

# Non-equilibrium ionization in photoionized haloes: implications for shock stability and absorption-line signatures

Itai Bromberg<sup>1</sup>,<sup>1</sup>★ Kartick C. Sarkar,<sup>2</sup> Orly Gnat<sup>1</sup> and Yuval Birnboim<sup>1</sup>

<sup>1</sup>Centre for Astrophysics and Planetary Science, Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

<sup>2</sup>Raman Research Institute, Bangalore 560 080, India

Accepted 2026 July 7. Received 2026 June 24; in original form 2026 May 1

## ABSTRACT

We investigate the impact of non-equilibrium ionization (NEI) and the metagalactic radiation-field on the thermal evolution, virial shock stability, and absorption signatures of gas surrounding galaxies. Using 1D, spherically symmetric hydrodynamical simulations with an extended version of the HYDRA code, we follow dark-matter growth, gas dynamics, time-dependent ionization, and cooling in the presence of the UV background. We explicitly track all ions of H, He, C, N, O, Ne, Mg, Si, S, and Fe in haloes of mass  $10^{11}$ – $10^{13} M_{\odot}$  from  $z = 100$  to  $z = 0$ . Without a UV background, NEI enhances post-shock cooling due to underionized gas, reducing pressure support and raising the minimum mass for stable shock formation. Including the UV background pre-ionizes the intergalactic medium (IGM), suppressing NEI, and restoring the collisional equilibrium threshold. We find that the IGM temperatures deviate from thermal equilibrium due to adiabatic expansion and collapse, while ionization remains close to equilibrium in the presence of a UV background, except in transient rapidly cooling regions where NEI occurs. We compute absorption columns of O VI, C IV, and H I, showing that, in our spherical models, the photoionized gas outside the shocked halo may produce substantial warm-ion columns extending beyond  $R_{\text{vir}}$ , including O VI column densities comparable to observed values. Our models indicate weak halo-mass dependence and extended distributions. We also find that  $z \gtrsim 3$  haloes can produce C IV ( $N_{\text{C,IV}} \sim 10^{13-15} \text{ cm}^{-2}$ ) and H I ( $N_{\text{H,I}} \sim 10^{15-17} \text{ cm}^{-2}$ ) columns out to  $\sim 10R_{\text{vir}}$ . Our results highlight the role of the UV background in regulating the thermal state and observable signatures of the gas surrounding galaxies. This emphasizes the importance of accounting for IGM contributions when interpreting circumgalactic absorption-line observations.

**Key words:** galaxies: general – galaxies: haloes – intergalactic medium – ultraviolet: galaxies.

## 1 INTRODUCTION

In the present-day Universe, a large fraction of the baryonic matter resides outside galaxies, in the diffuse gas that fills the regions between and around them. This gas forms the intergalactic medium (IGM) and the circumgalactic medium (CGM). In this work, we use the term CGM to refer to the shock-heated gas inside galactic haloes, and IGM to refer to the cooler, infalling gas outside the shock radius. The CGM plays a central role in galaxy evolution, regulating both the accretion of gas onto galaxies and the ejection of material through outflows (J. X. Prochaska et al. 2011; J. Tumlinson, M. S. Peeples & J. K. Werk 2017).

Early theoretical work (M. J. Rees & J. P. Ostriker 1977; J. Silk 1977; S. D. M. White & M. J. Rees 1978) established that hot gaseous haloes can naturally form as intergalactic gas is shock-heated while accreting into the gravitational potential wells of forming galaxies. The ensuing thermal and chemical evolution of this gas is shaped by a complex interplay of hydrodynamics, radiative cooling, and ionization processes. As a result, the CGM,

confined within the virial shock radius, becomes a multiphase and dynamically rich environment (e.g. J. M. Shull, J. Tumlinson & M. L. Giroux 2003; J. K. Werk et al. 2016; J. Tumlinson et al. 2017; B. D. Oppenheimer et al. 2018; T. J. Cooper et al. 2021; M. Fumagalli 2024; Sameer et al. 2024).

Observationally, the distribution of ionized gas around galaxies has been extensively probed through both UV absorption-line studies and X-ray observations. Since the accretion-shock radius is generally not directly measured in individual galaxy haloes, these observations are typically interpreted relative to the theoretically estimated virial radius,  $R_{\text{vir}}$ , which provides an approximate scale for the extent of the halo. Early work with the COS-Halos survey (J. Tumlinson et al. 2011) revealed widespread O VI absorption in the CGM, with relatively flat column density profiles out to  $\sim R_{\text{vir}}$ . Subsequent studies, including IMACS (S. D. Johnson, H.-W. Chen & J. S. Mulchaey 2015) and CGM<sup>2</sup> (K. Tchernyshyov et al. 2022), have extended these measurements and suggest that O VI absorption may persist to large impact parameters, in some cases well beyond the virial radius, although with varying covering fractions (Z. Qu et al. 2024a). Additional observations (J. N. Burchett et al. 2019; M. Ng et al. 2019; Y. Zheng et al. 2019) further indicate that the CGM is extended

\* E-mail: [itai.bromberg@mail.huji.ac.il](mailto:itai.bromberg@mail.huji.ac.il)

and multiphase, with contributions from both warm and hot gas. Complementary X-ray studies (A. Gupta et al. 2012; Y. Yao et al. 2012; F. Nicastro et al. 2013) provide evidence for a pervasive hot halo component surrounding galaxies. However, the radial extent and physical origin of this gas remain uncertain: while some studies suggest that highly ionized material is largely confined within the virial radius (e.g. S. D. Johnson et al. 2015; Z. Qu et al. 2024b), others find evidence for substantial ionized gas extending well into the surrounding IGM (J. X. Prochaska et al. 2011; K. Tchernyshyov et al. 2022). In this work we are interested in ions such as C IV, N V, and O VI, which can arise in either photoionized or collisionally ionized gas, and distinguish them from hotter X-ray ions such as O VII and O VIII. Following common usage in the CGM literature, we refer to these species as ‘warm ions’.

Our theoretical understanding of the CGM is built upon a combination of complementary modelling approaches that span a wide range of physical scales and levels of complexity. Cosmological simulations provide a global framework for studying galaxy formation and the CGM in its cosmological context, capturing structure growth and environmental diversity, but necessarily rely on subgrid prescriptions to model unresolved processes such as star formation, feedback, and microphysical effects (e.g. D. Nelson et al. 2018; B. D. Oppenheimer et al. 2018; S. H. Ho, C. L. Martin & J. Schaye 2021; S. Appleby et al. 2023; C. Strawn et al. 2024). These simulations demonstrate that high-ion absorption can extend beyond the central shock-heated halo. In ILLUSTRIS, D. Nelson et al. (2018) found that O VI columns persist to impact parameters of several Mpc because of contributions from nearby haloes and the IGM. Similarly, EAGLE simulations predict slowly declining or nearly flat O VI column-density profiles with impact parameter, including contributions from gas beyond the virial radius (B. D. Oppenheimer et al. 2018; S. H. Ho et al. 2021), while SIMBA predicts substantial contributions to O VI absorption from gas outside the central halo (S. Appleby et al. 2023).

At the other extreme, low-dimensional models and simulations focus on specific physical processes, allowing for efficient exploration of parameter space and clearer interpretation of how individual mechanisms – such as cooling, photoionization, or feedback – shape the observable properties of the CGM (e.g. J. Stern et al. 2016; Y. Faerman, A. Sternberg & C. F. McKee 2017; M. McQuinn & J. K. Werk 2018; Z. Qu & J. N. Bregman 2018; J. Stern et al. 2018, 2019; Y. Faerman, A. Sternberg & C. F. McKee 2020). In such models, O VI around galaxies is traditionally attributed to hot, collisionally ionized gas within the CGM.

In our previous work (I. Bromberg et al. 2025, hereafter **B + 25**), we augmented that interpretation, showing that a significant fraction of these detections may instead originate in cooler, photoionized gas in the surrounding IGM, particularly for lower-mass haloes. In that work, we used a spherically symmetric analytic toy-model of halo formation to demonstrate that absorption by highly ionized species arises naturally in the IGM as it responds to the UV background radiation field.

The **B + 25** model, which assumed ionization equilibrium, showed that photoionized IGM alone could account for a significant fraction of the observed O VI columns. This contribution arises from gravitational focusing, which enhances the gas density in the IGM outside of the shock radius, and allows warm ions to form under photoionization equilibrium (PIE) conditions. These findings emphasized the importance of including the contribution of the IGM when interpreting CGM absorption-line observations. However, the treatment in **B + 25** was highly ide-

alized, relying on a static CGM model and neglecting several key physical processes, including full hydrodynamical evolution and departures from ionization equilibrium. Both effects are expected to influence the thermal history, ionization structure, and resulting observable signatures of the gas.

Using one-dimensional spherical hydrodynamic and  $N$ -body simulations Y. Birnboim & A. Dekel (2003, hereafter **BD03**) showed that haloes must satisfy a stability criterion ( $M_{\text{vir}} \gtrsim 10^{12} M_{\odot}$ ) for a virial shock to form around them. In their model, radiative cooling was computed using collisional ionization equilibrium (CIE) tables from R. S. Sutherland & M. A. Dopita (1993). They found that halo masses below a critical mass  $\approx 3 \times 10^{11} M_{\odot}$  (with the exact threshold depending on metallicity and redshift; see A. Dekel & Y. Birnboim 2006), a stable shock cannot be maintained. In these systems, rapid cooling behind the shock reduces the post-shock pressure support, causing the shock to collapse or preventing an extended shocked region from developing in the first place. This behaviour has significant implications for CGM observational signatures, particularly in unstable haloes with masses below the critical mass. In such cases, the shock radius lies well within the virial radius – potentially, as low as the edge of the galaxy – so that a substantial fraction of the gas within  $R_{\text{vir}}$  remains part of the infalling IGM rather than a virialized hot halo. These results highlight the importance of a consistent hydrodynamic treatment in modelling the structure and evolution of the CGM and its surrounding environment.

In rapidly evolving regions, such as shock fronts and post-shock cooling layers, where heating or cooling proceeds faster than ionization or recombination, non-equilibrium ionization (NEI) becomes significant. In these conditions, the ion fractions of key species such as O VI and C IV can deviate substantially from their equilibrium values, as ionization and recombination processes lag behind the thermal evolution. This affects both emission and absorption-line diagnostics. In addition, departures from equilibrium can modify radiative cooling rates, with cooling enhanced in underionized gas and suppressed in overionized gas (O. Gnat & A. Sternberg 2007; O. Gnat & A. Sternberg 2009; K. C. Sarkar, A. Sternberg & O. Gnat 2021a; K. C. Sarkar, O. Gnat & A. Sternberg 2021b). Consequently, time-dependent ionization modelling may be important for accurately predicting ion abundances and thermal evolution, and for interpreting CGM observations in environments undergoing rapid heating and cooling transitions.

However, if the gas is exposed to an ambient ionising radiation field, such as the metagalactic UV background, the impact of non-equilibrium ionization may be significantly reduced (E. O. Vasiliev 2011; B. D. Oppenheimer & J. Schaye 2013; O. Gnat 2017). In such cases, although NEI processes remain important for a physically consistent description of the gas, their effect on observable signatures may be limited (B. Arya, K. C. Sarkar & S. Sethi, in preparation).

