



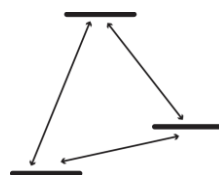
RAMAN RESEARCH INSTITUTE
LIGHT & MATTER PHYSICS GROUP

Signal transfer using an atomic microwave-to-optical antenna

by

Pradosh Kumar Nayak

Supervisor – Prof. Andal Narayanan



A thesis submitted for the degree of
Doctor of Philosophy

to

Jawaharlal Nehru University-New Delhi



DEDICATION

To my dearest parents and supportive brother-in-law (Mr. Satya Ranjan Nayak), whose constant love, encouragement, and belief in my potential have been my pillars of strength throughout my life.

DECLARATION

I, ***Pradosh Kumar Nayak*** (Enrolment No.:RRI/2016/013), hereby declare that the work presented in this thesis titled ‘***Signal transfer using an atomic microwave-to-optical antenna***’ is completely original. This thesis is composed independently by me at ***Raman Research Institute***, Bengaluru under the supervision of ***Prof. Andal Narayanan*** and is the result of my own work unless otherwise stated. I further declare that the subject matter presented in this thesis has not formed earlier the basis for the award of any degree, diploma, membership, associateship, fellowship or any other similar title of any university or institution. I also declare, this thesis has been checked through the plagiarism software **Turnitin**.

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CERTIFICATE

This is to certify that the work contained in the thesis titled ‘*Signal transfer using an atomic microwave-to-optical antenna*’, submitted by *Pradosh Kumar Nayak* (Enrolment No.:RRI/2016/013) to the Jawaharlal Nehru University for the award of the degree of *Doctor of Philosophy (Ph.D.)* in Physical Sciences, is the bonafide record of original research work carried out by the candidate under my guidance and supervision at Raman Research Institute (RRI), Bengaluru, India. The results embodied in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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ACKNOWLEDGEMENT

Before I begin, I want to mention a few individuals who made this thesis and my time at RRI possible.

Foremost, I would like to express my sincere gratitude to Prof. Andal Narayanan for giving me a valuable opportunity to work in an exciting research field. I am incredibly thankful for her constant guidance, mentorship, encouraging discussions, support, and patience.

I acknowledge Prof. Fabien Bretenaker and Prof. Barry C Sanders for their meaningful discussions, which led to active collaboration. I thank Prof. Reji Philip and Prof. Arun Roy for being an essential part of my thesis advisory committee.

I thank the honorable director, the administrative department of RRI for facilitating the scientific environment, and the RRI faculty for all the encouragement and support. Special thanks to M. S. Meena for all the technical support for my experimental work. I would also like to thank Raghunathan, Somashekar, Mugundhan, Keerthipriya, Raghavendra, and the EEG department for sharing the spectrum analyzer and other instruments.

I am thankful to all the students and faculty members of the LAMP group for sharing optical components and instruments, including Ibrahim and all members of the workshop department, particularly for machining the copper microwave cavity and other support.

Sincere gratitude to senior lab members Asha and Adwaith for their guidance, inspiration, and immersive discussions. I Would also like to thank Megha for designing the Aluminium microwave cavity. I would especially like to thank Saaswath for his kind nature, helpful discussions, and assistance in the lab.

I wish to thank Savitha, Shiva, and Manju for their support. I sincerely thank the library staff for providing a wholesome, informative environment and their help, especially Meera, M. Manjunath, Manjunath K., Nagaraj, and Vani.

Gratitude to all members of the administrative department, especially Naresh, C. S.

R. Murthy, Marisa, Radha, Vidya, Subbu, Sasidharan, and Harini. I sincerely thank Sachin, Muneeswaran, and others from the civil and electrical departments. My thanks to all members of the computer department, account section, purchase section, canteen, gardeners, security and transport. I am thankful to the cooks of the Vyalikaval hostel mess for providing us with homely food and caring behavior. I am deeply grateful for friends like Ashutosh, Ashish, Vishnu, Saumya, Harikrishna, Dipak, Chandeshwar, Sebanti, Anand, Swarnak, Deepak, Abhisek, Arsalan, Rashmi, and others for the fantastic time spent together with rock-solid support and motivation. I want to thank the medical staff of RRI, particularly Dr. Archana and Dr. Baliga, for their round-the-clock assistance.

I am forever grateful to my parents, teachers, relatives, and acquaintances for their support, knowingly and unknowingly. Lastly, I wholeheartedly thank Roshni for her patience, support, and care.

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Synopsis

Electromagnetic waves at microwave and optical frequencies are the two most dominating channels used for information processing and data communication in today's world. The data transmission at microwave frequencies requires less complex circuitries but the signal attenuates as it transmits over the lossy cables. The loss can be upto few dB/m at 3 GHz. Apart from attenuation the signal also interferes with other unwanted signals which collectively results erroneous information at the receiving end and hence less fidelity. On the contrary the optical signal can transmit through optical fibre over long distances without losing much of signal level. There is also no interference for optical signal as it travels through optical fibre which results less erroneous information at the receiving end as compared to microwave signal.

The emerging field of quantum technology (QT) has applications in microscopy, quantum communication, positioning and, sensing [1, 2]. The most fundamental building block for quantum information processing is a qubit. Over the past decades there are many platforms such as cold trapped ions [3], neutral atoms [4], solid-state defects [5] and supeconducting qubit [6] which has contributed for the development of qubits for quantum information processing around the world. But the most promising QT for making quantum states of electromagnetic fields is superconducting qubits, which operate in the GHz frequency regimes.

But the lossy nature of microwave cables limits the transmission of this quantum signal over long distances. On the contrary, optical photons containing quantum information can be transmitted over very long distances through optical fibre without suffering much loss. Hence coherent conversion of signals from microwave to optical is a fast emerging field of study.

In this thesis we investigate the coherent transduction of microwave signal into the optical signal using dilute room temperature atomic system as the transducer. The various aspects covered in the thesis are: microwave to optical signal conversion, a specific digital communication protocol based on such a conversion, group delays associated with microwave to optical pulse transfer and simultaneous generation of three-wave and six-wave mixing based microwave to optical conversion. These three main results forms the core of the work done for this thesis and forms three chapters. The introduction chapter intro-

duces the specific case of atomic structures and electromagnetic fields used in this thesis and also traces relevant literature. The concluding chapter puts forth the significance of our results and outlines future work that can emerge from this thesis.

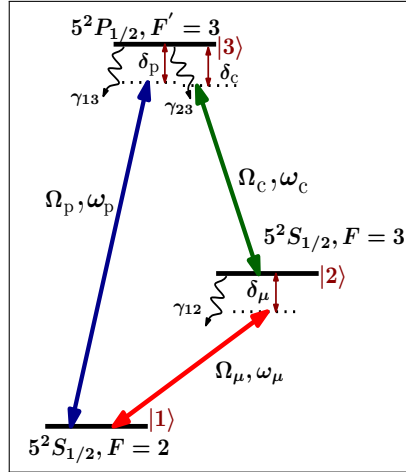


Figure 1: Hyperfine energy levels in a Δ configuration in ^{85}Rb . δ_p , δ_c and δ_μ are the single photon detunings for probe, coupling and microwave fields, respectively. Ω_p , Ω_c and Ω_μ denotes the Rabi frequencies of the probe, coupling, and microwave fields, respectively.

Digital communication using a cyclic level scheme in an atomic radio-over-fibre device.

In this project we experimentally demonstrate that, dilute ensembles of neutral, room temperature atoms can be used as an receiver and transmitter for signal [7, 8, 9]. For the signal encoding purpose we implemented both Binary phase shift keying(BPSK) and Four phase shift keying(4-PSK) protocol in our system. The information signal is encoded in the phase of the microwave field also called carrier signal by using above said protocol. The phase modulated microwave signal and an optical coupling laser field interact with Rubidium(^{85}Rb) atoms and an optical intensity modulated probe field gets generated through a second order nonlinear interaction. We use a cyclic three-level scheme in ^{85}Rb atoms to achieve this as shown in Fig. 4.1. The importance of our scheme is that the cyclic, closed interaction of the atomic levels with electromagnetic fields makes our system inherently sensitive to the phase of the microwave field. This enables our system to directly encode a phase modulated digital signal in the microwave field and decode it as intensity

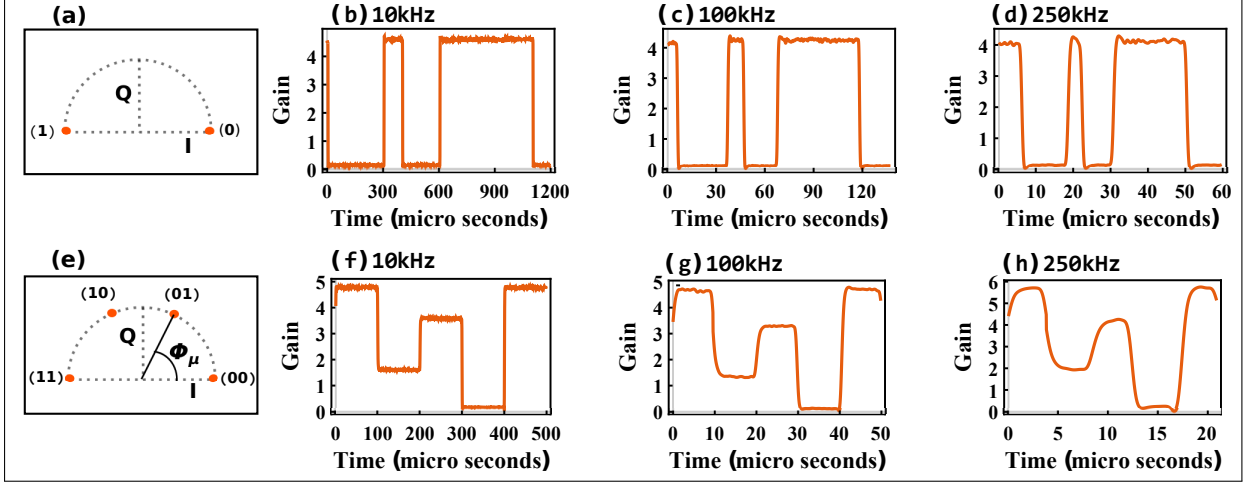


Figure 2: Constellation diagrams for (a) BPSK, (e) C-4PSK. Gain vs time plots showing demodulated intensity changes for symbol frequencies (b & f) 10 kHz, (c & g) 100 kHz, (d & h) 250 kHz for BPSK and C-4PSK respectively. Here $\delta_p = \delta_c$, $\delta_p - \delta_c - \delta_\mu = 0$ and microwave power is 500 mW.

modulation [10] in the optical field. The optical output gets down converted using optical heterodyne setup and detected with a high band-width photo-detector followed by a spectrum analyzer. The decoded optical signal with the corresponding bits is shown in Fig. 2. We measure the correlation of our demodulated optical signal with an ideal template and establish a viable signal bandwidth of about 1 MHz. Our atom based scheme also enables phase dependent amplification [11] of the demodulated optical signal through a hybrid second order non-linearity. Thus our phase dependent atomic transmitter-receiver has inherent features of demodulation, radio- to-optical conversion, and amplification. The ground states used in our scheme are quantum memory storage spin states, which makes our scheme inherently suitable for applications involving communication and storage and retrieval of quantum signals.

Microwave-driven generation and group delay control of optical pulse from an ultra-dilute atomic ensemble.

In this work we have theoretically investigated the non-linear interaction between three level atomic system with an Continuous wave(CW) optical field and a Gaussian microwave pulse. The hybrid second order non-linearity of the system aids for the generation of an optical field/pulse. The importance of our system is that the absolute phase of the non-linearity can be controlled by the intensity of the

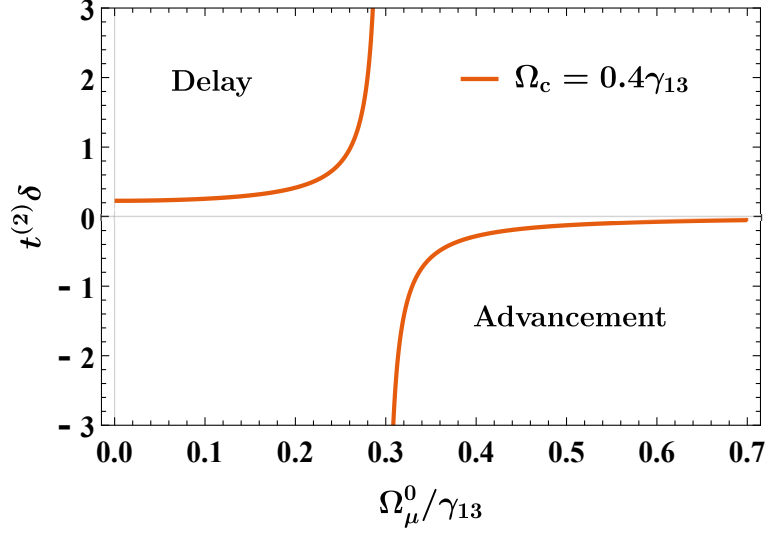


Figure 3: Group delay due to generation phase as function of the microwave amplitude, for $\delta = 0.3 \times 10^6$ Hz, no. density ($\mathcal{N}$) = 10^6 cm $^{-3}$ and $\gamma_{13} = 5\pi \times 10^6$ rad/s.

microwave field [12]. This handle over absolute phase can delay or advance the generated pulse inside the atomic medium as shown in fig. 3, where $t^{(2)}$ is the group delay of the generated pulse, and δ and Ω_μ^0 are the spectral width and amplitude of the input pulse. More importantly the delay and advancement of the pulse is atom number density independent. As a result, with dilute sample, optical pulse with few average number of photons can be made on demand whose delay can be controlled via microwave field intensity [13]. The pulse on-demand can be tailored by switching on both the fields. Our analysis exploits the hybrid second order non-linearity under conditions of electromagnetically induced transparency to generate the optical pulse. This atomic scheme can thus serve a controllable delay line for both classical and quantum signal processing.

Simultaneous observation of six wave and three wave mixing process

In this work we have investigated the simultaneous presence of both six wave [14] and three wave generation process in room temperature ^{85}Rb atoms. The scheme utilizes the same three level system with the connected fields, shown in Fig. 4.1. The difference between the current scheme as compared to previous schemes is that we have deliberately broken the three photon resonance condition which was satisfied in previous schemes. The breaking of three photon resonance ensures the six wave mixing (SWM) and three wave mixing (TWM) process can be frequency resolved

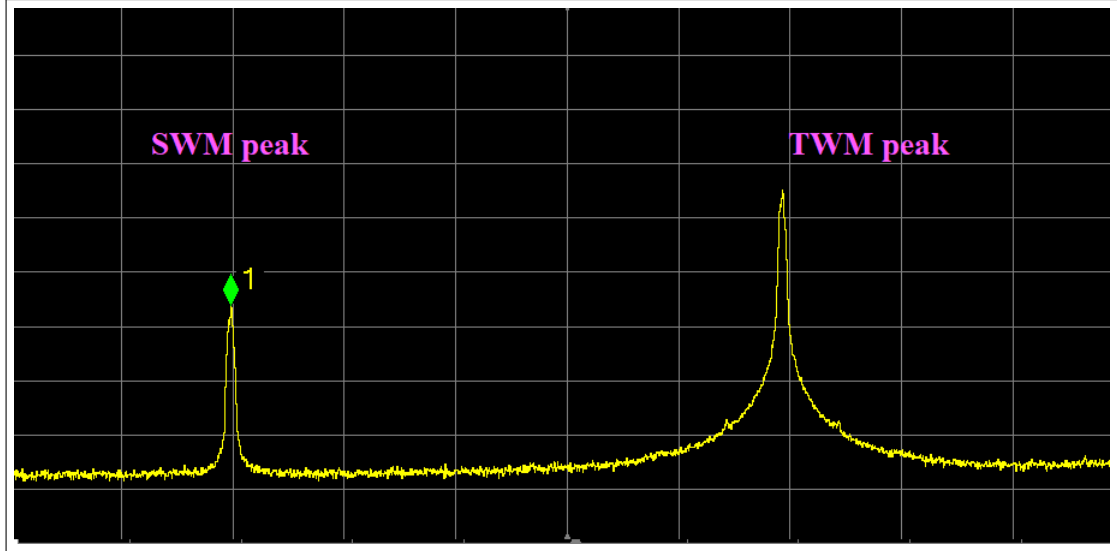


Figure 4: SWM and TWM signal peaks.

and observed without any ambiguity as shown in fig. 4. In this project we have simultaneously observed generation of six wave mixing process along with earlier reported three wave mixing [15] process in a cyclic three level atomic system. The three photon resonance gets violated by connecting a seed probe which is detuned by Δ_p from the corresponding resonance. The dependence of SWM generation with optical and microwave field is studied.

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List of publications

Journal publications

Publications included in the thesis

1. **K. N. Pradosh**, K. V. Adwaith, M. S. Meena, and Andal Narayanan, “Digital communication using a cyclic level scheme in an atomic radio-over-fiber device,” *Applied Physics Letters* **118**(6), 064001 (2021)
2. J. K. Saaswath, **K. N. Pradosh**, K. V. Adwaith, Barry C. Sanders, Fabien Bretenaker, and Andal Narayanan, “Microwave-driven generation and group delay control of optical pulse from an ultra-dilute atomic ensemble,” *Optics Express* **29**(11), 15940-15952 (2021)
3. **K. N. Pradosh**, J. K. Saaswath, and Andal Narayanan , “Simultaneous three-wave and six-wave mixing of microwave and optical fields in an atomic medium ,” *Optics Express* **31**(11), 18318-18326 (2023).

Publications not included in the thesis

1. K. V. Adwaith, **K. N. Pradosh**, J. K. Saaswath, Fabien Bretenaker, and Andal Narayanan, “Microwave controlled ground state coherence in an atom-based optical amplifier,” *OSA Continuum* **4**(2), 702–710 (2021)
2. Asha Karigowda, K. V. Adwaith, **K. N. Pradosh**, S. Sudha, Barry C. Sanders, Fabien Bretenaker, and Andal Narayanan, “Phase-sensitive amplification of an optical field using microwaves,” *Optics Express* **27**(22), 32111–32121 (2019)

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Chapter 1

Introduction

1.1 Overview

The science of light-matter interactions has seen a tremendous transformation since the invention of the laser. The interplay between this coherent, quasi-monochromatic, and highly directional light with the natural atoms, as well as artificial atoms, has garnered much attention in the last decades. These Light-matter interactions are at the core of our understanding of nature, illuminating the fundamental principles governing matter's behavior at atomic and subatomic levels. This understanding has laid the foundation for quantum information and made progress in quantum computing and quantum communication. In the domain of sensing and metrology, laser technology, imaging, spectroscopy, and quantum technology, the study of light-matter interactions is essential. These studies have already witnessed many ground-breaking discoveries and are also becoming a workhorse for future advancements.

The substantial progress made so far in this field aids in materializing quantum bits for quantum computing purposes. Though there are many platforms for qubit implementation, like trapped ions [1, 2], defects in solids [3], and cold neutral atomic systems [4], one of the most promising platforms is the superconducting qubits [5, 6, 7]. These versatile qubits have been shown to have long coherence times [8], strong qubit interactions [9] due to high nonlinearity, suitability for integration and scaling, and ease of readout and measurements, which makes it a prime platform over many others for quantum information processing. These superconducting qubits operate in the microwave frequency regime that aligns with most of the modern-day electronics, making these qubits easier for generation,

manipulation, and detection.

Even though the manipulation of microwave signals is not much of a difficulty, it faces challenges for long-distance communication. The lossy nature of radio-frequency(RF) cables makes the signal transmission inefficient. In addition, microwave signals suffer interference from other unwanted signals as well as get corrupted by thermal photons at room temperature while propagating inside the RF cable. This leads to erroneous data at the receiving station. In contrast, optical signals can travel through an optical fiber to long distances without suffering much loss. In addition, interference with noisy signals is absent in optical communication, and the optical signal is immune to thermal photons, making it a suitable candidate for signal transmission.

Using optical communication to transfer quantum information in superconducting architecture can avoid the problems faced by its microwave counterpart, leading to lesser bit flip error and high-fidelity [10] information transfer. However, the implementation requires a microwave-to-optical [11, 12] transducer.

