

CHAPTER - II

THEORY OF LIGHT PROPAGATION ALONG THE OPTIC AXIS OF A CHOLESTERIC LIQUID CRYSTAL-INFINITELY THICK SPECIMEN

INTRODUCTION

The effect of a plate of anisotropic material such as a crystal, on a collimated beam of polarised light may be represented mathematically as a linear transformation of the components of the electric vector of the light. The effect of a retardation plate, or an anisotropic absorber, or of a crystal possessing optical activity may be represented as a matrix operator which operates on the electric vector of the incident light. A plane wave of light characterised by the phases and amplitudes of the two transverse components of the electric vector, can be represented by a two matrix, with matrix elements in general complex. The above ideas were first developed in detail with various applications by R.C. Jones in a series of papers (1941 - 56) and the mathematical method has come to be known as the Jones calculus. *We shall discuss CS theory of the*
~~We shall now develop a theory of the~~ optical properties of an infinitely thick cholesteric liquid crystal for light propagation along the optic axis by the application of the Jones calculus.

Normal rotatory dispersion :

We consider here a helically arranged pile of birefringent

layers, neglecting at first the effect of reflection. It is known that each layer is only a few angstroms thick. If the pitch is comparable to the wave length of light the angle between adjacent layers will be of the order 10^{-2} radians. Now, the question arises whether such thin layer may be looked upon as birefringent plates. But it is well known that when an electromagnetic wave passes through a sheet of isotropic scattering material it acquires an additional phase $-q_0$ where $-iq_0$ is the complex amplitude scattered by the layer in the exact forward direction, when a plane wave of unit amplitude falls on it. This is equivalent to ascribing a refractive index to the layer. The argument may be extended to anisotropic layers as well.

We define a quantity γ which represents half the phase difference between waves polarised along the two principal axes after passing through a layer of thickness p

$$\text{i.e. } \gamma = \pi/\lambda (n_1 - n_2) \dots\dots\dots(1)$$

From the available data for pure cholesteric substances $(\Delta n) = (n_1 - n_2)$ is of the order of 0.075. Thus γ is 10^{-4} or less i.e. an order of magnitude smaller than θ .

We apply the Jones calculus to such a system assuming the light to be incident normal to the layers i.e. along Oz.

Let the principal axes of the first layer be inclined at an angle β with respect to OX and OY. If the Jones retardation matrix with respect to the principal axes is

$$G = \begin{bmatrix} e^{iV} & 0 \\ 0 & e^{-iV} \end{bmatrix}$$

where V is defined by (1).

The retardation matrix with respect to OX and OY is

$$J_1 = S G S'$$

where

$$S = S(\beta) = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$$

is the rotation matrix and $S' = S^{-1}$ is the transpose of S . So that $SS' = S'S = E$ the unit matrix (Jones 1941, Hurwitz and Jones 1941, see also Ramachandran and Ramaseshan 1961).

Now if D_0 is the complex, two dimensional column vector with respect to OX and OY representing the incident light, the emergent light vector after passing through the layer is

$$D_1 = J_1 D_0$$

When, one is interested only in the state of polarisation of the emergent beam, the factor $\exp(iV)$, where $V = \pi(n_1 + n_2)p/\lambda$ may be neglected. Let D_1 be now incident on a second

biefringent layer whose principal axes are inclined at 2θ with respect to OX and OY. The Jones matrix for this layer is

$$S(2\theta)GS'(2\theta) = S^2GS'^2$$

and the emergent vector is

$$\begin{aligned} D_2 &= S^2GS'^2D_1 = S^2GS'^2SGS' D_0 \\ &= S^2GS'S'SGS'D_0 \\ &= S^2 (GS')^2 D_0 \\ &= J_2 D_0 \end{aligned}$$

where $J_2 = S^2(GS')^2$ is the appropriate Jones matrix for this system of two layers. In general, if a pile of 'n' layers is considered, where the principal axes of the a^{th} layer are inclined at $m\theta$ with respect to OX, OY ($m = 1, 2, 3..n$) the Jones matrix for the pile is evidently

$$J_n S^n (GS')^n$$

We now need to calculate J_n explicitly. We adopt a direct proof given by Chandrasekhar and Srinivasa Rao (1968).

