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Scaling properties of non-linear gravitational clustering

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ABSTRACT

Hamilton et al. recently proposed the idea that the growth of density perturbations in an expanding universe is governed by a general scaling law, and showed agreement with existing numerical simulations. We examine the possible origin of this scaling behaviour in more detail. The underlying equations of motion are cast in a suggestive form, and motivate a conjecture that the scaled pair velocity, $h(x, a) \equiv -[v/(\dot{a}x)]$, depends on the expansion factor a and comoving coordinate x only through the density contrast $\overline{\xi}(x, a)$ (the two-point correlation averaged over a sphere of radius x). This leads naturally to the proposed scaling law – the true non-linear density contrast is a universal function of the density contrast $\overline{\xi}_{\rm L}(l, a)$, computed in the linear theory and evaluated at a scale l which is derived to be $l = x(1+\overline{\xi})^{1/3}$. Apart from basing the proposed scaling form on an explicit dynamical hypothesis, this gives a convenient solution for the scaling function in terms of the input pair velocity. Possibilities for further elaboration of this approach in interpreting simulations of non-linear gravitational clustering are briefly discussed.

Key words: galaxies: clustering – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

It is generally believed that large-scale structures in the Universe formed through the growth of small inhomogeneities via gravitational instability. One convenient measure of the inhomogeneities present in the Universe at any time t is provided by the mean-square fractional fluctuation $\sigma^2(x, t) = \langle (\delta M/M)_x^2 \rangle$ in the mass contained within a sphere of comoving radius x at cosmic epoch t. Another closely related measure is the mean value of the two-particle correlation function ξ within a sphere of radius x, which is written as $\overline{\xi}(x)$. When $\sigma^2 \ll 1$, the evolution of inhomogeneities can be understood using linear perturbation theory in an expanding background. The non-linear phase, when σ^2 grows beyond a value of order unity, is usually handled by numerical simulations, and analytic methods seem to have very limited validity. Our understanding of gravitational instability would increase significantly if the results of numerical simulations could be understood in the framework of simple physical concepts and analytic approximations. Much of the pioneering work on analytic approaches, especially in the self-similar case, is described in the text by Peebles (1980, henceforth P). One interesting recent suggestion is due to Hamilton et al. (1991, henceforth HKLM). Their proposal is that $\overline{\xi}(x, t)$ is a universal function of $\overline{\xi}_{\rm L}(l, t)$ which is the initial value of $\overline{\xi}$ at a different scale l related to x, extrapolated to the epoch t using *linear* theory, i.e. growing as the square of the expansion factor a. The relation between the two scales is postulated to be $l = x[1+\overline{\xi}(x, t)]^{1/3}$. For fixed l, HKLM term $x(t)$ a 'conserved pair surface', because, by construction, the mean number of neighbours of a given particle within l in the linear regime equals the number within x in the non-linear regime. In their treatment, the starting assumption is that 'the time evolution of the physical radius x of a conserved pair surface is a universal function of the cosmic scale factor, suitably scaled'. They then show that this leads to the scaling form for $\overline{\xi}$ which is consistent with the earlier numerical simulations of Efstathiou et al. (1988), use these to extract the scaling function $\overline{\xi}(\overline{\xi}_{\rm L})$, and apply the result to observations. For reasons elaborated below, we feel that there is room for improvement in the understanding of this scaling law for $\overline{\xi}$, and this is the aim of our paper, which is organized as follows. The next section reviews clustering in an expanding universe and stresses certain properties of the resulting equations. As is well known, they admit the possibility of a similarity solution in which the statistical properties of density fluctuations at different times are identical apart from a change of spatial scale. This solution is used to motivate a relation between the non-linear and linear density contrasts which is applicable even in the general non-self-similar case. The simplest relation does not work very well, but brings out

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the key problem of the transition between the linear and extreme non-linear regimes, i.e. the relationship between x and l. In Section 3, which contains the main results of the paper, this matching is *deduced* from the pair continuity equation and a physically motivated ansatz for the pair velocity. Section 4 discusses implications and possibilities for extension of our theory.