In the current research scenario, microwave-to-optical conversion is a critical component for the transfer of quantum information. Quantum computers constructed from microwave-driven qubits rely entirely on the efficient conversion [13] of quantum signals from the microwave to the optical domain. All the architectures for microwave-to-optical conversion are based on sum frequency or difference frequency generation schemes. In such schemes, an input microwave photon combines with an optical coupling photon, which gives rise to another optical photon. There have been several platforms that have successfully demonstrated microwave-to-optical conversion during the last ten years.

Electro-optical transducers [14, 15] rely on the electro-optical nonlinearities for converting microwave photons into optical photons. These transducers require resonators with high-quality factors to do the conversion.

Also, mechanical resonators [16, 17, 18] can interact with microwave and optical photons inside an optical cavity in an optomechanical system. The micro-motion of these resonators driven by microwave photons changes the cavity properties. Using the optomechanical coupling produced by the resonator, the information can be coherently transferred from the microwave to the optical domain.

In [19], authors reported microwave-to-optical conversion in an atomic system with a conversion efficiency of about 5%. In a similar system [13], the conversion efficiency of

82% with 1 MHz bandwidth is also reported recently. Both of these systems are based on cold atomic samples and use Rydberg levels to couple microwaves.

This thesis investigates a potential mechanism for signal transfer from the microwave to the optical domain using coherent interactions between electromagnetic fields with room-temperature atomic vapor. Our analysis involves interactions of two optical fields with one microwave field in the presence of three hyperfine levels of ^{85}Rb in D1 manifold. Because of the closed nature of the interactions, the system is phase-sensitive, allowing one optical field's intensity to be regulated by the phase of the microwave field. This phase-to-intensity mapping enables the radio-over-fibre device's ability to encode information on a phase-modulated microwave field and decode it in an optical field. By using the second-order nonlinearity provided by the atoms, the microwave pulse can also be up-converted to an optical pulse through the process of three-wave mixing (TWM). This optical pulse can also be delayed or advanced inside the medium. In addition to the three-wave mixing process, the system exhibits a six-wave mixing process.

1.1.1 Three level system

All the works presented in this thesis are based on a common platform of a three-level atomic system. In this section, a brief idea about the atomic three-level system is given.

A three-level system consists of three different energy levels and is connected to each other by laser fields. Generally, states having the lowest and highest energy are called ground and excited states, respectively. The energy level between the ground and the excited state is called the intermediate or metastable state. Depending on the geometrical configuration of these states, three-level systems can be classified as Lambda (Λ), Vee (V), or Ladder systems as shown in Fig. 1.1. The interaction between laser fields and the energy

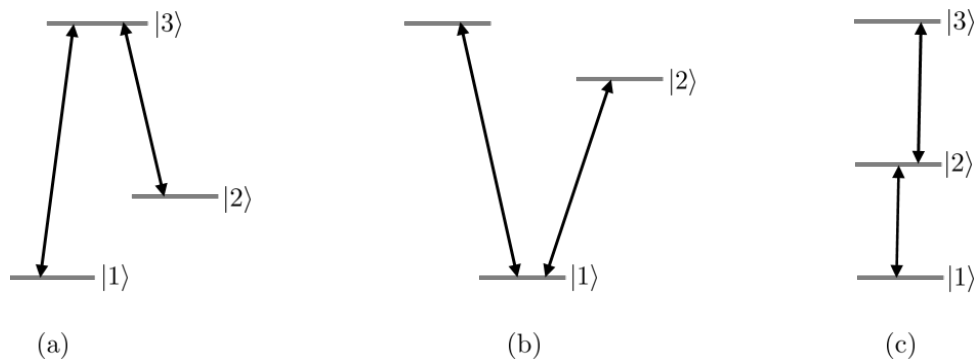


Figure 1.1: Three-level system in (a) Lambda, (b) Vee, and (c) Ladder configuration.

levels alters the population at each level and establishes the coherence between the levels, resulting in various fascinating effects. As the problems discussed in this thesis use the lambda (Λ) system, we will restrict our discussion to the said configuration.

The optical properties of an atom are linked to its intrinsic energy levels and can be manipulated by inducing coherence between atomic states. This can lead to quantum interference between the excitation pathways. Electromagnetically induced transparency (EIT) [20] is a quantum interference effect where a medium that is generally opaque to a certain frequency of light (Probe laser) becomes transparent due to the coherence induced by another laser field (Coupling laser). In the event that an atom can make a transition from its ground state to its excited state via more than one pathway, the transition probability is equal to the squared sum of the probability amplitudes of all of those pathways. The destructive interference between these probability amplitudes modifies the linear susceptibility of the probe laser and makes it transparent. This transparency window is an effect due to the creation of dark states. The dark state is a superposition of ground and intermediate state, which gets decoupled to the excited state. The atoms in the dark state get trapped and can not be excited with a probe laser. This quantum interference also modified the real part of the linear susceptibility, which results in slowing down of light pulse [21, 22, 23]. More importantly, in the same transparency window, the nonlinear susceptibility [24] shows constructive interference and makes it a suitable platform for nonlinear interactions. The coupling between the two ground states of the Λ system by an electromagnetic field makes it look like a Delta (Δ). This Δ atomic system has fascinating properties that we will cover in this thesis.

1.2 Motivation and objectives

Developing a suitable platform that transduces the signal from microwave to optical frequency is the motivation behind the challenges discussed in this thesis. Though there are platforms that convert microwave signals into optical, the most straightforward and simple approach is a three-level atomic system. The three hyperfine energy levels of the neutral ^{85}Rb atom in the D1 manifold are a suitable platform to achieve the transduction process. Though the even-order nonlinear processes are naturally absent in this kind of atomic system due to its centrosymmetric nature, the magnetic dipole interactions be-

tween a pair of energy levels break the symmetry. As a result, processes like TWM can be demonstrated by these systems, which upconvert the microwave field to an optical field that is separated by five orders of magnitude in frequency from the former one. It also addresses how the cyclic and coherent nature of the interactions between the electromagnetic fields in the presence of the three-level system gives rise to information bit transfer across the frequency domain for communication purposes. In the past, these three-level systems have shown to be very promising candidates for slowing down the light pulse [25], making them useful for quantum memory applications. Making accessible pulse on demand by utilizing the second-order nonlinearity provided by these atoms is also a motivating objective. The process of six-wave mixing is also explored in this thesis.

The goals of this thesis, in brief, are

- Experimental demonstration of a radio-over-fibre device in a three-level Δ atomic system at room temperature.
- Theoretical investigation of an atomic system where the delay time of an optical pulse propagating inside the medium can be tailored, making it a pulse-on-demand device.
- Perform an experiment to observe higher-order nonlinear processes, such as the six-wave mixing process in a dilute sample of atomic vapor.

1.3 Organisation of the thesis

This thesis consists of an introduction chapter, three main chapters covering experimental work as well as theoretical calculations, followed by a chapter for summary and future work.

Chapter 2 experimentally investigates a novel communication platform for transferring information across microwaves to the optical domain. In this communication system, the neutral dilute sample of atoms plays the role of an antenna, which receives and sends the information bits. The hybrid second-order nonlinearity mediated by these room temperature ^{85}Rb vapor couples one microwave photon to an optical photon and results in a generation of another optical photon in a process called three-wave mixing (TWM) [26].

The closed cyclic interaction of this atomic system turns the response of the system phase-dependent [27, 28] and makes the system that can correlate the phase of the input signal to the intensity of the output signal. Taking advantage of this phase-to-intensity mapping, our communication scheme uses microwave photons as the carrier signal whose phase gets modulated according to the information bits. The information bits get demodulated and seen as an intensity-modulated optical field, which gets detected by optical heterodyne measurement. Both binary phase shift keying(BPSK) and four phase shift keying(4-PSK) are addressed as communication protocols/schemes in this work. The tunability, as well as the role of the intensity of the carrier signal, is also discussed. The chapter concludes by correlating the recovered data with an ideal template that validates the legitimacy of this atomic vapor-mediated communication scheme.

Chapter 3 theoretically investigates the creation and group-delay control of few-photon optical pulses in a cyclic atomic level system that interacts with an optical and a microwave field. In order to produce an optical pulse, our method utilizes the hybrid second-order nonlinearity possessed by the atom. The research work starts with solving the master equation for the three-level system, followed by solving the wave equation for understanding pulse propagation inside the medium. The results explain through microwave intensity control of the absolute phase of the second-order nonlinearity, the produced pulse can be either delayed or advanced. The significance of the results shows this control over the generated pulse's group delay is not dependent on number density. Thus, employing ultra-dilute atomic samples, our method is perfectly suited for the creation and management of few-photon optical pulses. Our findings will make it possible to create tiny atomic interface systems that can be used as delay channels with adjustable delay for both classical and quantum signal processing.

In Chapter 4, we experimentally examine and theoretically explain the generation of both six-wave mixing (SWM) and three-wave mixing (TWM) phenomena in a dilute sample of ^{85}Rb atoms. Both the generation process results in optical frequency transduced from an idler microwave field. The presence of both SWM and TWM signals opens dual channels for the microwave-to-optical transduction process hence making this platform an ideal platform for signal multiplexing. The breaking of the three-photon resonance condition plays a major role in detecting both the generation process as frequency-separated signals as well as observing coherent population oscillation. The dependency of idler and

pump power in the generation of both SWM and TWM processes is also compared.

1.4 Some basics of nonlinear quantum optics

1.4.1 Theory of wave mixing process

This section explains the basics of the nonlinear wave mixing process, in particular, the sum frequency generation process. The equations used in "Nonlinear Optics" by Robert Boyd [29] served as the foundation for this section. In Fig. (1.2) the frequency of two input fields given as ω_1 and ω_2 interacts inside a medium which has non-zero second order nonlinearity($\chi^{(2)}$). The interaction results in the generation of a new field at frequency $\omega_1 + \omega_2 = \omega_3$. In this derivation we have assumed the electromagnetic fields are monochromatic, the nonlinear medium does not contain free charges or free currents, and the medium is nonmagnetic. Then, the Maxwell equations can be written as

$$\nabla \cdot \mathbf{D} = 0 \quad (1.4.1.1a)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (1.4.1.1b)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1.4.1.1c)$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial \mathbf{D}}{\partial t} \quad (1.4.1.1d)$$

where $\mathbf{E}, \mathbf{B}$ stands for the electric and magnetic fields inside the medium. The $\mathbf{D}$ denotes the displacement vector, defined as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (1.4.1.2)$$

here ϵ_0 implies the permittivity of free space and $\mathbf{P}$ denotes the polarization vector.

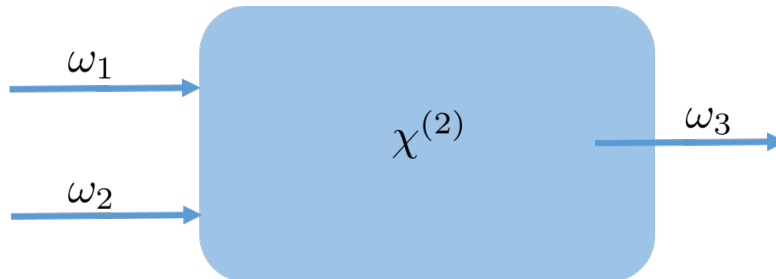


Figure 1.2: Sum frequency generation process

Now taking the curl of the Eq. (1.4.1.1c) and substituting Eq. (1.4.1.1d) and using the identity $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$, we obtain

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad (1.4.1.3)$$

Where c is the speed of light in vacuum, $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$.

The total polarization ($\mathbf{P}$) in the Eq. (1.4.1.3) is a composition of linear ($\mathbf{P}_L = \epsilon_0 \chi^{(1)} \mathbf{E}$) and nonlinear ($\mathbf{P}_{NL}$) terms. After decomposing $\mathbf{P}$, the wave Eq. (1.4.1.3) equation becomes

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} (1 + \chi^{(1)}) \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2} \quad (1.4.1.4)$$

Where $\chi^{(1)}$ is the linear susceptibility of the medium.

The above Eq. (1.4.1.4) is a inhomogeneous wave equation. The term that appears on the right-hand side of the equation acts as a source term that arises from the nonlinear response of the system. Without the source term, the solution to the Eq. (1.4.1.4) is a plane wave at frequency ω_3 and traveling with a speed of c/n in $+z$ direction is given by

$$\mathbf{E}_3(z, t) = A_3 e^{i(k_3 z - \omega_3 t)} + \text{C.C.} \quad (1.4.1.5)$$

and where the amplitude of the wave A_3 is constant.

The input electric fields travelling in $+z$ direction can be written as

$$\begin{aligned} \mathbf{E}_1(z, t) &= A_1 e^{i(k_1 z - \omega_1 t)} + \text{C.C.} \\ \mathbf{E}_2(z, t) &= A_2 e^{i(k_2 z - \omega_2 t)} + \text{C.C.} \end{aligned} \quad (1.4.1.6)$$

where c.c. represents complex conjugate, $k_i = n_i \omega_i / c$ and $n_i, (i = 1, 2)$ is the refractive index of the medium. A_i represents the amplitude of the field.

We represent the source term in Eq. (1.4.1.4) as

$$\mathbf{P}_{NL}(z, t) = P_3 e^{-i\omega_3 t} + \text{C.C.} \quad (1.4.1.7)$$

where

$$P_3 = 2\epsilon_0\chi^{(2)}A_1A_2e^{i(k_1+k_2)z} \quad (1.4.1.8)$$

Now, we can write the solution of the Eq. (1.4.1.4) in the same form as in Eq. (1.4.1.5), except A_3 is now a slowly varying function of z . This is valid when the nonlinear source term is small.

After substituting Eq. (1.4.1.5), Eq. (1.4.1.7) and Eq. (1.4.1.8) into the wave Eq. (1.4.1.4) and dropping the C.C. terms from each side, we obtain

$$\begin{aligned} \left(\frac{d^2 A_3(z, t)}{dz^2} + 2ik_3 \frac{dA_3(z, t)}{dz} - k_3^2 A_3(z, t) \right) + \frac{\omega_3^2}{c^2} (1 + \chi^{(1)}) A_3(z, t) \\ = -\frac{2\omega_3^2}{c^2} \chi^{(2)} A_1 A_2 e^{i\Delta k z}, \end{aligned} \quad (1.4.1.9)$$

where

$$\Delta k = k_1 + k_2 - k_3, \quad (1.4.1.10)$$

is the wave vector mismatch.

As we know $k_3^2 = \omega_3^2(1 + \chi^{(1)})/c^2$, so the third and fourth term on the left-hand side cancel each other. The first term on the left-hand side of Eq (1.4.1.9) is much much smaller compared to the second term. This approximation indicates that the fractional change in A_3 is much smaller than unity within a distance of the order of a wavelength. The approximation is called the slowly varying amplitude approximation (SVAA) and is valid when

$$\left| \frac{d^2 A_3}{dz^2} \right| \ll \left| k_3 \frac{dA_3}{dz} \right| \quad (1.4.1.11)$$

After using SVAA the Eq. (1.4.1.9) becomes

$$\frac{dA_3(z, t)}{dz} = \frac{i\omega_3^2 \chi^{(2)}}{c^2 k_3} A_1 A_2 e^{i\Delta k z} \quad (1.4.1.12)$$

The Eq. (1.4.1.12) is called the coupled-amplitude equation, as it shows the variation of the amplitude of ω_3 as a result of its coupling to ω_1 and ω_2 fields.

After exiting the nonlinear medium of length (L), the amplitude of the sum frequency (ω_3) wave can be written down as

$$\begin{aligned} A_3(L) &= \frac{i\omega_3^2 \chi^{(2)} A_1 A_2}{c^2 k_3} \int_0^L e^{i\Delta k z} dz \\ &= \frac{i\omega_3^2 \chi^{(2)} A_1 A_2}{c^2 k_3} \left(\frac{e^{i\Delta k L} - 1}{i\Delta k} \right) \end{aligned} \quad (1.4.1.13)$$

For phase-matched condition ($\Delta k = 0$), the generated field amplitude can be written as

$$\lim_{\Delta k \rightarrow 0} A_3(L) = \frac{i\omega_3^2 \chi^{(2)} A_1 A_2}{c^2 k_3} L \quad (1.4.1.14)$$

Assuming both input fields as undepleted wave and for a perfect phase-matched condition, the intensity of the generated field (ω_3), is given by

$$\begin{aligned} I_3 &= \lim_{\Delta k \rightarrow 0} 2n_3 \epsilon_0 c |A_3|^2 \\ &= \frac{2n_3 \epsilon_0 \omega_3^4 |\chi^{(2)}|^2 |A_1|^2 |A_2|^2}{c^4 k_3^2} L^2 \end{aligned} \quad (1.4.1.15)$$

1.4.2 Difference frequency generation and parametric amplification

In this section, we discuss the difference frequency generation [29] and also the parametric amplification of a signal field in a nonlinear medium. The Fig.1.3 depicts a lossless nonlinear medium where waves at frequencies ω_1 and ω_2 interacts to generate another wave at the difference frequency $\omega_3 = \omega_1 - \omega_2$. We have assumed the wave at ω_1 is a undepleted field.

We represent the input fields as

$$\begin{aligned} \mathbf{E}_1(z, t) &= A_1 e^{i(k_1 z - \omega_1 t)} + \text{C.C.} \\ \mathbf{E}_2(z, t) &= A_2 e^{i(k_2 z - \omega_2 t)} + \text{C.C.} \end{aligned} \quad (1.4.2.1)$$

and generated output field as

$$\mathbf{E}_3(z, t) = A_3 e^{i(k_3 z - \omega_3 t)} + \text{C.C.} \quad (1.4.2.2)$$

The nonlinear source term for the difference frequency generation can be written as

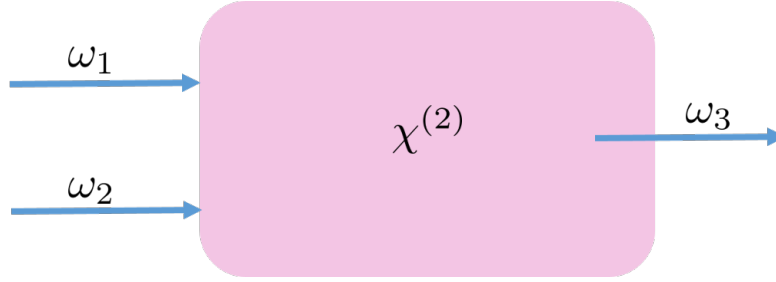


Figure 1.3: Difference frequency generation

$$\mathbf{P}_{\text{NL}}(z, t) = P_3 e^{-i\omega_3 t} + \text{C.C.} \quad (1.4.2.3)$$

where

$$P_3 = 2\epsilon_0 \chi^{(2)} A_1 A_2^* e^{i(k_1 - k_2)z} \quad (1.4.2.4)$$

Substituting Eq. (1.4.2.2), Eq. (1.4.2.3) and Eq. (1.4.2.4) into the wave Eq. (1.4.1.4), we obtain, the wave equation for $A_3(z, t)$ as

$$\frac{dA_3(z, t)}{dz} = \frac{i\omega_3^2 \chi^{(2)}}{c^2 k_3} A_1 A_2^* e^{i\Delta k z}, \quad (1.4.2.5)$$

Now the nonlinear source term for $A_2(z, t)$ can be defined as

$$P_2(z, t) = 2\epsilon_0 \chi^{(2)} A_1 A_3^* e^{i(k_1 - k_3)z} \quad (1.4.2.6)$$

In a similar fashion as $A_3(z, t)$ the wave equation for $A_2(z, t)$ can be derived as

$$\frac{dA_2(z, t)}{dz} = \frac{i\omega_2^2 \chi^{(2)}}{c^2 k_2} A_1 A_3^* e^{i\Delta k z}, \quad (1.4.2.7)$$

where

$$\Delta k = k_1 - k_2 - k_3 \quad (1.4.2.8)$$

The above written Eq. (1.4.2.5) and Eq. (1.4.2.7) are coupled-amplitude equations describing the difference frequency generation at ω_3 and ω_2 with ω_1 as an undepleted field.

