

CHAPTER 6

HYDRODYNAMIC INSTABILITIES IN SMECTIC- C

6.1 Introduction

In chapter five, we have discussed cases in which there is either macroscopic flow or dynamics of the director in a single domain of smectic- C . The hydrodynamic theory of Sm- C developed in chapter four incorporates coupling between the different hydrodynamic variables, viz. velocity, layer spacing and the $\mathbf{c}$ vector. The main problem regarding experimental and theoretical investigations of Sm- C has been the complexity arising from such couplings. In this chapter, a linear stability analysis of instabilities is carried out and solutions of the equations of hydrodynamics that describe these instabilities are discussed. This brings out the intimate coupling between the hydrodynamic variables [1]. To reduce the complexity of the problems, the earlier investigations [2-5] assumed that : (i) The layers are flat. To achieve this, external counter torques were introduced by these investigators to ensure such a condition. (ii) The flow along the layer normal is nearly nonexistent. This means

that permeation process is not operating. Though these assumptions appear reasonable, they lead to mathematical inconsistencies and an incomplete description of even well-known instabilities like the Freedericksz transition.

In this chapter, certain field and flow induced instabilities are discussed. Our analysis leads to a more complex behaviour in each case. Incidentally, even though we work with an asymmetric stress tensor, the linearized version of our theory is similar to the theory of hydrodynamics of smectic-C formulated by Martin, Parodi and Pershan (RIPP) [6]. As in the case of uniaxial nematics, the only difference is in the choice of forces and fluxes in developing the constitutive equations. This choice merely depends on the type of the problem one is tackling. For instance, the approach of Ericksen, Leslie and Parodi (ELP) [7-9] in formulating the theory of hydrodynamics of uniaxial nematics is instructive when one considers field and flow induced instabilities; but, the MPP theory is useful to study only small fluctuations and wave propagation [10].

6.2 Field induced instabilities

In recent times, there has been considerable interest [3-5,11] in the dynamics of $\mathbf{c}$ vector reorientation under the destabilizing influence of a field. The interest has been spurred on by the possibility of fast-switching chiral smectic-C electro-optic devices. Though we shall be studying the effect of a magnetic field on a sample of aligned Sm-C, the results can be extended to the effect of an electric field on surface stabilized ion-free chiral smectic-C liquid crystals.

We consider the sample to be confined between plates at $y = \pm(l/2)$ with the layers in the so-called bookshelf geometry, *i.e.*, perpendicular to the plates and along the X-Y plane. At the plates, the $\mathbf{c}$ vector is considered to be rigidly anchored along the X-axis. The geometry is depicted in Figure 6.1.

A magnetic field is applied along the Y direction and acts as a destabilizing

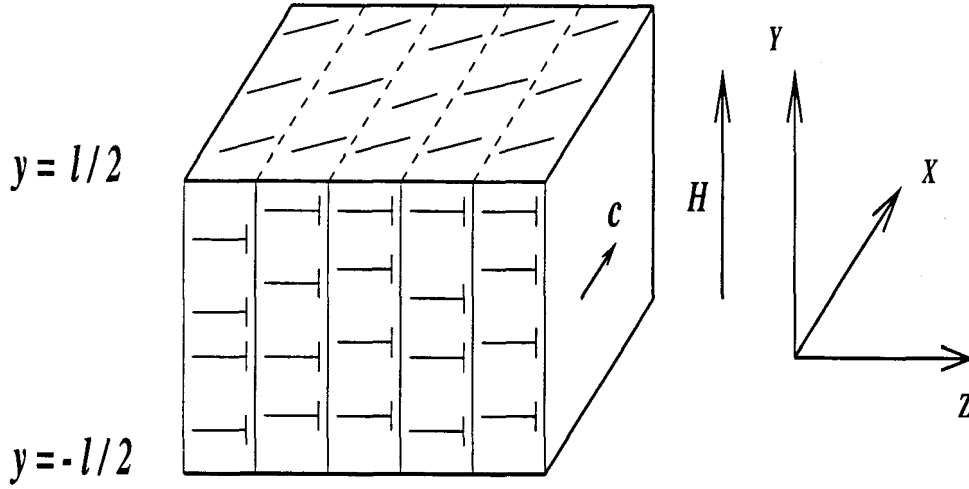


Figure 6.1: A schematic representation of the geometry for Freedericksz transition.

field on $\mathbf{c}$ provided the appropriate diamagnetic anisotropy of this biaxial system is positive. An instability occurs at a threshold field, H_c , at which the $\mathbf{c}$ vector will develop a component along the Y-axis. This in turn leads to a transient flow called "backflow" in the transverse direction, *i.e.*, X-axis. As pointed out by Carlsson and coworkers [13-51], Barratt and Duffy [11], here also, as in nematics [12, 13], for $H > H_c$, the effect of backflow is to alter the director profile and the response time of switching during this transient state. The linear stability analysis of these authors which is similar to that used in uniaxial nematics assumes that the layers are flat and that the flow exists only along the X direction. Though flow in the direction normal to the layers has also been considered by Barratt and Duffy [11], their analysis is incomplete since the effects of permeation have been ignored. We have incorporated permeation and reanalyzed the problem. We assume that the hydrodynamic variables are functions of Y and vary with time as ef^t where f is the inverse of a relaxation time. When $f > 0$, the perturbation grows and the stationary state becomes unstable. Generally, in view of small velocities, the acceleration terms associated with macroscopic flow may be neglected compared to viscous terms. Since

