenitors are single stars. As will be seen later, such low speeds *per se* are also difficult to achieve in close binary systems in which accretion has played a role. We shall return to this point later on, and discuss now only the high-velocity class.

As noted earlier, there is no other satisfactory explanation to date for the high velocities observed, and we are left again with the only alternative that all pulsars had an

origin in binary systems. At the IAU symposium on pulsars, van den Heuvel (1981) had concluded on similar considerations that the majority of pulsars were likely to come from binaries. Another indication in this direction is as follows. The majority of evolving stars today occur in binary or multiple systems (Abt & Levy 1976, 1978). The difficulty of getting rid of the angular momentum in gas clouds which are collapsing to form stars appears to strongly favour the formation of binary or multiple systems (Bodenheimer 1981). Many O and B stars which could be expected to explode later as supernovae and leave behind a pulsar are in 'runaway' binaries which acquired their velocities on the explosion of their companions (Stone 1979). Indeed, according to this author, there is little or no evidence that single stars of this category exist.

We shall therefore proceed on the assumption that all high-velocity pulsars acquire their velocities due to their release from binary systems which are disrupted by symmetric supernova explosions.

VI

Now, if pulsar velocities observed today are indeed a consequence of their having been in binary systems, what were the characteristics of those binary systems?

For this purpose let us take the pulsar velocity as it is observed today and trace backwards to the binary orbit which gave rise to it. We assume that all the supernova explosion did was to reduce the mass of the exploding star on a timescale very short compared to the orbital period, without affecting the positions and velocities of the component stars of the binary. We shall be interested in the case when the resulting orbit is unbound, *i.e.*, the amount of mass ejected by the supernova is more than half of the initial total mass of the binary. The speeds are very soon expected to approach their asymptotic values which correspond to a separation between the components large enough to allow neglect of the gravitational attraction between them. Later passage through the interstellar medium is hardly expected to affect the velocities. We thus need a relation between velocities just after (or what is the same thing, just before) the explosion and the asymptotic velocities.

Towards this aim, we must digress to mention a remarkable property of binaries which are disrupted by symmetric explosions. To an observer situated in the same frame of reference as the centre of mass of a binary system with circular orbits, a sudden expulsion of mass from one of the stars in a symmetric way leaves the speed of the remnant of this star *unchanged for all time*. Remarkably, this is true irrespective of the amount of mass lost, *i.e.*, whether as a consequence the system remains bound, or is disrupted. The reason for this surprising property can be understood in terms of a hodograph, which is a representation of the motion of a particle in velocity space just as the orbit is such a representation in coordinate space.

It is well known that the hodograph *i.e.*, the path in velocity space, is a circle in the Keplerian case, even though both the speed and direction are changing continuously as in an elliptic orbit (Goldstein 1980). Except for circular orbits, the centre of the hodograph is not at the origin. For a parabolic orbit, the hodograph is still a full circle but it passes through the origin. For a hyperbolic orbit, it shrinks to an arc bigger than a semicircle whose centre is displaced from the origin by more than its radius. If the observer is in a frame of reference whose origin is at the centre of the hodograph, clearly he will see a constant speed. The surprising thing is that when one of the stars in a binary system suddenly loses mass, the centre of mass of the remaining two-body system,

whether bound or not, moves in exactly such a way in the original centre of mass frame, that the origin in velocity space continues to remain at the centre of *this* star's hodograph. The radius of the hodograph also remains unchanged. The derivation of this unexpected result is given in the Appendix.

Returning to our main objective, the implication of this extraordinary property is that the progenitor of a pulsar had precisely the *same* speed in orbit (assumed to be circular) just before the explosion, as the pulsar has today far away in space and time from this event. Let us ignore for the moment the possibility of orbits being elliptical. We can then conclude that the observed distribution of space velocities for the pulsars whose births disrupted the binaries was *exactly* the distribution of the orbital speed of their progenitors. Note that this is not true for the other object in the binary system whose mass did not undergo any sudden change. In all of the above, we have neglected the motion of the binary system as a whole before the explosion, which is a relatively minor perturbation.