In order to solve the above coupled differential equations we differentiate Eq. (1.4.2.5) with respect to z and substitute the complex conjugate of Eq. (1.4.2.7) so as to get rid of the dA_2^*/dz term from the right-hand side of the Eq. (1.4.2.5). Then, the obtained equation is

$$\begin{aligned} \frac{d^2 A_3}{dz^2} &= \frac{\omega_2^2 \omega_3^2 \chi^{(2)^2}}{k_2 k_3 c^4} |A_1|^2 A_3, \\ &= \eta^2 A_3, \end{aligned} \quad (1.4.2.9)$$

where

$$\eta^2 = \frac{\omega_2^2 \omega_3^2 \chi^{(2)^2}}{k_2 k_3 c^4} |A_1|^2, \quad (1.4.2.10)$$

and η is called the coupling constant.

The solution to the Eq. (1.4.2.9) can be written as

$$A_3(z) = u \sinh \eta z + v \cosh \eta z, \quad (1.4.2.11)$$

where u and v are the constants and their values can be found out from boundary conditions.

We assume the boundary conditions as

$$A_3(z = 0) = 0 \quad \text{and} \quad A_2(z = 0) = A_2(0). \quad (1.4.2.12)$$

Now the solution to the Eq. (1.4.2.9) and Eq. (1.4.2.7) with the above boundary

conditions can be written down as

$$A_3(z) = i \left(\frac{n_2 \omega_3}{n_3 \omega_2} \right)^{1/2} \frac{A_1}{|A_1|} A_2^*(0) \sinh \eta z, \quad (1.4.2.13a)$$

$$A_2(z) = A_2(0) \cosh \eta z. \quad (1.4.2.13b)$$

As can be seen from the Eq. (1.4.2.13), both the fields ω_2 and ω_3 grows exponentially with respect to z . From Eq. (1.4.1.12), we can understand the presence of ω_2 field aids in generating the ω_3 field through difference frequency generation. Also the ω_3 field stimulates the generation of ω_2 field. So the generation of ω_2 field reinforces ω_3 field to generate and vice versa, which leads to each field growing exponentially. In the language of nonlinear optics, the ω_2 field is called the signal, and the ω_3 field is called the idler. The signal field gets amplified through a difference-frequency generation process, which is a parametric process. Hence, this process is called parametric amplification. As a by-product of this process, an idler field is also gets generated.

1.4.3 Optical heterodyne detection

Optical heterodyning [30] is a technique to extract information in optical measurement and communication systems. In this technique, two optical signals with different frequencies interfere with each other. The resulting beat signal is then detected, which contains information about the original signal.

Optical heterodyning uses a reference signal called local oscillator (LO) to measure the unknown spectral characteristics of the second optical source called signal. Generally, the frequency difference between LO and signal is much less as compared to the frequency of any optical source. The two sources are mixed together on a beam splitter or in an optical coupler where they interfere with each other. The beat notes contain both the sum and the difference frequencies produced as a result of interference. The beat note contains the information about the signal, which gets detected with a fast photodetector.

To understand the principle of optical heterodyning, let us consider two optical fields

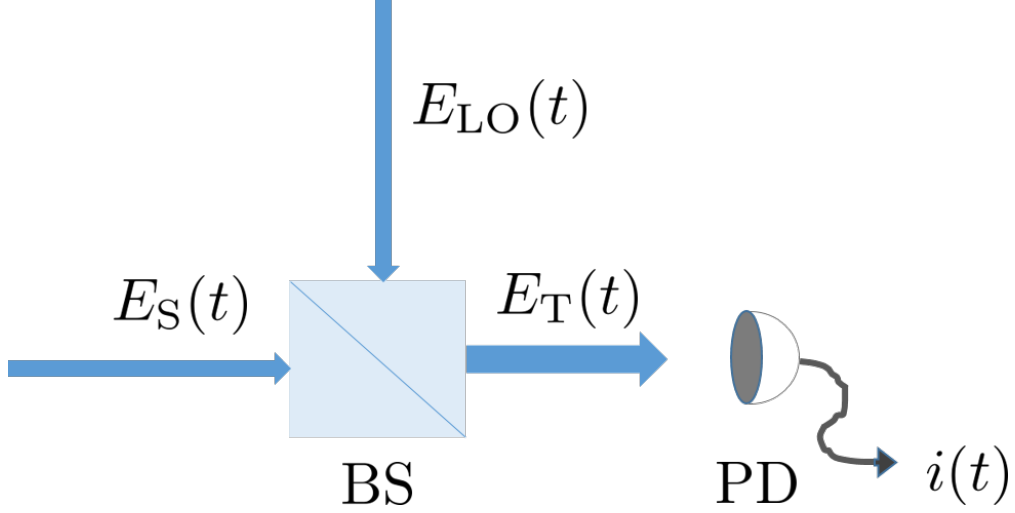


Figure 1.4: An optical heterodyne setup. BS-Beam splitter, PD-Photodetector.

falling on the photodetector after passing through the beam splitter as shown in Fig. (1.4). The electric field of the two fields is defined as

$$E_S(t) = \sqrt{P_S(t)} \cos(\omega_S t + \phi_S(t)) \quad (1.4.3.1a)$$

$$E_{LO}(t) = \sqrt{P_{LO}} \cos(\omega_{LO} t + \phi_{LO}(t)), \quad (1.4.3.1b)$$

where ω_S , ϕ_S , P_S and ω_{LO} , ϕ_{LO} , P_{LO} represents the angular frequency, phase and optical power of the signal and LO respectively. The fields are normalized such that their magnitude squared are optical powers.

After passing through the beam splitter, the spatially overlapped optical fields then fall on a photodetector, where the interference is identified. The total field $E_T(t)$ at the photodetector can be written as

$$E_T(t) = E_S(t) + E_{LO}(t). \quad (1.4.3.2)$$

A photodetector is a nonlinear device with respect to the optical field and follows the principle of square law detection. Therefore, in a photodetector, we detect the optical field's power rather than the field itself. The output of the photodetector is photocurrent, $i(t)$, and can be represented as

$$i(t) = \mathcal{R}|E_T|^2 \quad (1.4.3.3)$$

where $\mathcal{R}$ is the responsivity of the photodetector.

The total optical power P can be calculated as

$$P = |E_S(t) + E_{LO}(t)|^2$$

$$= \frac{P_S(t)}{2} + \frac{P_{LO}(t)}{2} + 2\sqrt{P_S(t)}\sqrt{P_{LO}(t)}\cos((\omega_S - \omega_{LO})t + \Delta\phi(t)) + \text{H.F.}, \quad (1.4.3.4)$$

where H.F. stands for high-frequency terms and $\Delta\phi(t) = \phi_S - \phi_{LO}$.

The H.F. includes frequency components that fall within the optical frequency range. There is no photodetector that can follow these first oscillations; as a result, these high-frequency components get filtered out. Now, the photocurrent can be represented as

$$i(t) = \mathcal{R} \left(\frac{P_S(t)}{2} + \frac{P_{LO}(t)}{2} + 2\sqrt{P_S(t)}\sqrt{P_{LO}(t)}\cos((\omega_S - \omega_{LO})t + \Delta\phi(t)) \right). \quad (1.4.3.5)$$

The first two terms of the right-hand side of Eq. (1.4.3.5) are the D.C. averaged power and corresponds to the intensity of E_S and E_{LO} . The crucial mixing term is the third one, which contains information about the optical signal. Thus, in the heterodyne method, with the help of a local oscillator, the spectral information of a high-frequency optical field gets transferred to a low-frequency $(\omega_S - \omega_{LO})$ RF field, which can be measured with electronics.

It is evident from Eq. (1.4.3.5) that the difference frequency term is proportional to the product of signal and LO electric fields. The more the LO amplitude, the more the amplitude of the mixing term. As a result, LO provides a gain in the photocurrent, which is crucial in the detection of weaker signals.

The phase information of an optical field is lost in the case of direct photodetection. In contrast, heterodyne detection preserves the phase information and permits the same to be detected. Any phase shift in the optical signal will result in an equal amount of change in the mixing term, eventually resulting in changing the value of the mixing term. This change in photocurrent can be measured and back-calculated to the amount of phase shift in the optical field. In phase measurement, typically, both the signal and LO are derived from a common laser source in order to maintain coherence in their phase.

In our experiment, the electro-optic modulator creates a band of laser fields that are frequency-separated. These beams co-propagate and fall on a photodetector after interact-

ing with the atoms placed inside the microwave cavity. Now, to make direct measurements in one of the beams is impossible until we make it spatially separated from other beams. To resolve this issue, we take the advantage of optical heterodyne. In the experiment, a part of the laser beam is diverted using a beam splitter. This diverted laser beam is then passed through a Double-pass acoustic-optic modulator system, where the frequency of the laser beam is changed by a few MHz. The beam is then called LO and mixes with all the copropagating beams. These co-propagating frequency-separated beams then mix with the LO and produce beat notes at respective downconverted frequencies after falling on a photodetector. These down-converted beat notes can be observed on a spectrum analyzer.

1.4.4 Microwave cavity

As stated earlier, the three hyperfine levels of ^{85}Rb in the D1 manifold form the basis of Δ system in all our experiments, which can be seen in Fig. (1.5). The two ground states connect to a common excited state by two optical fields (Ω_p , and Ω_c in Fig.), and those transitions are electric dipole allowed. In contrast, the transition between the two ground states is electrically forbidden but magnetically (magnetic dipole) allowed as can be dictated by selection rules. We all know the magnetic dipole transition strength is several orders weaker as compared to its electric dipole counterpart. So, to manipulate atoms between ground states, we require high magnetic field strength.

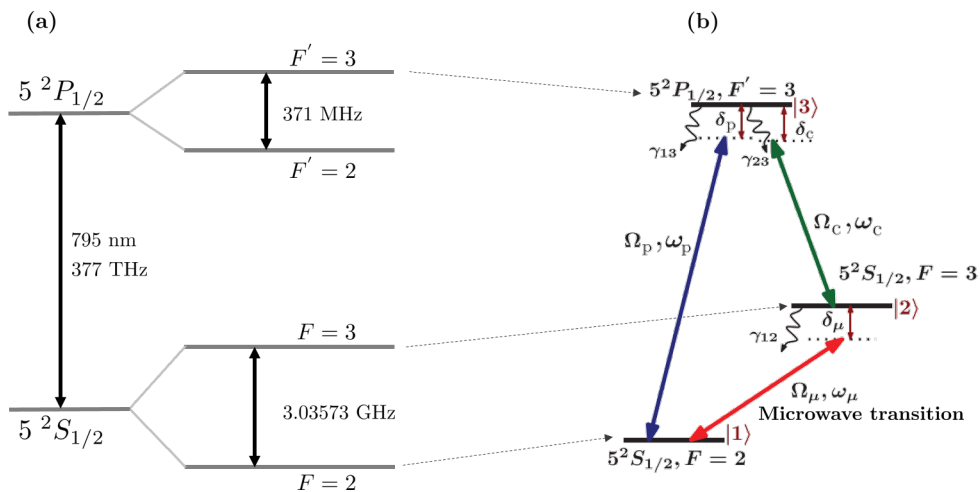


Figure 1.5: Fig.(a) depicts the energy levels of the ^{85}Rb in the D1 manifold. Fig. (b) illustrates the Δ system formed when the lasers are coupled with the appropriate atomic energy levels.

A cavity resonator amplifies the electromagnetic wave that bounces back and forth in-

side it due to constructive interference. Hence, it enhances the magnetic field several times and, as a result, strengthens the magnetic dipole transition. Some commonly used cavities are rectangular, cylindrical, and spherical cavities. Among all three, the cylindrical cavity is easy to construct and gives a better quality factor(Q).

The most important parameter of a cavity is the resonance frequency. A cavity can only excite certain frequencies of the electromagnetic wave called modes. The fundamental mode is the frequency of the lowest order. These modes depend on the dimension of the cavity as well as the material of the cavity. The dependence of resonant frequencies on the cavity dimensions can be found in the mode chart. Fig. (1.6) depicts the mode chart of a cylindrical resonator [31]. In our experiment, the cylindrical cavity [32] resonates at

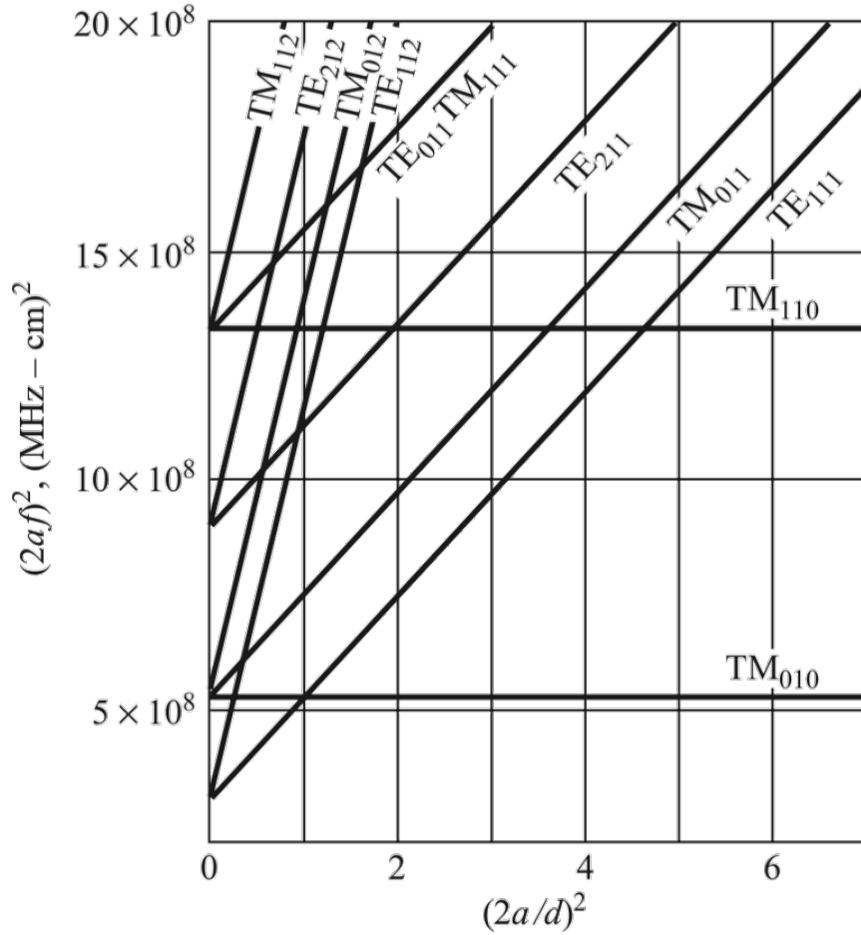


Figure 1.6: Mode chart of a circular cylindrical cavity. Inner radius= a , length= d . Picture courtesy - Microwave engineering by David M Pozar [31].

3.03573 GHz, which is the energy difference between the two ground states of the ^{85}Rb as can be seen in Fig. 1.5. Apart from the temporal modes, the transverse modes of the cavity are also crucial as they explain the spatial distribution of the electric field and

magnetic field of the resonant electromagnetic wave inside the cavity. These modes are the solutions of the Maxwell wave equation. In our case, the TE_{011} mode floods the cavity as it has a maximum value of magnetic field intensity at the center of the symmetry axis of the cylinder, which is essential to the magnetic dipole transition. We place the Rb vapor cell at the center of the axis for all our experiments, as can be seen in Fig. (2.2) of this thesis.

The material of the cavity plays a major role in designing and deciding the resonance frequency. The suitable material is one that has very high conductivity and has less skin depth so as to reduce the conductive losses and provide better isolation. Aluminum and copper are the best choices for this.

The second most important parameter of a resonator is the quality factor or Q value. The Q factor is defined as the ratio of initial energy stored in the cavity to the energy lost in a one-radian cycle of oscillation. The Q value is also a measure of the strength of damping of the oscillations captured inside it. The higher the Q value, the lower the energy dissipation. As a result, the electromagnetic wave can oscillate sustainably for more periods of time inside the cavity. Q value also indicates the linewidth of the cavity, which can be defined as given in Eq. (1.4.4.1)

$$Q = \frac{\text{Resonant Frequency}}{\text{Linewidth of the cavity}}. \quad (1.4.4.1)$$

For e.g., a cavity having a Q value of 10000 and resonating at 3 GHz will have a linewidth of 300 kHz. The Q factor is a dimensionless parameter. The Q factor of a cavity can be obtained by measuring either S_{21} or S_{11} parameter of the scattering matrix. In our lab, we determine the Q factor by measuring the S_{11} parameter with the help of a vector network analyzer (VNA).

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Chapter 2

Digital communication using a cyclic level scheme in an atomic radio-over-fiber device

2.1 Introduction

Signal communication is the process of transmitting and receiving information through signals. Depending on the nature of the signal it can either be classified as analog communication or digital communication. In analog communication, data is communicated through an analog signal between sending and receiving stations. An analog signal is a continuous-time signal containing information. In contrast, digital communication uses digitized signals that consist of discrete/quantized values of the continuous signal. Though analog communication requires less bandwidth for transmission as compared to digital communication but the latter has more advantages than the former one. The digital signal requires less power for transmission in comparison to an analog signal. The analog signal is generally susceptible to noise and a small change in voltage level in the analog signal can produce significant error in information processing, whereas the digital signal is immune to noise because of its better signal-to-noise ratio (SNR) [1].

In communication the signal can be sound, light, or radio waves. For transmitting data over long distances, the information signal requires a high-frequency carrier signal. The carrier signal gets modulated according to the information signal before getting transmitted. The imprinting of information on a carrier signal can be achieved by modulating the

amplitude (AM), phase (PM) or frequency (FM) of the carrier signal. The communication system can be separated into microwave communication and optical communication based on the frequency of the carrier signal.

Because of its faster data transfer speeds, large bandwidth and resistance to electromagnetic interference, optical communication has long dominated microwave communication. A significant quantity of data may be transmitted across great distances with minimal loss via optical communication which makes use of light waves, specifically lasers. Microwave communication, in contrast, makes use of radio waves or microwaves, which have a limited bandwidth and can be interfered by other electromagnetic sources, resulting in a slower and less dependable data transport. The presence of thermal photons at microwave frequencies adds unwanted noise in the received information resulting poor SNR. Additionally, optical communication is more secure than microwave communication because it uses fiber optic lines, which lessens the possibility of signal interception or interference. In response to this, optical communication has become more prevalent for large-scale data transmission in industries including telecoms, data centers and internet connectivity.

The use of "radio-over-fiber" (RoF) [2] technology offers a very promising solution to avoid losses and interference that happen in communicating through microwaves. In this technology, RF signal is encoded in laser light and sent via optical fiber. Optical fibers are a better option than metallic coaxial cables as they have incredibly low levels of loss and optimal electrical separation. Generally, the RoF architectures include a signal source, an RF-to-optical encoder, a fiber-optical network, a photodetecting stage, and demodulation electronics.

The RoF architecture involves modulation of laser light in order to imprint information on it. In this work, we have shown a neutral atom-based RoF platform that does not require direct modulation of laser light. We have demonstrated a neutral atom can be a sensitive optical antenna that enables the direct encoding of information in free space. As a result, no electrical contacts are required at the receiver end, allowing for a more straightforward and adaptable RoF architecture.

Sensing of RF and microwave fields with alkali metal atoms through the existence of appropriate energy levels in these frequency regimes is a rapidly growing area of study [3, 4]. The exquisite frequency selectivity offered by the discrete atomic levels

makes it a natural filter, thus retrieving weak signals from ambient noise. In recent years, these atomic systems exploit the versatile electromagnetically induced transparency (EIT) effect [5, 6] available at optical frequencies, to effect a radio-over-fibre conversion of RF and microwave fields to the optical regime. Specifically, RF field sensing and communication between microwave and optical frequencies have been demonstrated using these alkali atoms [7, 8, 9, 10, 11]. These experiments have opened an entirely unexplored area of RF signal processing and transmission, with atoms occupying the central stage in multiple roles acting like a sensor, mixer and a radio-over-fibre device. In the past few years there has been a very interesting parallel development of utilizing cyclic energy level schemes available in Rb atoms to connect microwave and optical frequencies in a coherent and phase dependent manner [12]. The EIT effect has been utilized in this scheme as well and demonstrations of microwave activated optical switches [13], generation of optical fields [14], phase dependent amplification of optical fields controlled by the microwave phase [15] and audio modulation of microwave carrier and its optical read-out has already been done [16]. However a practical implementation of a digital communication protocol with a cyclic system has not been demonstrated until now.