2 SELF-SIMILAR CLUSTERING

This section reviews some known aspects of clustering in a framework somewhat different from, although equivalent to, the standard approach. Consider a set of particles, all of mass m, interacting via Newtonian gravity in an expanding universe at the critical density. The equation of motion for the ith particle is

$$\ddot{\boldsymbol{r}}_i = -\sum_{j\neq i} \frac{Gm\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|^3} \equiv -\nabla\Phi, \qquad \boldsymbol{r}_{ij}=\boldsymbol{r}_i-\boldsymbol{r}_j,$$

$$\nabla^2\Phi = 4\pi Gm\sum \delta(\boldsymbol{r}-\boldsymbol{r}_i). \tag{1}$$

Here $\boldsymbol{r}_i$ stands for the proper coordinate related to the comoving coordinate $\boldsymbol{x}_i$ by the expansion factor $a(t)$: $\boldsymbol{r}_i = a(t)\boldsymbol{x}_i$. In writing a common potential Φ for all the particles, we implicitly ignore the restriction $j\neq i$ and hence work in the continuum or fluid limit. Choosing $[a(t)/a_0] = (t/t_0)^{2/3}$, the expansion factor in a matter-dominated $\Omega = 1$ universe, one can derive the equations of motion in terms of the comoving coordinates $\boldsymbol{x}_i$. These have two features (P): an extra repulsive force $\propto \boldsymbol{x}$ cancelling the long-range part of gravity (which has become subsumed in the deceleration of the expansion); and a time-dependent damping term proportional to $\dot{x}$. It is useful to transform these equations so that the time dependence disappears. Introducing new dimensionless time and space coordinates τ and $\boldsymbol{q}_i$ via

$$\tau \equiv \ln(t/T), \qquad \boldsymbol{q}_i = \boldsymbol{x}_i/L, \qquad L^3 = Gmt_0^2/a_0^3, \tag{2}$$

with arbitrary constants T and a_0, and transforming equations (1), we find

$$\frac{\mathrm{d}^2\boldsymbol{q}_i}{\mathrm{d}\tau^2} + \frac{1}{3}\frac{\mathrm{d}\boldsymbol{q}_i}{\mathrm{d}\tau} = -\nabla_q U, \tag{3}$$

with

$$\nabla_q^2 U = 4\pi\sum_j \delta(\boldsymbol{q}-\boldsymbol{q}_j) - \frac{2}{3}. \tag{4}$$

Equations (3) and (4) have several noteworthy features, which we list below.

(1) The compensating negative background density, of $(-1/6\pi)$ in the units chosen, cancels the long-range part of the potential arising from the mean density ρ_0. The usual assumption made in the analysis of the Jeans instability is thus justified *in these coordinates*. One obtains exponential growth in $\tau = \ln t$, i.e. power-law growth in t. More precisely, the analysis without the damping term would have led to modes $\exp(\gamma\tau)$ with $\gamma^2 = 4\pi G\rho_0 = 2/3$. With this term, one obtains $\gamma^2 + \gamma/3 = 2/3$, i.e. $\gamma = 2/3$ or -1. The damping term has ensured that the growth and decay rates of the two modes are *not* equal, and one recovers the correct $t^{2/3}$ and t^{-1} growing and decaying modes.

(2) The damping term is also needed to recover other known results. For example, two bound bodies spiral inwards in $\boldsymbol{q}$ with a shrinking period in τ, this being just an alternative description of a bound system with a fixed size in $\boldsymbol{r}$ and period in t. This regime operates in regions with densities much greater than the mean, so that the repulsion does not contribute. It is also instructive to look at the free evolution of a test particle in an otherwise uniform universe. Writing $p_i = \mathrm{d}q_i/\mathrm{d}\tau$, one gets $\mathrm{d}p_i/\mathrm{d}\tau = -p_i/3$, i.e. $p_i \propto \exp(-\tau/3) = t^{-1/3}$. This is nothing but the well-known result (expressed in terms of p) that the peculiar velocity of a particle decays as the inverse of the scale factor.