we are interested in the growth of the instability, we shall be concerned with the linearized force and torque equations, *i.e.*, equations (4.26-4.30) of chapter four. The velocity field is taken as $\mathbf{v} = (v_x, 0, v_z)$. Then the equations reduce to:

$$f a_1 \phi_{,y} + a_2 v_{x,yy} + a_3 v_{z,yy} = 0 \quad (6.1)$$

$$f a_4 \phi_{,y} + a_3 v_{x,yy} + a_5 v_{z,yy} = A_{21} u_{,yyyy} + C_2 \phi_{,yyy} \quad (6.2)$$

$$f u - v_z = -\lambda_p (A_{21} u_{,yyyy} + C_2 \phi_{,yyy}) \quad (6.3)$$

$$f a_6 \phi - a_1 v_{x,y} - a_4 v_{z,y} = -h \phi - B_2 \phi_{,yy} - C_2 u_{,yyy} \quad (6.4)$$

where $a_1 = -\frac{1}{2}(\gamma_1 + \gamma_3)$, $a_2 = \frac{1}{2}(\alpha_2 + \alpha_6 - \gamma_3 - \frac{1}{2}\gamma_1)$, $a_3 = \frac{1}{4}(2\alpha_8 - \gamma_2)$, $a_4 = -\frac{1}{2}\gamma_2$, $a_5 = \frac{1}{2}(\alpha_2 + \alpha_4)$, $a_6 = \gamma_1$, and $h = \chi_a H^2$. The diamagnetic anisotropy associated with the $\mathbf{c}$ vector is given by χ_a and is assumed to be positive. Here, and in the rest of this chapter, we have used the notation $\phi_{,y} = \partial\phi/\partial y$ and $\phi_{,yy} = \partial^2\phi/\partial y^2$.

Equations (6.1-6.4) have to be solved with the boundary conditions $0 = v_x = v_z = u = u_{,y} = 0$ at the plates. The last condition implies that the layers are clamped at the plates. This condition of layer clamping takes care of one more boundary condition necessary to completely solve the fourth-order differential equation. It should also be noted that if we assume the layers to be flat or the velocity normal to the layers to be zero, we would get an overdetermined set of equations. This is true even if the coupling constant is nonexistent, *i.e.*, $C_2 = 0$. Hence, the earlier investigations of Carlsson et al. [3] and, Barratt and Duffy [11] obtained under such assumptions are not realistic. The analysis presented here corrects this error.

6.2.1 Effect of C_2

It was shown in chapter four that the term involving C_2 is allowed by the monoclinic symmetry of smectic-C. It couples distortions in the $\mathbf{c}$ vector with layer

curvature. The equations of equilibrium (6.3,6.4) couple these variables. We shall now discuss the effects of C_2 on the instability of the $\mathbf{c}$ vector in the external field, H.

The solution that satisfies the equations of equilibrium and the boundary conditions is

$$\phi = A \left(\cosh ky - \cosh \frac{kl}{2} \right) \quad (6.5)$$

$$u = B \left(\sinh ky - \frac{2y}{d} \sinh \frac{kl}{2} \right) + C y \left(y^2 - \frac{l^2}{4} \right) \quad (6.6)$$

where

$$B = -AC_2/(A_{21}k)$$

$$C = [2AC_2/(A_{21}kl^2)] (k \cosh(kl/2) - (2/l) \sinh(kl/2))$$

and the critical field at which the instability sets in is given by the relation

$$h_c = - (A_{21}B_2 - C_2^2) k_c^2 / A_{21}$$

where k_c is a solution of the transcendental equation

$$12 C_2^2 + (A_{21}B_2 - C_2^2) (k_cl)^2 = [24C_2^2/(k_cl)] \tanh(k_cl/2)$$

It is to be noted that the sign of C_2 does not change the end results and that the threshold is reduced because of this coupling parameter. Hence, when distortions in the $\mathbf{c}$ vector field is coupled to layer curvature, the bare elastic constant, B_2 , gets replaced by a smaller effective elastic constant, B_2^{effec} . Another feature of this solution is that if the wavenumber k_c which will be imaginary can be extracted experimentally, and if it is not equal to πi ($i = \sqrt{-1}$), then it will unequivocally establish that $C_2 \neq 0$. An indication of layer curvature in the static situation also establishes the same result.