In this work, we reexamine the findings of **B + 25** using a more realistic and physically complete model. Instead of the analytic, dark-matter-only toy model, we employ a one-dimensional simulation within a  $\Lambda$ CDM cosmology, which more accurately captures halo growth and self-consistently follows the gas hydrodynamics, including shock formation. We build upon the Lagrangian code HYDRA introduced in **BD03**, extending it to include additional physical ingredients. First, we incorporate a time-dependent ionization network that follows the non-equilibrium ion fractions, as well as the associated cooling and heating rates, in the evolving gas. For this purpose, we adopt the ionization

network presented in K. C. Sarkar et al. (2021a, hereafter S + 21). Secondly, we include the effects of the redshift-dependent metagalactic UV background, which photoionizes and heats the gas.

The enhanced version of HYDRA allows us to toggle between models with and without a UV background, and with either equilibrium or non-equilibrium ionization, enabling a systematic exploration of their impact on shock stability and observable signatures. We simulate haloes across a range of masses, solving the coupled equations of hydrodynamics, radiative cooling and heating, and ionization evolution. This approach allows us to follow the spatial and temporal evolution of the gas density, temperature, and ionization state both within and beyond the shock radius.

The goals of this study are threefold: (1) to evaluate the impact of departures from ionization equilibrium on the stability of virial shocks; (2) to assess the role of the UV background in regulating shock stability; and (3) to evaluate the resulting absorption-line column densities and to quantify the contribution of in-falling IGM gas to absorption-line observations. As we use one-dimensional simulations, this work is not intended to model the full multiphase CGM, but rather to isolate how ionization physics alone affects shock stability and large-scale ion distributions.

The structure of the paper is as follows. In Section 2, we describe our model, including the governing hydrodynamic and ionization equations, the implementation of the UV background, the numerical methods, and the explored parameter space. In Section 3, we present our results regarding shock stability and examine the roles of non-equilibrium ionization and the UV background. In Section 4, we analyse the resulting thermal and ionization structure of the gas in and around haloes. In Section 5, we investigate the absorption-line signatures of our models, focusing on the roles of NEI and UV radiation and demonstrating the significant contribution of the IGM to high-ion column densities. In Section 6, we discuss the main limitations of our model, and in Section 7 we summarize our conclusions.

## 2 METHOD

We use HYDRA (BD03, A. Dekel & Y. Birnboim 2006; Y. Birnboim, A. Dekel & E. Neistein 2007), a one-dimensional, spherically symmetric Lagrangian hydrodynamics code, to simulate the evolution of gas and dark matter in spherical haloes. The code follows the radial evolution of concentric shells of gas and collisionless dark matter under gravity, while solving the Euler equations for the gas. We also couple the hydrodynamics to a time-dependent ionization network (S + 21), allowing us to follow the non-equilibrium ionization state of the gas when desired. This enables us to compute the corresponding radiative cooling and photoabsorption heating due to the metagalactic UV background.

In the following subsections, we describe the method used to follow the evolution of the dark matter (Section 2.1), gas hydrodynamics (Section 2.2), ionization states (Section 2.3), and the ionizing background radiation (Section 2.4). We then present the suite of simulations used to study the stability of, and absorption-line signatures around, spherical haloes (Section 2.5).

### 2.1 The evolution of the dark matter

As described above, in HYDRA, dark matter is represented by collisionless shells and evolves under the influence of gravity. The gravitational force includes contributions from both gas and dark matter, with a small central smoothing length introduced

to avoid a numerical singularity at  $r = 0$ . Angular momentum is included through a centrifugal term in the equation of motion, with a specific angular momentum assigned to each shell and conserved during the evolution. The simulations adopt a  $\Lambda$  cold dark matter ( $\Lambda$ CDM) cosmology with  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ ,  $h = 0.7$ , and  $\sigma_8 = 1$ , and include cosmic expansion due to the cosmological constant. The equation of motion reads,

$$\frac{d^2 r}{dt^2} = -\frac{GM_{\text{DM}}(< r) + GM_{\text{gas}}(< r)}{(r + a)^2} + \frac{j^2}{R^3} + h^2 \Omega_\Lambda r \quad (1)$$

where  $M_{\text{DM}}(< r)$  and  $M_{\text{gas}}(< r)$  are the total dark matter and baryonic mass inside radius  $r$ , and the softening length.<sup>1</sup>,  $a$  ( $= 50$  pc), prevents a singularity at  $r = 0$ . The term  $j$  is the specific angular momentum of the shell, providing centrifugal support. The last term ( $h^2 \Omega_\Lambda r$ ) represents the repulsive acceleration due to the cosmological constant, where  $h$  is the dimensionless Hubble parameter, and  $\Omega_\Lambda$  is the cosmological constant density parameter. The equation of motions of the dark matter, as well as of the baryons (Section 2.2), are solved by using a fourth order Runge–Kutta scheme. If the higher order corrections exceed some threshold fraction over the second-order correction, the simulation reverts to the beginning of the time-step and proceeds with a smaller time-step to ensure convergence in time-steps.

As in BD03, A. Dekel & Y. Birnboim (2006), and Y. Birnboim et al. (2007), the simulations are initialized at a high redshift ( $z = 100$ ) with a small spherically symmetric density perturbation embedded in an expanding cosmological background, which is reflected in the initial velocity profile. The initial gas and dark matter distributions follow the same perturbation profile, which is taken to be proportional to the linear two-point correlation function of the assumed cosmology, representing a typical fluctuation in a Gaussian random field. The amplitude of the perturbation determines the collapse time of different mass shells.

### 2.2 Evolution of the gas

For the gas, HYDRA follows the hydrodynamic properties of the baryonic shells. It solves the following set of one dimensional Lagrangian hydrodynamic equations, for each shell,

$$\begin{aligned} \frac{d^2 r}{dt^2} = & -\frac{1}{\rho} \nabla(P + \sigma) + \\ & -\frac{G[M_{\text{DM}}(< r) + M_{\text{gas}}(< r)]}{(r + a)^2} + \\ & + \frac{j^2}{r^3} + h^2 \Omega_\Lambda r \end{aligned} \quad (2)$$

$$\rho = \frac{dM_{\text{gas}}}{4\pi r^2 dr} \quad (3)$$

$$\frac{de}{dt} = \frac{P + \sigma}{\rho^2} \frac{d\rho}{dt} - q \quad (4)$$

$$P = (\gamma - 1)\rho e. \quad (5)$$

where,  $P$ ,  $e$ ,  $\rho$ ,  $q$ , and  $\gamma$  denote the gas pressure, specific internal energy, gas mass density, net radiative cooling rate per unit mass,

<sup>1</sup>In 1D spherical calculations the softening length affects only the innermost region and has a negligible impact on the halo-scale ( $r \gg a$ ) evolution.

and the adiabatic index, respectively. Here,  $\sigma$  is the artificial viscosity used to capture shocks (BD03).

The hydrodynamic evolution is computed using an operator-splitting approach. At each time-step, the hydrodynamic equations are first advanced without the radiative source term  $q$ , updating the gas density, velocity, and internal energy due to the pressure and gravity forces. The thermodynamic state of the gas is then propagated over the full time-step at constant density while the ionization network is evolved (see Section 2.3). This yields the updated internal energy (temperature), ion fractions and the corresponding radiative cooling and photoabsorption heating rate  $q$ .

We consider 1000 baryonic shells, and 10 000 dark matter shells in each simulation to properly resolve the virial shock formation.

### 2.2.1 Metallicity profile

An important parameter in the calculation of radiative losses is the gas metallicity, since the radiative cooling rates depend strongly on the abundance of heavy elements. This, in turn, affects the evolution of the ionization network (see Section 2.3). HYDRA does not self-consistently track the metallicity of the gas. Instead, the metallicity is prescribed as a function of position. We assume a metallicity profile consisting of an infalling IGM metallicity ( $Z_{\text{IGM}}$ ) and a CGM metallicity ( $Z_{\text{CGM}}$ ):

$$Z(r) = \begin{cases} Z_{\text{CGM}} = 0.3 & , \quad r \leq r_s \\ Z_{\text{IGM}} = 0.1 & , \quad r > r_s \end{cases} \quad (6)$$

where  $r_s$  is the shock radius. While the metallicity in and around haloes remains an active area of research, both observations and simulations of the CGM generally find metallicities in the range  $0.1\text{--}1 Z_{\odot}$  (N. Lehner et al. 2019; C. B. Wotta et al. 2019; A. W. S. Cook et al. 2025; S. Grayson et al. 2025; B. D. Oppenheimer et al. 2025), with simulations typically favouring the lower end of this range. Our choice of  $Z_{\text{CGM}} = 0.3 Z_{\odot}$ , which lies near the middle of this range, therefore provides a representative value for the shocked-halo gas. Metallicity estimates for gas outside the virial radius, extending out to the turnaround radius, are more limited. However, numerical simulations suggest typical values of  $Z \sim 0.1 Z_{\odot}$  in these regions (S. Grayson et al. 2025; B. D. Oppenheimer et al. 2025; B. Arya et al., in preparation).

In our simulations, the metallicity is initially set everywhere to  $Z = Z_{\text{IGM}}$ . To model the enrichment of halo gas, HYDRA tracks (using a temperature threshold, see Section 3) the shock radius,  $r_s$ , as a function of time and assigns a metallicity  $Z_{\text{CGM}}$  to a gas shell once it crosses the shock radius. This is a phenomenological prescription, used to separate the IGM and CGM metallicities. After a Lagrangian shell is shocked, its metallicity is kept fixed at  $Z_{\text{CGM}}$  for the remainder of the simulation, even if the shock later recedes and the shell temporarily moves to  $r > r_s$ . This treatment is physically motivated, as gas that has been enriched is not expected to lose its metal content if the shock becomes temporarily unstable. In practice, such cases are rare and typically last only a few time-steps, with negligible impact on our results.

### 2.3 Time-dependent ionization and cooling

For tracking the evolution of the non-equilibrium ionization, radiative cooling, and photoabsorption heating, we use the ionization network (IN) presented in S + 21. Here we use the IN to obtain the time-dependent ionization fractions of H, He, C, N, O,

Ne, Mg, S, Si, and Fe in each baryonic shell (spanning a network of 104 ions). The non-equilibrium ion fractions are then used to calculate the associated radiative cooling and photoabsorption heating rates. For the IN, we solve the following set of coupled differential equations,

$$\frac{dx_{k,i}}{dt} = S_{k,i}(\{x\}, Z, z, T) \quad (7)$$

where  $x_{k,i}$  is the ion fraction of ion  $i$  of element  $k$ , and  $S_{k,i}$  is the source term for an ion  $i$  of element  $k$ . The source term  $S_{k,i}$  takes into account the total radiative and dielectric recombination, collisional ionization, photoionization, Auger ionization, and charge-transfer reactions. Details of the calculation of the source term ( $S_{k,i}$ ) and the solution of this set of equations are described in (S + 21).

The IN also calculates, for each shell and time, the radiative cooling and photoabsorption heating rates for a given set of (non-equilibrium) ion fractions, metallicity, density, and the radiation field. The radiative cooling  $\mathcal{L}$  ( $\text{erg s}^{-1} \text{cm}^{-3}$ ) is comprised of free-free emission, collisionally excited- and recombination-line emissions, and is given by:

$$\mathcal{L} = n_e \sum_{k,i} n_{k,i} \Lambda_{k,i}(T) \quad (8)$$

where  $n_e$  and  $n_{k,i}$  are the electron and ion densities, respectively, and  $\Lambda_{k,i}$  is the ion radiation efficiency taken from pre-computed tables from CLOUDY-17 (G. J. Ferland et al. 2017, S + 21 and references therein). The heating includes the photoabsorption-heating (PH) and charge transfer (CT) terms:

$$\mathcal{H} = H_{\text{ph}} + H_{\text{ct}}. \quad (9)$$

The calculation of  $H_{\text{ph}}$  and  $H_{\text{ct}}$  follows equation (15) in (S + 21).