Let $A = GS'(\theta)$

$$\begin{aligned} &= \begin{bmatrix} e^{i\gamma} \cos \theta & e^{i\gamma} \sin \theta \\ -e^{-i\gamma} \sin \theta & e^{-i\gamma} \cos \theta \end{bmatrix} \quad \text{By matrix multiplication} \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (\text{Say}) \end{aligned}$$

Alma we see that $ad - bc = 1$ (2)

Therefore A is unimodular. The characteristic equation

$$|(A - \lambda E)| = \lambda^2 - \lambda(a + d) + 1 = 0$$

gives the eigen values

$$\left[\frac{a+d}{2} \pm \sqrt{\left(\frac{a+d}{2}\right)^2 - 1} \right]$$

which reduced to $\exp (\pm i\theta)$

by putting

$$\left[(a + d)/2 \right] = \cos \theta \quad \text{.....(3)}$$

Evidently

$$\begin{bmatrix} -b \\ a - e^{i\theta} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -b \\ a - e^{-i\theta} \end{bmatrix} \text{ may taken as the eigen vecto}$$

belonging respectively to the eigen values $e^{i\theta}$ and $e^{-i\theta}$.

Hence, with

$$T = \begin{bmatrix} -b & -b \\ a - e^{i\theta} & a - e^{-i\theta} \end{bmatrix}$$

we have

$$T^{-1} A T = \begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix}$$

and

$$A T = T \Lambda$$

$$A = T \Lambda T^{-1}$$

$$A^n = (T \Lambda T^{-1})^n = T \Lambda^n T^{-1}$$

: 6 :

$$= \begin{bmatrix} -b & -b \\ a-e^{i\theta} & a-e^{-i\theta} \end{bmatrix} \begin{bmatrix} e^{in\theta} & 0 \\ 0 & e^{-in\theta} \end{bmatrix} \begin{bmatrix} \frac{a-e^{i\theta}}{\Delta} & b/\Delta \\ \frac{a-e^{-i\theta}}{\Delta} & -b/\Delta \end{bmatrix}$$

where $\Delta = \det T = a^2 - 2ib \sin \theta$

Thus

$$A^n = \frac{1}{\sin \theta} \times \begin{bmatrix} a \sin \theta - \sin(n-1)\theta & b \sin \theta \\ \frac{-\sin n \theta (a^2 - 2a \cos \theta + 1)}{b} & \sin(n+1)\theta - a \sin n \theta \end{bmatrix}$$

Using (a) and (3) and the relation

$$\sin(n+1)\theta + \sin(n-1)\theta = 2 \sin n \theta \cos \theta$$

we get

$$A^n = \frac{\sin n \theta}{\sin \theta} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \frac{\sin(n-1)\theta}{\sin \theta} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= U(Z) A_{n-1} - U(Z) E_{n-2} \dots \dots \dots (4)$$

Where

$$U(Z)_n = \frac{\sin(n+1)\theta}{\sin \theta}$$

Now in the present problem $\cos \theta = (a+d)/2$ gives

$$\cos \theta = (a+d)/2 = \cos \beta \cos \gamma \dots (a)$$

The same result was derived by Abeles (1950) using the Lagrange-Sylvester interpolation formula. Born and Wolf (1959) have also derived the above result using an identity and have used it in discussion the light propagation in periodically stratified media.

Now the factor A in (4) is the effect of a single layer. Therefore, as far as the total effect of 'n' layers is concerned, 'n' being large, without any sensible error, we can put $A \approx S'$ i.e. $\exp(iV)$ is very large nearly = 1 in A.