6.2.2 Dynamic response

To bring out the main features of the dynamics of c vector reorientation, we need not explicitly include C_2 since the end results are corrected only slightly, and the form of the solution is not altered. A solution of the differential equations (6.1-6.4) satisfying the already specified boundary conditions is obtained by assuming the following ansatz :

$$\begin{aligned}\phi &= \sum_{i=1}^3 \eta_i \left(\cosh k_i y - \cosh \frac{k_i l}{2} \right) \\ v_x &= \sum_{i=1}^3 \mu_i \left(\sinh k_i y - \frac{2y}{l} \sinh \frac{k_i l}{2} \right) \\ v_z &= \sum_{i=1}^3 \nu_i \left(\sinh k_i y - \frac{2y}{1} \sinh \frac{k_i l}{2} \right) \\ u &= \sum_{i=1}^3 \xi_i \left(\sinh k_i y - \frac{2y}{l} \sinh \frac{k_i l}{2} \right)\end{aligned}$$

Then, it is found that

$$\mu_i = -\frac{f}{k_i X_i} [(a_2 a_4 - a_1 a_3) (f + \lambda_p A_{21} k_i^4) - a_1 A_{21} k_i^2] \eta_i \quad (6.7)$$

$$\nu_i = -\frac{f}{k_i X_i} [(a_2 a_4 - a_1 a_3) (f + \lambda_p A_{21} k_i^4)] \eta_i \quad (6.8)$$

$$\xi_i = -\frac{f}{k_i X_i} (a_2 a_4 - a_1 a_3) \eta_i \quad (6.9)$$

$$\eta_2 = \frac{X_2}{X_1} \frac{k_2}{k_1} \frac{M_2}{M_1} \eta_1 \quad (6.10)$$

$$\eta_3 = \frac{X_3}{X_1} \frac{k_3}{k_1} \frac{M_3}{M_1} \eta_1 \quad (6.11)$$

where

$$X_i = (a_2 a_5 - a_3^2) (f + \lambda_p A_{21} k_i^4) - a_2 A_{21} k_i^2$$

$$Z_i = k_i \cosh(k_i l/2) - (2/d) \sinh(k_i l/2)$$

$$Q_i = (a_1^2 a_5 - 2a_1 a_3 a_4 + a_2 a_4^2) (f + \lambda_p A_{21} k_i^4) - a_1^2 A_{21} k_i^2$$

$$M_1 = Z_2 k_3^4 \sinh(k_3 l/2) - Z_3 k_2^4 \sinh(k_2 l/2)$$

$$M_2 = Z_3 k_1^4 \sinh(k_1 l/2) - Z_1 k_3^4 \sinh(k_3 l/2)$$

$$M_3 = Z_1 k_2^4 \sinh(k_2 l/2) - Z_2 k_1^4 \sinh(k_1 l/2)$$

and the unknowns k_i and f are obtained from the following:

$$X_i \left(f a_6 + h + B_2 k_i^2 \right) + f Q_i = 0 \quad (6.12)$$

and

$$\sum_{i=1}^3 M_i \left[(f a_6 + h) k_i X_i \cosh \frac{k_i l}{2} + \frac{2f}{l} Q_i \sinh \frac{k_i l}{2} \right] = 0 \quad (6.13)$$

Equation (6.12) is a sixth order polynomial in k , and this gives the six roots of the form $\pm k_i$. In the domain of the parameters chosen by us, the roots, k_i , are either imaginary or real. We have to find that f which fixes these roots as well as satisfy equation (6.13). We make use of the Rayleigh criterion to find the suitable solution, that is, choose the set of roots with the largest value of f .

With the onset of the instability, there exists curvature distortion of the layers and flow along the layer and normal to the layer, that is, both along X and Z axes. Transverse permeation flow normal to the layers is essentially coupled to layer distortions. Thus, these results predict a different structure when compared with the earlier investigations [3, 11]. Further, there exists analytical solutions of the linear equations satisfying the boundary conditions with two real values of k and an imaginary k . This is not surprising since the inclusion of layer distortions brings in additional length scales associated with permeation. It should be noted that in this analysis, there are three wavenumbers as compared to one in the previous studies. The previous investigators have stated that their solutions are physical only within a range of values of the parameters. Due to the complexity of the equations (6.12, 6.13), we are not able to derive an analytical expression for such a range of parameters. Parameters outside this range have not been considered since they

lead to the unphysical answer of f being positive even for zero field. *i.e.*, the initial bookshelf state would be 'mechanically unstable' even in the absence of a magnetic field. With the same values of the parameters in the permitted range, the growth rate f deduced by this analysis is larger than that of the previous work [3] in all cases. But, it is not clear whether the increase of f is due to the additional resistance provided by layer curvature, or whether it is just a consequence of the extra viscosity coefficients that exist in this general theory. There are other important consequences of this analysis.

(i) We note that the distortion of the c vector has features similar to that found in earlier studies [3]. That is, the effects of backflow become important at higher values of the field, and the variation of the backflow effects with a particular ratio of the material parameters is as in earlier works [3, 11]. And, similarly the imaginary k saturates with the field at a particular value. An interesting result is that the ϕ profile exhibits distortions of opposite senses near the plates and near the centre. The profiles of the ϕ distortions for two different $h(= \chi_a H^2)$ values are shown in Figures 6.2(a) and 6.3(a).