We invoke the IN module at the end of every HYDRA hydrodynamic step. We evolve the ion fractions and energy (after cooling or heating) using the coupled set of ionization equations and the energy equation. We sub-cycle the hydrodynamic time-step to reduce integration errors, and if the equations become stiff the code reverts to analytic matrix inversion for these ions. Since the IN calculation dominates the computation time, we implemented a parallelized MPI-based solution where at every ionization time-step each shell is assigned a task for the calculation. On completion, the main ('hydro') thread collects the results and assigns a new shell to that thread until all shells are done. We use between 32 and 64 nodes per calculation, yielding a considerable acceleration in computation time.

Another mode of calculation is the equilibrium model, in which ion fractions are set to their equilibrium values ( $dx/dt = 0$ ) at the local temperature and density. The temperature is then evolved using the corresponding equilibrium cooling rate, integrated with subcycled time-steps.

### 2.4 UV background evolution

We include the UV background radiation field as a function of redshift using the tables from F. Haardt & P. Madau (2012, hereafter, HM12), employing linear interpolation between tabulated redshifts. The spectral range is discretized into 40 frequency bins, spanning an energy range of 5-2, 500 eV, while maintaining the primary ionization edges of H, He, C, N, and O. A list of frequencies used in this work is given in (S + 21). Using an isochoric gas-cooling test (cf. O. Gnat & A. Sternberg 2007), we verified that

**Table 1.** Summary of the physical scenarios explored in this work.

| Model    | UV background | Ionization      |
|----------|---------------|-----------------|
| Eq       | No            | Equilibrium     |
| nEq      | No            | Non-equilibrium |
| Eq + UV  | Yes           | Equilibrium     |
| nEq + UV | Yes           | Non-equilibrium |

this reduced frequency sampling does not noticeably affect the ion fractions or the overall cooling and heating rates.

### 2.5 Simulation suite

We compute the coupled hydrodynamic, thermal, and ionization-state evolution for haloes with present-day ( $z = 0$ ) dark-matter masses  $M_h = 10^{11}, 3 \times 10^{11}, 10^{12}, 3 \times 10^{12}$ , and  $10^{13} M_\odot$ . For each halo mass, we consider four different physical scenarios. The first scenario is similar to that considered in BD03 (albeit with the newer cooling rates): the ionization states are assumed to be in ionization equilibrium and no UV background radiation is included. We refer to this scenario as ‘Eq’. We then consider departures from equilibrium ionization by following the time-dependent ion fractions and cooling rates, but still without a background radiation field. These models are referred to as ‘nEq’. Next, we examine the impact of the UV background radiation on halo evolution, both in the equilibrium case (‘Eq + UV’) and in the non-equilibrium case (‘nEq + UV’). We therefore consider four models: ‘Eq’, ‘nEq’, ‘Eq + UV’, and ‘nEq + UV’, as summarized in Table 1. For the equilibrium models, the ion fractions depend only on the local density, temperature, and ionizing radiation field, rather than being history-dependent as implied by equation (7).

## 3 RESULTS: SHOCK STABILITY

A stable shock is a shock that is supported by sufficient thermal pressure. The shock, however, may lose the thermal energy via radiative cooling and may not be able to sustain itself against the gravitational pull of the halo. Such unstable shocks are often noticed in low-mass haloes ( $M_h \lesssim 3 \times 10^{11} M_\odot$ ) where the virial velocity is such that the resulting shock temperature is  $\lesssim 10^5$  K, and therefore, the shock suffers from extreme radiative cooling. The stability of the shock is discussed in detail in BD03. This work is an extension of BD03, but includes deviations from ionization equilibrium, and the redshift-dependent UV background from HM12. In BD03, assuming equilibrium cooling, an analytic criterion for shock formation was derived as a function of infall velocity, density, and metallicity, which can be mapped to halo mass and redshift in a cosmological context. In contrast, with the inclusion of time-dependent non-equilibrium processes, the criterion for shock formation can no longer be derived analytically and must instead be determined numerically.

Fig. 1 shows results from the simulations of all masses and modes. Each panel shows the evolution of a single halo, in terms of its shells’ radii at each redshift. Different columns are for different halo masses, and different rows are for the four different physical scenarios. The diagrams are colour-coded by the gas temperature at each radius and redshift. The white dashed line displays the theoretical virial radius inferred from a spherical overdensity

criterion<sup>2</sup> We use a temperature threshold,  $T_{\text{th}} = 2 \times 10^5$  K, to continuously identify the shocked region during the simulation. The shock front tracking is shown by the dashed black-pink lines in Fig. 1. The identified shock radius closely follows the outer boundary of the hot, post-shock gas in all cases. The gas infall halts at  $\sim 0.1, r_{\text{vir}}$  due to the prescribed angular momentum. This angular-momentum-supported region corresponds to the stellar and gaseous disc.

### 3.1 Equilibrium ionization (No UV background, ‘Eq’)

In this scenario, ionization and radiative cooling are calculated assuming collisional ionization equilibrium (CIE). This is the same assumption made in BD03. The evolution of the gas trajectories and temperature for this scenario is shown in the top row of Fig. 1, with different panels corresponding to different final halo masses. With no background radiation, the IGM gas, which initially expands with the Hubble flow and eventually turns around and contracts, remains cold, at  $T \sim 10$ – $100$  K, and evolves only through adiabatic processes.

The temperature only increases when a shock forms. This can be seen in the top row of Fig. 1, where the appearance of the red/orange regions marks the formation of shock-heated gas. In lower-mass haloes these shocks are often unstable and short-lived: radiative cooling removes the post-shock pressure support, causing the shock to recede and the gas to cool again. In the  $M_h = 3 \times 10^{11} M_\odot$  case, a shock forms intermittently and remains unstable for much of the evolution, although a more persistent shock appears at later times. By contrast, haloes with  $M_h \gtrsim 10^{12} M_\odot$  develop stable virial shocks that persist and maintain a hot halo, consistent with the results of BD03. Note that for these high masses, the stable shock radius also lies close to the theoretical virial radius inferred from the spherical overdensity criterion. For the  $M_h = 10^{11} M_\odot$  halo, no virial shock develops because cooling is too efficient. It is dominated by Ly  $\alpha$  emission and strong metal-line cooling, and prevents the shock from forming where it would otherwise be expected.

At high masses ( $\gtrsim 3 \times 10^{11} M_\odot$ ), we occasionally observe transient cooling-flow-like behaviour in the temperature. The top-right panel is an excellent example of such a flow where several baryonic shells cool and clump together to produce a colder and denser set of shells that finally fall down to the galactic disc. Although this picture is not exactly the same as in (J. Stern et al. 2019), it shows a strong resemblance.

### 3.2 Non-equilibrium ionization (No UV background, ‘nEq’)

The temperature evolution in the non-equilibrium case is shown in the second row of Fig. 1. As in the equilibrium case, the appearance of red regions indicates the formation of shock-heated gas. Comparing the equilibrium and non-equilibrium cases reveals several important differences for the lower mass haloes. In the non-equilibrium row, the lower-mass haloes contain more cold (dark-blue) gas than their equilibrium counterparts, indicating more efficient cooling. For the  $3 \times 10^{11} M_\odot$  halo, no shock forms

<sup>2</sup>The virial radius at each time is defined as the radius of the shell whose mean enclosed density exceeds the virial overdensity. For a  $\Lambda$ CDM universe with  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$  the virial overdensity at  $z = 0$  is  $\Delta_v \approx 340$ . See G. L. Bryan & M. L. Norman (1998), equation (6) therein.

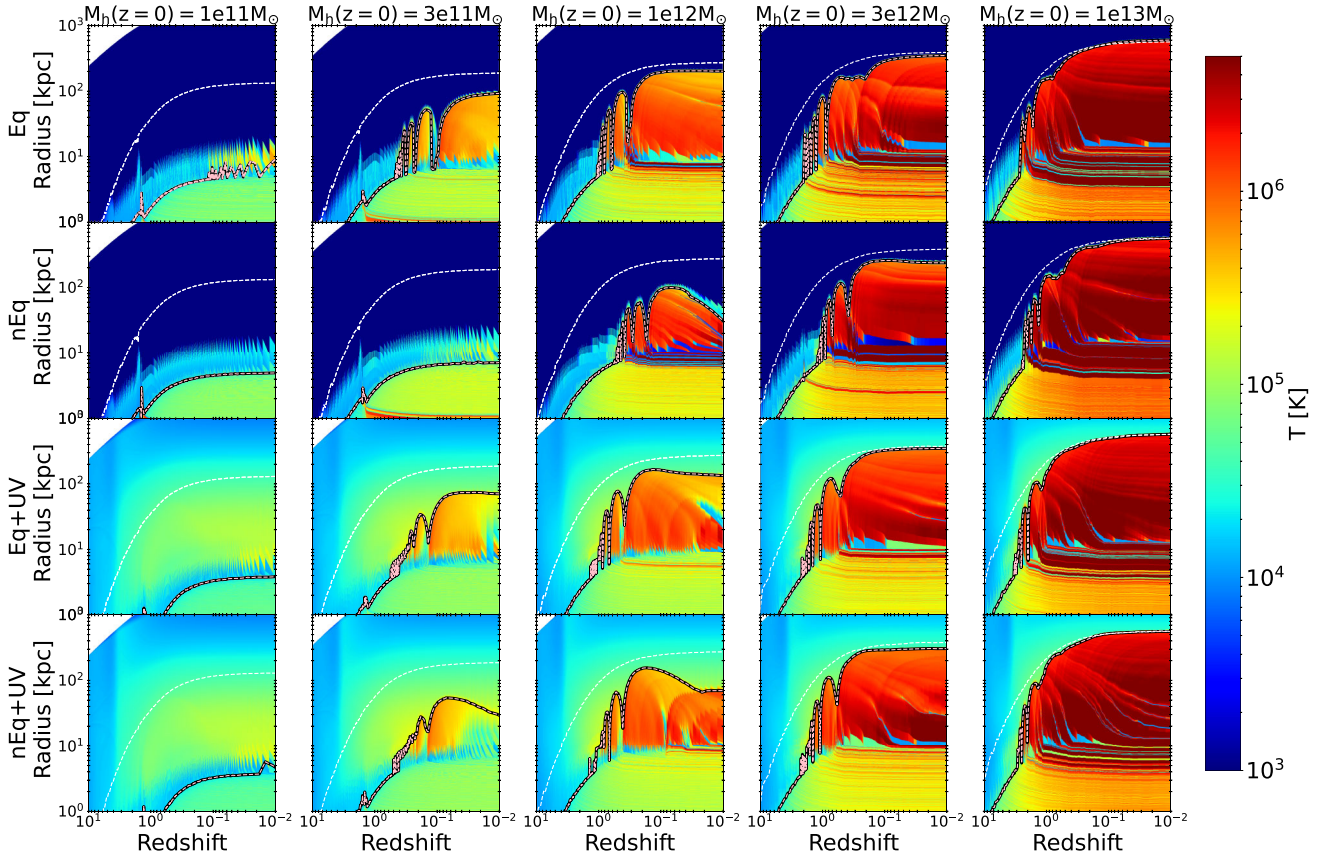


**Figure 1.** Evolution of gas shells in haloes of different masses and physical scenarios. Each row shows a different physical model (‘Eq’, ‘nEq’, ‘Eq+UV’, ‘nEq + UV’, from top to bottom). Each column is for a different final halo mass ( $10^{11}$ ,  $3 \times 10^{11}$ ,  $10^{12}$ ,  $3 \times 10^{12}$ ,  $10^{13} M_\odot$  from left to right). Each panel shows the trajectories of Lagrangian gas shells in radius–redshift space, colour-coded by gas temperature. The thin white dashed line indicates the virial radius, and the thick black-pink dashed line indicates the shock radius.

in the time-dependent case, whereas in the equilibrium case a shock appears at late times. For the  $10^{12} M_\odot$  halo, a shock does form in the non-equilibrium model, but its radius decreases with time and does not approach  $R_{\text{vir}}$  as it does in the equilibrium case. For haloes with  $M \gtrsim 3 \times 10^{12} M_\odot$ , the evolution is qualitatively similar in the two models. This indicates that in the time-dependent case, the critical mass for stable shock formation is higher than in CIE, with stable shocks appearing only for masses  $\gtrsim 10^{12} M_\odot$  in our simulations.