Further, $\sin \theta$ and β are also small, though of an order of magnitude greater than V we may take

$$\begin{aligned}\sin \theta &\approx \sin \beta \approx \beta \\ \cos \theta &\approx \cos \beta \approx 1\end{aligned}$$

On simplification

$$(GS')^n \approx \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

Hence

$$\begin{aligned}J_n &= S^n (GS')^n \approx \begin{bmatrix} \cos n\beta & -\sin n\beta \\ \sin n(\beta-\theta) & \cos n(\beta-\theta) \end{bmatrix} \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos n(\beta - \theta) & -\sin n(\beta - \theta) \\ \sin n(\beta - \theta) & \cos n(\beta - \theta) \end{bmatrix} \dots\dots(6)\end{aligned}$$

J is just a rotation matrix with the angle of rotation as $\alpha = n(\phi - \theta)$. Therefore we see that the effect of 'n' layers is to produce a rotation of the incident vector in the positive direction.

Since β and γ are small, we have from (5)

$$\begin{aligned} (1 - \frac{\theta^2}{2!} + \dots) &= (1 - \frac{\beta^2}{2!} + \dots) (1 - \frac{\gamma^2}{2!} + \dots) \\ &\approx 1 - \frac{\beta^2}{2!} - \frac{\gamma^2}{2!} \end{aligned}$$

Therefore $\theta^2 \approx (\beta^2 + \gamma^2)$

$$\begin{aligned} \text{Hence } \alpha &= n \left[\beta - (\beta^2 + \gamma^2)^{1/2} \right] \\ &= n \left[\beta - \beta(1 + \gamma^2/\beta^2)^{1/2} \right] \\ &\approx n \left[\beta - \beta(1 + \gamma^2/2\beta^2) \right] \\ &= - \frac{n\gamma^2}{2\beta} \end{aligned}$$

If 'n' is the number of layers per turn of the helix of pitch p , $n\beta = 2\pi$ and $np = P$

Substituting from (1) the rotation per pitch is

$$\alpha = - \frac{\pi(n_1 - n_2)^2 p^2}{4 \lambda^2}$$

and the rotatory power in radians per unit length is

$$\rho = - \frac{\pi(n_1 - n_2)^2 P}{4 \lambda^2} \dots\dots(7)$$

The negative sign indicates that the sign of rotation is opposite to that of the helical twist or the structure. This formula and also that derived by Jones (1941) to demonstrate, some applications of his theory under the name 'Sohncke's theory of optical activity' by assuming a few layers in the initial calculation and then extrapolating for large number of layers. This formula is identical with the one derived by Mauguin (1911) and also equivalent to the de Vries equation when $\lambda^2 \ll 1$

An elementary theory of reflection :

The results of the theory discussed in the previous section show that the liquid crystal behaves as if it has circular birefringence; in other words right and left circular vibrations travel without change of form but with slightly different velocities.

We shall now give a simple mathematical interpretation of the reflection of circular light from such a structure: Let right circular light given by $D_0 = \frac{1}{1}$ referred to OX and OY, be incident normally on the surface of the liquid crystal. We shall suppose that the structure is right handed i.e. β is positive. After passing through m molecular layers, each of thickness 'p', the light vector will be $D_0 \exp(-i\phi)$ where $\phi = 2\pi n_d p / \lambda$, n_d = the refractive index for right circular light and λ the wavelength in vacuum.

To calculate the reflection at the boundary between m^{th} and $(m + 1)^{\text{th}}$ layer we resolve the light vector along the principal axes of the m^{th} layer which are inclined at an angle $m\theta$ with respect to OX and OY. The resolved components will be given by

$$\begin{bmatrix} \cos m\theta & \sin m\theta \\ -\sin m\theta & \cos m\theta \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \exp(i - i\phi) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \exp[i(m\theta - \phi)] \dots (8)$$

Since the principal axes of the $(m + 1)^{\text{th}}$ layer are rotated slightly with respect to those of the m^{th} layer one of the components of (8) will, on emerging from the m^{th} layer, meet a 'rarer' medium, whereas the other component will meet a 'denser' medium. Thus one component will get reflected without any change of phase and other with a change of phase of π . As a consequence, the sense of circular polarization remains the same after reflection. Referred to the principal axes of the m^{th} layer, the reflected wave on reaching the surface will be