(3) Particles at the boundary of an underdense region feel the strong repulsion coming from the locally uncompensated negative background term, which pushes them further out until they enter the domain of repulsion of an adjacent such region.

(4) The damping term suggests a possible way of interpreting the success of adhesion models (Bagla & Padmanabhan 1994).

(5) The translation invariance in τ would have led to a conventional interacting system of particles *in the absence of the damping term*. Its presence reminds us that we should not be trying to apply statistical mechanics in the usual way, quite apart from the well-known difficulties associated with the singular short-range behaviour of Newtonian gravity.

(6) The usual calculation (multiplication by $\dot{\boldsymbol{q}}$ and integration) which leads to the conservation of energy for a time-independent potential will now show, because of the damping term, that the 'energy' in these coordinates always decreases, being constant only in the trivial and unstable unperturbed case in which all 'velocities' and 'accelerations' are zero. This decreasing 'energy' can be viewed as the driving force behind the continued evolution with formation of bound structures and evacuation of voids. In fact, the form of behaviour closest to a steady state that is known for this system is self-similar clustering (P).

We now describe the self-similar situation. Let us suppose that the power spectrum at some epoch has the form $P(k)\propto k^n$, at sufficiently large scales that are in the linear regime. Let $\sigma^2(x, t)$ denote the variance in the fractional density fluctuation smoothed over a sphere of radius x, and $\sigma_{\mathrm{L}}^2(x, t)$ the same quantity in the linear regime. Then $\sigma_{\mathrm{L}}^2(x) \propto k^3P(k) \propto x^{-(3+n)}$ at the early, linear stage. Since σ grows as a in the linear theory, it follows that $\sigma_{\mathrm{L}}^2(a, x) \propto a^2x^{-(n+3)}$. This evolution is self-similar with $\sigma_{\mathrm{L}}^2 \propto s^{-(n+3)}$ and $s = (x/t^\alpha)$; $\alpha = [4/3(3+n)]$. In the extreme non-linear case, bound structures with fixed proper radius $r = a(t)x$ will not participate in the cosmic expansion. At a fixed r, $\sigma_{\mathrm{NL}}^2 = \langle\langle \delta\rho/\rho_{\mathrm{b}})_r^2\rangle$ must now grow as $(a^6/a^3) = a^3$. The reason is that $\delta\rho$ for a bound object is fixed, while the a^6 factor occurs because of the facts that the background density ρ_{b} decreases as a^{-3} and $\sigma^2 \propto \rho_{\mathrm{b}}^{-2}$. The a^3 factor in the denominator arises from the fact that, of all samples of proper radius r used in computing the variance, only a fraction proportional to a^{-3} will contain bound objects and contribute. Thus, in the extreme non-linear regime, $\sigma_{\mathrm{NL}}^2(x, t) \propto a^3(t)F[a(t)x]$. Assuming a power law for F and matching the linear and non-linear expressions at, say,

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$\sigma = \sigma_c$, the standard result is obtained (P) that $\sigma_{NL}^2 \propto a^3[ax]^{-\gamma}$ with $\gamma = 3(n+3)/(n+5)$. Thus

$$\sigma_L^2(a, x) \propto a^2(t) x^{-(n+3)}, \qquad \sigma_{NL}(x, a) = \sigma_L(l, a), \quad l = x \quad (\text{for } \sigma_{NL} \leq \sigma_c). \tag{5}$$