These differences arise from the time-dependent nature of the ionization calculation. Following the non-equilibrium ion fractions alters the thermal evolution of the gas because the cooling rates depend on the instantaneous ion populations. In particular, the gas entering the shock is significantly underionized relative to the shock temperature, making cooling more efficient than in the equilibrium case. This is because the lower ionization species that remain present at the shock temperature, are more efficiently excited by the hot thermal electrons. The enhanced cooling reduces the post-shock pressure support, making the shock more difficult to sustain.

### 3.3 Equilibrium ionization with UV background (‘Eq + UV’)

The third row in Fig. 1 shows the evolution of haloes in photoionization equilibrium with the redshift-dependent UV background.

The ionizing background significantly alters the thermal and ionization properties of the IGM gas. Instead of remaining cold and evolving only through adiabatic processes, the gas temperature and ionization state are now also affected by photoionization and photoabsorption heating. As a result, the typical IGM temperatures increase to  $\sim 10^4$ – $10^5$  K, with the exact equilibrium temperature depending on the gas density and redshift.

We note that in our simulations the gas temperature does not always correspond to the standard thermal equilibrium solution obtained by equating the radiative cooling and photoabsorption heating rates. This is because adiabatic processes associated with cosmic expansion and gravitational collapse also affect the thermal evolution of the gas. In particular, adiabatic cooling due to cosmic expansion often dominates in the low-density IGM, keeping the gas temperature near  $10^4$  K. As the gas collapses and approaches the virial radius, adiabatic compression raises its temperature above the standard thermal equilibrium value (e.g. B. Arya et al., in preparation). In both cases, the deviation from equilibrium is at most a factor of a few.

The red and orange regions again mark the formation of shock-heated gas. Despite the substantial differences in the IGM properties between the runs with and without a UV background, the shocked halo properties remain broadly similar, with comparable shock-formation epochs and overall shock stability. This is expected because the properties of the shock-heated gas are dominated by collisional pro-

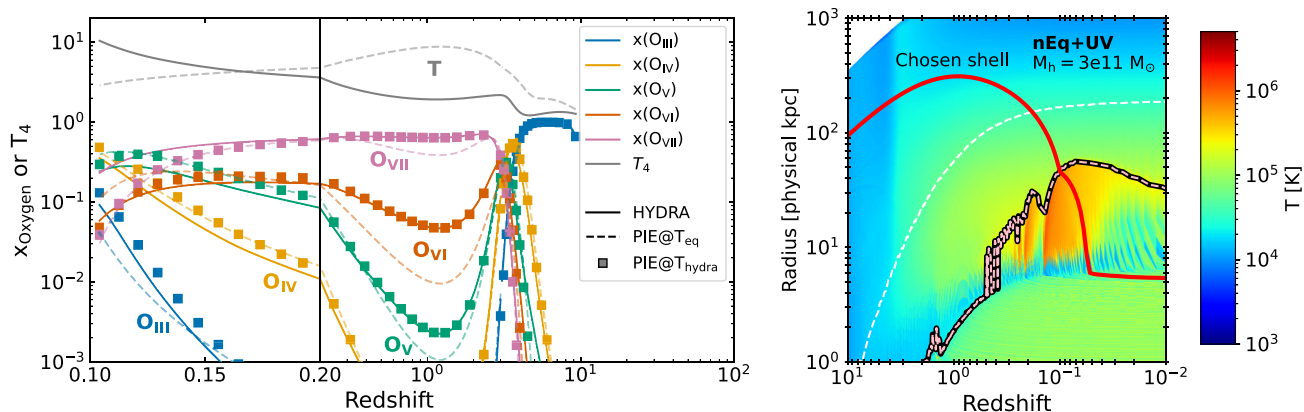


**Figure 2.** Right:  $r - z$  trajectory of a gas shell (no. 441) in a  $3 \times 10^{11} M_{\odot}$  nEq + UV halo. The thin white dashed line indicates the virial radius, and the thick black-pink dashed line indicates the shock radius. Left: Oxygen ion fractions vs. redshift for the same shell (left section focuses on late times). Grey curves show  $T_4 \equiv T/(10^4 \text{ K})$ ; coloured curves show ion fractions. Solid lines: HYDRA; dashed: PIE at  $T_{\text{eq}}$ ; squares: PIE at  $T_{\text{HYDRA}}$ .

cesses at the virial temperature rather than by the background radiation.

### 3.4 Non-equilibrium ionization & UV radiation ('nEq + UV')

Finally, in the last row of Fig. 1, we display the temperature evolution in haloes exposed to the UV background radiation in the non-equilibrium case. This scenario is likely the most realistic for astrophysical haloes.

As in the Eq + UV case, the impact of the UV background on the IGM properties is clearly apparent: the temperature evolution of the IGM gas closely matches that of the equilibrium model, since departures from ionization equilibrium have little effect on the slowly evolving IGM.

When the UV background is included, departures from equilibrium also no longer play a significant role in shaping the structure and stability of the CGM, unlike in the non-equilibrium model without radiation. This occurs because the UV background pre-ionizes the IGM gas as it approaches the virial shock. As a result, the infalling gas is much less under-ionized than in the radiation-free case, and the associated cooling rates are reduced, approaching their equilibrium values. This allows thermal pressure to build up more steadily behind the shock.

### 3.5 Minimal mass for shock formation

We refer to the lowest halo mass in our grid that develops a persistent, spatially extended shocked region (see Fig. 1) as the critical mass. In our simulations, the critical mass is only bracketed by the discrete halo masses considered (spaced by 0.5 dex), and we therefore quote it to order-of-magnitude accuracy. Across all models, the transition spans approximately one mass interval (0.5 dex), but its location shifts depending on the ionization physics.

With this in mind, while non-equilibrium effects alone enhance post-shock cooling and appear to shift the critical mass for shock formation to higher values ( $M_{\text{h}} \gtrsim 10^{12} M_{\odot}$ ), the inclusion of the UV background largely suppresses these non-equilibrium effects by pre-ionizing the infalling gas. As a result, the critical mass for stable shock formation returns to  $M_{\text{h}} \sim 3 \times 10^{11} M_{\odot}$ , independent of whether the ionization state is treated in equilibrium or not. Thus, by pre-ionizing the infalling IGM, the UV

background moderates the non-equilibrium cooling behind the shock and brings the critical mass back to the equilibrium value found by BD03.

## 4 RESULTS: IGM TEMPERATURE AND ION FRACTIONS

In this section, we discuss the thermal state and ionization structure of the infalling IGM gas in our simulations. We focus on the models that do include the UV background, as we consider those more realistic.

Two aspects are of particular interest. First, we demonstrate that the temperature of the IGM gas generally does not follow the standard photoabsorption-heating equals radiative-cooling thermal equilibrium solution. Instead, adiabatic effects associated with cosmic expansion and gravitational compression modify the thermal evolution, producing temperatures that deviate from the usual equilibrium curve. Secondly, we show that when the UV background is included, the low-density IGM is a natural site for the production of high ions. As we show in Section 5.1, these IGM ions may significantly contribute to UV absorption around galaxies (see also B + 25).

In Fig. 2 (left panel), we show the evolution of the gas temperature ( $T_4$ , grey) and oxygen ion fractions (coloured curves) as a function of redshift for a representative gas shell as it expands with the universe, turns around, and eventually approaches the virial shock at  $z \simeq 0.1$ . This is shell no. 441 in the  $3 \times 10^{11} M_{\odot}$  nEq + UV model. The right panel shows the shell trajectory in radius-redshift space. The left panel shows the evolution starting at  $z = 10$ . To highlight the final stages of the shell's evolution, the interval  $0.1 < z < 0.2$  is expanded on the left-hand side of the panel, while the evolution from  $z = 0.2$  to  $z = 10$  is shown on the right-hand side. The solid curves show the results of our HYDRA simulation. The dashed curves show the corresponding results assuming standard thermal and photoionization equilibrium (PIE): Specifically, the dashed grey curve shows the equilibrium temperature,  $T_4 \equiv T/(10^4 \text{ K})$ , obtained by requiring that photoabsorption heating balances radiative cooling at the local gas density for the redshift-dependent radiation field. The dashed coloured curves show the corresponding photoionization equilibrium ion fractions evaluated at this equilibrium temperature. We

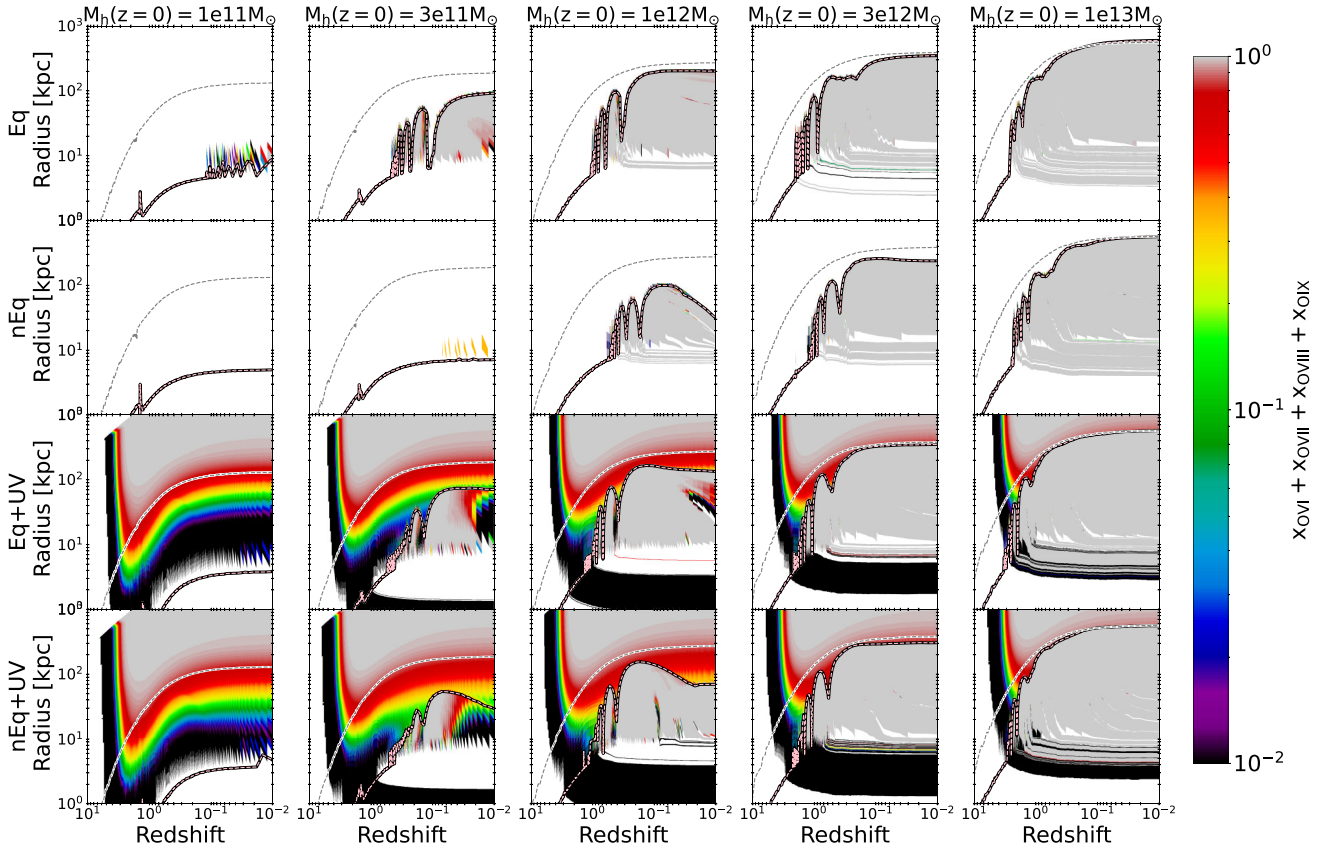


**Figure 3.** Same as Fig. 1, but colour-coded by the sum of the ion fractions of O VI, O VII, O VIII, and O IX. This sum represents the ionization state of the gas. The thin white/grey dashed line indicates the virial radius, and the thick black-pink dashed line indicates the shock radius.

also plot, with coloured squares, the PIE abundances evaluated at the temperature obtained in our HYDRA simulations.