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \exp[i(m\theta - 2\phi)] \dots (9)$$

neglecting a factor representing the reflection coefficient of the layer. Transforming back to OX and OY (9) becomes

$$\begin{bmatrix} \cos m\theta & -\sin m\theta \\ \sin m\theta & \cos m\theta \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \exp[i(m\theta - 2\phi)] \\ = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \exp[2i(m\theta - \phi)] \dots (10)$$

which is a right circular wave propagated along the negative direction of Oz. When $\lambda = a_d P$, $2\pi n_d p/\lambda = \beta$, a
 $\phi = m\beta$ since $np = P$ and $n\beta = 2\pi$ where 'n' is the number of molecular layers per turn of the helix. The phase factor $\exp 2i(m\beta - \phi)$ in (10) then becomes unity, irrespective of the values of m, and the waves reflected by the successive layers will all co-operate to give rise to a strong interference maximum. On the other hand, if the structure is left handed, i.e. β is negative, $(m\beta - \phi)$ does not vanish when $\lambda = n_d P$. Therefore, the reflected waves from different layers will not be in phase and the vibration will be transmitted unchanged.

It has been assumed in (9) and (10) that the reflection coefficients for the two linearly polarized components are the same. This may be quite true and it is possible that right circular light acquires a slight ellipticity after reflection from a single layer. However, if the amplitude of the waves reflected by different layers of the helical structure are added, the resultant reflected wave will evidently be circularly polarised.

As the average reflection coefficient may be calculated approximately as follows: For an arbitrary direction of the electric vector, the reflection coefficient at the boundary between the m^{th} and $(m+1)^{\text{th}}$ layers of the pile of helically

twisted birefringent layers is

$$|q| = \frac{n'_{m+1} + 1 - n'_m}{n'_{m+1} + 1 + n'_m} \dots\dots(11)$$

where n'_{m+1} and n'_m are the refractive indices of $(m+1)^{th}$ and m^{th} layer for a given direction of the electric vector.

The general expression for the variation of n' with direction is given by the well known equation

$$1/n'^2 = \cos^2 \alpha / n_1^2 + \sin^2 \alpha / n_2^2 \dots\dots(12)$$

(see eg. Ramachandran and Ramaseshan, 1961) where n_1 and n_2 are the principal refractive indices. We observe that when the birefringence is not too large, (12) may be represented quite accurately by the relation $n'_m = n - 1/2(\Delta n) \sin(2m\theta) \dots\dots(13)$

where $n = 1/2 (n_1 + n_2)$ and $n = n_1 - n_2$.

If we assume that

$$n'_{m+1} - n'_m \approx \beta (\Delta n)$$

$$|q| = \frac{\beta (\Delta n)}{2n}$$

The reflection coefficient per thickness P of the liquid crystal is then

$$Q = n|q| = \pi(\Delta n)/n \quad \dots\dots(14)$$

Since $n\beta = 2\pi$, where 'n' is the number of molecular layers per pitch of the helix. A better approximation, is to make use of the full expression (13). Substituting (13) in (11) and averaging over the different layers, we have

$$|q| \approx \frac{\beta (\Delta n)}{n\pi}$$

$$\text{and } Q = n|q| = 2(\Delta n)/n \quad \dots\dots\dots(15)$$

Dynamical Theory of reflection :

The complete solution of the problem has to take into account of multiple reflections. It can be done by setting up difference equations in a manner closely similar to that used by Darwin (1914) in his dynamical theory of X-ray diffraction. Such a procedure is valid because, circularly polarized components travel in the medium without change of form and the interference of multiply reflected waves with one another and with the primary wave can be evaluated directly.