In this experimental work we demonstrate two digital phase modulation schemes. A binary phase-shift keying (BPSK) and a customised 4 phase-shift keying (C-4PSK) is implemented on a 3 GHz microwave carrier and its optical intensity demodulation readout is demonstrated at 795 nm. Our experiment uses three atomic hyperfine levels interacting with one microwave and two optical fields in a closed and cyclic fashion in ^{85}Rb atoms at room temperature. The noteworthy feature in our communication protocol as compared to Rydberg atom based ones is that our cyclic atomic scheme is sensitive to the relative phase of fields it interacts with. Thus it can naturally encode a signal in the phase of a microwave field and retrieve it as an intensity change in the optical field without the need for an external demodulator.

Utilising this feature, we demonstrate a viable signal bandwidth of about 1 MHz and a carrier frequency change of about 15 MHz. More importantly, our atomic system amplifies the demodulated optical signal through a hybrid second-order nonlinearity involving a magnetic dipole and an electric dipole transition. This amplification increases the contrast of the output optical signal and makes it sensitive to small input phase changes. Furthermore our system utilises for its communication protocol, a dark state which is

also a quantum memory state [17] making this scheme ideal for storage of microwave [18] quantum signals and its coherent conversion to optical regimes [19]. Thus our experiment is one of the very few direct digital phase modulation and demodulation schemes to be implemented in an amplifying atomic radio-over-fibre device.

2.2 Experimental details

For our experiment we use three hyperfine energy levels in the D1 manifold of ^{85}Rb in a Λ configuration. These levels are cyclically connected by two optical and one microwave field forming a phase sensitive closed Λ system also called as a Δ system. This is shown in Fig. 2.1.

The strong coupling laser connecting the $5^2S_{1/2}$, $F = 3$ ($|2\rangle$) metastable state and

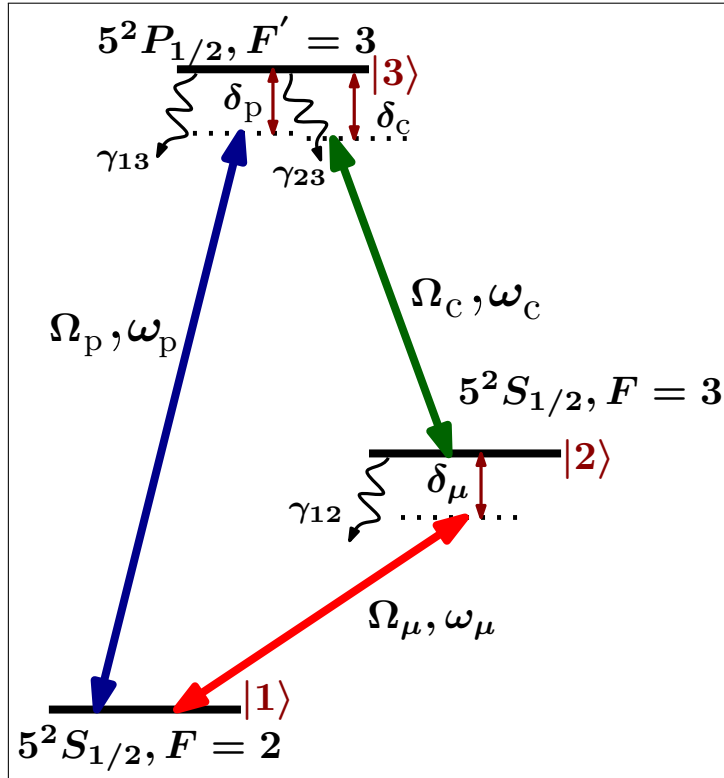


Figure 2.1: Hyperfine energy levels in a Δ configuration in ^{85}Rb used for our communication scheme. The detunings of probe, coupling and microwave fields are represented by δ_p , δ_c and δ_μ respectively. The Rabi frequencies of the probe, coupling and microwave fields are denoted as Ω_p , Ω_c and Ω_μ respectively

$5^2P_{1/2}$, $F' = 3$ ($|3\rangle$) excited state is derived from an External Cavity Diode Laser (ECDL), tuned at 795 nm. The $5^2S_{1/2}$, $F=2$ ($|1\rangle$) ground state is connected to the excited state

$|3\rangle$ by a weak probe beam, derived from the coupling laser after passing through a phase modulated Electro-Optic Modulator(EOM). The EOM is driven by a microwave source which also is the source of the carrier microwave signal in our experiment as shown in Fig. 2.2. The derivation of probe beam from coupling beam ensures phase coherence

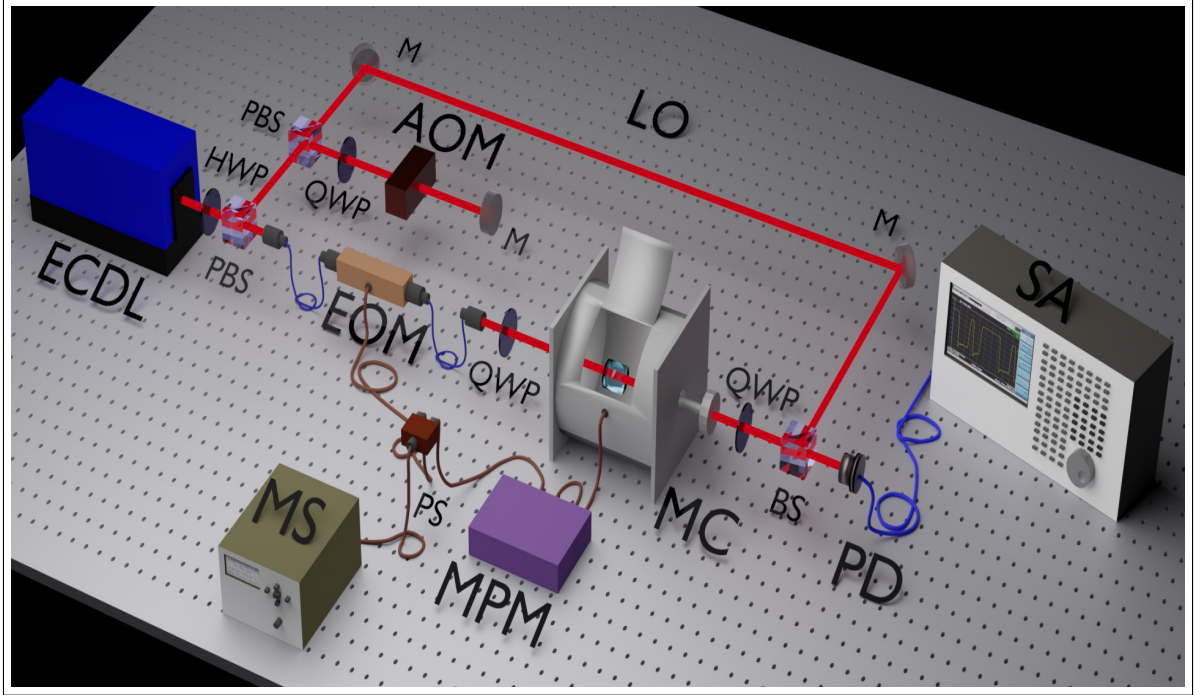


Figure 2.2: Experimental Setup. Here ECDL —External Cavity Diode Laser, MC —Microwave Cavity, MS —Microwave Source, MPM —Microwave Phase Modulator, SA —Spectrum Analyzer, PD —Photo-Detector, EOM —Electro Optic Modulator, AOM —Acusto Optic Modulator, PS —Power Splitter, QWP —Quarter Wave Plate, HWP —Half Wave Plate, M —Mirror, PBS —Polarising Beam Splitter, BS —Beam Splitter, LO —Local Oscillator.

between the two fields. The ground state $|1\rangle$ and the metastable state $|2\rangle$ is coupled to the oscillating magnetic field of microwave carrier enabling magnetic dipole transition between the states as dictated by selection rules. The 1 cm Rb vapour cell, which is our atomic antenna in the experiment, is placed inside a microwave cavity resonating at 3.0357 GHz. The cavity is designed to support TE_{011} mode to further strengthen the weak magnetic dipole interaction between the microwave field and atoms. The microwave excitation is fed into the cavity through a small loop antenna placed inside the cavity. The measured quality factor and the line width of the microwave cavity is 10946 ± 274 and 277.467 ± 6.814 kHz respectively at 3.0357 GHz. The loop antenna couples about 84 % of the input microwave power to the cavity at resonance. Thus the atoms inside the cavity interact with the two optical fields and the microwave field as shown in Fig. 2.2. The Rabi frequencies associated with coupling, probe and microwave fields are Ω_c , Ω_p ,

Ω_μ respectively.

We use optical heterodyning technique to separate the probe beam exiting the cavity from other co-propagating optical fields. The heterodyne technique consists of mixing the fields coming out of the experimental cell with that of a reference local oscillator (LO) optical beam. In our experiment the LO is derived from the coupling beam(ω_c) after passing it through a double pass set-up which up-shifts ω_c by 120 MHz with the help of an acousto-optic modulator(AOM) giving a beam at ($\omega_c + 120$ MHz). The LO beats with the output beams coming out of the vapour cell through mixing on a beam-splitter. The desired optical probe beam which is at $\omega_p = \omega_c + 3.0357$ GHz is then separated from other beams and appears at a down converted rf frequency ($3035.7 - 120$) MHz which is then detected by a high bandwidth photo detector. The output of the photo-detector is recorded in a spectrum analyser.

We define δ_p , δ_c and δ_μ as the detunings of the probe, coupling and the microwave field from their respective atomic transition energy levels as shown in Fig. 2.1.

The co-propagating coupling and probe laser powers are $160 \mu\text{W}$ and $20 \mu\text{W}$ respectively with a minimum beam diameter of 1 mm. The three-photon resonance defined as $\delta_p - \delta_c - \delta_\mu = 0$ is satisfied throughout our experiment. This ensures that time-independent steady state values for populations and coherences exist in our scheme [20]. Under an unchanging three photon resonance, our digital communication protocol is tested for various values of two-photon detuning between the optical fields given by $\delta_2 = \delta_p - \delta_c$. Under two-photon resonance given by $\delta_2 = 0$, our cyclic atomic system has a dark state which enables transparency for the probe field due to the EIT effect which reduces absorption. Hence the effective bandwidth for our communication is decided by the EIT bandwidth.

The cyclic and closed nature of interaction of the three electromagnetic fields with their respective transitions makes the populations and coherences dependent on the relative phase between all the three fields. This relative phase $\Delta\phi$ is defined as $\Delta\phi = (k_p - k_c)z + \phi_\mu$. The first term represents the propagation phases of the coupling and probe fields and the second term is the absolute phase of the standing wave microwave field inside the microwave cavity. In our experiment we have changed the relative phase $\Delta\phi$ by changing the phase ϕ_μ of the microwave field through a digital phase modulator.

We give below a very brief explanation of the nature of hybrid second-order non-linearity in our cyclic atomic system which enables our digital communication scheme.

The polarization induced in the atoms by probe, coupling and microwave electromagnetic fields is given by [21]

$$P \approx \epsilon_o E_{0p} (\chi_p^{(1)}(\omega_p) e^{-i\omega_p t} + c.c)/2 + \epsilon_o E_{0c} B_{0\mu} (\chi_p^{(2)}(\omega_c, \omega_\mu) e^{-i((\omega_c + \omega_\mu)t + \Delta\phi)} + c.c)/4 \quad (2.2.0.1)$$

Here E_{0p} , and E_{0c} represent the electric field amplitudes of the probe and coupling optical fields and $B_{0\mu}$ represents the magnetic field amplitude of the microwave field with its phase ϕ_μ . The combination $E_{0c} B_{0\mu}$ represents the hybrid second-order non-linearity which generates a response at the sum frequency $\omega_c + \omega_\mu = \omega_p$. The induced transparency effects seen in our system is captured by the $\chi_p^{(1)}$ response and the generation of probe frequency resulting in probe amplification comes from the $\chi_p^{(2)}$ term.

The magnetic dipole interaction breaks the centre of inversion symmetry present in our atomic system, ensuring a non-zero three-wave mixing nonlinearity which enables interaction between the strong coupling and microwave field. This hybrid second order non-linearity is shown to generate a coherent optical probe field in our earlier experiment [14]. It is also shown by us experimentally that the generated probe field can coherently superimpose on a seed probe beam and give rise to a phase dependent amplification and de-amplification of the probe field [15].

For explicit calculation of nonlinearity the following method can be adopted. We can obtain steady state expressions for the density matrix elements ρ_{ij} , which represent populations in various atomic states and coherences between a pair of states due to interaction with electromagnetic fields. This can be obtained from solving the master equation representing the atomic interaction with the electromagnetic fields and the bath [20]. Using the steady state expressions of ρ_{ij} the induced atomic polarization can be written as

$$P = N(d_{13}\rho_{13}^{(1)} e^{-i\omega_p t} + d_{13}\rho_{13}^{(2)} e^{-i(\omega_c + \omega_\mu)t} + d_{23}\rho_{23} e^{-i\omega_c t} + c.c) \quad (2.2.0.2)$$

where N is the atom density.

Comparing Eqs. (2.2.0.1) & (2.2.0.2), from steady state expression of $\rho_{13}(\Omega_p, \Omega_c, \Omega_\mu, \delta_\mu, \delta_c)$ we can identify coefficients of Ω_p ($\rho_{13}^{(1)}$) as $\chi_p^{(1)}$ and coefficients of $\Omega_c \Omega_\mu$ ($\rho_{13}^{(2)}$) as $\chi_p^{(2)}$. Thus we can get a handle on the three-wave mixing nonlinearity $\chi_p^{(2)}$ and its dependence on $\Omega_c, \Omega_\mu, \delta_\mu$, & δ_c .

2.3 Results and discussions

To demonstrate the microwave phase dependent intensity changes in the optical probe field we performed an initial experiment where we change the phase ϕ_μ of our microwave field continuously over a full period. The resultant intensity changes in the optical probe

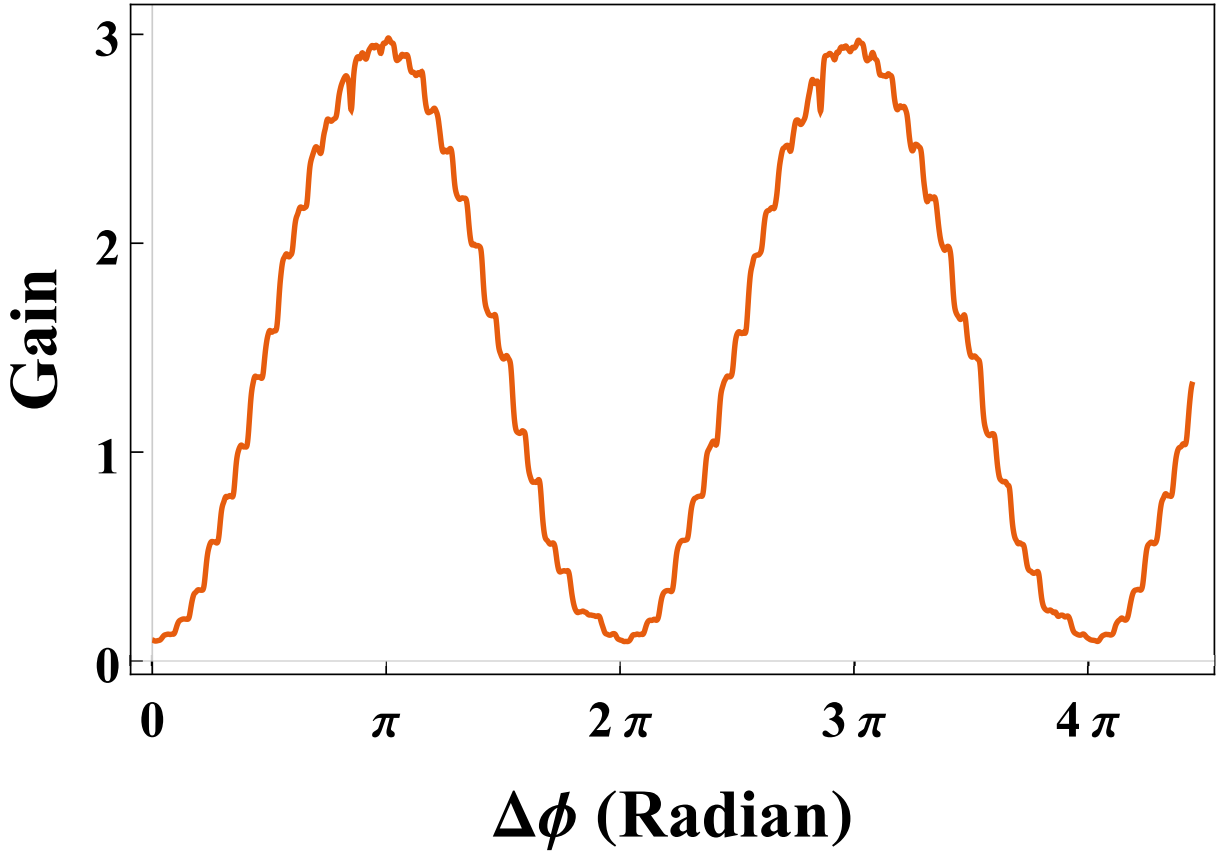


Figure 2.3: Normalized transmitted probe intensity (Gain) vs relative phase($\Delta\phi$). The data is taken at $\delta_2 = 0$ and $\delta_3 = 0$ for a microwave power of 500 mW.

field is shown in Fig. 2.3. The intensity of the output optical probe field is normalized with its intensity when the probe is far-detuned from its resonance, by about 1 GHz. Thus any value which is more than unity in our plot in Fig. 2.3 signifies amplification of the optical probe field and values less than unity signify de-amplification.

We now use this microwave phase dependent optical intensity changes to encode information in the communication channel. We choose appropriate microwave phase values where a one-to-one correspondence of the output probe intensity to the input microwave carrier phase exists. We have implemented both BPSK and C-4PSK using this phase-to-

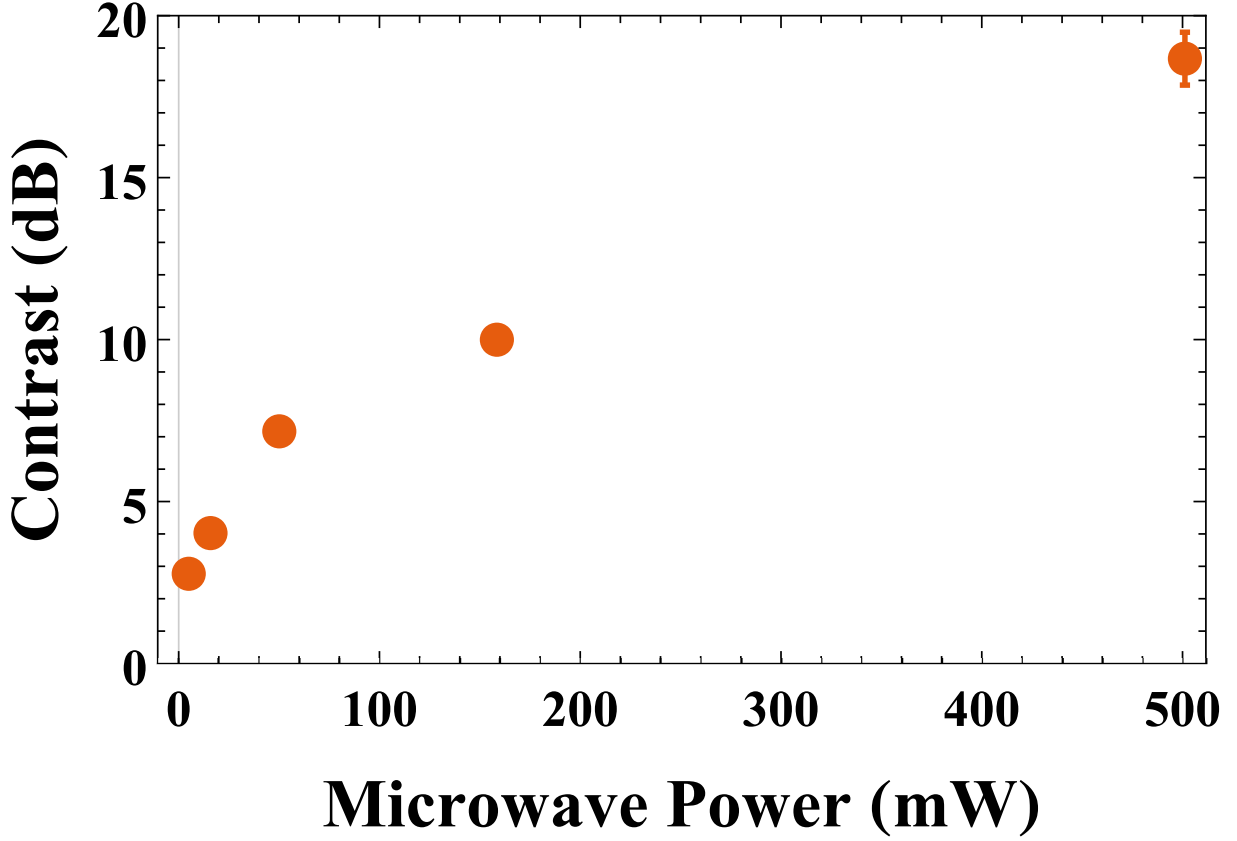


Figure 2.4: Optical probe contrast vs microwave power for a symbol frequency of 250 kHz. The data is taken at $\delta_2 = 0$ and at $\delta_3 = 0$ values.

intensity mapping.