(ii) It has been generally assumed in previous investigations that the flow in the direction normal to the layers is negligible due to a large viscosity in that direction. This is a reasonable approximation at low fields. Even our calculations of scaled velocities as shown in Figures 6.2(b) and 6.2(c) bear out this fact. But, it is found that as the field is increased, the transient flow velocity in the two directions can become comparable. This is apparent in the scaled velocities shown in Figures 6.3(b) and 6.3(c). It should be emphasized that the amplitude of the profiles cannot be extracted from this theory just as the amplitude of distortion in a Freedericksz transition is not obtainable from a linear theory. It is to be mentioned that our calculations are for certain values of the parameters since experimental values are

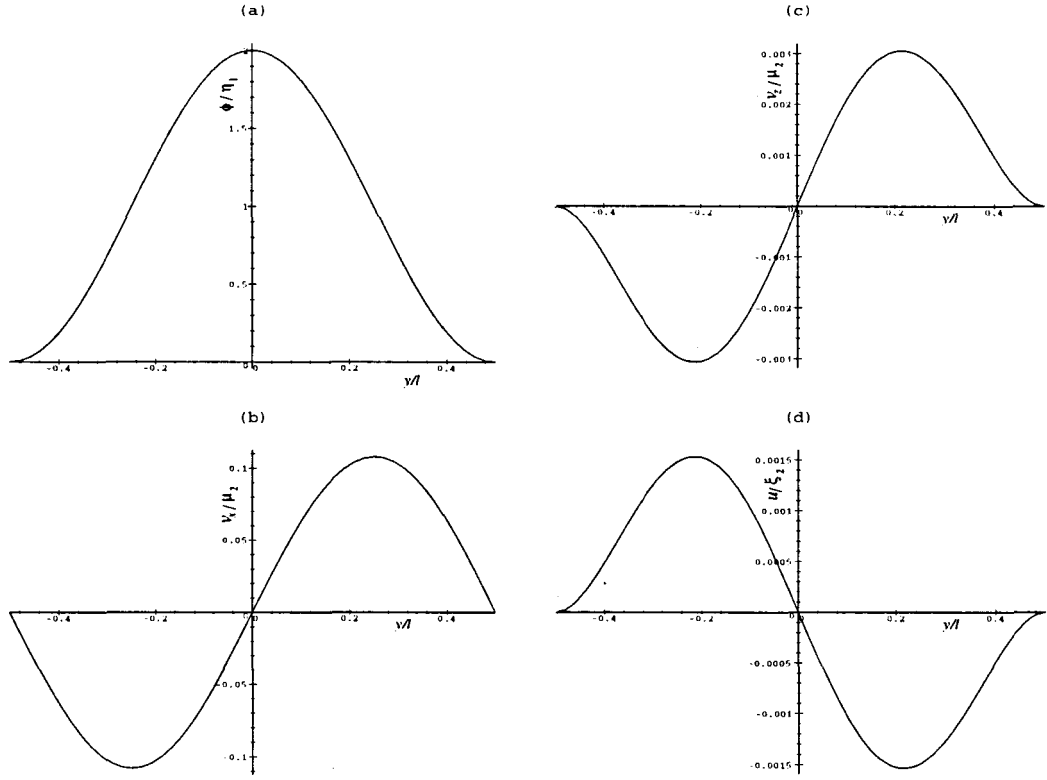


Figure 6.2: The threshold h_c for the instability is $h_c \simeq 10 \text{ ergs cm}^{-3}$. Here, $h = 50 \text{ ergs cm}^{-3}$. The separation between the plates is $l = 10^{-3} \text{ cm}$. The velocity normal to the layers is much smaller than that along the layers. (a) The angle of $\mathbf{c}$ with x-axis, ϕ (b) v_x (c) v_z (d) u , the layer displacement. Here, v_x, v_z and u have been normalized by η_1, μ_2, μ_2 and ξ_2 respectively to give dimensionless quantities. In this case, with $i = \sqrt{-1}$, $f = 49.5760 \text{ sec}^{-1}$, $k_1 d = \pm 6.2985i$, $k_2 d = \pm 0.6422$, $k_3 d = \pm 122474.4871$. Here, $\lambda_p = 10^{-14} \text{ gm}^{-1} \text{ cm}^3 \text{ sec}$, $A_{21} = 3 \times 10^{-6} \text{ dynes}$, $B_2 = 10^{-6} \text{ dynes}$, $a_1 = 2 \text{ poise}$, $a_2 = 5 \text{ poise}$, $a_3 = 2.3419 \text{ poise}$, $a_4 = 1 \text{ poise}$, $a_5 = 1.1169 \text{ poise}$, $a_6 = -1.01 \text{ poise}$.

still lacking. Apart from this, it should also be pointed out that the symmetry of the equations do not allow the flow along the layer normal to be in the nature of a plug-flow. It can be seen that if ϕ is an even function of y , then the other quantities v_x, v_z and u will be odd functions, and these features are clearly shown in Figures 6.2 and 6.3. However, for practical situations, the other permitted solution with odd ϕ and even v_x, v_z and u are improbable.