Now for the purpose of developing this theory, we shall first regard the liquid crystal, as consisting of a set of parallel plates spaced at a distance 'P' apart. Each plane therefore replaces the n-molecular layers, per turn of the

helix of pitch P . Clearly $-iQ$ represents the over all effect of the reflections from the n -molecular layers, and is related to $-iq$, the reflection coefficient of a single molecular layer, in exactly the same way as the structure factor F of a crystal is related to the scattering factor f of an atom. Since F of a crystal is related to the scattering factor f of an atom. Since Darwin's theory can be applied to any crystal merely by substituting F for f , this model should be adequate for deducing the multiple-reflection characteristics of the liquid crystal. Here the reflection curve is derived as a function of wave length instead of the Bragg angle in the original theory. The structure is assumed to be right handed and that right circular light is incident normally. Let T_r and S_r be the complex amplitudes of the primary and reflected waves respectively at a point just above the plane. (The top most plane is designated by the serial number zero). The difference equations, suitably modified to the optical case, assuming that absorption is negligible can be written as (see eg. James (1954))

$$S_r = -iQ T_r + \exp(-i\phi) S_{r+1} \dots\dots\dots(16)$$

$$T_{r+1} = \exp(-i\phi) T_r - iQ \exp(-2i\phi) S_{r+1} \dots\dots(17)$$

where $\phi = 2\pi n_d P / \lambda$ and n_d is the refractive index for right circularly polarized light. The reflection coefficient is here taken to be the same on both sides of the plane.

Replacing r by $r-1$ in (17) and using (16) and (17) we obtain

$$T_{r+1} + T_{r-1} = [\exp(\phi) + \exp(-\phi) + Q^2 \exp(-\phi)] T_r \dots (18)$$

We assume a solution in the form

$$T_{r+1} = x T_r \dots (19)$$

Here x is independent of ' r '. Hence x must satisfy

$$x + 1/x = \exp(\phi) + \exp(-\phi) + Q^2 \exp(-\phi) \dots (20)$$

Substituting for T_{r+1} from (19) in (17) and using (16) we get

$$S_{r+1} = \frac{x - \exp(-\phi)}{x \exp(-\phi) - \exp(-2\phi) - Q^2 \exp(-2\phi)} S_r \dots (21)$$

from (20) and (21) we have

$$S_{r+1} = x S_r \dots (22)$$

As we have seen earlier the reflection condition is

$$n_d P = \lambda_0 \text{ or } \phi_0 = 2\pi$$

Let us put

$$\phi = 2\pi \lambda/\lambda_0 = 2\pi + \epsilon$$

where

$$\epsilon = -2\pi (\lambda - \lambda_0) 1/\lambda_0 \dots\dots(23)$$

which is a small quantity in the neighbourhood of reflection.

We have therefore from (20)

$$x + 1/x = \exp(i\epsilon) + \exp(-i\epsilon) + Q^2 \exp(-i\epsilon) \dots\dots(24)$$

This suggests that the neighbourhood of reflection we may put

$$x = \exp(-\xi) \quad (-i\epsilon_0) \quad (-\xi) \dots\dots(25)$$

where ξ is small and may be complex. From (24) and (25) we have

$$\xi \approx \pm (Q^2 - \epsilon^2)^{1/2} \dots\dots\dots(26)$$

Using (22), (16) and (26) on putting $V = 0$ we obtain the ratio of the reflected to the incident amplitudes

$$\frac{S_0}{T_0} = - \frac{Q}{\epsilon \pm (Q^2 - \epsilon^2)^{1/2}} \dots\dots\dots(27)$$

When $-Q < \epsilon < Q$, (27) may be expressed as

$$\frac{S_0}{T_0} = - \frac{Q}{\epsilon \pm (Q^2 - \epsilon^2)^{1/2}} \dots\dots\dots(28)$$

and hence

$$R = |S_0/T_0|^2 = 1$$

The reflection is therefore total within this range. Outside this range the reflection decreases rapidly and symmetrically on either side. It will be seen from (23) that when λ is greater than λ_0 , ϵ is negative and so the negative value of the square root in the denominator of (27) has to be taken, because R can never exceed unity. Similarly when λ is less than λ_0 the positive value has to be taken.