In our experiment the contrast is defined as the difference between maximum and minimum values of probe transmission. To find the optimal contrast for implementing BPSK and C-4PSK communication, we examine the contrast of the probe field for different values of input microwave power to the cavity. These results are shown in Fig. 2.4 and we choose a microwave carrier power of 500 mW for the experiment which gives the highest contrast.

In BPSK, we modulate the microwave carrier phase at various modulation frequencies also called symbol frequencies. The modulation phase ϕ_μ of the microwave carrier encodes two binary states ‘0’ and ‘1’ corresponding to two phases which are π radian apart as shown in Fig. 2.5(a). These two phases are chosen to ensure maximum contrast in the probe transmission. As explained before our atomic system inherently demodulates and amplifies the encoded signal to optical intensity changes. This is shown in Fig. 2.5(b)-(d) for various input symbol frequencies.

For our C-4PSK communication scheme, we select microwave carrier phases as 0,

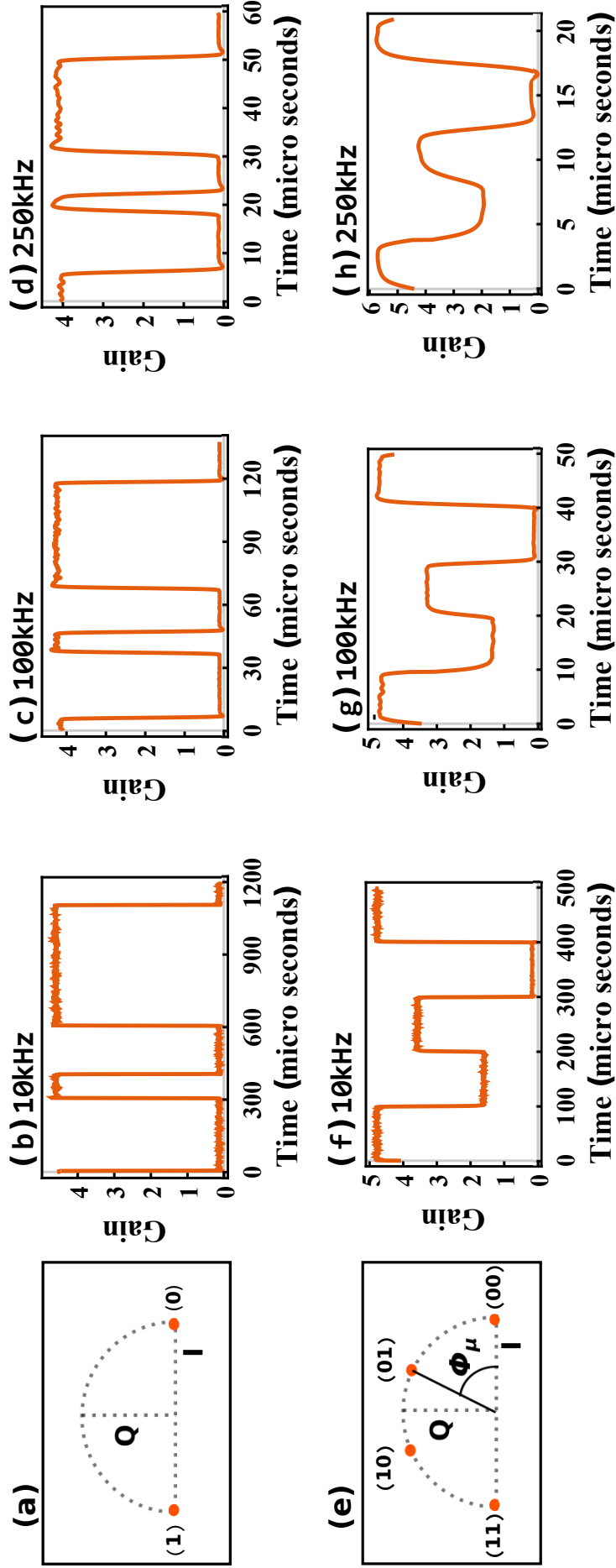


Figure 2.5: Constellation diagrams for (a) BPSK, (e) C-4PSK. Gain vs time plots showing demodulated intensity changes for symbol frequencies (b & f) 10 kHz, (c & g) 100 kHz, (d & h) 250 kHz for BPSK and C-4PSK respectively. Here $\delta_2 = 0$, $\delta_3 = 0$ and microwave power is 500 mW.

0.361π , 0.638π and π to encode information. This is shown in Fig. 2.5(e). These phases correspond to the four distinct and equispaced values in the output probe intensity and ensures demodulation with minimum error. The output probe intensity changes are shown for various modulation frequencies in Fig. 2.5(f)-(h). Thus Fig. 2.5 demonstrates the phase-to-intensity demodulation and phase dependent amplification between microwave and optical frequencies for both BPSK and C-4PSK communication protocol. We can demonstrate faithful demodulation up to a symbol frequency of 750 kHz. Beyond this frequency the output optical probe power's contrast between different states reduces. This restriction mainly arises from the EIT bandwidth and the resonance bandwidth of our microwave cavity. We also find that on decreasing the microwave power the contrast between different states in the plots of Fig. (2.5) decreases.

In Fig. 2.6 we show the transmitted probe signal as a function of microwave carrier

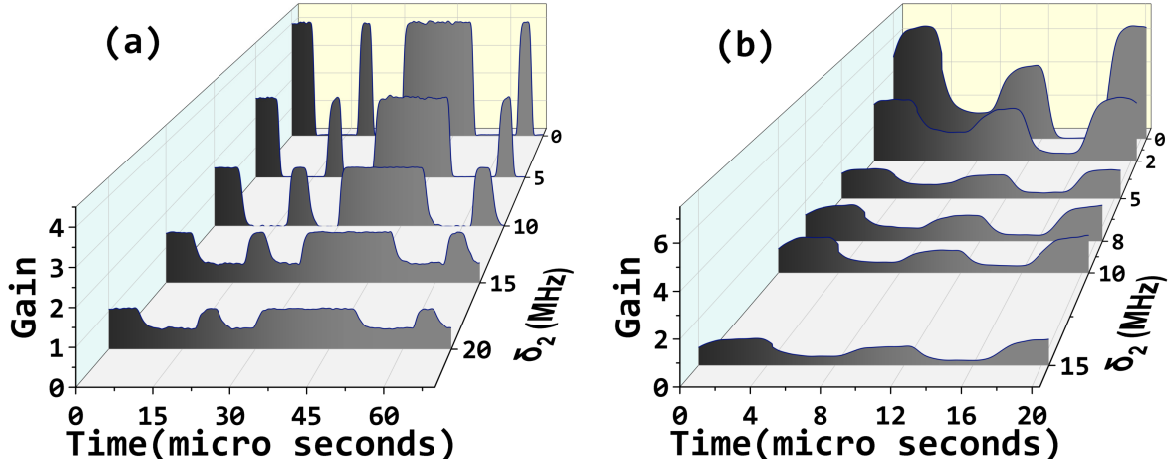


Figure 2.6: Gain vs time plot with various δ_2 values for (a) BPSK, (b) C-4PSK for $\delta_3 = 0$ and microwave power of 500 mW.

frequency change. We have changed systematically the microwave detuning δ_μ in various runs of our experiment to find the effective carrier frequency change of our communication protocol. It is to be noted that changing δ_μ in our experiment effectively changes δ_2 .

The plots in Fig. 2.6(a) give the transmitted probe power for various two-photon detuning (δ_2) values for BPSK scheme. These plots indicate that the mapping of microwave phase to optical intensity is univocal upto a carrier frequency detuning of 15 MHz. As expected, the contrast reduces with increasing detuning due to the violation of two photon resonance condition. Our C-4PSK communication protocol also show similar trends as that for BPSK. This is shown in Fig. 2.6(b).

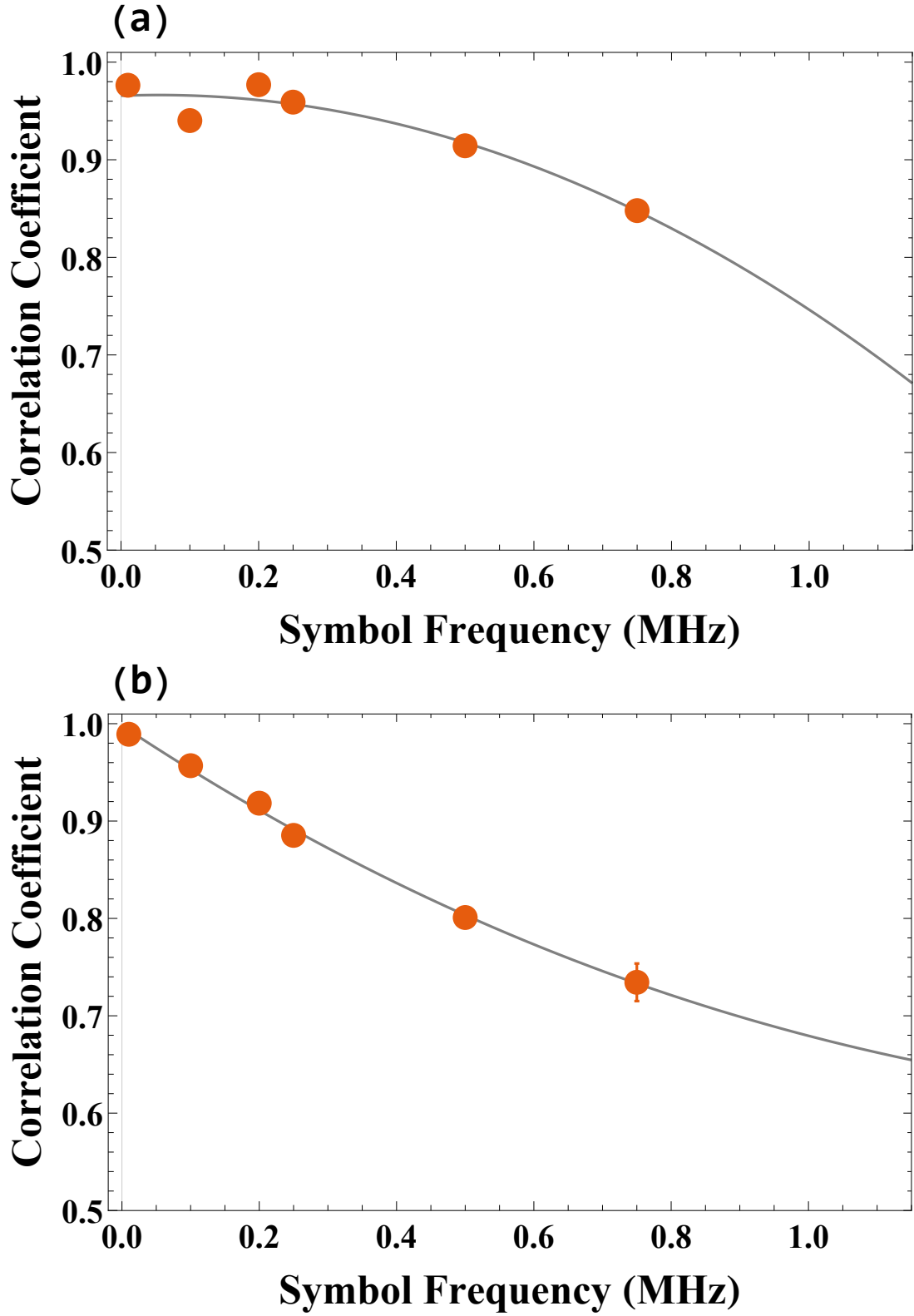


Figure 2.7: Correlation coefficient vs symbol frequency plot for (a) BPSK, (b) C-4PSK at $\delta_2 = 0$, $\delta_3 = 0$ and microwave power of 500 mW. The continuous line is the fitted model for experimental data.

In-order to understand the error performance of our atomic receiver we correlate the demodulated output with an ideal template and obtain the correlation coefficient for

different symbol frequencies. The correlation plots for BPSK and C-4PSK is shown in Fig. 2.7.

We have performed measurements for both BPSK and C-4PSK upto a symbol frequency of 750 kHz. From the fitted model to the plots the 3 dB frequency roll-off measured as $\frac{1}{\sqrt{2}}$ value of unity is seen to be 1.08 MHz for BPSK and 0.85 MHz for C-4PSK. The relatively high value of correlation coefficient is also maintained upto a carrier frequency change of 15 MHz.

2.4 Conclusion

We have experimentally implemented BPSK and C-4PSK communication protocols using a cyclic scheme in a room temperature ^{85}Rb atomic vapor system. The microwave phase dependent nature of output optical probe intensity and the amplification resulting from a hybrid three wave mixing nonlinearity makes our atomic radio-over-fibre system act simultaneously as a demodulator and an amplifier. We demonstrate a symbol frequency bandwidth of about 1 MHz and a microwave carrier frequency range of 15 MHz for both our communication protocols. The atomic states used in our scheme are also quantum memory storage states making our system inherently suitable for future radio-over-fibre conversion of quantum signals.

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Chapter 3

Microwave-driven generation and group delay control of optical pulse from an ultra-dilute atomic ensemble

3.1 Introduction

Generating few-photon and single-photon [1, 2, 3, 4] pulses-on-demand is an important resource for many applications in quantum enabled technologies. Several practically realized quantum key distribution protocols use attenuated laser pulses [5, 6] whereas quantum repeaters and quantum computing modules use single-photon pulses-on-demand [2, 3, 4, 7]. Such few-photon on-demand sources are traditionally implemented through controllable semiconductor sources such as quantum dots and color centers in crystals [8].

Dilute atomic sample-based controllable generation, storage and transmission of single-photon pulses has been experimentally realized [9, 10]. Such atom-based generation schemes relies on the Electromagnetically Induced Transparency (EIT) effect and is important for practical realization of quantum networks. A large number of EIT-based systems have been studied for controlling the group delay of pulses. In these systems control of propagation dynamics of an input pulse as subluminal [11, 12, 13, 14, 15] and superluminal [16, 17, 18, 19] propagation have been both experimentally and theoretically investigated. In addition there are studies that provide a knob to change the propagation of light from subluminal to superluminal [18, 19, 20, 21, 22, 23, 24, 25], giving control for continuous tunability of group delay. Observation of nonlinear generation of subluminal

and superluminal pulses using EIT-based schemes in atomic vapours [26, 27, 28] has been experimentally investigated too. These studies are of particular importance in realising optical delay lines [29], optical coherence tomography [30] and pulse-jitter corrections [27].

In the above mentioned studies of EIT-based generation and control of photon pulses done so far, efficiency of the process is directly proportional to number density and spatial extent of the sample. This scaling limits the prospect of integration of these systems with chip-based integrated circuit devices, which are being increasingly used as a platform for realising scalable quantum technological architectures [31].

In this work a atomic level scheme interacting with an optical and a microwave field is proposed for generation and group-delay control of few-photon optical pulses. More importantly, the group delay is independent of atom no. density and spatial extent of the generating sample. Our analysis exploits a hybrid second order-nonlinearity under conditions of electromagnetically induced transparency in a dilute sample of cyclic closed three-level system to generate an optical pulse. As the probe pulse is generated by a sum-frequency process between pump field photons, it can be made on-demand by tailoring the time when one of the pump fields, either the optical field or the microwave field, is switched on.

We show that group delay of the optical pulse can be controlled by the intensity of the pump microwave field. This tunable control has the characteristics of subluminal and superluminal propagation of light but arises from a different physical effect. The intensity of the pump field alters the absolute phase for different frequencies present within the generated optical pulse resulting in changes in the group delay. Most significant of all, this control is shown to be number density-independent and hence can be achieved with few atoms resulting in few photons per pulse. For experimentally feasible parameters, we show that a group delay of the order of $1\ \mu\text{s}$ can be achieved with an ultra-dilute density of $10^6\ \text{cm}^{-3}$. At such densities, we show that the generated optical pulse has approximately a mean photon number of one. Sample lengths as small as $100\ \mu\text{m}$ have been shown to introduce changes in group delay paving the way for the integration of such systems with chip-based photonic architectures.

3.2 Deriving Hamiltonian

For the theoretical analysis, we consider a three-level system with the energy level and the connected laser fields as shown in Fig. 3.1

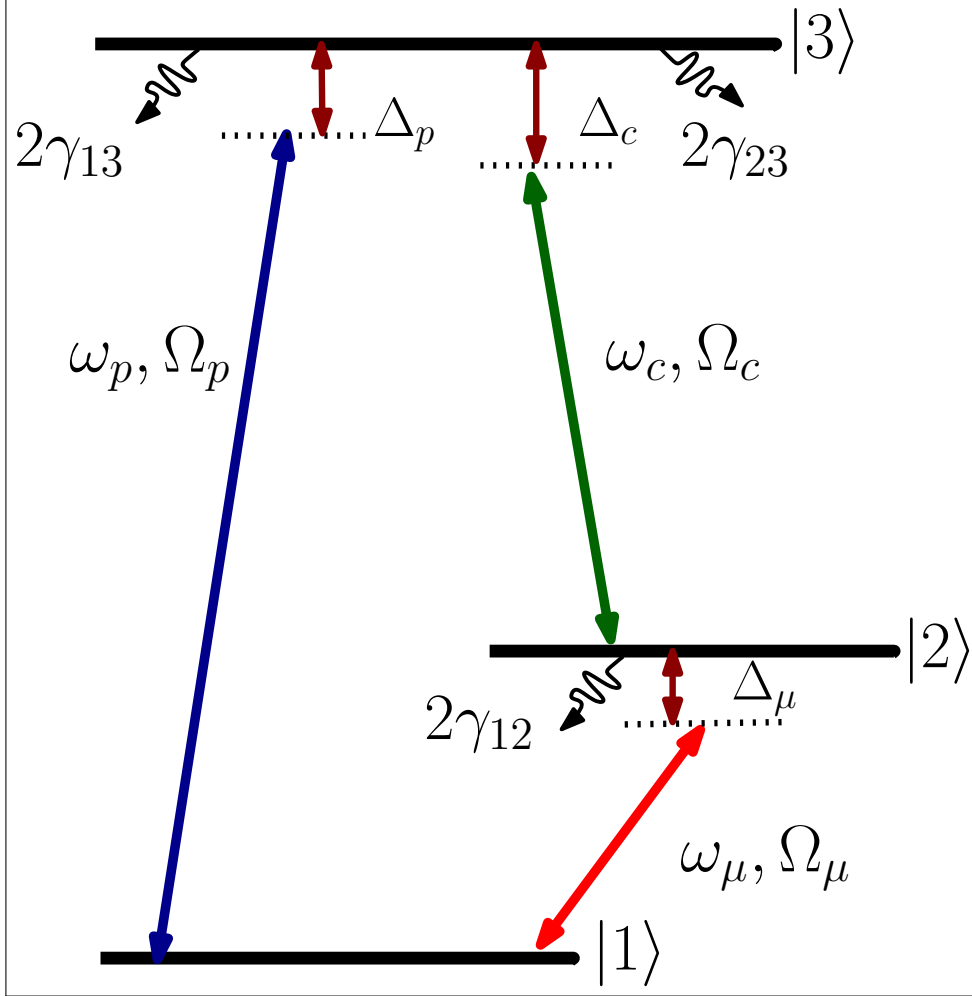


Figure 3.1: Schematic diagram of Δ atomic system. $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ are electric dipole-allowed transitions and are coupled by probe (Ω_p) and coupling (Ω_c) fields respectively. $|1\rangle \rightarrow |2\rangle$ is a magnetic dipole allowed transition and is connected by a microwave field (Ω_μ).