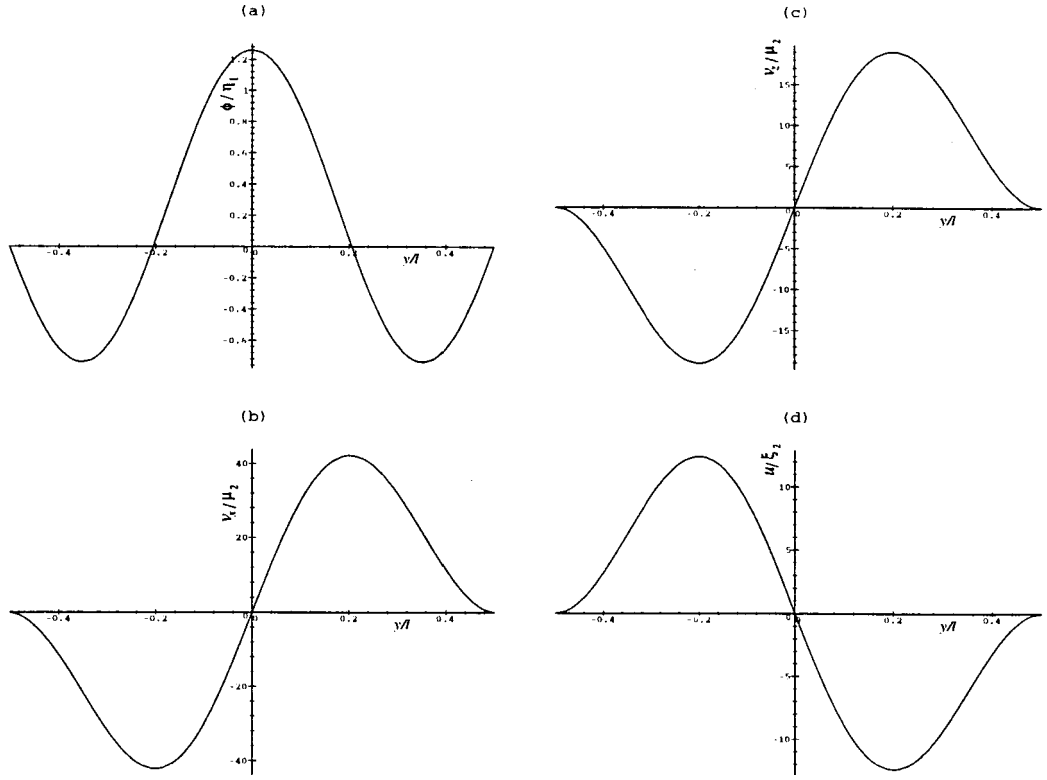


Figure 6.3: Here, $h = 200 \text{ ergs cm}^{-3}$, and rest of the parameters are the same as in Figure 6.2. (a) ϕ (b) v_x (c) v_z (d) u . It can be seen in (a) that the orientation of the $\mathbf{c}$ vector near the walls is opposite to that near the centre. And, from (b) and (c), it can be seen that the velocities are comparable. The normalisation of the quantities is as described in Figure 6.2. In this case, $f = 602.3026 \text{ sec}^{-1}$, $k_1 d = \pm 8.90622$, $k_2 d = \pm 3.1336$, $k_3 d = 122474.4866$ where $i = \sqrt{-1}$.

6.3 Flow induced instabilities

In the rest of this chapter, instabilities that can arise in flow geometries will be discussed. One interesting flow geometry discussed in detail in literature has been the simple shear flow without surface anchoring, *i.e.*, with one plate moving relative to the other. It has been shown [10, 14] that in this case there exists a stationary state with the smectic layers parallel to the plates and $\mathbf{c}$ vector either parallel or antiparallel to the direction of main velocity. Further, as to which direction $\mathbf{c}$

adopts depends on whether the product of the shear rate and a particular viscosity coefficient is positive or negative. On the other hand, again under no anchoring conditions, when the layers are perpendicular to the plates, and the shear plane is parallel to the smectic layers, a different stationary state is obtained. Here, $\mathbf{c}$ is aligned everywhere at a particular angle $\phi = \frac{1}{2} \cos^{-1}(-\gamma_1/\gamma_3)$ with respect to the flow direction. γ_1 and γ_3 are viscosity coefficients. Obviously, this state occurs only when the ratio of the viscosity coefficients, $|\gamma_1/\gamma_3| \leq 1$, otherwise the phenomenon of tumbling occurs where $\mathbf{c}$ continuously rotates from one plate to the other. There are interesting instabilities in these geometries that have not been considered so far. These two geometries have been considered but with surface anchoring: and thus, bringing in elastic distortions in the $\mathbf{c}$ vector and layers.

6.3.1 Shear plane normal to the layers

We consider the layers to be parallel to the plates. The $\mathbf{c}$ vector is aligned everywhere along the Y axis, the direction of shear flow. This state is described by $\mathbf{c} = (0, 1, 0)$ and $\mathbf{v} = (0, 2sz, 0)$. The geometry is shown in Figure 6.4.