In the discussion above we have assumed the liquid crystal to consist of a set of parallel planes with a reflection coefficient $-iq$. Clearly $-iq$ represents the over all effect of reflection from n -molecular layers and is related to $-iq$ the reflection coefficient of a single molecular layer as follows

$$Q = q \left[1 + \exp(-2i\delta) + \exp(-2i\delta) - \exp(-2ni\delta) \right]$$

$$= q \left[\frac{1 - \exp(-2in\delta)}{1 - \exp(-2i\delta)} \right]$$

where

$$\delta = 2\pi n_0 P / n \lambda = \phi / n$$

Therefore

$$|Q|^2 = q^2 \sin^2 n\delta / \sin^2 \delta = n^2 q^2 (\sin n\phi / \phi)^2$$

In the neighbourhood of reflection

$$Q = Q_0^2 (\sin \epsilon / \epsilon)^2$$

Spectral width of total reflection and primary extinction.

The range of total reflection extends from $\epsilon = +Q$ to $\epsilon = -Q$ i.e. from

$$\lambda_0 - \frac{Q\lambda}{2\pi} \quad \text{to} \quad \lambda_0 + \frac{Q\lambda}{2\pi}$$

The spectral width of reflection ($\Delta\lambda$) is therefore

$$\Delta\lambda = \frac{Q\lambda}{\pi} \approx \frac{Q\lambda_0}{\pi}$$

Now using (14) we get $\Delta\lambda = P\Delta n$ in exact agreement with the de Vries theory. The better approximation (15) gives

$$= \frac{2P\Delta n}{\pi}$$

In the solution of difference equation we have assumed that the amplitude of the primary beam is changed by a factor $\exp(-\xi)$ on passing from one plate to the next. Within the total reflection Q^2 is greater than ϵ^2 so that ξ is real and primary extinction takes place. To evaluate the extinction coefficient we first calculate the mean value of ξ within the total reflecting range.

$$\xi = \frac{1}{Q} \int_0^Q (Q^2 - \epsilon^2)^{1/2} d\epsilon = \pi Q/4$$

The intensity falls by a factor $\exp(-2\xi)$ on passing through a thickness P as by $\exp(-\pi Q/2P)$ per unit length.

Outside the region of total reflection ξ is imaginary

and primary extinction vanishes. We shall give some theoretically expected magnitudes of spectral width of total reflection and primary extinction for some typical values of the constants involved. Taking $Q_0 = 0.1$ and

$\lambda_0 = 6 \times 10^{-5}$ cms and $n_d = 1.8$ and $P = 4000$ Å the width of reflection turns out as $\sqrt{191}$ Å. The extinction coefficient turns out as 3.92×10^3 . Thus the intensity is reduced to about 1/1000 of its incident value after passing through a thickness of about 13μ .

Anomalous rotatory dispersion : In the region of normal dispersion the optical rotation per thickness P of the liquid crystal is given by

$$1/2 (\phi_d - \phi_l) = \pi P / \lambda (n_d - n_l) = -\pi V^2 / 2\beta$$

where ϕ_d, ϕ_l are the phase retardations and n_d and n_l the refractive indices for right and left circular vibrations. (The convention adopted in these books for right and left circular vibrations here is that a clockwise rotation as seen along the direction of propagation of light is taken as positive). In the region of reflection the dynamical theory shows that the right circular component suffers an additional phase retardation and also under certain circumstances attenuation as it travels through the medium. Left circular light on the other hand, exhibits normal behaviour throughout and as a consequence the rotatory dispersion is anomalous around the reflection region.

Case 1 : Region of total reflection.

According to (25) the amplitude of the right circular wave as it passes from one place to the next is given by

$$T_{r+1} = x T_r$$

where

$$x = \exp(-\xi) \exp(-i\phi_0)$$

$$\xi = (Q^2 - e^2)^{1/2}$$

$$\phi_0 = \phi_d - c = \frac{2\pi n_d P}{\lambda} + 2\pi(\lambda - \lambda_0)/\lambda$$