This three-level scheme can also be realized using the hyperfine energy levels of ^{85}Rb atom in D1 manifold. The three-level system consists of a ground state $|1\rangle$, a metastable state $|2\rangle$ and an excited state $|3\rangle$. The resonance frequency of the transition between the levels is defined as ω_{ij} where $(i, j) \in (1, 2, 3)$ and $i \neq j$. Two optical fields, called probe (ω_p) and coupling (ω_c), couple the electric dipole permitted transitions $|1\rangle \leftrightarrow |3\rangle$ and $|3\rangle \leftrightarrow |2\rangle$. A microwave field connects the magnetic dipole allowed transition $|2\rangle \leftrightarrow |1\rangle$. The rabi frequencies and detunings of the probe, coupling and microwave fields are defined as (Ω_p, Δ_p) , (Ω_c, Δ_c) and (Ω_μ, Δ_μ) respectively. $2\gamma_{13}$, $2\gamma_{23}$ and $2\gamma_{12}$ are the decay rates

from the respective basis states.

The bare atom hamiltonian is defined as

$$\mathbf{H}_0 = \begin{pmatrix} \hbar\omega_{01} & 0 & 0 \\ 0 & \hbar\omega_{02} & 0 \\ 0 & 0 & \hbar\omega_{03} \end{pmatrix}, \quad (3.2.0.1)$$

where ω_{0i} , for $i \in (1, 2, 3)$ are the frequency of the bare atomic energy level with respect to the reference level.

We express the electric field and magnetic field of the probe, coupling and microwave fields as

$$\mathbf{E}_p(t) = \vec{\epsilon}_p E_p \cos(\omega_p t + \phi_p), \quad \mathbf{B}_p(t) = \vec{b}_p B_p \cos(\omega_p t + \phi_p), \quad (3.2.0.2)$$

$$\mathbf{E}_c(t) = \vec{\epsilon}_c E_c \cos(\omega_c t + \phi_c), \quad \mathbf{B}_c(t) = \vec{b}_c B_c \cos(\omega_c t + \phi_c), \quad (3.2.0.3)$$

$$\mathbf{E}_\mu(t) = \vec{\epsilon}_\mu E_\mu \cos(\omega_\mu t + \phi_\mu), \quad \mathbf{B}_\mu(t) = \vec{b}_\mu B_\mu \cos(\omega_\mu t + \phi_\mu), \quad (3.2.0.4)$$

respectively. ω_i and ϕ_i are the frequency and initial phase of the respective laser fields, where $i \in (p, c, \mu)$. ϵ_i and b_i are the polarization vectors of the electric field and magnetic field respectively.

With the above definitions the interaction Hamiltonian($\mathbf{H}_{\text{int}}$) for the three level system can be defined as

$$\begin{aligned} \mathbf{H}_{\text{int}} &= -\vec{\mathbf{d}} \cdot \vec{E}_T(t) - \vec{\boldsymbol{\mu}} \cdot \vec{B}_T(t), \\ &= -\vec{\mathbf{d}} \cdot (E_p(t) + E_c(t) + E_\mu(t)) - \vec{\boldsymbol{\mu}} \cdot (B_p(t) + B_c(t) + B_\mu(t)) \end{aligned} \quad (3.2.0.5)$$

where $\mathbf{E}_T$ and $\mathbf{B}_T$ are the total electric field and magnetic field of all the interacting field respectively. $\vec{\mathbf{d}}$ and $\vec{\boldsymbol{\mu}}$ are the electric and magnetic dipole moment operator respectively.

We can write the above Hamiltonian with the eigenbasis of $\mathbf{H}_0$ in matrix form as

$$\mathbf{H}_{\text{int}} = - \begin{pmatrix} 0 & \vec{\boldsymbol{\mu}}_{12} \cdot \vec{B}_T(t) & \vec{\mathbf{d}}_{13} \cdot \vec{E}_T(t) \\ \vec{\boldsymbol{\mu}}_{21} \cdot \vec{B}_T(t) & 0 & \vec{\mathbf{d}}_{23} \cdot \vec{E}_T(t) \\ \vec{\mathbf{d}}_{31} \cdot \vec{E}_T(t) & \vec{\boldsymbol{\mu}}_{32} \cdot \vec{B}_T(t) & 0 \end{pmatrix} \quad (3.2.0.6)$$

By using rotating wave approximation, we can safely remove the field components in the

Hamiltonian $\mathbf{H}_{int}$, which are insignificant. After modification, we can write the Hamiltonian as

$$\mathbf{H}_{int} = -\frac{1}{2} \begin{pmatrix} 0 & \vec{\mu}_{12} \cdot \vec{b}_\mu B_\mu e^{i(\omega_\mu t + \phi_\mu)} & \vec{d}_{13} \cdot \vec{\epsilon}_p E_p e^{i(\omega_p t + \phi_p)} \\ \vec{\mu}_{21} \cdot \vec{b}_\mu B_\mu e^{-i(\omega_\mu t + \phi_\mu)} & 0 & \vec{d}_{23} \cdot \vec{\epsilon}_c E_c e^{i(\omega_c t + \phi_c)} \\ \vec{d}_{31} \cdot \vec{\epsilon}_p E_p e^{-i(\omega_p t + \phi_p)} & \vec{d}_{32} \cdot \vec{\epsilon}_c E_c e^{-i(\omega_c t + \phi_c)} & 0 \end{pmatrix} \quad (3.2.0.7)$$

Now, we can write the total Hamiltonian as $\mathbf{H}_t = \mathbf{H}_0 + \mathbf{H}_{int}$.

The above Hamiltonian needs simplification as we can see all the non-zero terms in $\mathbf{H}_t$ are time-dependent. To reduce the no. of time-dependent terms in the Hamiltonian, we need to transform it to a new basis. We define the matrix for unitary transformation ($\mathbf{H}_t \xrightarrow{U} \mathbf{H}'_t$) as

$$\mathbf{U}(t) = \begin{pmatrix} e^{-i(\omega_\mu t + \phi_\mu)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\omega_c t + \phi_c)} \end{pmatrix} \quad (3.2.0.8)$$

In order to derive the transformation equation for $\mathbf{H}_t \xrightarrow{U} \mathbf{H}'_t$ we have to make sure that $\mathbf{H}'_t$ must obey the Schrodinger equation,

$$\begin{aligned} \mathbf{H}'_t |n'\rangle &= i\hbar \frac{\partial |n'\rangle}{\partial t} \\ &= i\hbar \frac{\partial (\mathbf{U} |n\rangle)}{\partial t} \\ &= i\hbar \left(\mathbf{U} \frac{\partial |n\rangle}{\partial t} + \frac{\partial \mathbf{U}}{\partial t} |n\rangle \right) \\ &= i\hbar \left(\frac{-i\mathbf{U}}{\hbar} \mathbf{H}_t |n\rangle + \frac{\partial \mathbf{U}}{\partial t} |n\rangle \right) \\ &= \left(\mathbf{U} \mathbf{H}_t \mathbf{U}^\dagger + i\hbar \frac{\partial \mathbf{U}}{\partial t} \mathbf{U}^\dagger \right) |n'\rangle, \end{aligned} \quad (3.2.0.9)$$

where $|n\rangle$ and $|n'\rangle$ are the basis vector of $\mathbf{H}_t$ and $\mathbf{H}'_t$ respectively. They are related by $|n'\rangle = \mathbf{U}(t)|n\rangle$.

Thus the transformation equation for $\mathbf{H}'_t$ is,

$$\mathbf{H}'_t = \mathbf{U} \mathbf{H}_t \mathbf{U}^\dagger + i\hbar \frac{\partial \mathbf{U}}{\partial t} \mathbf{U}^\dagger \quad (3.2.0.10)$$

By using Eq. (3.2.0.1, 3.2.0.7, 3.2.0.8) in Eq. (3.2.0.9) we get,

$$\begin{aligned}
\mathbf{H}'_t &= -\frac{1}{2} \begin{pmatrix} -2\hbar\omega_{01} & \vec{\mu}_{12} \cdot \vec{b}_\mu B_\mu & \vec{d}_{13} \cdot \vec{\epsilon}_p E_p e^{-i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} \\ \vec{\mu}_{21} \cdot \vec{b}_\mu B_\mu & -2\hbar\omega_{02} & \vec{d}_{23} \cdot \vec{\epsilon}_c E_c \\ \vec{d}_{31} \cdot \vec{\epsilon}_p E_p e^{i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} & \vec{d}_{32} \cdot \vec{\epsilon}_c E_c & -2\hbar\omega_{03} \end{pmatrix} \\
&\quad + \hbar \begin{pmatrix} \omega_\mu & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\omega_c \end{pmatrix} \\
&= -\frac{1}{2} \begin{pmatrix} -2(\hbar\omega_{01} + \hbar\omega_\mu) & \vec{\mu}_{12} \cdot \vec{b}_\mu B_\mu & \vec{d}_{13} \cdot \vec{\epsilon}_p E_p e^{-i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} \\ \vec{\mu}_{21} \cdot \vec{b}_\mu B_\mu & -2\hbar\omega_{02} & \vec{d}_{23} \cdot \vec{\epsilon}_c E_c \\ \vec{d}_{31} \cdot \vec{\epsilon}_p E_p e^{i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} & \vec{d}_{32} \cdot \vec{\epsilon}_c E_c & 2(-\hbar\omega_{03} + \hbar\omega_c) \end{pmatrix}
\end{aligned} \tag{3.2.0.11}$$

where $\phi_{rel} = \phi_p - \phi_c - \phi_\mu$ is the relative phase difference between the interacting fields.

As we already know, the addition of any constant times unitary matrix can not change the time evolution of a system, we add $-\hbar\omega_{02}$ times Identity matrix (I) in $\mathbf{H}'_t$ to simplify it further. We got the new $\mathbf{H}'_t$

$$\begin{aligned}
\mathbf{H}'_t &= -\frac{1}{2} \begin{pmatrix} -2(\hbar\omega_{01} + \hbar\omega_\mu) & \vec{\mu}_{12} \cdot \vec{b}_\mu B_\mu & \vec{d}_{13} \cdot \vec{\epsilon}_p E_p e^{-i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} \\ \vec{\mu}_{21} \cdot \vec{b}_\mu B_\mu & 2\hbar\omega_{02} & \vec{d}_{23} \cdot \vec{\epsilon}_c E_c \\ \vec{d}_{31} \cdot \vec{\epsilon}_p E_p e^{i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} & \vec{d}_{32} \cdot \vec{\epsilon}_c E_c & 2(-\hbar\omega_{03} + \hbar\omega_c) \end{pmatrix} \\
&\quad - \hbar\omega_{02} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= -\frac{1}{2} \begin{pmatrix} 2(-\hbar\omega_{01} - \hbar\omega_\mu + \hbar\omega_{02}) & \vec{\mu}_{12} \cdot \vec{b}_\mu B_\mu & \vec{d}_{13} \cdot \vec{\epsilon}_p E_p e^{-i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} \\ \vec{\mu}_{21} \cdot \vec{b}_\mu B_\mu & 2(-\hbar\omega_{02} + \hbar\omega_{02}) & \vec{d}_{23} \cdot \vec{\epsilon}_c E_c \\ \vec{d}_{31} \cdot \vec{\epsilon}_p E_p e^{i((\omega_c + \omega_\mu - \omega_p)t - \phi_{rel})} & \vec{d}_{32} \cdot \vec{\epsilon}_c E_c & 2(-\hbar\omega_{03} + \hbar\omega_c + \hbar\omega_{02}) \end{pmatrix}
\end{aligned} \tag{3.2.0.12}$$

Below we define few quantities which will further simplify the Hamiltonian. Those are

$$\Delta_p = \omega_p - \omega_{31} \rightarrow \text{Detuning of probe field from its resonance,} \quad (3.2.0.13)$$

$$\Delta_c = \omega_c - \omega_{32} \rightarrow \text{Detuning of control field from its resonance,} \quad (3.2.0.14)$$

$$\Delta_\mu = \omega_\mu - \omega_{21} \rightarrow \text{Detuning of microwave field from its resonance,} \quad (3.2.0.15)$$

as shown in Fig.(3.1).

$$\Delta_3 = \Delta_p - \Delta_c - \Delta_\mu = \omega_p - \omega_c - \omega_\mu, \text{ is called the three photon detuning.} \quad (3.2.0.16)$$

$$\Omega_p = \frac{\vec{\epsilon}_p \cdot \vec{d}_{31} E_p}{2\hbar} \rightarrow \text{Rabi frequency of the probe field.} \quad (3.2.0.17)$$

$$\Omega_c = \frac{\vec{\epsilon}_c \cdot \vec{d}_{32} E_c}{2\hbar} \rightarrow \text{Rabi frequency of the control field.} \quad (3.2.0.18)$$

$$\Omega_\mu = \frac{\vec{b}_\mu \cdot \vec{\mu}_{21} B_\mu}{2\hbar} \rightarrow \text{Rabi frequency of the microwave field.} \quad (3.2.0.19)$$

After using the above definitions the Eq. (3.2.0.12) reduces to

$$\mathbf{H}'_t = \hbar \begin{pmatrix} \Delta_\mu & -\Omega_\mu^* & -\Omega_p^* e^{i(\Delta_3 t + \phi_{rel})} \\ -\Omega_\mu & 0 & -\Omega_c^* \\ -\Omega_p e^{-i(\Delta_3 t + \phi_{rel})} & -\Omega_c & -\Delta_c \end{pmatrix}. \quad (3.2.0.20)$$

The above Hamiltonian is simplified and contain only two time-dependent terms which are complex conjugate to each other.

3.3 Density matrix formalism

Now, as the state of the three-level system can be a mixed state we require density matrix to represent the state of the system, which can be defined in $|n\rangle$ basis as

$$\rho = \sum_n P_n |n\rangle \langle n|, \quad (3.3.0.1)$$

where P_n is the probability of finding the system in state $|n\rangle$.

The time evolution of the system can be explained by using the von Neumann equation which is

$$\dot{\rho} = \frac{-i}{\hbar} [\mathbf{H}_t, \rho] \quad (3.3.0.2)$$

Similarly in the transformed basis ($|n'\rangle$) the von Neumann equation for time evolution will be,

$$\dot{\sigma} = \frac{-i}{\hbar}[\mathbf{H}'_t, \sigma], \quad (3.3.0.3)$$

where σ is the transformed density operator and is related to ρ as

$$\sigma = \sum_{n'} P_{n'} |n'\rangle \langle n'| = \mathbf{U} \rho \mathbf{U}^\dagger \quad (3.3.0.4)$$

Till now, we have not included any dissipation in our three-level system. As we know, dissipation can arise due to spontaneous emission of a photon, collision between atoms, etc., which can eventually de-populate the atom in one of the energy levels and also can create phase decoherence. In this section, we will account for the population decay in the three-level system.

Taking dissipation into consideration, the master equation can be described as

$$\dot{\sigma} = \frac{-i}{\hbar}[\mathbf{H}'_t, \sigma] + \sum_{k \in (p, c, \mu)} \Lambda(\mathbf{L}_k) \sigma \quad (3.3.0.5)$$

where Λ is called the Lindblad super-operator, which takes dissipation into account and is defined as,

$$\Lambda(\mathbf{L}_k) \sigma = \mathbf{L}_k \sigma \mathbf{L}_k^\dagger - \frac{(\sigma \mathbf{L}_k^\dagger \mathbf{L}_k + \mathbf{L}_k^\dagger \mathbf{L}_k \sigma)}{2} \quad (3.3.0.6)$$

The $\mathbf{L}_k$'s are atomic states projection operators defined as,

$$\mathbf{L}_p = \begin{pmatrix} 0 & 0 & \sqrt{2\gamma_{13}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad 2\gamma_{13} \text{ is the spontaneous decay rate from } |3\rangle \rightarrow |1\rangle \quad (3.3.0.7)$$

$$\mathbf{L}_c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{2\gamma_{23}} \\ 0 & 0 & 0 \end{pmatrix}; \quad 2\gamma_{23} \text{ is the spontaneous decay rate from } |3\rangle \rightarrow |2\rangle \quad (3.3.0.8)$$

$$\mathbf{L}_\mu = \begin{pmatrix} 0 & \sqrt{2\gamma_{12}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad 2\gamma_{12} \text{ is the spontaneous decay rate from } |2\rangle \rightarrow |1\rangle \quad (3.3.0.9)$$

With this, we can write the first and second term on the right-hand side of Eq. (3.3.0.6) as

$$\begin{aligned} & \sum_{k \in (p, c, \mu)} \mathbf{L}_k \boldsymbol{\sigma} \mathbf{L}_k^\dagger \\ &= \begin{pmatrix} 2\gamma_{12}\sigma_{22} + 2\gamma_{13}\sigma_{33} & 0 & 0 \\ 0 & 2\gamma_{23}\sigma_{33} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \quad (3.3.0.10)$$

and

$$- \sum_{k \in (p, c, \mu)} \frac{(\boldsymbol{\sigma} \mathbf{L}_k^\dagger \mathbf{L}_k + \mathbf{L}_k^\dagger \mathbf{L}_k \boldsymbol{\sigma})}{2} \quad (3.3.0.11)$$

$$= \begin{pmatrix} 0 & -\gamma_{12}\sigma_{12} & -\gamma_{23}\sigma_{13} - \gamma_{13}\sigma_{13} \\ -\gamma_{12}\sigma_{21} & -2\gamma_{12}\sigma_{22} & -\gamma_{23}\sigma_{23} - \gamma_{13}\sigma_{23} - \gamma_{12}\sigma_{23} \\ -\gamma_{23}\sigma_{31} - \gamma_{13}\sigma_{31} & -\gamma_{23}\sigma_{32} - \gamma_{13}\sigma_{32} - \gamma_{12}\sigma_{32} & -2\gamma_{23}\sigma_{33} - 2\gamma_{13}\sigma_{33} \end{pmatrix}. \quad (3.3.0.12)$$

Replacing Eq. (3.2.0.20, 3.3.0.10 and 3.3.0.12) in Eq. (3.3.0.5) we obtain the evolution equation for density matrix element as

$$\dot{\sigma}_{33} = i\Omega_c\sigma_{23} + i\Omega_p e^{-i(\Delta_3 t + \phi_{rel})}\sigma_{13} - i\Omega_c^*\sigma_{32} - i\Omega_p^* e^{i(\Delta_3 t + \phi_{rel})}\sigma_{31} - 2(\gamma_{13} + \gamma_{23})\sigma_{33} \quad (3.3.0.13a)$$

$$\dot{\sigma}_{22} = i\Omega_c^*\sigma_{32} + i\Omega_\mu\sigma_{12} - i\Omega_c\sigma_{23} - i\Omega_\mu^*\sigma_{21} + 2\gamma_{23}\sigma_{33} - 2\gamma_{12}\sigma_{22} \quad (3.3.0.13b)$$

$$\dot{\sigma}_{32} = -\gamma\sigma_{32} + i\Omega_p e^{-i(\Delta_3 t + \phi_{rel})}\sigma_{12} + i\Omega_c(\sigma_{22} - \sigma_{33}) - i\Omega_\mu^*\sigma_{31} \quad (3.3.0.13c)$$

$$\dot{\sigma}_{31} = -\alpha\sigma_{31} + i\Omega_p e^{-i(\Delta_3 t + \phi_{rel})}\sigma_{11} + i\Omega_c\sigma_{21} - i\Omega_p e^{-i(\Delta_3 t + \phi_{rel})}\sigma_{33} - i\Omega_\mu\sigma_{32} \quad (3.3.0.13d)$$

$$\dot{\sigma}_{21} = -\beta\sigma_{21} - i\Omega_p e^{-i(\Delta_3 t + \phi_{rel})}\sigma_{23} + i\Omega_c^*\sigma_{31} + i\Omega_\mu\sigma_{11} - i\Omega_\mu\sigma_{22} \quad (3.3.0.13e)$$

with

$$\sigma_{11} = 1 - \sigma_{33} - \sigma_{22} \quad (3.3.0.14)$$

where

$$\alpha = \gamma_{13} + \gamma_{23} - i(\Delta_c + \Delta_\mu) \quad (3.3.0.15)$$

$$\beta = \gamma_{12} - i\Delta_\mu \quad (3.3.0.16)$$

$$\gamma = \gamma_{13} + \gamma_{23} + \gamma_{12} - i\Delta_c \quad (3.3.0.17)$$

In the above derivation the dissipation due to atomic collisions is not taken into consideration as we can assume our atomic sample is very dilute which is indeed the case in our experimental setup.