To describe the onset of the instability in this case, we undertake a linear stability analysis. We assume $\mathbf{v} = (2v_x(z), 2sz, 0)$ and $\mathbf{c} \simeq (\phi(z), 1, 0)$. The hydrodynamic equations (4.26-4.30) do not lead to either layer distortion or permeation flow. As usual, we assume a e^{ft} time dependence. Then the relevant governing equations become

$$(fb_1 - b_2s) \phi_{,z} + b_3 v_{x,zz} = 0 \quad (6.14)$$

$$B_3 \phi_{,zz} - (b_1s - fb_4) \phi - b_1 v_{x,z} = 0 \quad (6.15)$$

where $b_1 = \gamma_2$, $b_2 = \alpha_6 + \alpha_9$, $b_3 = \alpha_2 + \alpha_4$, $b_4 = \gamma_1$. We impose $v_x = 0$ and rigid

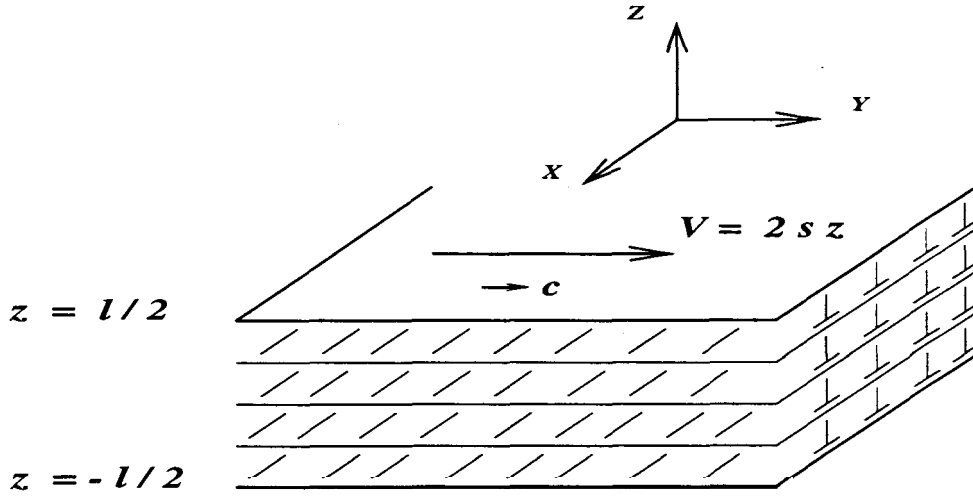


Figure 6.4: A schematic representation of the geometry with shear flow of the layers which are parallel to the walls and are sliding past each other. The shear plane is perpendicular to the layers. The nail representation is used and the nail head indicates that the $\mathbf{c}$ vector goes into the plane of the paper.

anchoring for $\mathbf{c}$ at the boundaries. The solution takes the form

$$\phi = A \left[\cos kz - \cos k \frac{l}{2} \right] \quad (6.16)$$

$$v_x = B \left[\sin kz - \frac{2z}{l} \sin k \frac{l}{2} \right] \quad (6.17)$$

with $B = -A(\lambda b_1 - b_2 k)/(b_3 k)$. The values of k and f are obtained from

$$s b_1 (b_1^2 - b_2 b_4) \left[1 - \frac{2}{kl} \tan k \frac{l}{2} \right] = B_3 k^2 \left[b_3 b_4 + \frac{2}{kl} b_1^2 \tan k \frac{l}{2} \right] \quad (6.18)$$

$$f (b_3 b_4 + b_1^2) = b_3 \left[B_3 k^2 + b_1 s \left(1 + \frac{b_2}{b_3} \right) \right] \quad (6.19)$$

It can be seen that for $s > 0$ ($s < 0$), there will be an instability above a threshold value of the shear only if $b_1 = \gamma_2 < 0$ ($b_1 > 0$).

Above this threshold shear there exists a transverse *flow*, v_x , but unlike in

Freedericksz transition, this is *not a transient phenomenon*. Also, well-above the threshold, the twist distortion of the c vector near the centre is of opposite sense to the twist near the plates. A schematic representation of such a c vector distortion is depicted in Figure 6.5. It must be remembered that when the c vector is aligned everywhere perpendicular to the shear plane, *i.e.*, along the X -axis, the instability has no threshold at all.

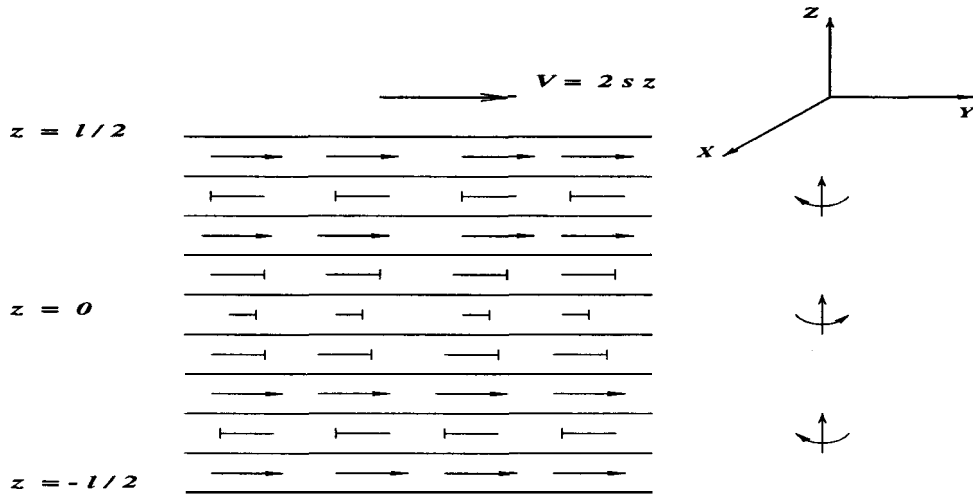


Figure 6.5: A schematic representation of the distortion in c associated with the geometry described in Figure 6.4. It can be seen that the twist distortion near the walls is opposite in sense to the twist distortion near the centre.