Below are the formulae for the observed speeds of the pulsar and its companion in terms of the progenitor speed in the circular orbit.

$$V_{\text{psr}} = V_{\text{prog}} \quad (1)$$

$$V_{\text{comp}} = \left[1 + (2\alpha - 1) \left(\frac{M_1 + M_2}{M_2} \right)^2 \right]^{1/2} V_{\text{prog}} \quad (2)$$

and

$$V_{\text{prog}} = \frac{438 M_2}{[(M_1 + M_2)a]^{1/2}} \text{ km s}^{-1} \quad (3)$$

with

$$\alpha \geq 1/2, \quad (4)$$

where M_1 is the mass of the pre-supernova object and M_2 that of the companion, in solar masses. The separation a between M_1 and M_2 is in solar radii. The fraction of the original total mass lost in the explosion is α . Equation (4) just expresses the fact that the resulting orbit is unbound. It may be noted in passing that Equation (1) in other forms has been derived before (see *e.g.*, Gott, Gunn & Ostriker 1970), but its message seems to have escaped attention so far.

VII

As noted earlier, observations provide us only with transverse velocities. To estimate the space velocities we have multiplied the data of Lyne, Anderson & Salter (1982) by $(3/2)^{1/2}$ to allow for the radial components in a statistical sense. The range of space velocities for the high-velocity pulsars thus obtained is 100–450 km s⁻¹ with a mean of 270 km s⁻¹.

We can now use this to learn something about the binary systems from which these pulsars originated. The explosion which releases a pulsar as a single object can either be the first or the second explosion in the binary system. There are many systems (*i.e.*, X-ray binaries) containing a neutron star which must have clearly survived the first explosion, and at least one case (*i.e.*, PSR 1913 + 16) which survived two (we are ignoring possible neutron star formation by accretion on to white dwarfs). Still, let us consider release of the pulsar in the first explosion. Since more than one-half of the total mass has to be lost in the explosion to disrupt the binary, and assuming the mass of a neutron star to be $1.4 M_{\odot}$, we have

$$M_1 \geq 2.8 + M_2. \quad (5)$$

This leads to

$$a \leq \left(\frac{438}{V_{\text{psr}} \text{ km s}^{-1}} \right)^2 \frac{M_2^2}{2.8 + 2M_2} R_{\odot}. \quad (6)$$

It is clear that the separation required to give the observed velocities is very small (*e.g.*, consider a typical $M_2 = 5$; then $a \leq 5R_{\odot}$ for $V_{\text{psr}} = 270 \text{ km s}^{-1}$). We also know that it is only in the case of very wide binaries that the individual stars can evolve just as if they were single. In such wide binaries the velocities would be clearly too low. On the other hand, if the stars were as close as required by the high velocities then mass transfer is inevitable, and the primary will become the less massive of the two before it reaches the supernova stage (Paczynski 1971; van den Heuvel 1976). The ensuing explosion cannot then unbind the system, and we will have a binary system containing one neutron star which can later become an X-ray source and/or a recycled pulsar.

Thus we must now consider the release of pulsars due to the second explosion in a binary. In this case we have the additional constraint that the mass of the companion is now $\sim 1.4 M_{\odot}$. We can go to Fig. 5 which depicts, for $M_2 = 1.4$, the dependence of the separation of the stars on the total mass as given by Equation 3. Considering that speeds in the range $100\text{--}450 \text{ km s}^{-1}$ are to be accounted for, we see that the separations have to be very small, and the dense cores of the collapsing stars have to be very compact indeed. This implies that the evolution of the binary system between the first and second SN explosions *must* go through a common envelope phase. The consequent tidal