3.4 Steady state solutions

The above transformed density-matrix equations are written in the rotating frame. These transformed density matrix (σ) is related to the original density matrix (ρ) in terms of matrix elements as

$$\rho_{32} = \sigma_{32} e^{-i(\omega_c t + \phi_c)}, \quad (3.4.0.1a)$$

$$\rho_{31} = \sigma_{31} e^{-i((\omega_c + \omega_\mu)t + \phi_c + \phi_\mu)}, \quad (3.4.0.1b)$$

$$\rho_{21} = \sigma_{21} e^{-i(\omega_\mu t + \phi_\mu)}. \quad (3.4.0.1c)$$

Now, in order to understand the propagation of probe field inside the nonlinear medium we have to solve the wave equation, which require the calculation of susceptibilities for probe field.

So writing scalar polarization of the atoms, induced by electromagnetic fields to first order in probe amplitude, we get

$$P(t) = \frac{\epsilon_0 E_p (\chi_p^{(1)}(\omega_p) e^{-i\omega_p t} + \text{c.c.})}{2} + \frac{\epsilon_0 E_c E_\mu (\chi_p^{(2)}(\omega_c, \omega_\mu) e^{-i(\omega_c + \omega_\mu)t} + \text{c.c.})}{4}, \quad (3.4.0.2)$$

where $\chi_p^{(1)}(\omega_p)$, $\chi_p^{(2)}(\omega_p)$ are the linear and second-order susceptibilities of the probe field.

Writing polarization in terms of rotated density matrix elements by using Eq. (3.4.0.1)

$$P(t) = \mathcal{N} (d_{13} \sigma_{31} e^{-i(\omega_c + \omega_\mu)t} + d_{32} \sigma_{32} e^{-i\omega_c t} + \text{c.c.}), \quad (3.4.0.3)$$

where $\mathcal{N}$ is the number density of atoms.

The susceptibilities of probe $\chi_p^{(1)}$, $\chi_p^{(2)}$ are obtained by calculating σ_{13} to first order in probe amplitude [20]. Towards this, we expand density-matrix elements σ_{jk} of (3.3.0.13) as

$$\sigma_{jk} = \sigma_{jk}^{(0)} \Omega_c \Omega_\mu + \sigma_{jk}^{(1)} \Omega_p e^{-i(\Delta_3 t + \phi_{\text{rel}})} + \sigma_{jk}^{(2)} \Omega_p^* e^{i(\Delta_3 t + \phi_{\text{rel}})} \quad (3.4.0.4)$$

$$(j, k \in \{1, 2, 3\}).$$

The perturbative expansion (3.4.0.4) is substituted in Eqs. (3.3.0.13), and we neglect terms of order Ω_p^2 as $\Omega_p \ll \Omega_\mu$ & Ω_c . The 15 unknowns $\sigma_{jk}^{(0)}, \sigma_{jk}^{(1)}, \sigma_{jk}^{(2)}$ (three for each five equations) are found by solving the resulting linear equations for steady state as can be found in the appendix. Then the required susceptibilities are obtained by substituting Eq. (3.4.0.4) for σ_{31} in Eq. (3.4.0.3) and comparing it with Eq. (3.4.0.2), giving us

$$\chi_p^{(2)}(\omega_c + \omega_\mu, \omega_c, \omega_\mu) = \left(\frac{4\mathcal{N}d_{13}d_{12}\mu_{23}}{\epsilon_0\hbar^2} \right) \sigma_{31}^{(0)}(\Omega_c, \Omega_\mu, \Delta_c, \Delta_\mu), \quad (3.4.0.5)$$

$$\chi_p^{(1)}(\omega_p, \omega_p) = \left(\frac{2\mathcal{N}d_{13}^2}{\epsilon_0\hbar} \right) \sigma_{31}^{(1)}(\Omega_c, \Omega_\mu, \Delta_c, \Delta_\mu, \Delta_p). \quad (3.4.0.6)$$

In the succeeding section, $\chi_p^{(1)}$ and $\chi_p^{(2)}$ are used to study the group delay and temporal profile of the generated probe pulse.

3.5 Derivation of generated pulse profile

In this section we study the Δ system without the input probe field and investigate the effect of various parameters on group delay of a generated probe pulse, generated through the hybrid second order-nonlinearity $\chi_p^{(2)}$. Towards this we use a modified Δ scheme given in Fig. 3.2. In this scheme the input microwave field ($\Omega_\mu(\Delta_\mu)$) is a pulse which gets up-converted to a probe (Ω_p) pulse (not shown in figure) in presence of a coupling field (Ω_c).

The pulse propagation can be analysed by solving Maxwell's equation given by

$$\frac{d^2 E(z, t)}{dz^2} = \frac{1}{c^2} \frac{d^2 E(z, t)}{dt^2} + \mu_0 \frac{d^2 (P(z, t))}{dt^2} \quad (3.5.0.1)$$

where $E(z, t)$ is the electric field propagating in the medium and $P(z, t)$ is the source term/polarization of the medium and can be separated into linear (P_L) and nonlinear (P_{NL}) part.

As the microwave pulse is composed of many frequencies and so will be the generated pulse, we have to solve the wave equation for probe pulse in frequency domain which is

given in Eq. (3.5.0.2)

$$\frac{d^2 E_p(z, \omega)}{dz^2} = -\frac{\omega^2}{c^2} E_p(z, \omega) - \mu_0 \omega^2 (P_L + P_{NL}) \quad (3.5.0.2)$$

where $E_p(z, \omega)$ is the pulse amplitude in frequency domain

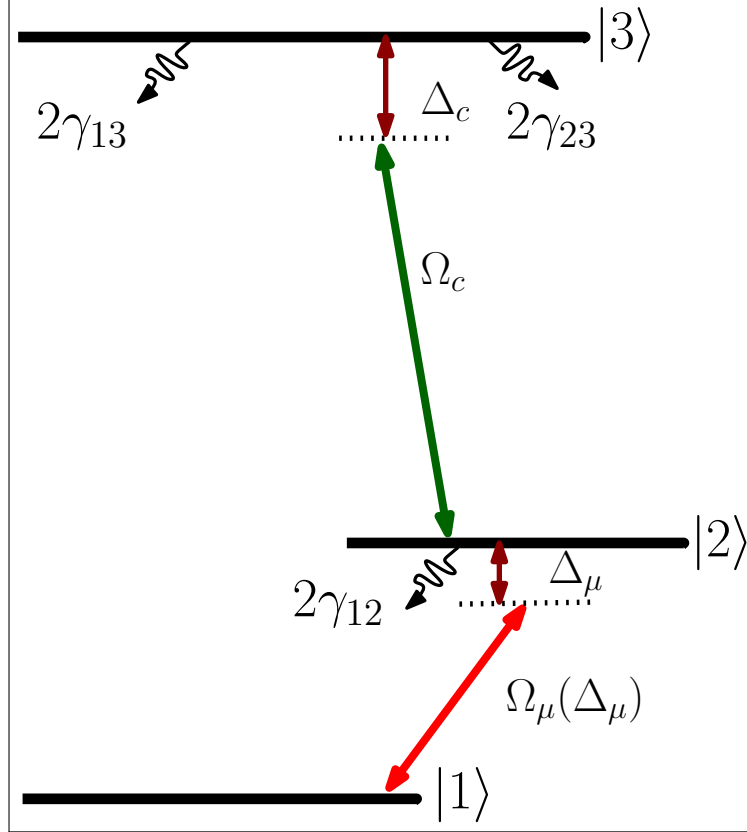


Figure 3.2: Pulsed microwave field $\Omega_\mu(\Delta_\mu)$ and a CW coupling field Ω_c is used to observe pulsed probe generation through the nonlinear susceptibility $\chi_p^{(2)}$.

As the up-conversion discussed here is a sum-frequency generation process, the nonlinear source term is the second-order polarization for probe pulse, which can be written in time domain [32] as

$$P^{(2)}(z, t) = \epsilon_0 \int_{-\infty}^{\infty} \frac{d\omega_{c'}}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_{\mu'}}{2\pi} \chi^{(2)}(\omega_{c'}, \omega_{\mu'}) E_c(z, \omega_{c'}) E_\mu(z, \omega_{\mu'}) e^{-i\omega_\sigma t}, \quad (3.5.0.3)$$

where $\omega_\sigma = \omega_{c'} + \omega_{\mu'}$ and $\omega_{c'}, \omega_{\mu'}$ are the frequencies used for up-conversion.

but according to the Fourier transformation

$$P^{(2)}(z, \omega) = \int_{-\infty}^{\infty} P^{(2)}(z, t) e^{i\omega t} dt. \quad (3.5.0.4)$$

So, substituting Eq. (3.5.0.3) in Eq. (3.5.0.4), we get

$$P^{(2)}(z, \omega) = \epsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega_{c'}}{2\pi} \frac{d\omega_{\mu'}}{2\pi} \chi^{(2)}(\omega_{c'}, \omega_{\mu'}) E_c(z, \omega_{c'}) E_{\mu}(z, \omega_{\mu'}) e^{-i\omega_{\sigma}t} e^{i\omega t} dt \quad (3.5.0.5)$$

by using the property of Fourier transformation of complex exponential function

$$\begin{aligned} P^{(2)}(z, \omega) &= \epsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega_{c'}}{2\pi} \frac{d\omega_{\mu'}}{2\pi} \chi^{(2)}(\omega_{c'}, \omega_{\mu'}) E_c(z, \omega_{c'}) E_{\mu}(z, \omega_{\mu'}) \delta(\omega_{\sigma} - \omega) \\ &= \epsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega_{c'}}{2\pi} \frac{d\omega_{\mu'}}{2\pi} \chi^{(2)}(\omega_{c'}, \omega_{\mu'}) E_c(z, \omega_{c'}) E_{\mu}(z, \omega_{\mu'}) \delta(\omega_{\mu'} - (\omega - \omega_{c'})) \\ &= \epsilon_0 \int_{-\infty}^{\infty} \frac{d\omega_{c'}}{2\pi} \chi^{(2)}(\omega_{c'}, \omega - \omega_{c'}) E_c(z, \omega_{c'}) E_{\mu}(z, \omega - \omega_{c'}) \end{aligned} \quad (3.5.0.6)$$

but $E_c(\omega_{c'})$ is a monochromatic field which can be written as

$$E_c(\omega_{c'}) = E_c(\omega_c) \delta(\omega_{c'} - \omega_c) \quad (3.5.0.7)$$

after inserting Eq. (3.5.0.7) in Eq. (3.5.0.6), we obtain

$$P^{(2)}(z, \omega) = \epsilon_0 \chi^{(2)}(\omega_c, \omega - \omega_c) E_c(\omega_c) E_{\mu}(\omega - \omega_c) \quad (3.5.0.8)$$

We have assumed both the coupling and microwave fields are undepleted fields. So, we can write, $E_c(z, \omega_{c'}) = E_c(\omega_{c'}) = E_c(0, \omega_{c'}) e^{ik_c z}$ and $E_{\mu}(z, \omega_{\mu'}) = E_{\mu}(\omega_{\mu'}) = E_{\mu}(0, \omega_{\mu'}) e^{ik_{\mu} z}$.

Now let us assume the solution of wave Eq. (3.5.0.2) is

$$E_p(z, \omega) = A(z, \omega) e^{ik_p z}, \quad (3.5.0.9)$$

where $A(z, \omega)$, k_p is the amplitude of the envelope and propagation vector, respectively.

Substituting Eq. (3.5.0.9) in Eq. (3.5.0.2), we obtain

$$\begin{aligned}
& \left(\frac{d^2 A(z, \omega)}{dz^2} + 2ik_p \frac{dA(z, \omega)}{dz} - k_p^2 A(z, \omega) \right) e^{ik_p z} \\
&= -\frac{\omega^2}{c^2} A(z, \omega) e^{ik_p z} \\
&\quad - \omega^2 \mu_0 \left(\epsilon_0 \chi_p^{(1)} A(z, \omega) e^{ik_p z} + \epsilon_0 \chi_p^2(\omega_2; \omega - \omega_c) E_c(0, \omega_c) E_\mu(0, \omega - \omega_c) e^{i(k_c + k_\mu)z} \right)
\end{aligned} \tag{3.5.0.10}$$

Which then simplifies to

$$\begin{aligned}
& \left(\frac{d^2 A(z, \omega)}{dz^2} + 2ik_p \frac{dA(z, \omega)}{dz} - k_p^2 A(z, \omega) \right) \\
&= -\frac{\omega^2}{c^2} (1 + \chi_p^{(1)}) A(z, \omega) - \frac{\omega^2}{c^2} \chi_p^2(\omega_2; \omega - \omega_c) E_c(0, \omega_c) E_\mu(0, \omega - \omega_c) e^{i\Delta k z}
\end{aligned} \tag{3.5.0.11}$$

where

$$\Delta k = k_c + k_\mu - k_p, \tag{3.5.0.12}$$

is called the phase mismatch.

we can drop $d^2 A(z, \omega)/dz^2$ on the left-hand side of Eq. (3.5.0.11) as it is very small compared to the second term. This approximation is called slowly varying amplitude approximation (SVAA) and is valid when

$$\left| \frac{d^2 A(z, \omega)}{dz^2} \right| \ll \left| k_p \frac{dA(z, \omega)}{dz} \right| \tag{3.5.0.13}$$

After SVAA and using $k_p^2 = \frac{\omega^2}{c^2} (1 + \chi_p^{(1)}(\omega))$, the Eq. (3.5.0.11) becomes

$$\begin{aligned}
2ik_p \frac{dA(z, \omega)}{dz} &= -\frac{\omega^2}{c^2} \chi_p^{(2)}(\omega_c, \omega - \omega_c) E_c(0, \omega_c) E_\mu(0, \omega - \omega_c) e^{i\Delta k z} \\
\Rightarrow \frac{dA(z, \omega)}{dz} &= \frac{i\omega^2}{2c^2 k_p} \chi_p^{(2)}(\omega_c, \omega - \omega_c) E_c(0, \omega_c) E_\mu(0, \omega - \omega_c) e^{i\Delta k z}
\end{aligned} \tag{3.5.0.14}$$

and

$$A(z, \omega) = \frac{i\omega^2 \chi_p^{(2)}(\omega_c, \omega - \omega_c) E_c(0, \omega_c) E_\mu(0, \omega - \omega_c)}{2c^2 k_p} \frac{(e^{i\Delta k z} - 1)}{i\Delta k} \quad (3.5.0.15)$$

For a perfect phase matched condition, where $\Delta k = 0$, the pulse profile will be

$$\begin{aligned} \lim_{\Delta k \rightarrow 0} A(z, \omega) &= \frac{i\omega^2 \chi_p^{(2)}(\omega_c, \omega - \omega_c) E_c(0, \omega_c) E_\mu(0, \omega - \omega_c)}{2c^2 k_p} \lim_{\Delta k \rightarrow 0} \frac{(e^{i\Delta k z} - 1)}{i\Delta k} \\ &= \frac{i\omega^2 \chi_p^{(2)}(\omega_c, \omega - \omega_c) E_c(0, \omega_c) E_\mu(0, \omega - \omega_c)}{2c^2 k_p} z \end{aligned} \quad (3.5.0.16)$$

The temporal pulse profile of probe electric field amplitude by means of Fourier transformation can be written as

$$E_p(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(z, \omega) e^{i(k_p z - \omega t)} d\omega \quad (3.5.0.17)$$

where $A(z, \omega)$ is the pulse amplitude in the frequency domain as derived. With the help of susceptibility expressions given in Eqs. (3.4.0.5) and (3.4.0.6), we calculate the temporal profile for a generated probe pulse. To ensure pulse-probe generation, we employ an input Gaussian pulsed microwave field,

$$\Omega_\mu(z = 0, \Delta_\mu) = (2\Omega_\mu^0 \sqrt{2\pi}/\delta) e^{-\Delta_\mu^2/2\delta^2} \quad (3.5.0.18)$$

driving $|1\rangle \leftrightarrow |2\rangle$, where $2\Omega_\mu^0$ and δ are the amplitude and spectral width of the envelope, respectively. The coupling field in Fig. 3.2 is the same as the one in Fig. 3.1, a CW with $\Delta_c = 0$. The input fields are taken to be propagating along the positive z -direction. The vapour cell has length l and extends from $z = 0$ to l , and the input fields are defined at the entrance to the cell at $z = 0$.

Due to $\chi_p^{(2)}$ the pulsed microwave field combines with the CW optical coupling field to give rise to a pulsed probe field through a sum-frequency generation process. By conservation of energy, the generated probe detuning (Δ_p) will be equal to the microwave detuning, i.e., $\Delta_p = \Delta_\mu$. Such a generation was theoretically studied [33] and already experimentally observed by us [34].

We have assumed the input microwave pulse and the CW optical coupling fields are

undepleted pump fields. The magnitudes of the propagation vectors of the coupling, microwave, and probe (at center frequency ω_{31}) fields inside the vapor cell are

$$k_c^2 = \frac{\omega_{32} n_c^2}{c^2} = \frac{\omega_{32}^2 (1 + \chi_c^{(1)})}{c^2}, \quad k_\mu^2 = \frac{\omega_{21} n_\mu^2}{c^2} = \frac{(\omega_{21} + \Delta_\mu)^2 (1 + \chi_\mu^{(1)})}{c^2}, \quad (3.5.0.19)$$

$$k_p^2 = \frac{\omega_{31} n_p^2}{c^2} = \frac{(\omega_{31} + \Delta_p)^2 (1 + \chi_p^{(1)})}{c^2}, \quad (3.5.0.20)$$

where $\chi_c^{(1)}, \chi_\mu^{(1)}$ are the linear susceptibilities of the coupling and microwave fields. The wave-vector mismatch of the generated probe field is defined as $\Delta k = (k_c + k_\mu) - k_p$. We assume a length l for the cell such that $\Delta k l \ll 1$ over the bandwidth of the microwave pulse. For brevity, the arguments of the following functions are suppressed: $\Omega_\mu(\Delta_p) = \Omega_\mu$, $k_p(\Delta_p) = k_p$ and $\chi_p^{(2)}(\Delta_p) = |\chi_p^{(2)}| e^{i\phi^{(2)}}$. Under the assumptions stated above, substituting Eq. (3.5.0.16) in Eq. (3.5.0.17) and using Eq. (3.2.0.17-3.2.0.19), the temporal envelope (Ω_p^g) of the generated probe pulse at $z = l$ and with $\omega = \omega_{31} + \Delta_p$ in the slowly varying amplitude approximation [35] is

$$\Omega_p^g(l, t) = \int_{-\infty}^{\infty} \frac{i(\omega_{31} + \Delta_p)^2 l \Omega_\mu \Omega_c |\chi_p^{(2)}| e^{i\phi^{(2)}} e^{i(k_p l - (\omega_{31} + \Delta_p)t)}}{K c^2 k_p} d\Delta_p, \quad (3.5.0.21)$$

where $K = 2\pi\mu_{21}d_{32}/\hbar d_{31}$. For simplicity, the microwave pulse bandwidth is assumed to be small enough such that only the effect of central frequency is considered for estimation of $\chi_p^{(1)}$. As can be seen from Eq. (3.5.0.21), the dynamics of the generated probe pulse profile depends on the complex ($\mathbb{C}$) quantities $\chi_p^{(1)}$ and $\chi_p^{(2)}$. We study in-detail the effects of linear and non-linear susceptibility on the amplitude and group delay of the generated probe pulse.