6.3.2 Shear plane parallel to the layers

In this case, the layers are normal to the plates in the bookshelf geometry as shown in Figure 6.6. The shear plane is parallel to the layers. With $|\gamma_1/\gamma_3| \leq 1$ and no c vector anchoring at the walls, it can be readily verified that with $\mathbf{v} = (2sy, 0, 0)$ and $\mathbf{c} = (\cos\phi, \sin\phi, 0)$, the continuum equations lead to a shear independent c vector orientation given by

$$\cos 2\phi_0 = -\frac{\gamma_1}{\gamma_3} \quad (6.20)$$

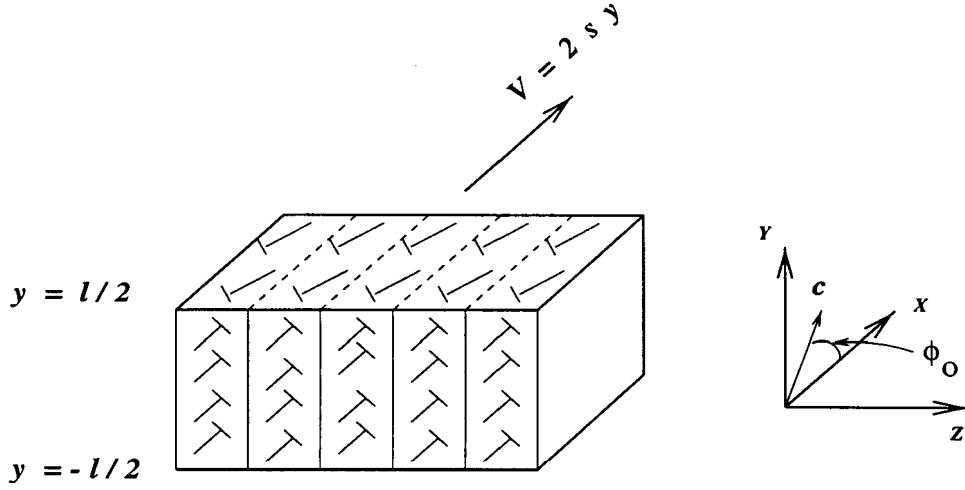


Figure 6.6: A schematic representation of the geometry with the shear flow along the layers which are perpendicular to the walls. The shear plane is parallel to the layers. Here, we show the undistorted sample with the wall anchoring of $\mathbf{c} = (\cos \phi_0, \sin \phi_0, 0)$.

In this flow, the layers are flat and parallel to xy -plane. It is easy to see that this simple solution is realisable with the same surface alignment of $\mathbf{c} = (\cos \phi_0, \sin \phi_0, 0)$ at the walls. Hence, at the boundary and elsewhere, the $\mathbf{c}$ orientation is $\mathbf{c} = (\cos \phi_0, \sin \phi_0, 0)$. Here, we examine the stability of this solution. We shall assume spatial variations only along the y -axis. The analysis is similar to that in field-induced instabilities. We take $\mathbf{v} = (2sy + 2v_x, 0, 2v_z)$ and $\mathbf{c} = (\cos(\phi_0 + \delta\phi), \sin(\phi_0 + \delta\phi), 0)$. Once again the instability involves a distortion in the $\mathbf{c}$ vector and also distortion of the layers due to transverse permeation flow given by $\mathbf{v}_\perp$. It should be noted that the velocity along the direction of shear will not be linear but corrected by a nonlinear term. To estimate the threshold shear rate, s_c , we assume that the perturbations have a $e^{f t}$ time dependence. The instability is assumed to be stationary whereby the principle of exchange of stabilities is valid and $f = 0$ at threshold [15].

The linearized hydrodynamic equations (4.26-4.30) in this case yield

$$s c_1 u_{,yy} + c_2 v_{x,yy} + s c_3 \phi_{,y} + c_4 v_{z,yy} = 0 \quad (6.21)$$

$$s c_5 u_{,yy} + c_6 v_{x,yy} + s c_7 \phi_{,y} + c_8 v_{z,yy} + c_9 u_{,yyyy} = 0 \quad (6.22)$$

$$-2v_z = \lambda_p c_9 u_{,yyyy} \quad (6.23)$$

$$s c_{10} u_{,y} + s c_{11} \phi + c_{12} v_{z,y} + c_{13} \phi_{,yy} = 0 \quad (6.24)$$