As discussed in Section 3.3, adiabatic processes associated with cosmic expansion and gravitational collapse affect the thermal evolution of the gas. This is seen by comparing the solid and dashed grey curves. During the expansion phase, the gas temperature remains lower than the thermal equilibrium value (as seen at higher redshift), while during collapse it rises above the equilibrium value (visible at lower redshift). Since the gas temperature departs from the thermal equilibrium value, the associated ion fractions also deviate from the standard equilibrium predictions (solid vs. dashed curves). In addition, Fig. 2 demonstrates that as the gas approaches the accretion shock, departures from ionization equilibrium further shift the ion fractions away from the equilibrium values corresponding to the simulated temperature (solid curves versus squares), but the deviations remain moderate.

A second aspect that we wish to highlight is the prevalence of high ions in and around galactic haloes. We illustrate this by considering the evolution of the combined abundance of high oxygen ions, O VI + O VII + O VIII + O IX. Fig. 3 shows the evolution of this high-ion abundance in the same radius–redshift parameter space used in Fig. 1. As expected, high ions are abundant inside the shock-heated CGM for all halo masses and physical scenarios due to efficient collisional ionization. However, since the different scenarios differ in whether and when extended shocks form, they also differ in the amount of highly ionized oxygen produced. In the IGM the different models show markedly dif-

ferent behaviour. When no UV radiation is included, the IGM remains cold and largely devoid of high ions, so any contribution to high-ion absorption around galaxies arises from the CGM. By contrast, when the UV background is included, photoionization drives the ionization state of the IGM to much higher values, producing significant abundances of high ions outside  $R_{\text{vir}}$  (seen in grey), with substantial contributions between  $R_{\text{vir}}$  and  $r_s$  for haloes with  $M_h \gtrsim 3 \times 10^{11} M_\odot$ . We do not see much difference in the overall ionization state between the Eq + UV and nEq + UV cases. The pre-ionization reduces the non-equilibrium cooling enhancement behind the shocks. This gives rise to IGM contributions to UV absorption around galaxies (see B + 25, and Section 5).

## 5 RESULTS: OBSERVATIONAL SIGNATURES

In this section, we study the absorption-line signatures that arise in our spherical halo models. As in the halo stability analysis presented above, we now examine how departures from ionization equilibrium and the presence of a UV background affect the metal absorption properties of galactic environments. Various factors, including the position and stability of the virial shock, as well as the evolving thermal and ionization state of the gas, affect the observable metal–ion column densities and the relative contributions of the CGM and the IGM to each absorption line. Here, we compute the column densities produced in our spherical models for the different physical scenarios considered in this work. For each ion, the column density is obtained by integrat-

ing the corresponding ion number density along a line of sight through the spherical model at impact parameter  $b$ . Specifically, for an element  $m$  in ionization state  $i$ ,

$$N_i^m(b) = 2 \int_b^{r_{\max}} \frac{n^m(r) x_i^m(r)}{\sqrt{1 - (b/r)^2}} dr, \quad (10)$$

where  $x_i^m(r)$  is the ion fraction,  $n^m(r) = n_H(r) A^m Z(r)$  is the number density of element  $m$ ,  $A^m$  is the solar abundance of that element, and  $Z$  is the metallicity. The factor of two accounts for the two sides of the spherical halo along the line of sight. Since we focus on the ions produced outside the galactic disc, we do not integrate over radii smaller than  $0.1 R_{\text{vir}}$ . Following B + 25 and D. Nelson et al. (2018), we truncate the integrated column densities at a maximal radius,  $r_{\max} = 3$  Mpc. As in B + 25, we define the CGM column as the contribution arising from the shock-heated gas within the shock radius,  $r_s$ .

In Section 5.1, we examine O VI columns, a commonly used tracer of CGM gas. In Section 5.2, we focus on the hydrogen Ly $\alpha$  column densities, a primary indicator of IGM absorption systems. Finally, in Section 5.3 we consider C IV absorption.

## 5.1 O VI column densities

In Fig. 4, we present the O VI column densities obtained from our simulations. In the top row we show a snapshot at  $z = 0.2$  of the integrated O VI column density as a function of the scaled impact parameter,  $b/R_{\text{vir}}$ . We evaluate the projected columns at  $z = 0.2$  because this redshift is representative of the low-redshift observational samples used for comparison, such as COS-Halos (J. K. Werk et al. 2016) and CGM<sup>2</sup> (K. Tchernyshyov et al. 2022).

Each panel corresponds to a different halo mass. The corresponding stellar masses, derived using the stellar-to-halo mass relation of G. Girelli et al. (2020), are indicated at the top of each panel. As in B + 25, we truncate the integrated column densities at a radius of 3 Mpc. The sharp drop in column density seen on the right-hand side of the highest-mass panels is caused by this finite integration limit, and does not reflect a physical drop in the ion abundance. Adopting a common truncation radius well outside  $R_{\text{vir}}$  allows a consistent comparison between haloes of different masses.

The different curves represent the physical scenarios considered in this work: Eq (dashed blue), Eq + UV (solid orange), nEq (dashed green), and nEq + UV (solid red). For the nEq + UV case, we also show – with red dash-dotted lines – the column densities arising from the hot halo alone (i.e. integrating the column only out to  $r_s$ ). We refer to this as the ‘CGM-column’. Note that while the shock radius roughly coincides with the virial radius for haloes more massive than  $\sim 3 \times 10^{12} M_\odot$ , for haloes with  $3 \times 10^{11} M_\odot < M < 10^{12} M_\odot$  the shock radius lies inside  $R_{\text{vir}}$ . In our decomposition, the photoionized gas between  $r_s$  and  $R_{\text{vir}}$  is included in the total projected column but assigned to the IGM contribution. For comparison, the solid grey curves show the column densities predicted by the simplified toy model of B + 25. Observational detections (filled circles) and upper limits (empty triangles) from the K. Tchernyshyov et al. (2022) compilation of CGM surveys are also plotted. This compilation spans  $0 < z < 0.6$ , with a median redshift of  $z \simeq 0.25$ .

Fig. 4 confirms that the inclusion of a UV background alters the state of the infalling IGM. In simulations without a UV background, the IGM remains cold and largely neutral, resulting in very low abundances of highly ionized species. Correspondingly, the O VI columns outside the shocked halo drop below  $10^{12} \text{ cm}^{-2}$ ,

and the sharp decline in the column-density profile marks the shock radius. When a UV background is included, however, the IGM becomes warm and significantly ionized, producing substantial populations of high ions even outside the shocked halo. The transition at  $r_s$  therefore appears more subtly as a change in the slope of the column-density profile rather than as a sharp cutoff.

The impact of the UV radiation is clearly reflected in the predicted O VI column densities. Models that include the UV background (red, orange) reproduce the observed columns reasonably well<sup>3</sup>, whereas simulations without an ionising background (blue, green) significantly underpredict the observations at large ( $> r_s$ ) impact parameters. Furthermore, when the UV radiation is included, the difference between the total nEq + UV columns (solid red) and the shocked-halo-only columns (dash-dotted red) demonstrates that photoionized gas outside  $r_s$  significantly enhances the total O VI columns. The metagalactic radiation field therefore creates an extended, photoionized component surrounding the halo that contributes substantially to the absorption along the line of sight.

The top panels of Fig. 4 also show the O VI column densities produced solely within the shocked halo for the nEq + UV model (dash-dotted red curves). For the higher-mass haloes, shown in the three right-hand panels, the halo contribution alone cannot reproduce the typical observed O VI columns, even at impact parameters  $b < r_s$ . In the  $10^{13} M_\odot$  halo, the predicted halo contribution is lower than the observations by roughly an order of magnitude. For the  $1-3 \times 10^{12} M_\odot$  haloes, the CGM-only columns reach only the level of the weakest observed absorption systems. Our one-dimensional  $\Lambda$ CDM hydrodynamical simulations therefore confirm the conclusions of B + 25, where a simplified toy model was used to explore the contribution of the surrounding IGM to the observed warm-ion absorption around galaxies. The results of B + 25 are shown for comparison by the grey curves.<sup>4</sup> Keeping the model limitations in mind (see Section 6), we find that in our spherical, no-feedback models, gas outside the shock radius can produce O VI column densities comparable to observations.

Also apparent in the figure is that the O VI column densities in the Eq + UV and nEq + UV models are nearly identical across all halo masses. This is true both for impact parameters within and outside the shock radius. This similarity arises because a substantial fraction of the projected column arises from photoionized gas outside the shock radius, where the ionization state is set primarily by the UV background. Since both models are exposed to the same radiation field, and the ionization and recombination time-scales are much shorter than the (cosmological) time-scales for changes in the gas temperature or radiation field, the gas remains close to ionization equilibrium. As a result, departures from equilibrium have little effect on the predicted ion fractions in the IGM. This is consistent with Fig. 2, which shows similar O VI fractions in the nEq (solid) and equilibrium (squares, dashed curves) cases.

<sup>3</sup>The quantitative agreement with the observed column densities depends on the assumed metallicities and UV background model, and should therefore be interpreted at the level of the overall agreement rather than as a unique fit to the data (see Section 6).

<sup>4</sup>Note that the metallicity adopted by B + 25 was three times lower than in our models, compensating for the higher densities produced by their matter-dominated Einstein–de Sitter cosmology, which contains roughly three times more matter.