Inside the totally reflecting range ξ is real and therefore the medium becomes highly circularly dichroic. It is readily seen that the ellipticity χ produced per thickness P is given by

$$\tan \chi = \frac{1 - \exp(-\xi)}{1 + \exp(\xi)} = \tanh(\xi/2)$$

$$\text{or } \chi \simeq \xi/2$$

The azimuth of the major axis of the ellipse after passing through a thickness P is

$$\begin{aligned} \alpha &= \frac{1}{2}(\phi_0 - \phi_1) = \frac{\pi P}{\lambda}(n_d - n_1) + \frac{\pi(\lambda - \lambda_0)}{\lambda} \\ &= -\frac{n \sqrt{2}}{2\beta} + \frac{\pi(\lambda - \lambda_0)}{\lambda} \end{aligned}$$

Therefore the rotatory power in radians per cm.

$$= \frac{-\pi(n_1 - n_2)^2 P}{4 \lambda^2} + \frac{\pi(\lambda - \lambda_0)}{P \lambda}$$

which is valid within the range $\lambda_0 - Q \lambda / 2\pi < \lambda < \lambda_0 + Q / 2\pi$

$$\text{and } \rho = \frac{-18(n_1 - n_2)^2 P}{4 \lambda^2} + \frac{18(\lambda - \lambda_0)}{P \lambda} \dots\dots(29)$$

expressed in degrees

Case 2 :

Outside the region of total reflection.

Outside the totally reflection range $= 1(\epsilon^2 - Q^2)^{1/2}$

and may be positive or negative depending on whether ϵ given by (23) is positive or negative. Therefore

$$\begin{aligned} \alpha &= \frac{1}{2} \left[(\epsilon^2 - Q^2)^{1/2} + \delta_0 - \delta_1 \right] \\ &= \frac{1}{2} \left[(\epsilon^2 - Q^2)^{1/2} + \delta_0 - \epsilon - \delta_1 \right] \\ &= -\frac{n\sqrt{2}}{2\beta} - \frac{\epsilon}{2} \left[1 - (1 - Q^2/\epsilon^2)^{1/2} \right] \\ &= -\frac{n\sqrt{2}}{2\beta} + \frac{\pi(\lambda - \lambda_0)}{\lambda} \left[1 - (1 - Q^2/\epsilon^2)^{1/2} \right] \end{aligned}$$

Hence the rotatory power in radians per cm.

$$= \frac{-(n_1 - n_2)^2 P}{4 \lambda^2} + \frac{\pi(\lambda - \lambda_0)}{P \lambda} \left[1 - (1 - Q^2/\epsilon^2)^{1/2} \right]$$

and the rotatory power in degrees/mm. turns out to be

$$= \frac{-18(n_1 - n_2)^2 P}{4 \lambda^2} + \frac{18(\lambda - \lambda_0)}{P \lambda} \left[1 - (1 - Q^2/\epsilon^2)^{1/2} \right]$$

We give here theoretical rotatory dispersion and reflection

curve for some typical values of the constants, taking into consideration the relation between Q_0 and (Δn) and also the variation of (Δn) with λ (Fig.1). *

$$\lambda_0 = 6000 \text{ \AA}^0; \quad n_d = 1.5$$

$$P = 4000 \text{ \AA}; \quad Q_0 = 0.1 \text{ (or } (\Delta n) = 0.075 \text{ at } \lambda_0).$$

The layer birefringence is assumed to vary linearly from 0.1 at 4000\AA to 0.075 at 6000 \AA and extrapolated to longer wavelengths. The curve gives the rotatory power (in degrees/mm) of a right handed structure using the standard convention according to which a clockwise rotation as seen by an observer looking at the source of light is taken positive. Except that the theory gives an expression for the rotatory power inside the region of reflection, the curves are similar to what is expected from the de Vries equation. The rotatory power inside the region of reflection cannot, however, be measured for a very thick specimen, since one of the circular components is completely attenuated. Nevertheless, the result is theoretically significant in that it brings out how the phase difference between the two opposite circular waves (or equivalently how the azimuth of the major axis of the elliptically polarized wave) varies with λ within the reflection band.

Fig.1 Theoretical rotatory dispersion and reflection curves for an infinitely thick specimen.