We have taken $\varphi_1 = \sin \phi_0$ and $\varphi_2 = \cos \phi_0$, $c_1 = -\gamma_2 \varphi_1 - 2\alpha_8 \varphi_1 - 3\alpha_{10} \varphi_1 \varphi_2^2$, $c_2 = -\gamma_3 (\varphi_2^2 - \varphi_1^2) - \frac{\gamma_1}{2} + \alpha_2 + \alpha_6 + 2\alpha_{13} \varphi_1^2 \varphi_2^2$, $c_3 = 4\gamma_3 \varphi_1 \varphi_2 + 4\alpha_{13} \varphi_1 \varphi_2 (\varphi_2^2 - \varphi_1^2)$, $c_4 = \alpha_8 \varphi_2 - \frac{\gamma_2}{2} \varphi_2 + \alpha_{10} \varphi_2 \varphi_1^2$, $c_5 = -(\gamma_3 - \alpha_6 + \alpha_9 - 2\alpha_{13} \varphi_1^2) \varphi_1 \varphi_2$, $c_6 = -\frac{\gamma_2}{2} \varphi_2$, $c_7 = \frac{\gamma_2}{2} \varphi_1 - \alpha_8 \varphi_1 + 2\alpha_{10} (2\varphi_1 \varphi_2^2 - \varphi_1^3)$, $c_8 = \alpha_2 + \alpha_4 + (\alpha_6 + \alpha_9) \varphi_1^2$, $c_9 = -(A_{11} \varphi_1^2 \varphi_2^2 + A_{21} \varphi_2^4 + A_{12} \varphi_1^4)$, $c_{10} = \gamma_2 \varphi_1$, $c_{11} = -4\gamma_3 \varphi_1 \varphi_2$, $c_{12} = \gamma_2 \varphi_2$, $c_{13} = B_1 \varphi_1^2 + B_2 \varphi_2^2$.

Here, we have neglected the elastic coupling between distortions in the c vector and those of the layers, *i.e.*, $C_2 = 0$. These equations are similar to equations (6.1-6.4) and the solutions for v_x, v_z, ϕ and u are of the same form. The two relations that yields the threshold shear are :

$$\begin{aligned} & s (c_2 c_7 - c_3 c_6) \left[s c_{10} - \frac{1}{2} \lambda_p c_9 c_{12} k f \right] - \\ & (s c_{11} + c_{13} k_i^2) \left[s (c_2 c_5 - c_1 c_6) c_2 c_9 k_i^2 - \frac{1}{2} \lambda_p c_9 (c_2 c_8 - c_4 c_6) k f \right] = 0 \end{aligned} \quad (6.25)$$

$$\sum_{i=1}^3 P_i Q_i R_i = 0 \quad (6.26)$$

where

$$P_i = s c_{10} - \frac{1}{2} \lambda_p c_9 c_{12} k_i^4,$$

$$R_i = (2/l) \sinh \frac{1}{2} k_i l - [s c_{11} / (s c_{11} + c_{13} k_i^2)] k_i \cosh \frac{1}{2} k_i l,$$

$$Q_1 = Z_2 k_3^4 \sinh \frac{1}{2} k_3 l - Z_3 k_2^4 \sinh \frac{1}{2} k_2 l,$$

$$Q_2 = Z_3 k_1^4 \sinh \frac{1}{2} k_1 l - Z_1 k_3^4 \sinh \frac{1}{2} k_3 l,$$

$$Q_3 = Z_1 k_2^4 \sinh \frac{1}{2} k_2 l - Z_2 k_1^4 \sinh \frac{1}{2} k_1 l.$$

$$\text{where } Z_i = k_i \cosh \frac{1}{2} k_i l - (2/l) \sinh \frac{1}{2} k_i l.$$

From the analysis, it is seen that the threshold shear rate varies inversely as the square of the sample thickness and *increases* with increasing λ_p . The latter implies that as the viscosity *reduces* along the z-axis, a larger shear rate is required to make the stationary state unstable. It should be emphasized that here also, the transverse permeation flow and layer curvature are *not transient* in this flow induced instability.

References

- [1] Sreejith Sukumaran and G. S. Ranganath, Phys. Rev. E 57, 5597 (1998)
- [2] T. Carlsson, F. M. Leslie, and N. A. Clark, Phys. Rev. E 51, 4509 (1995)
- [3] T. Carlsson, N. A. Clark, and Z. Zou, Liquid Crystals 15, 461 (1993)
- [4] Z. Zou, N. A. Clark, and T. Carlsson, Phys. Rev. E 49, 3021 (1994)
- [5] Z. Zou and N. Clark, Phys. Rev. Lett. 75, 1799 (1995)
- [6] P. C. Martin, O. Parodi, and P. S. Pershan, Phys. Rev. A 6, 2401 (1972)
- [7] J. L. Ericksen, Trans. Soc. Rheol. 5, 23 (1961)
- [8] F. M. Leslie, Adv. Liq. Cryst. 4, 1 (1979)
- [9] O. Parodi, J. Phys. (Paris) 31, 581 (1970)
- [10] P. G. de Gennes and J. Prost, The Physics of Liquid Crystals, Second edition (Clarendon Press, Oxford, 1993).
- [11] P. J. Barratt and B. R. Duffy, Liquid Crystals 21, 865 (1996)
- [12] F. Brochard, P. Pieranski, and E. Guyon, Phys. Rev. Lett. 28, 1681 (1972)
- [13] P. Pieranski, F. Brochard, and E. Guyon, J. Phys. (Paris) 34, 35 (1973)
- [14] F. M. Leslie, I. W. Stewart, and M. Nakagawa, Molec. Cryst. Liq. Cryst. 198, 443 (1991)
- [15] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability (Clarendon Press, Oxford, 1961)