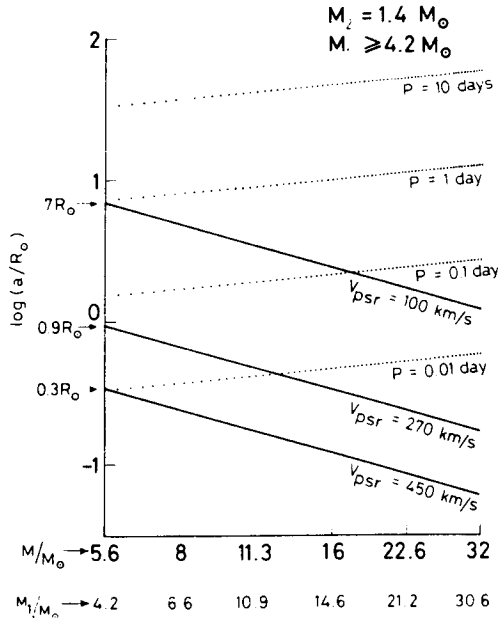


Figure 5. Plot of separation in solar radii ($a/R_{\odot}$) between the stars *vs.* the total mass in solar masses ($M/M_{\odot}$) of the binary system, for different values of V_{psr} , as derived from Equation 1. The companion neutron star mass M_2 is assumed to be $1.4 M_{\odot}$. Numbers with arrows along the Y-axis indicate maximum separations appropriate for the observed range of velocities; they correspond to the minimum value of $M = 5.6 M_{\odot}$ required to disrupt the binary. Dotted lines are of constant binary orbital period and are labelled by P_d , the period in days.

interactions would inescapably lead to circularization of the orbits, incidentally justifying fully our earlier assumption of circular orbits prior to the explosion.

It is interesting that our conclusion that a common envelope phase is inevitable, drawn here purely on the basis of pulsar proper motions, accords well with the conclusions of numerous studies undertaken from the point of view of stellar evolution in binaries (Tutukov 1981; van den Heuvel 1981). However, it still remains to explain the magnitudes of the observed pulsar velocities for both classes.

VIII

Binaries disrupted by the second explosion will produce two pulsars. The young one will have a lower velocity and a high magnetic field and the old recycled one a higher velocity and low magnetic field. As noted earlier, this is exactly the opposite of what is observed and is an indication that the velocities of our two classes of pulsars require different scenarios to explain them. Indeed, the low velocities are too low to follow from Equation (3) for masses appropriate for massive close binary systems.

The above contradiction is a result of our identification of low-velocity low-field pulsars as recycled ones. As one possible attempt at a unified explanation for the two classes, let us consider the possibility that the magnetic fields are low at birth and build up later with age, as has been suggested by Blandford *et al.* (1983). In such a case, it will be natural for low velocity pulsars to have low magnetic fields, since they are the young ones. The large spin-down ages ($\geq 10^7$ yr) for these are then just a reflection of their low fields. Now, if we take typical observed low and high velocities to be 50 and 250 km s^{-1} , this is a ratio of 1:5. Through the relationships between asymptotic velocities and pre-explosion velocities (Equations 1 and 2), we can then immediately infer that the progenitor of the young pulsar is required to have a mass of $\geq 9 M_{\odot}$. Such a massive pre-supernova core is considered unrealistic (see van den Heuvel in this volume).

We thus see that the observed pulsar velocities cannot be simply explained using the magnetic field build-up theory. Incidentally this theory also runs into difficulty when confronted with the data on the Crab pulsar. In such a theory it is easily shown that the characteristic age must always be greater than half the exponential field growth timescale. Blandford *et al.* (1983) invoke a growth time of 10^5 yr which is clearly incompatible with a characteristic age of 1000 years. If this timescale is arbitrarily assumed to be as short as 2000 years, the period of the Crab pulsar should then have changed by less than 3 ms since 1054 AD. This would raise anew the problem of the energy source of the Crab nebula, which was put to rest on the basis that the initial pulsar period was ~ 16 ms. Neither can we consider the Crab Pulsar to have been the first born one in a binary so that its field could have been built up to the present value during the several million years before the second explosion. The reason is again its present observed characteristic age. If there was no accretion during the binary phase, then as mentioned just above its characteristic age must be greater than half of the field growth timescale, *i.e.*, $\sim 10^5$ yr; on the other hand, if accretion did spin it up, then as mentioned earlier in the context of recycling, its characteristic age must be $\geq 10^7$ yr. (We thank D. Bhattacharya for pointing out some of the above.)