One can restate this as a differential equation for σ_{NL} which admits generalization to the case in which the spectrum is not a pure power law. Note that, in the extreme non-linear regime,

$$\frac{\partial \ln \sigma}{\partial \ln a} = \frac{3}{(n+5)}, \qquad \frac{\partial \ln \sigma}{\partial \ln x} = \frac{3(n+3)}{2(n+5)}. \tag{6}$$

Eliminating n,

$$\frac{\partial \ln \sigma}{\partial \ln a} - \frac{\partial \ln \sigma}{\partial \ln x} = \frac{2}{3}. \tag{7}$$

Such an equation can be interpreted by saying that, at each epoch and scale, one is evolving σ using the result of the self-similar solution for the currently and locally applicable value of n. This equation is assumed to hold for $\sigma > \sigma_c$. For $\sigma < \sigma_c$ we use the rather trivial evolution equation of the linear theory:

$$\frac{\partial \ln \sigma}{\partial \ln a} = 1. \tag{8}$$

Equations (7) and (8) are to be interpreted as an initial-value problem. At some very early epoch (say, at $a = a_{rec}$) we are given the density contrast $\sigma(x, a_{rec})$. We will assume that the initial epoch is chosen sufficiently early that $\sigma_{rec} \ll 1$ at all relevant scales. With this initial condition, the above equations can be integrated forward for all further times, thereby giving $\sigma(x, a)$. This approach clearly does not require σ to be a power law.

Equation (7) can easily be integrated, the general solution for $\sigma > \sigma_c$ being

$$\ln \sigma_{NL}(x, a) = \frac{3}{2} \ln\left(\frac{a}{a_{rec}}\right) + F(ax), \tag{9}$$

where F is a function to be determined by matching σ_{NL} and σ_L at $\sigma = \sigma_c$. Since the linear density contrast evolves as $\sigma_L \propto a$, it follows that we can relate σ_{NL} to σ_L as follows:

$$\left[\frac{\sigma_{NL}(x, a)}{\sigma_c}\right] = \left[\frac{\sigma_L(l, a)}{\sigma_c}\right]^{3/2}, \qquad l = x\left[\frac{\sigma_{NL}(x, a)}{\sigma_c}\right]^{3/2} \quad (\text{for } \sigma_{NL} > \sigma_c),$$
$$\sigma_{NL}(x, a) = \sigma_L(l, a), \quad l = x \quad (\text{for } \sigma_{NL} \leq \sigma_c). \tag{10}$$

This shows that, when $\sigma > \sigma_c$, σ_{NL} at a point x is determined by σ_L at a point $l = x(\sigma_{NL}/\sigma_c)^{2/3}$. In other words, non-linear evolution does introduce the transfer of information from larger to smaller comoving scales in this approximation.

Equation (7) is merely the simplest one that can be written down in which the power-law index n is eliminated. It does not make a smooth transition from linear to non-linear scales, and does not provide us with a value for σ_c. Nevertheless the solution (10) suggests a very simple ansatz. It may be possible to take the true density contrast, $\sigma_{NL}(x, a)$, to be a universal function of $\sigma_L[f(x, \sigma_{NL}), a]$, where $f(x, \sigma_{NL}) \simeq x$ for $\sigma_{NL} \ll 1$ and $f(x, \sigma_{NL}) \propto x\sigma_{NL}^{2/3}$ for $\sigma_{NL} \gg 1$. In such a more realistic theory, we expect f to vary smoothly from one limit to the other. Note that the conserved pair surface of HKLM corresponds to $f = x(1+\sigma^2)^{1/3}$ and satisfies these two limits. Clearly, this is only one possible way of interpolating between the two regimes; $f(x, \sigma) = x(1+10\sigma^6)^{1/9}$ would do the same. Knowledge of the relation between x and l in the two extreme limits leaves considerable freedom in interpolating between them. In the next section, the scaling relation is obtained from the equation governing the evolution of density perturbations, and the appropriate form of f is *deduced*. The function $\sigma_{NL}(\sigma_L)$ remains undetermined, but is expressed in terms of the pair velocity.