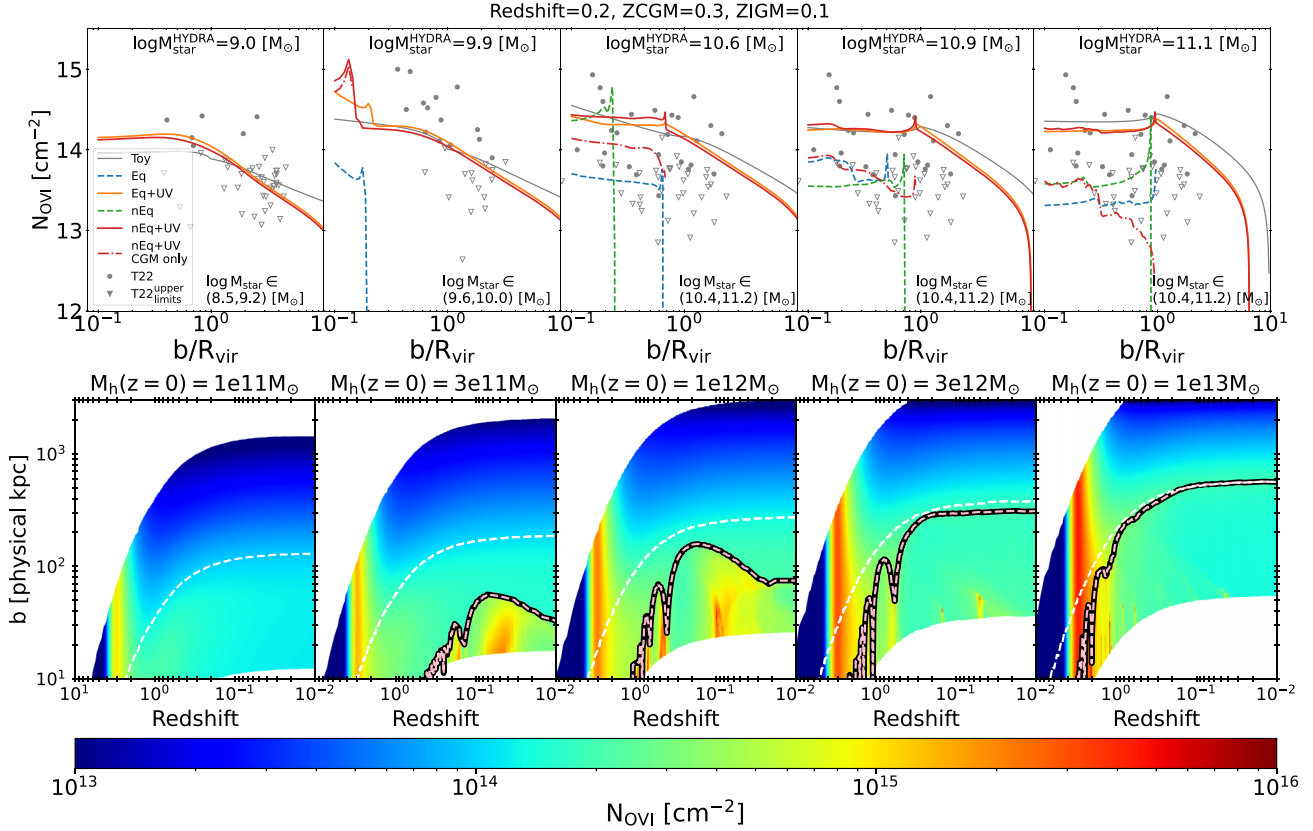


**Figure 4.** O VI columns. Top row: O VI columns at  $z = 0.2$  versus scaled impact parameter,  $b/R_{\text{vir}}$ , for different halo masses. Curves show the Eq (blue), Eq + UV (orange), nEq (green), and nEq + UV (solid red) models, the CGM-only contribution ( $r < r_s$ ) for nEq + UV (dash-dotted red), and the toy model of **B + 25** (grey). The sharp drop in column density seen on the right-hand side of the highest-mass panels is caused by the finite integration limit,  $r_{\text{max}} = 3$  Mpc, and does not reflect a sharp drop in the ion abundance. The stellar mass shown at the top of each panel is derived using G. Girelli et al. (2020). Markers show detections (filled circles) and upper limits (empty triangles) from K. Tchernyshyov et al. (2022), for galaxies in the stellar-mass range indicated at the bottom of each panel. Bottom row: Total projected line-of-sight O VI columns (colour bar), for the nEq + UV model, in the redshift–impact-parameter plane. Thin white dashed line shows the virial radius; thick black-pink dashed line shows the shock radius, as in Fig. 1. Since we focus on ions produced outside the galactic disc, we do not integrate over radii smaller than  $0.1R_{\text{vir}}$ ; the corresponding excluded impact parameters appear as the white region at the bottom of the panels.

Crossing the shock front could, in principle, be a site of significant departures from ionization equilibrium, since the infalling IGM may be underionized with respect to the shock temperature. However, as shown in Fig. 1, the UV background pre-ionizes the IGM, and the warm photoionized gas does not undergo significant cooling after passing through the shock. Further downstream, departures from equilibrium could arise if the shocked gas cools on a time-scale shorter than the recombination time. At  $z = 0.2$ , however, Fig. 1 shows no substantial cooling behind the shock. We therefore do not expect large differences between the Eq + UV and nEq + UV cases. And indeed, Fig. 4 confirms that the predicted O VI column-density profiles are nearly identical in the two models, even inside  $r_s$ .

The bottom row in Fig. 4 shows the predicted O VI column densities across the full  $r$ - $z$  parameter space of Fig. 1 for the nEq + UV case. As before, different panels are for different masses, and the virial and shock radii are indicated in each panel. Here, the colour scheme shows the O VI column density corresponding to each  $r$ - $z$  combination, where the radial coordinate represents the projected sightline through the halo, i.e. the observational impact parameter,  $b$ . Since these are projected columns, even sightlines with  $b < r_s$  or  $b < R_{\text{vir}}$  include gas at larger radii along the line of

sight, including material outside the shock radius and beyond the virial radius. The lower panels therefore show where large total columns are expected observationally, while the upper panels are needed to separate the shocked-halo contribution from the contribution of gas outside  $r_s$ .

Fig. 4 shows that readily observable O VI column densities,  $N_{\text{OVI}} \gtrsim 10^{13} \text{ cm}^{-2}$ , are ubiquitously produced in our simulations, extending to several times the virial radius. The overall O VI column density varies only weakly with halo mass, reflecting the dominant contribution of the IGM. The O VI columns become particularly large at  $z = 2$ – $4$ , reaching  $N_{\text{OVI}} \gtrsim 10^{15} \text{ cm}^{-2}$ , during the peak of cosmic star formation (when the UV background is maximal), as indicated by the orange-red vertical strip in the figure. Such column densities are well above the typical detection limits of optical surveys probing high-redshift intervening O VI/metal absorbers (e.g. J. Schaye et al. 2000; J. Bergeron et al. 2002; B. Carswell, J. Schaye & T.-S. Kim 2002; R. A. Simcoe, W. L. W. Sargent & M. Rauch 2002; R. A. Simcoe, W. L. W. Sargent & M. Rauch 2004; N. Lehner et al. 2014), and are consistent with observations of highly ionized gas at  $z \gtrsim 2$ . These systems are generally interpreted as high-redshift IGM absorbers, although N. Lehner et al. (2014) argue that optically thick absorbers may also trace

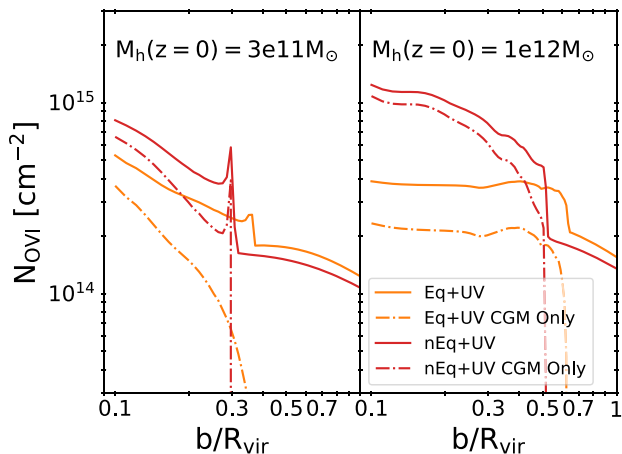


**Figure 5.**  $N_{\text{OVI}}$  versus impact parameter, as in Fig. 4, for  $3 \times 10^{11}$  and  $10^{12} M_{\odot}$  haloes at  $z = 0.08$ , where transient ‘cooling flow’ behaviour is observed.

the galactic and circumgalactic gas of high-redshift galaxies. The abrupt transition from low ionization to high ionization oxygen corresponds to the reionization of the Universe, akin to the well-known hydrogen reionization.

Finally, we note that while the O VI column densities are generally not strongly affected by departures from ionization equilibrium, this is not always the case. As discussed above, when the UV background pre-ionizes the infalling IGM, departures from equilibrium near the shock radius remain small. In most cases, the shocked gas remains hot, preventing significant non-equilibrium effects associated with rapid cooling. This is the case for  $z = 0.2$ , displayed in the upper row of Fig. 4. However, Fig. 1 shows that rapid cooling can occasionally develop in the post-shock layers for the non-equilibrium models. For example, a large cool region is visible in the  $3 \times 10^{11} M_{\odot}$  halo at  $z < 0.1$ , and a localized cooling flow appears in the  $10^{12} M_{\odot}$  halo at  $z \sim 0.08$ . In such regions, the cooling time may become shorter than the recombination time, leading to departures from ionization equilibrium (e.g. O. Gnat & A. Sternberg 2007; O. Gnat & A. Sternberg 2009). These ‘flows’ arise from instabilities, and their detailed properties are likely numerical in origin. We defer a more robust analysis of these features to future work.

Under such ‘cooling-flow’ conditions, recombination may lag behind cooling, enhancing the columns of high ions. Such enhancements are visible in the lower panel of Fig. 4 for the relevant halo masses and redshifts. In Fig. 5, we present the O VI column density as a function of impact parameter for the relevant halo masses ( $3 \times 10^{11}$  and  $10^{12} M_{\odot}$ ) at  $z = 0.08$ , comparing the Eq + UV (orange) and nEq + UV (red) models. As before, solid curves show the total integrated column density, while dash-dotted curves indicate the shocked gas (‘CGM-only’) contribution. The figure demonstrates that the halo columns differ between the equilibrium and non-equilibrium models, producing differences of a factor of 2-3 in the total observed columns.

## 5.2 H I column densities

Ly  $\alpha$  absorption is one of the primary tracers of absorber redshift in the IGM. It is particularly useful at high redshifts, where metal lines are less common due to the low metallicity of the early Universe. Fig. 6 (top panel) shows the H I column-density maps

from our simulations. At  $z \sim 3$ , our predicted column densities,  $N_{\text{HI}} \sim 10^{15-17} \text{ cm}^{-2}$ , are consistent with the range of H I columns observed in high-redshift Ly  $\alpha$  forest and H I -selected absorber samples (e.g. A. Songaila & L. L. Cowie 1996; G. C. Rudie et al. 2013; N. Lehner et al. 2022). At lower redshifts,  $z \lesssim 1$ , our models predict  $N_{\text{HI}} \lesssim 10^{14.3} \text{ cm}^{-2}$  in the nearby IGM ( $R_{\text{vir}} \lesssim b \lesssim 3R_{\text{vir}}$ ). These columns are comparable to weak Ly  $\alpha$  absorbers found in low- and intermediate-redshift galaxy-absorber studies, particularly at large impact parameters, although stronger galaxy-associated H I absorption is also observed (e.g. J. X. Prochaska et al. 2017; S. Dutta et al. 2026a, b).

In the CGM, neutral hydrogen is commonly associated with cool, photoionized gas that coexists with warmer and hotter phases in a multiphase medium (e.g. J. X. Prochaska et al. 2017; J. Tumlinson et al. 2017; T. J. Cooper et al. 2021; Z. Qu et al. 2023; S. Borthakur et al. 2024). Our simplified spherical simulations do not model this full multiphase structure, and therefore produce relatively small amounts of H I, with little variation in  $N_{\text{HI}}$  across the shocked halo. While the resulting H I column densities are consistent with the lower end of detections in the COS-Halos survey (J. X. Prochaska et al. 2017,  $N_{\text{HI}} \sim 10^{14-19} \text{ cm}^{-2}$ ), they do not reproduce the higher observed values. This is expected, since the strongest CGM H I absorbers are associated with cooler, denser, photoionized gas phases not captured by our model.