IX

Consequently, we conclude that the types of evolution which lead to the low and high velocity pulsars in Fig. 3 are different. In particular, the binaries which give birth to

pulsars do not produce a pair such that one of them belongs to the low velocity class and the other to the high velocity one. Nor as was noted earlier, can the low velocities be explained by assuming that pulsars originate from single stars. Therefore the magnitudes of the low velocities must be related to some special property of the binary systems which produce them.

It was proposed by Radhakrishnan (1984) that the low velocity pulsars could be explained if there occur enough binary systems consisting of a neutron star and a companion which eventually disappears in a non-explosive manner. The single neutron star left behind will then have all the characteristics of a 'low-velocity' pulsar. The spiralling-in of the neutron star (Paczynski 1976; Taam, Bodenheimer & Ostriker 1978) during the common envelope phase could result in such an outcome for certain initial conditions.

Returning to the high velocity class, the more likely result of the spiral-in is, however, an even closer binary than the initial one. Typically, the resultant binary will have a neutron star ($1.4 M_{\odot}$) and a He (or more evolved) star ($5 M_{\odot}$) in an orbit of period ~ 1.5 h, corresponding to a separation of $\sim 1.2 R_{\odot}$ (these numbers are based on discussions with E. P. J. van den Heuvel during and after the workshop, on the outcomes of the common-envelope evolution).

On disruption, the young pulsar will obviously have a high magnetic field. The old pulsar will also have a high magnetic field if the field decay timescale is of the order of (or longer than) the time interval between the two SN explosions. After the first SN, the He star takes $\sim 3.7 \times 10^6$ yr to overflow its Roche lobe, the spiral-in takes $\sim 10^3$ to 10^4 yr and the post spiral-in timescale for the evolved core to become an SN is at most 6×10^5 yr (for these numbers, see van den Heuvel, 1977 and these proceedings). The interval between the two SN explosions is thus $\leq 4 \times 10^6$ yr during which the magnetic field will hardly have decayed for generally believed values of the decay timescale.

This outcome of the spiral-in thus leads to two high-velocity high-field pulsars after the second SN explosion. Our high-velocity class of pulsars is then a mixture of two subclasses. If one could distinguish between them, an inverse correlation between velocities and magnetic field is predicted in this picture. As the spread in initial magnetic fields of pulsars is most probably greater than e , this prediction may be difficult to test. It should be pointed out here that half of these pulsars should also be 'recycled' ones. Note however that in this case accretion will not spin-up these high-field pulsars as much as it will low-field ones (see the equilibrium line in Fig. 2).

X

While concluding that binaries which produce pulsars must undergo a common envelope phase in Fig. 5, we had identified the velocities of high-field pulsars with the V_{psr} of Equation 1. This conclusion must be re-examined if the high-field pulsar sample is indeed a mixture. The data on high-field pulsars would contain both V_{psr} and V_{comp} of Equations 1 and 2, and it may not be possible to unscramble them. Due to selection effects, one or the other subclass may dominate the data. But, irrespective of the ratio, some conclusions can be drawn if both types are represented in the observed sample of high-velocity pulsars.