3 SCALING IN THE NON-SELF-SIMILAR CASE

Expressing the correlation function $\xi(x, t)$ as the Fourier transform of the power spectrum,

$$\xi(\boldsymbol{x}, t) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} P(\boldsymbol{k}, t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \tag{11}$$

we have $\bar{\xi}(\boldsymbol{x}, t)$, the average correlation within a sphere of radius x, defined by

$$\bar{\xi} = \frac{3}{x^3} \int_0^x \xi(t, y) y^2 \, dy. \tag{12}$$

The density contrast in a sphere of radius x is given by the following weighted average of ξ:

$$\sigma^2(x, t) = \frac{3}{x^3} \int_0^{2x} \xi(y, t) W(y) y^2 \, dy, \tag{13}$$

where $W(y) = [1-(y/2x)]^2[1+(y/4x)]$ equals 1 at $y = 0$, vanishes at $y = 2x$, and has the same integral as the (unity) weight in (12). With smoothly varying ξs, (12) and (13) are numerically quite close, which is why we have so far treated both $\bar{\xi}$ and σ^2 as nearly equivalent measures of density contrast for preliminary discussion.

We shall now obtain an equation satisfied by $\bar{\xi}$, following the treatment (P) for the evolution of the correlation function ξ. The mean number of neighbours within x of any particle is given by

$$N(x, t) = n \int_0^x 4\pi y^2 \, dy[1 + \xi(y, t)], \tag{14}$$

where n is the comoving number density. The conservation law for pairs hence implies that

$$\frac{\partial \xi}{\partial t} + \frac{1}{ax^2} \frac{\partial}{\partial x}[x^2(1+\xi)v] = 0, \tag{15}$$

where $v(t, x)$ denotes the mean relative velocity of pairs at separation x and epoch t. Using (12), we find

$$(1+\xi) = \frac{1}{3x^2} \frac{\partial}{\partial x}[x^3(1+\bar{\xi})]. \tag{16}$$

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Substituting this in (15), we obtain

$$\frac{1}{3x^2}\frac{\partial}{\partial x}\left[x^3\frac{\partial}{\partial t}(1+\bar{\xi})\right]=-\frac{1}{ax^2}\frac{\partial}{\partial x}\left\{\frac{v}{3}\frac{\partial}{\partial x}[x^2(1+\bar{\xi})]\right\}. \quad (17)$$

Integrating, we find

$$x^3\frac{\partial}{\partial t}(1+\bar{\xi})=-\frac{v}{a}\frac{\partial}{\partial x}[x^3(1+\bar{\xi})]. \quad (18)$$

The integration allows the addition of an arbitrary function of t on the right-hand side. We have set this function to zero so as to reproduce the correct limiting behaviour (see below). It is now convenient to change from t to a, thereby obtaining an equation for $\bar{\xi}$:

$$a\frac{\partial}{\partial a}[1+\bar{\xi}(a,x)]=\left(\frac{v}{-\dot{a}x}\right)\frac{1}{x^2}\frac{\partial}{\partial x}\{x^3[1+\bar{\xi}(a,x)]\} \quad (19)$$

or, defining $h(a,x)=-(v/\dot{a}x)$,

$$\left(\frac{\partial}{\partial \ln a}-h\frac{\partial}{\partial \ln x}\right)(1+\bar{\xi})=3h(1+\bar{\xi}). \quad (20)$$

This equation shows that the behaviour of $\bar{\xi}(a,x)$ is essentially decided by h, the dimensionless ratio between the mean relative velocity v and the Hubble velocity $\dot{a}x=(\dot{a}/a)x_{\rm prop}$, both evaluated at scale x. To understand the behaviour of this equation, let us consider its solutions in two limiting cases. In the extreme non-linear limit, peculiar motion is assumed to compensate for the Hubble expansion to form bound structures; hence $h=(v/-\dot{a}x)=1$, giving

$$a\frac{\partial}{\partial a}(1+\bar{\xi})-x\frac{\partial}{\partial x}(1+\bar{\xi})=3(1+\bar{\xi}). \quad (21)$$