## 5.3 C IV column densities

The C IV 1548 Å transition is a commonly observed absorption line in quasar spectra at  $z \sim 2-3$ , and is widely used to constrain the metallicity and ionization state of the IGM. Observations typically find C IV column densities of  $N_{\text{CIV}} \sim 10^{12-14} \text{ cm}^{-2}$  (A. Songaila & L. L. Cowie 1996; J. Schaye et al. 2000; R. A. Simcoe et al. 2011; E. Banerjee et al. 2023), often associated with partial Lyman limit systems (pLLSs;  $N_{\text{HI}} \sim 10^{16-17} \text{ cm}^{-2}$ ) and Lyman Limit Systems (LLSs;  $N_{\text{HI}} \sim 10^{17-19} \text{ cm}^{-2}$ ).

Our simulations reproduce such systems, as shown in the bottom panel of Fig. 6. At  $z \sim 2-3$ , C IV is abundant ( $N_{\text{CIV}} \sim 10^{13-15} \text{ cm}^{-2}$ ) in gas with  $N_{\text{HI}} \sim 10^{15-17} \text{ cm}^{-2}$ , consistent with observational constraints. In addition, C IV is present over a wide radial range, extending well beyond the virial radius and out to  $\sim 10R_{\text{vir}}$ , in agreement with the findings of E. Banerjee et al. (2023).

At lower redshifts ( $z \lesssim 1$ ), we find C IV columns of  $N_{\text{CIV}} \sim 10^{13-14} \text{ cm}^{-2}$  around low-mass haloes, primarily arising outside the shock radius. For low-mass haloes, virial shocks are often unstable, and when they do form they typically lie well within  $R_{\text{vir}}$ . As a result, even sightlines with small impact parameters ( $b < R_{\text{vir}}$ ) do not probe the virialized CGM, but instead intersect the infalling photoionized gas. For higher-mass haloes, the C IV column densities decrease by roughly an order of magnitude, but may still be detectable in absorption against background AGN.

## 6 CAVEATS

The hydrodynamical simulations presented in this work represent a significant step forward over the toy-model predictions of B + 25. In particular, the underlying cosmological framework is the  $\Lambda$ CDM model, which accounts for both the reduced matter density ( $\Omega_m = 0.3$ ) and the accelerated expansion driven by the cosmological constant. In addition, the hydrodynamical evolution of the gas is treated self-consistently, allowing us to follow shock formation and stability while properly capturing the associated jump conditions. Finally, departures from ionization

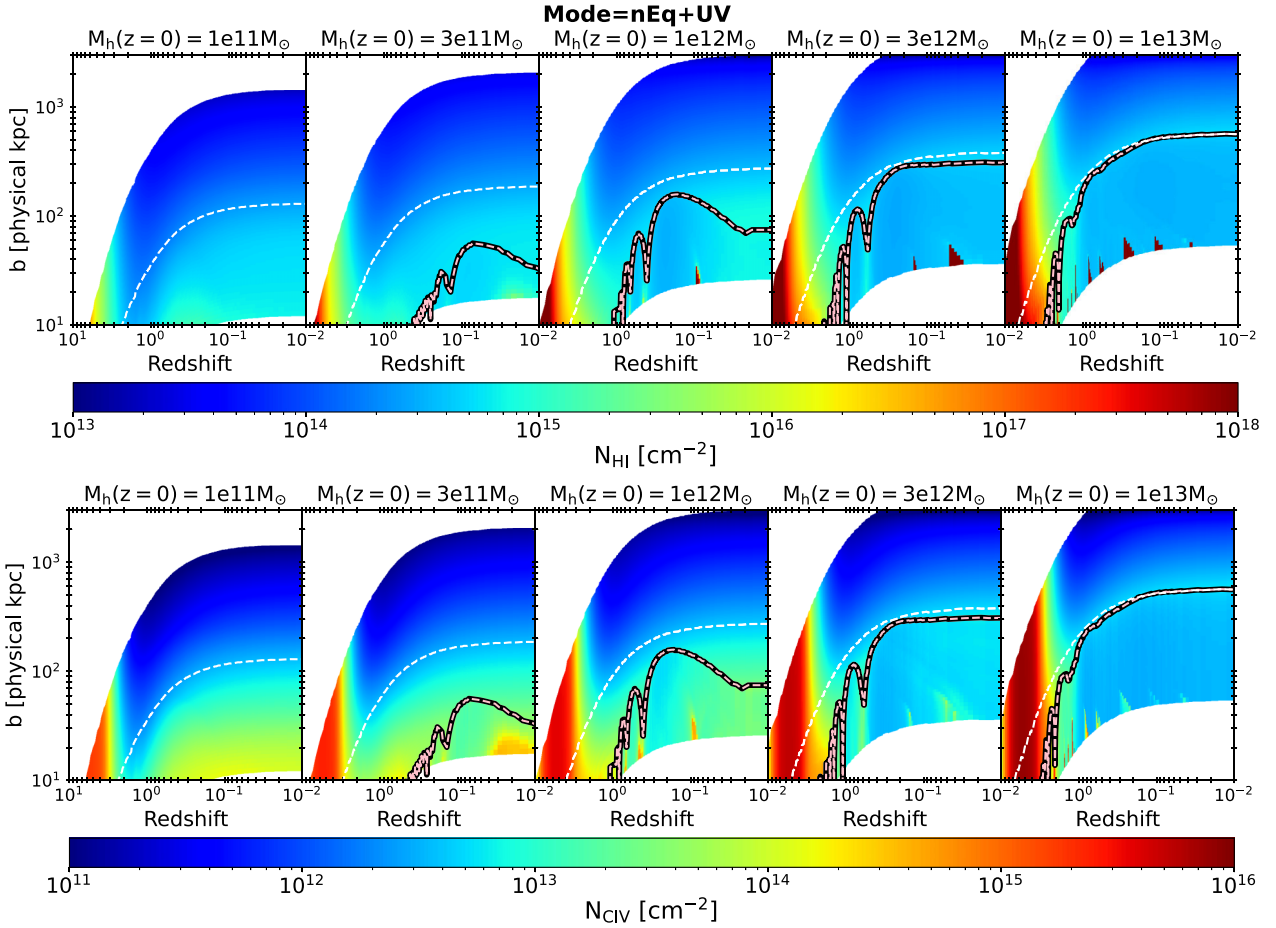


**Figure 6.**  $N_{\text{HI}}$  (top) and  $N_{\text{CIV}}$  (bottom) columns for the nEq + UV model across the full impact-parameter – redshift parameter space. Colour indicates the column density for each redshift–sightline (impact parameter) combination (see colourbar). The white dashed line indicates the virial radius, and the black-pink dashed line indicates the shock radius.

equilibrium and the corresponding non-equilibrium cooling are explicitly included.

Despite these improvements, our one-dimensional spherical simulations do not capture many of the complexities of real astrophysical haloes. In particular, departures from spherical symmetry, as well as internal sources that drive winds and inject energy and radiation, are not included in our models. We discuss these limitations below.

### 6.1 Effects of non-spherical dynamics

While our current one-dimensional framework provides a useful baseline for understanding halo thermodynamics, it is important to delineate the limitations imposed by the assumption of smooth, isotropic accretion. In the standard  $\Lambda$ CDM paradigm, dark matter haloes reside in highly anisotropic large-scale environments. The dynamical and radiative processes that govern galaxy formation are inherently non-spherical and often unstable, and lead to interactions between different thermal phases, producing a multiphase medium. Cold accretion filaments penetrate hot haloes, energetic outflows may be launched from galactic discs, and satellite systems continuously interact with the surrounding gas. Such ‘local’ processes play a key role in structure formation and significantly disturb the gas dynamics and com-

position. As a result, realistic galactic haloes are complex systems that can differ substantially from our idealized spherical models (see e.g. B. D. Oppenheimer et al. 2018). This work is not intended to model the full multiphase CGM, but rather to isolate how ionization physics alone affects shock stability and large-scale ion distributions.

### 6.2 Effects of supernovae/AGN feedback on virial shocks

Our simulations neglect the extensive impact of feedback processes. Central galactic outflows driven by star formation bursts or active galactic nuclei (AGNs) inject significant amounts of energy, momentum, and metal-enriched material into the CGM (V. Springel & L. Hernquist 2003; A. C. Fabian 2012). These feedback mechanisms can fundamentally alter the cooling function,  $\Lambda(T, Z)$ , by redistributing metals within and beyond the virial radius ( $R_{\text{vir}}$ ) and by suppressing catastrophic cooling flows (R. S. Sutherland & M. A. Dopita 1993; K. Schawinski et al. 2007; J. Tumlinson et al. 2017).

The interplay between filamentary inflows and feedback-driven outflows can also generate thermal instabilities and drive departures from ionization equilibrium, potentially modifying both the shock stability and absorption-line signatures (D. Fielding et al. 2017). Additionally, radiation generated by star-forming

regions can also suppress radiative cooling behind the virial shock (K. C. Sarkar et al. 2022). Taken together, these effects suggest that both the stability of the virial shock and the resulting absorption-line signatures may differ from those predicted by our idealized model. A more complete treatment therefore requires further investigation using more comprehensive hydrodynamic simulations and surveys. Incorporating energy and radiation feedback on the CGM physics will be the focus of future work. While this limitation is important, we note that the current state-of-the-art 3D cosmological simulations vary wildly in their subgrid and feedback prescriptions. This leads to significantly different predictions, except for those features that the models have been explicitly tuned to reproduce. We therefore consider this limitation to be shared by most galaxy formation simulations.

### 6.3 Sensitivity to the UV background and metallicity

Our calculations adopt the F. Haardt & P. Madau (2012) metagalactic UV background. Alternative UV-background synthesis models, such as E. Puchwein et al. (2019) and C.-A. Faucher-Giguere (2020), predict somewhat different photoionization and photoabsorption heating rates, typically differing by factors of a few ( $\lesssim 2$ -3). We also adopt a simplified metallicity prescription, with fixed values for the IGM and CGM. These values are observationally motivated but are not unique, and represent a broader range of plausible metallicities.

Both the UV background model and the adopted metallicity affect the thermal state and ionization balance of the gas. In the context of shock stability, varying either is therefore expected to shift the quantitative value of the transition mass for stable shock formation in all four computational modes. However, we expect the qualitative result to remain unchanged: non-equilibrium ionization in initially cold gas enhances post-shock cooling and reduces post-shock pressure support, making stable shocks more difficult to sustain, whereas pre-ionization by a metagalactic UV background mitigates this effect and restores shock stability.

The predicted metal-line column densities are also sensitive to these assumptions. In particular, in B + 25, we demonstrated that metal-ion columns scale approximately linearly with metallicity in both the CGM and IGM. These uncertainties affect both the normalization of the predicted columns and the inferred relative contributions of the shocked halo and surrounding gas. However, the metallicity exploration presented in B + 25 suggests that, over a broad range of plausible metallicities, photoionized gas outside the shocked halo can remain an important contributor to the observed high-ion columns. The main dynamical conclusions regarding shock stability, the role of non-equilibrium cooling, and the suppression of these non-equilibrium effects by UV pre-ionization are less sensitive to the exact metallicity choice.