Fig. 6 shows both $V_{\text{psr}} = \text{const.}$ and $V_{\text{comp}} = \text{const.}$ lines. Any point in this plane specifies a binary system of a particular total mass and a particular separation prior to the second SN explosion. Alternatively, the system is uniquely specified also by giving

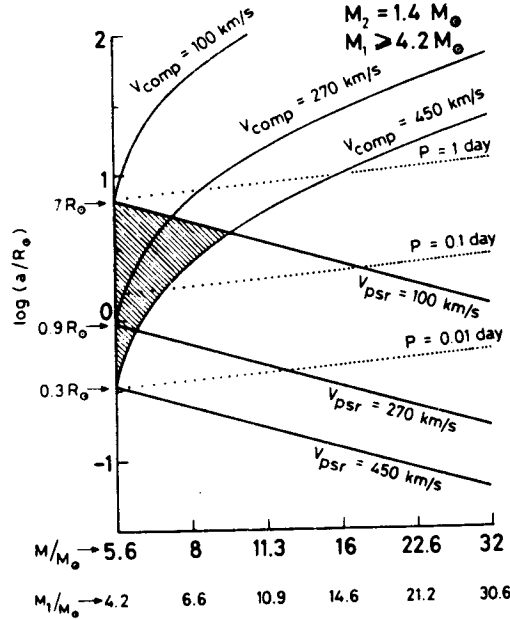


Figure 6. Same as Fig. 5 but including plots of separation $a/R_\odot$ vs. the total mass $M/M_\odot$ for different values of V_{comp} as derived from Equation 2. Shaded region indicates the values of a and M allowed by the proper-motion data as explained in the text.

V_{psr} and V_{comp} . The observational requirement that $100 \text{ km s}^{-1} \leq V_{\text{psr}}$, $V_{\text{comp}} \leq 450 \text{ km s}^{-1}$ restricts the relevant binary systems to the intersection of the two bands. These bands are bounded by $V_{\text{psr}} = 100, 450$ and $V_{\text{comp}} = 100, 450 \text{ km s}^{-1}$ lines. In Fig. 5 we had considered only the V_{psr} band. We see now that the binaries in question must lie in the triangular overlap between these two bands. If smaller ranges for either V_{psr} or V_{comp} are assumed, this region will shrink further. However, even the liberal ranges of Fig. 5 put severe constraints on the binaries, *e.g.*, if $M = 8 M_\odot$, the separation must lie between $2.4 R_\odot$ and $4.8 R_\odot$. In any case, $0.3 \leq a/R_\odot \leq 6.7$ and $5.6 \leq M/M_\odot \leq 9.5$. We conclude that pre-supernova masses must lie in the range $4 M_\odot$ to $8 M_\odot$.

Conclusions

Starting from the pulsar proper-motion data, we draw the following conclusions. All single pulsars were most likely born in and released from binary systems. The circumstances of their release determined their present velocities. Most of these binary systems must have undergone a common envelope phase of stellar evolution. Pulsars fall into two classes. One class consists of low-velocity pulsars which also have low magnetic fields and characteristic ages larger than 10^7 yr . These are old pulsars which were spun-up significantly during accretion from their companions. Pulsars in the other class have high velocities and high magnetic fields. They are produced in binary systems, having total masses between 5.6 and $9.4 M_\odot$ and separations less than $7 R_\odot$, prior to the second explosion which disrupts the system. As one of the two stars is already a neutron star, the velocity data enable us to derive limits of 4 to $8 M_\odot$ for the pre-supernova mass of the companion.

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Appendix

For Keplerian motion of two bodies of masses M_1 and M_2 we define the relative variables in the usual manner. The relative position is $\mathbf{r}$ such that in the centre of mass frame, M_1 is located at $\mathbf{r}_1 = (M_2/M)\mathbf{r}$ where $M = M_1 + M_2$. The relative velocity is $\mathbf{V}$. It is well known that the specific Lenz vector $\mathbf{\Lambda}$ is a constant of motion (Goldstein 1980), as of course is the specific angular momentum λ (i.e., the areal velocity), where

$$\lambda = \mathbf{r} \times \mathbf{V}, \quad (\text{A1})$$

and

$$\mathbf{\Lambda} = \mathbf{V} \times \lambda - \frac{GM}{r} \mathbf{r}. \quad (\text{A2})$$

The magnitude of $\mathbf{\Lambda}$ is GM_e , e being the eccentricity of the orbit.