This equation has the general solution $(1+\bar{\xi})=a^3F(ax)$ [compare with equation (9) earlier]. In the linear limit, we know that $h\ll 1$ (small peculiar velocities). Ignoring the second term on the left-hand side of (20), we obtain

$$\left(\frac{v}{-\dot{a}x}\right)=\frac{a}{3}\frac{\partial}{\partial a}(\bar{\xi}). \quad (22)$$

This is consistent with the linear theory result $\sigma_{\rm L}^2\propto a^2$ only if $(v/-\dot{a}x)=(2/3)\bar{\xi}$, which is the known limiting behaviour of the pair velocity for $\bar{\xi}\ll 1$ (P). Incidentally, this property singles out $\bar{\xi}$ over other measures of density contrast, such as σ, which weight the correlation function differently.

Equation (20) is exact, but not closed, in the sense that the scaled pair velocity h on the right-hand side is unknown in general. In the BBGKY framework, one derives equations for h that contain still higher (unknown) correlation functions. If, on the other hand, one wants to stop with $\bar{\xi}$, one needs to express $h(x,a)$ as a functional of $\bar{\xi}(x,a)$. The simplest possibility is a local function,

$$h(x,a)=H[\bar{\xi}(x,a)], \quad (23)$$

and this is consistent with the extreme linear limit $H=2\bar{\xi}/3$ and the extreme non-linear limit $H=1$. [This step can be viewed as analogous to using the local value of n in deriving (7) from (6) in Section 2.]

When $h(x,a)=H[\bar{\xi}(x,a)]$, it is possible to find a solution to (23) that reduces to the form $\bar{\xi}\propto a^2$ for $\bar{\xi}\ll 1$, as follows.

Let $A=\ln a$, $X=\ln x$ and $D(X,A)=(1+\bar{\xi})$. We define curves (characteristics) in the X, A, D space that satisfy

$$\left.\frac{{\rm d}X}{{\rm d}A}\right|_{\rm c}=-H(D[X,A]), \quad (24)$$

i.e. the tangent to the curve at any point (X, A, D) is constrained by the value of H at that point. Along this curve, the left-hand side of (20) is a total derivative, allowing us to write it as

$$\left[\frac{\partial D}{\partial A}-h(D)\frac{\partial D}{\partial X}\right]_{\rm c}=\left(\frac{\partial D}{\partial A}+\frac{\partial D}{\partial X}\frac{{\rm d}X}{{\rm d}A}\right)_{\rm c}\equiv\left.\frac{{\rm d}D}{{\rm d}A}\right|_{\rm c}=3HD. \quad (25)$$

This determines the variation of D along the curve. Integrating,

$$\exp\left[\frac{1}{3}\int\frac{{\rm d}D}{DH(D)}\right]=\exp(A+c)\propto a. \quad (26)$$

Squaring and determining the constant from the initial conditions at a_0, in the linear regime,

$$\exp\left[\frac{2}{3}\int_{\bar{\xi}(a_0,l)}^{\bar{\xi}(x)}\frac{{\rm d}\bar{\xi}}{h(\bar{\xi})(1+\bar{\xi})}\right]=a^2/a_0^2=\frac{\bar{\xi}_{\rm L}(l,a)}{\bar{\xi}(l,a_0)}. \quad (27)$$

We now need to fix the scale l. Equation (24) can be written, using equation (25), as

$$\frac{{\rm d}X}{{\rm d}A}=-H=\frac{1}{3D}\frac{{\rm d}D}{{\rm d}A}, \quad (28)$$

giving

$$3X+\ln D=\ln[x^3(1+\bar{\xi})]={\rm constant}.$$

Using the initial condition in the linear regime,

$$x^3(1+\bar{\xi})=l^3. \quad (29)$$

This shows that $\bar{\xi}_{\rm L}$ should be evaluated at $l=x(1+\bar{\xi})^{1/3}$. It can be checked directly that (29) and (27) satisfy (20).