## 7 CONCLUSIONS

In this paper, we build upon the one-dimensional, spherically symmetric Lagrangian hydrodynamics code HYDRA (BD03, A. Dekel & Y. Birnboim 2006; Y. Birnboim et al. 2007) and extend it to include the effects of the redshift-dependent metagalactic UV background and departures from ionization equilibrium (S + 21) on the thermal evolution, stability, and absorption-line signatures of galactic haloes.

To isolate the impact of these additional physical ingredients, we compare the evolution of several models. As a baseline, we

consider an equilibrium model without a UV background (Eq), similar to that presented in BD03. We then introduce each effect separately: a radiation-free model with non-equilibrium ionization and cooling (nEq), and an equilibrium model that includes the UV background (Eq + UV). Finally, we combine both effects in a single model (nEq + UV), which provides the most realistic representation of astrophysical haloes.

We summarize our main results and conclusions as follows:

(i) Using the equilibrium radiation-free model, we reproduce the results of BD03, and recover the existence of a critical halo mass  $\sim 3 \times 10^{11} M_{\odot}$ , below which stable shocks never form.

(ii) Non-equilibrium ionization alone (nEq) reduces virial shock stability. This is because as the cold, neutral infalling IGM is shocked, it is significantly underionized relative to the post-shock temperature, and the resulting enhanced cooling reduces the post-shock pressure support. This shifts the critical mass for stable shock formation to higher values  $\sim 10^{12} M_{\odot}$ .

(iii) The metagalactic UV background pre-ionizes and heats the infalling IGM, producing an extended layer of warm, ionized gas extending well beyond the virial radius.

(iv) We find that departures from equilibrium are largely suppressed once the UV background is included. When the pre-ionized IGM is shocked, the non-equilibrium cooling enhancement largely disappears, returning the critical mass to the BD03 value,  $\sim 3 \times 10^{11} M_{\odot}$ .

(v) We find that the temperatures in the infalling IGM are not given by the standard photoabsorption heating-cooling thermal equilibrium solution, but are instead modified by cosmic expansion and gravitational collapse.

(vi) We show that our simulations produce O VI column densities comparable to observations in and around galactic haloes. We confirm the results of B + 25, that (in spherical models) gas outside the shocked halo can produce O VI absorption comparable to observations. Furthermore, the O VI column densities depend only weakly on halo mass.

(vii) Non-equilibrium ionization effects are generally small in the presence of a UV background, and departures from equilibrium typically do not strongly affect the observational signatures, except in rapidly cooling transient regions.

(viii) We also find that at  $z \sim 3$ , the photoionized IGM around galaxies can give rise to H I and C IV columns of  $\sim 10^{15-17} \text{ cm}^{-2}$  and  $10^{13-15} \text{ cm}^{-2}$ , respectively, consistent with several blind absorption-line surveys.

Although our approach is highly idealized compared to full three-dimensional cosmological simulations, our framework allows us to isolate and control the underlying microphysics, providing a clearer understanding of the processes governing metal-line absorption around galaxies. By systematically incorporating additional physical ingredients, we assess the role of each component in shaping the thermal and ionization structure of the gas. Future extensions of this work – including feedback, metal transport, and local radiation fields – will enable a more complete and controlled exploration of these effects, complementing the statistical insights from large cosmological simulations.

## ACKNOWLEDGEMENTS

This research was supported by the ISRAEL SCIENCE FOUNDATION (ISF grant no. 2527/25). We thank the anonymous referee for the constructive review and valuable suggestions, which have significantly improved the manuscript.

**DATA AVAILABILITY**

The data underlying this article will be shared on reasonable request to the corresponding author.

**REFERENCES**

- Appleby S., Davé R., Sorini D., Cui W., Christiansen J., 2023, *MNRAS*, 519, 5514
- Banerjee E., Muzahid S., Schaye J., Johnson S. D., Cantalupo S., 2023, *MNRAS*, 524, 5148
- Bergeron J., Aracil B., Petitjean P., Pichon C., 2002, *A&A*, 396, L11
- Birnboim Y., Dekel A., 2003, *MNRAS*, 345, 349 (BD03)
- Birnboim Y., Dekel A., Neistein E., 2007, *MNRAS*, 380, 339
- Borthakur S. et al., 2024, preprint ([arXiv:2409.12554](https://arxiv.org/abs/2409.12554))
- Bromberg I., Sarkar K. C., Ashkenazy H., Gnat O., Birnboim Y., 2025, *ApJ*, 987, 131 (B + 25)
- Bryan G. L., Norman M. L., 1998, *ApJ*, 495, 80
- Burchett J. N. et al., 2019, *ApJ*, 877, L20
- Carswell B., Schaye J., Kim T.-S., 2002, *ApJ*, 578, 43
- Cook A. W. S., van de Voort F., Pakmor R., Grand R. J. J., 2025, *MNRAS*, 543, 1224
- Cooper T. J. et al., 2021, *MNRAS*, 508, 4359
- Dekel A., Birnboim Y., 2006, *MNRAS*, 368, 2
- Dutta S. et al., 2026a, preprint ([arXiv:2602.17593](https://arxiv.org/abs/2602.17593))
- Dutta S. et al., 2026b, *MNRAS*, 545, staf2080
- Fabian A. C., 2012, *ARA&A*, 50, 455
- Faerman Y., Sternberg A., McKee C. F., 2017, *ApJ*, 835, 52
- Faerman Y., Sternberg A., McKee C. F., 2020, *ApJ*, 893, 82
- Faucher-Giguère C.-A., 2020, *MNRAS*, 493, 1614
- Ferland G. J. et al., 2017, *RMxAA*, 53, 385
- Fielding D., Quataert E., McCourt M., Thompson T. A., 2017, *MNRAS*, 466, 3810
- Fumagalli M., 2024, preprint ([arXiv:2409.00174](https://arxiv.org/abs/2409.00174))
- Girelli G., Pozzetti L., Bolzonella M., Giocoli C., Marulli F., Baldi M., 2020, *A&A*, 634, A135
- Gnat O., 2017, *ApJS*, 228, 11
- Gnat O., Sternberg A., 2007, *ApJS*, 168, 213
- Gnat O., Sternberg A., 2009, *ApJ*, 693, 1514
- Grayson S., Scannapieco E., Comparat J., ZuHone J. A., Zhang Y., Shree-ram S., Bruggen M., Bulbul E., 2025, *ApJ*, 994, 89
- Gupta A., Mathur S., Krongold Y., Nicastro F., Galeazzi M., 2012, *ApJ*, 756, L8
- Haardt F., Madau P., 2012, *ApJ*, 746, 125
- Ho S. H., Martin C. L., Schaye J., 2021, *ApJ*, 923, 137
- Johnson S. D., Chen H.-W., Mulchaey J. S., 2015, *MNRAS*, 449, 3263
- Lehner N., O'Meara J. M., Fox A. J., Howk J. C., Prochaska J. X., Burns V., Armstrong A. A., 2014, *ApJ*, 788, 119
- Lehner N., Wotta C. B., Howk J. C., O'Meara J. M., Oppenheimer B. D., Cooksey K. L., 2019, *ApJ*, 887, 5
- Lehner N. et al., 2022, *ApJ*, 936, 156
- McQuinn M., Werk J. K., 2018, *ApJ*, 852, 33
- Nelson D. et al., 2018, *MNRAS*, 477, 450
- Ng M., Nielsen N. M., Kacprzak G. G., Pointon S. K., Muzahid S., Churchill C. W., Charlton J. C., 2019, *ApJ*, 886, 66
- Nicastro F. et al., 2013, *ApJ*, 769, 90
- Oppenheimer B. D., Schaye J., 2013, *MNRAS*, 434, 1043
- Oppenheimer B. D., Schaye J., Crain R. A., Werk J. K., Richings A. J., 2018, *MNRAS*, 481, 835
- Oppenheimer B. D. et al., 2025, *MNRAS*, 543, 2649
- Prochaska J. X., Weiner B., Chen H. W., Mulchaey J., Cooksey K., 2011, *ApJ*, 740, 91
- Prochaska J. X., Werk J. K., Worseck G. et al., 2017, *ApJ*, 837, 169
- Puchwein E., Haardt F., Haehnelt M. G., Madau P., 2019, *MNRAS*, 485, 47
- Qu Z., Bregman J. N., 2018, *ApJ*, 862, 23
- Qu Z. et al., 2023, *MNRAS*, 524, 512
- Qu Z. et al., 2024a, *ApJ*, 968, 8
- Qu Z. et al., 2024b, *ApJ*, 968, 8
- Rees M. J., Ostriker J. P., 1977, *MNRAS*, 179, 541
- Rudie G. C., Steidel C. C., Shapley A. E., Pettini M., 2013, *ApJ*, 769, 146
- et al., 2024, *MNRAS*, 530, 3827
- Sarkar K. C., Sternberg A., Gnat O., 2021a, *MNRAS*, 503, 5807 (S + 21)
- Sarkar K. C., Gnat O., Sternberg A., 2021b, *MNRAS*, 504, 583
- Sarkar K. C., Sternberg A., Gnat O., 2022, *ApJ*, 940, 44
- Schawinski K., Thomas D., Sarzi M., Maraston C., Kaviraj S., Joo S.-J., Yi S. K., Silk J., 2007, *MNRAS*, 382, 1415
- Schaye J., Rauch M., Sargent W. L. W., Kim T.-S., 2000, *ApJ*, 541, L1
- Shull J. M., Tumlinson J., Giroux M. L., 2003, *ApJ*, 594, L107
- Silk J., 1977, *ApJ*, 211, 638
- Simcoe R. A., Sargent W. L. W., Rauch M., 2002, *ApJ*, 578, 737
- Simcoe R. A., Sargent W. L. W., Rauch M., 2004, *ApJ*, 606, 92
- Simcoe R. A. et al., 2011, *ApJ*, 743, 21
- Songaila A., Cowie L. L., 1996, *AJ*, 112, 335
- Springel V., Hernquist L., 2003, *MNRAS*, 339, 289
- Stern J., Hennawi J. F., Prochaska J. X., Werk J. K., 2016, *ApJ*, 830, 87
- Stern J., Faucher-Giguère C.-A., Hennawi J. F., Hafen Z., Johnson S. D., Fielding D., 2018, *ApJ*, 865, 91
- Stern J., Fielding D., Faucher-Giguère C.-A., Quataert E., 2019, *MNRAS*, 488, 2549
- Strawn C. et al., 2024, *ApJ*, 962, 29
- Sutherland R. S., Dopita M. A., 1993, *ApJS*, 88, 253
- Tchernyshyov K. et al., 2022, *ApJ*, 927, 147
- Tumlinson J., Thom C., Werk J. K. et al., 2011, *Science*, 334, 948
- Tumlinson J., Peebles M. S., Werk J. K., 2017, *ARA&A*, 55, 389
- Vasiliev E. O., 2011, *MNRAS*, 414, 3145
- Werk J. K. et al., 2016, *ApJ*, 833, 54
- White S. D. M., Rees M. J., 1978, *MNRAS*, 183, 341
- Wotta C. B., Lehner N., Howk J. C., O'Meara J. M., Oppenheimer B. D., Cooksey K. L., 2019, *ApJ*, 872, 81
- Yao Y., Shull J. M., Wang Q. D., Cash W., 2012, *ApJ*, 746, 166
- Zheng Y., Peek J. E. G., Putman M. E., Werk J. K., 2019, *ApJ*, 871, 35

This paper has been typeset from a  $\mathrm{T}_{\mathrm{E}}\mathrm{X}/\mathrm{L}^{\mathrm{A}}\mathrm{T}_{\mathrm{E}}\mathrm{X}$  file prepared by the author.