One can write (carets denote unit vectors)

$$\mathbf{V} = \mathbf{V}_{\text{off}} + \frac{GM}{\lambda} \hat{\lambda} \times \hat{\mathbf{r}} \quad (\text{A3})$$

with

$$\mathbf{V}_{\text{off}} = \frac{\lambda \times \mathbf{\Lambda}}{\lambda^2}, \quad V_{\text{off}} = \frac{GM_e}{\lambda}. \quad (\text{A4})$$

This is the equation of the relative velocity hodograph. It is clearly a circle of radius GM/λ whose centre is off-set from the origin by V_{off} .

The individual velocities depend on the reference frame and are

$$\mathbf{V}_1 = \mathbf{V}_{\text{cm}} + \frac{M_2}{M} \mathbf{V} = \mathbf{V}_{\text{off}}(1) + \frac{GM_2}{\lambda} \hat{\lambda} \times \hat{\mathbf{r}}, \quad (\text{A5})$$

$$\mathbf{V}_2 = \mathbf{V}_{\text{cm}} - \frac{M_1}{M} \mathbf{V} = \mathbf{V}_{\text{off}}(2) - \frac{GM_1}{\lambda} \hat{\lambda} \times \hat{\mathbf{r}}. \quad (\text{A6})$$

The hodographs of individual velocities are also circles, and of radii GM_1/λ and GM_2/λ , the centres of which are displaced from the origin by $\mathbf{V}_{\text{off}}(1)$ and $\mathbf{V}_{\text{off}}(2)$ respectively, where

$$\mathbf{V}_{\text{off}}(1) = \mathbf{V}_{\text{cm}} + \frac{M_2}{M} \mathbf{V}_{\text{off}}, \quad (\text{A7})$$

and

$$\mathbf{V}_{\text{off}}(2) = \mathbf{V}_{\text{cm}} - \frac{M_1}{M} \mathbf{V}_{\text{off}}. \quad (\text{A8})$$

In a reference frame in which $\mathbf{V}_{\text{cm}} = -(M_2/M) \mathbf{V}_{\text{off}}$, V_1 will be constant in time.

Let us now consider two bodies in circular orbits, one of which (say M_1) suddenly loses an amount of mass $\Delta M = M_1 - M'_1 = \alpha(M_1 + M_2)$, in a spherically symmetric manner, without affecting any change in positions or velocities. For the resulting orbit

the eccentricity will be

$$e' = \frac{\alpha}{1 - \alpha}, \quad (\text{A9})$$

and thus

$$V'_{\text{off}} = GM(1 - \alpha)e'/\lambda' = GM\alpha/\lambda. \quad (\text{A10})$$

The velocity of the new centre of mass in the original centre of mass frame is

$$V'_{\text{cm}} = -\frac{\Delta M}{M(1 - \alpha)} V_{\text{li}}, \quad (\text{A11})$$

where V_{li} is the pre-explosion velocity of M_1 and is in the same direction as V'_{off} . Also $V_{\text{li}} = GM_2/\lambda$ and therefore

$$V'_{\text{cm}} = -\frac{M_2}{M(1 - \alpha)} V'_{\text{off}}.$$

Thus in the original centre of mass frame the speed of M'_1 is constant and equal to that of M_1 . If $\alpha > 1/2$ the final orbit is unbound and in this same frame the asymptotic speed of M_2 is related to that of M'_1 by

$$V_{2a} = [1 + (2\alpha - 1)(M/M_2)^2]^{1/2} V_{1a}, \quad (\text{A12})$$

where

$$V_{1a} = V_{\text{li}}, \quad (\text{A13})$$

Note that as $\alpha > 1/2$, $V_{2a} > V_{1a}$.

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