The final result can therefore be summarized by the equation [equivalent to equations (27) and (29)]

$$\bar{\xi}_{\rm L}(a,l)=\exp\left[\frac{2}{3}\int^{\bar{\xi}(a,x)}\frac{{\rm d}\mu}{H(\mu)(1+\mu)}\right], \quad (30)$$

with $\quad l=x[1+\bar{\xi}(a,x)]^{1/3}.$

Given the function $H(\bar{\xi})$, this relates $\bar{\xi}_{\rm L}$ and $\bar{\xi}$. The lower limit of the integral is chosen to give $\ln\bar{\xi}$ for small values of $\bar{\xi}$ in the linear regime.

Equation (30), deduced from the ansatz (26), is of the form proposed by HKLM. It expresses the density contrast in terms of the density contrast of the linear theory by a function that makes a smooth transition from the $\bar{\xi}\ll 1$ regime to the $\bar{\xi}\gg 1$ regime. This function is expressible in terms of the scaled pair velocity as a function of density contrast, $H(\bar{\xi})$. In our treatment, the relation (29) between the scales l and x in the linear and non-linear regimes emerges as a consequence, resolving the ambiguity noted in the previous section.

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The actual relation between σ_{NL} and σ_L depends on the form of $H(\bar{\xi})$. From the limiting behaviour discussed before, we know that $H(\bar{\xi}) \simeq (2/3)\bar{\xi}$ for small $\bar{\xi}$ and $H \simeq 1$ for $\sigma \gg 1$. The scaling function $\bar{\xi}(\bar{\xi}_L)$ for the non-linear density contrast depends crucially on the manner in which $H(\bar{\xi})$ reaches unity. We expect an overdense region to expand more slowly compared to the background, reach a maximum radius, collapse and virialize to form a bound structure. During the collapse phase, the average relative peculiar velocity will in general overshoot the Hubble expansion velocity. Only after virialization is complete will the pair velocity approach the asymptotic value of $(-\dot{a}x)$. In fact, numerical simulations (HKLM) show that $h(\sigma^2)$ has a single maximum with $H_{max} \simeq 1.5$–2; thus v overshoots $|v| = \dot{a}x$ before falling back to this value at about $\sigma^2 \simeq 8$–15.

4 CONCLUSIONS AND DISCUSSION

The basic proposal of this paper is that significant progress in understanding the evolution of the correlation function in the non-linear phase of gravitational clustering can be made via a scaling hypothesis. This assumption – that the pair velocity is a function only of the density contrast on the same scale – leads to the explicit formulae of Section 3. These agree with those proposed by HKLM but, in our view, are now placed in a more systematic framework. It will be interesting to test the hypothesis for a wider range of initial conditions than the power-law cases explored so far. Given that both the velocity and the density contrast are averages over a variety of environments, the emergence of such a relation is not obvious (except in the extreme linear and non-linear regimes), and deserves dynamical validation. It would be interesting to see whether the form of the function $H(\sigma^2)$ can be obtained from some approximate dynamical models: e.g. the spherical top hat, the Zeldovich approximation, the frozen flow model, etc. One would also like to investigate the conditions under which the universality is broken. For example, one can visualize two situations that start with the same input power spectrum, one having random phases (Gaussian fluctuations) and the other having non-Gaussian correlations. One does not, in general, expect the resulting correlation functions to match in the non-linear regime. Even in such situations, analysis in terms of deviations from 'universal' scaling might be an economical and physical way of compressing the information from a large ensemble of simulations.

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