3.6 Results and discussions

The amplitude of the generated probe pulse depends on $|\chi_p^{(2)}|$ and the imaginary part of $\chi_p^{(1)}$ ($\chi_{Ip}^{(1)}$). Under two-photon resonance between the generated probe pulse and the coupling pulse our system satisfies all the conditions necessary for showing EIT effect as seen in Fig. 3.3. At two-photon resonance, ($\Delta_p = 0$) the typical value of $\chi_{Ip}^{(1)}$ is around 10^{-7} . To make sure that the generated probe pulse suffers negligible absorption due to $\chi_{Ip}^{(1)}$, the generated probe bandwidth is chosen to be within the EIT window through

appropriate choices of δ and Ω_c values. Henceforth we assume that absorption of the generated pulse due to linear susceptibility is negligible. We therefore drop terms relating to $\chi_{\text{Ip}}^{(1)}$ from Eq. (3.5.0.21).

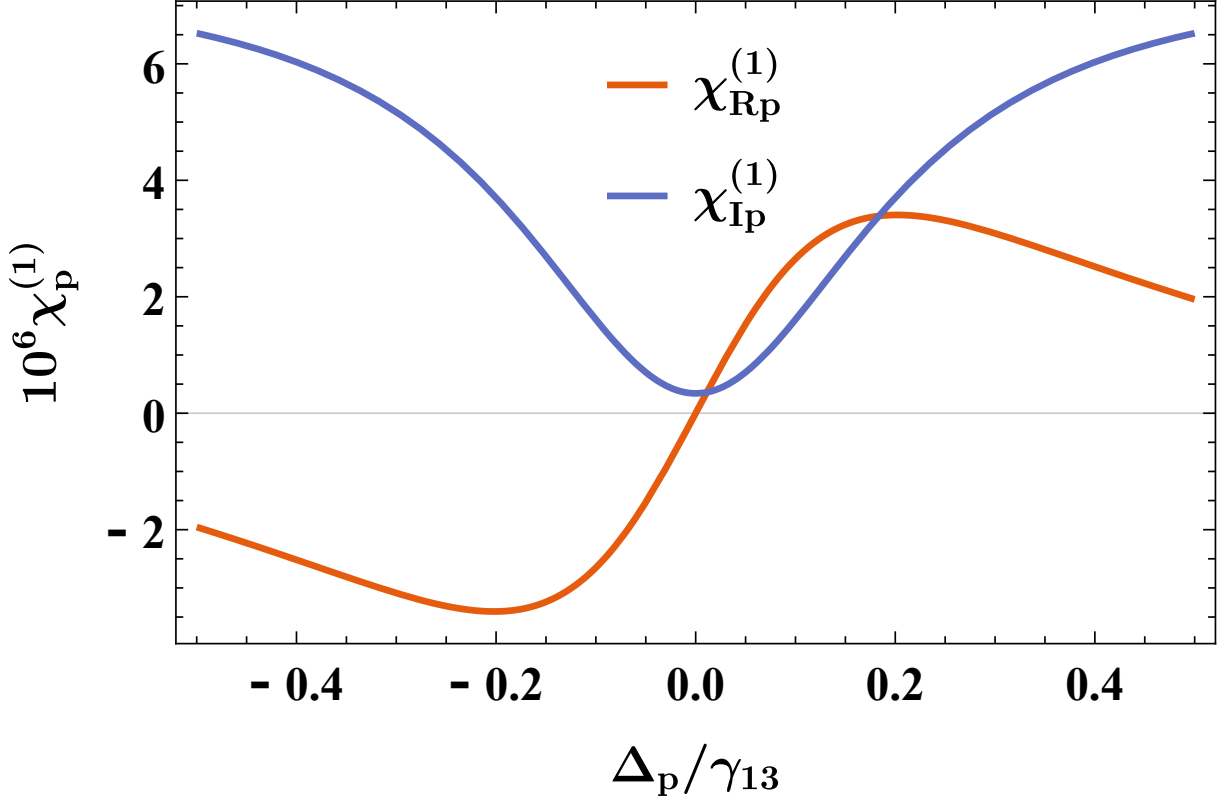


Figure 3.3: Real and imaginary parts of the linear susceptibility as a function of generated probe detuning, for $\Omega_c = 0.4\gamma_{13}$, $\Omega_\mu^0 = 0.004\gamma_{13}$, $\gamma_{12} = 0.004\gamma_{13}$, $\gamma_{13} = 5\pi \times 10^6$ rad/s and $\mathcal{N} = 10^8$ cm $^{-3}$.

Group delay of the generated probe with respect to a reference pulse is defined as the time difference in peak arrival time of the generated probe pulse and the reference pulse, both calculated at the end of the vapour cell. The peak here is defined as the peak of the generated optical pulse with maximum amplitude. The reference is a Gaussian pulse traversing the same length in vacuum given by

$$\Omega_r(l, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (\Omega_r^0 \sqrt{2\pi}/\sigma) e^{-\Delta_p^2/2\sigma^2} e^{i(\omega_{31} + \Delta_p)(l/c - t)} d\Delta_p, \quad (3.6.0.1)$$

where Ω_r^0 and σ are the amplitude and spectral width of the reference respectively.

This group delay depends on two qualitatively different spectral phases acquired by the generated probe as seen in Eq. (3.5.0.21). The first spectral phase is the traditional

propagation phase $\phi^{(1)} = k_p l$. Using Eq. (3.5.0.20) we derive an expression for $\phi^{(1)}$

$$\phi^{(1)} \approx \frac{(\omega_{31} + \Delta_p)(1 + \chi_{\text{Rp}}^{(1)}/2)l}{c}, \quad (3.6.0.2)$$

where $\chi_{\text{Rp}}^{(1)}$ is the real part of $\chi_p^{(1)}$. The second one is the generation phase, the initial phase probe pulse acquires during the generation process. This is defined as

$$\phi^{(2)} = \arg(\chi_p^{(2)}), \quad (3.6.0.3)$$

which is the phase of complex $\chi_p^{(2)}$.

Our main results presented below are due to the effect of $\phi^{(2)}$ on the group delay of the generated probe pulse. To calculate the group delay to first order, we perform a Taylor expansion of the phases $\phi^{(1)}$ and $\phi^{(2)}$ around $\Delta_p = 0$,

$$\phi^{(k)} \approx \phi^{(k)} \Big|_{\Delta_p=0} + \frac{\partial \phi^{(k)}}{\partial \Delta_p} \Big|_{\Delta_p=0} \Delta_p \quad (k \in \{1, 2\}). \quad (3.6.0.4)$$

We identify the pertinent temporal quantities in the above expansion and classify them as

$$t^{(1)} = \frac{\partial \phi^{(1)}}{\partial \Delta_p} \Big|_{\Delta_p=0} = \frac{n_g l}{c}, \quad (3.6.0.5)$$

$$n_g = 1 + (1/2) \chi_{\text{Rp}}^{(1)} \Big|_{\Delta_p=0} + (1/2) \frac{\partial \chi_{\text{Rp}}^{(1)}}{\partial \Delta_p} \Big|_{\Delta_p=0} \omega_{31}, \quad (3.6.0.6)$$

$$t^{(2)} = \frac{\partial \phi^{(2)}}{\partial \Delta_p} \Big|_{\Delta_p=0}. \quad (3.6.0.7)$$

Substituting Eqs. (3.6.0.2)-(3.6.0.7) in Eq. (3.5.0.21) we calculate the total group delay of the generated probe pulse with respect to the reference pulse. This is done by comparing the complex phases in Eqs. (3.5.0.21) and (3.6.0.1). Thus the total group delay (T_g) of the generated probe pulse with respect to the reference is given by

$$T_g = t^{(1)} + t^{(2)} - l/c. \quad (3.6.0.8)$$

In the above expression $t^{(1)}$ is the traditional group delay associated with c/n_g , the group velocity of the pulse. On the other hand, $t^{(2)}$ is the group delay associated with

the change of the generated probe initial phase with changing probe frequency. In the subsequent paragraphs we analyse each of the $t^{(1)}$ and $t^{(2)}$ contributions to the total group delay for different number densities of atoms.

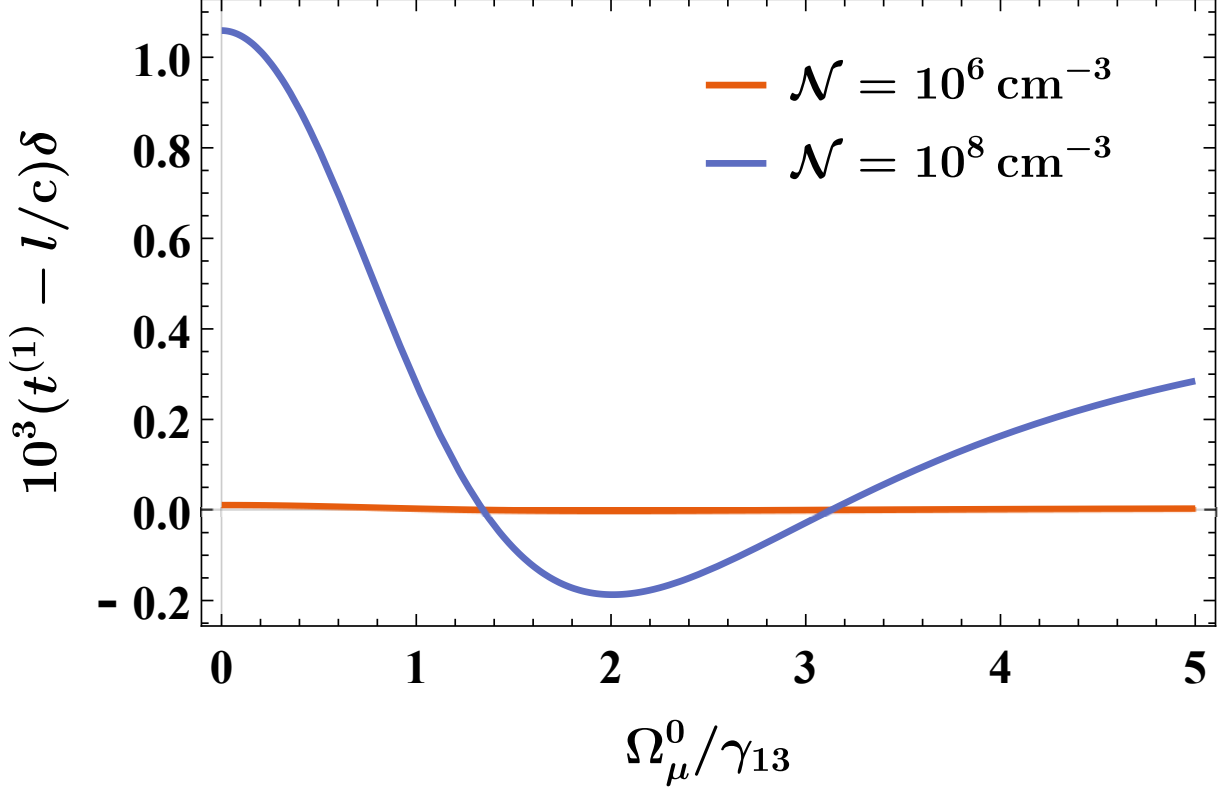


Figure 3.4: Group delay, due to the real part of linear susceptibility, as a function of microwave amplitude, for $\Omega_c = 3.2\gamma_{13}$, $\delta = 0.3 \times 10^6$ Hz, $\gamma_{13} = 5\pi \times 10^6$ rad/s and $l = 1$ cm.

There have been numerous studies on the group delay $t^{(1)}$ due to changes in group velocity of an input pulse traversing a coherently prepared atomic medium [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Some of these studies focused on developing a knob for changing the group velocity from subluminal to superluminal resulting in delay or advancement respectively of the peak of an input pulse [18, 19, 20, 21, 22, 23, 24, 25]. These changes in group delay are due to the real part of linear susceptibility and are number density-dependent as seen from Eq. (3.4.0.6).

To explicitly illustrate this density dependence, we show in Fig. 3.4 the group delay due to $\chi_{\text{Rp}}^{(1)}$ as a function of Ω_μ^0 for two different number densities. For a density value of $\mathcal{N} = 10^8 \text{ cm}^{-3}$ we see that the group delay changes from positive to negative as a function of Ω_μ^0 because of subluminal to superluminal change in group velocity. However by reducing the density by two orders to $\mathcal{N} = 10^6 \text{ cm}^{-3}$ we observe not only that the group delay remains practically constant with changing Ω_μ^0 , but also that its value is close

to zero indicating that the group velocity of the pulse numerically approximates its value in vacuum. Thus we note that any knob to change the value and sign of the group velocity of a pulse which is density-dependent is unlikely to work for low-density samples.

In contrast to the heavy dependence of $t^{(1)} - l/c$ on $\mathcal{N}$ and l , the group delay due to $t^{(2)}$ is independent of $\mathcal{N}l$, representing a density-independent group delay due to nonlinear generation. Mathematically $\mathcal{N}$ is a prefactor to both the real and imaginary part of $\chi_p^{(2)}$ (Eq. 3.4.0.5) and, since $\phi^{(2)}$ is the argument of $\chi_p^{(2)}$, it is independent of $\mathcal{N}$ leading to $t^{(2)}$ also being $\mathcal{N}$ independent. The physical origin of $\phi^{(2)}$ being independent of $\mathcal{N}$ is that this is the phase associated with the generated probe field by the atoms during the generation process. In the absence of atom-atom interaction this generation phase is independent of the number density.

For the purpose of generation of few-photons we henceforth will work with low-density samples. We show in our subsequent analysis how we can effect both an advancement and a delay in the peak of a few-photon pulse through $t^{(2)}$. The knob we use to change the sign of $t^{(2)}$ is the intensity of the microwave pulse.

We fix the input optical pump field (Ω_c) to be $0.4\gamma_{13}$ with its beam diameter taken to be 1 mm^2 and the spectral width of the input microwave pulse to be $\delta = 0.3 \times 10^6 \text{ Hz}$. The total energy in the generated pulse is given by

$$E = (b\hbar^2/d_{13}^2) \int_{-\infty}^{\infty} \varepsilon_0 c |\Omega_p^g(l, t)|^2 dt, \quad (3.6.0.9)$$

where b is the beam diameter. The number of photons in the pulse is estimated by dividing the total energy (E) by the energy of the central frequency photon ($\hbar\omega_{31}$). By this procedure we find that a few-photon pulse is generated at the output for $\mathcal{N} = 10^6 \text{ cm}^{-3}$ and $l = 1 \text{ cm}$. For these $\mathcal{N}$ and l values, the group-delay contribution from $(t^{(1)} - l/c)$ through linear susceptibility can be safely ignored as can be seen in Fig. 3.4.

This leads to minimising the effect of $\chi_{Rp}^{(1)}$ in Eq. (3.5.0.21), leading to the final generated probe profile as

$$\Omega_p^g(l, t) = \int_{-\infty}^{\infty} \frac{i(\omega_{31} + \Delta_p) l \Omega_c \Omega_\mu |\chi_p^{(2)}| e^{i\phi^{(2)}} e^{i(\omega_{31} + \Delta_p)(l/c - t)}}{Kc} d\Delta_p, \quad (3.6.0.10)$$

and its group delay with respect to the reference

$$T_g = t^{(2)}. \quad (3.6.0.11)$$

Now the group delay of the generated few-photon pulse with respect to the reference pulse originates entirely from the generation phase $\phi^{(2)}$ and not due to any propagation effect.

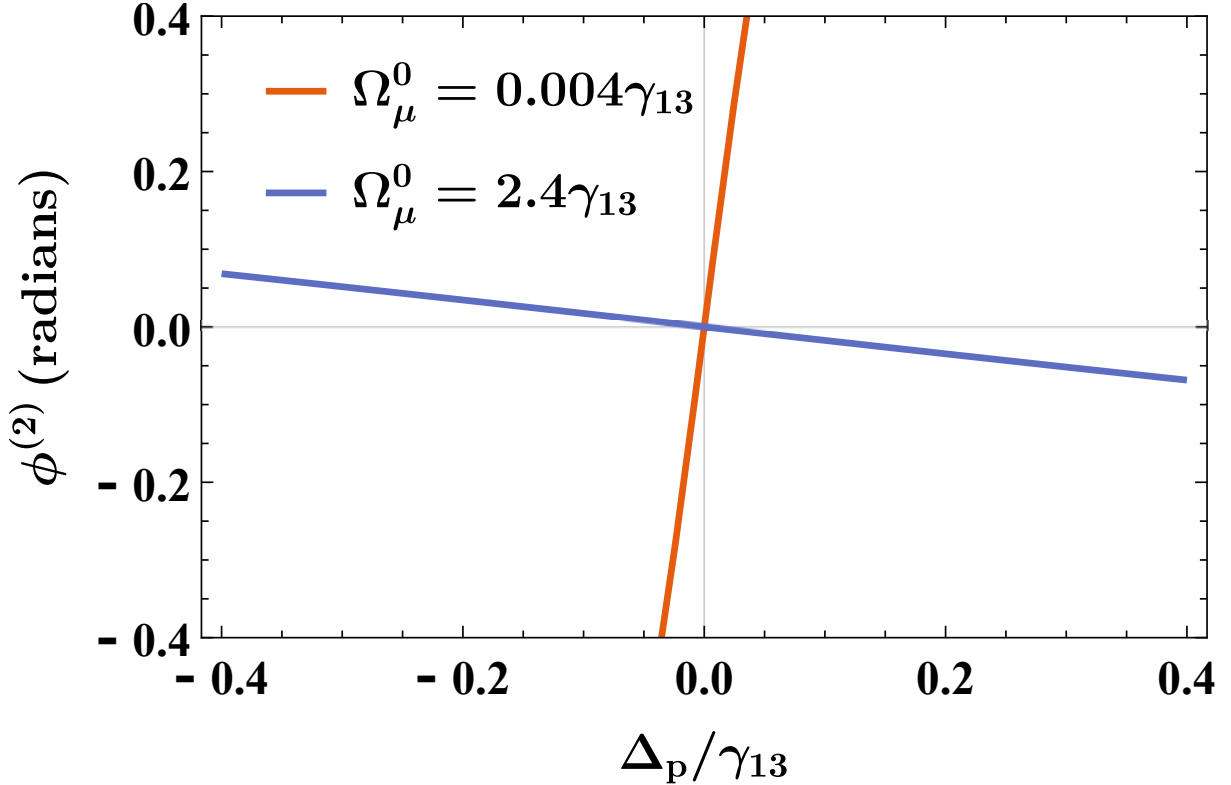


Figure 3.5: Generation phase as a function of the generated probe detuning, for $\Omega_c = 0.4\gamma_{13}$, $\mathcal{N} = 10^6 \text{ cm}^{-3}$ and $\gamma_{13} = 5\pi \times 10^6 \text{ rad/s}$.

As shown in Fig. 3.5 the atoms generate different frequencies of probe with different initial phases and this phase profile clearly depends on Ω_μ^0 . We can see that the slope around the $\Delta_p = 0$ value changes sign for two different Ω_μ^0 values. As $t^{(2)}$ is the slope of $\phi^{(2)}$ at $\Delta_p = 0$, it is clear from Fig. 3.5 that one can use Ω_μ^0 as a handle to delay or advance the generated probe pulse.

We will quantitatively estimate the group delay of the generated probe pulse through changes in pump field intensities. We plot in Fig. 3.6, $t^{(2)}$ as a function of Ω_μ^0 . This plot shows that Ω_μ^0 can be used as a knob for continuous tunability of $t^{(2)}$ resulting in delay or advancement in peak of the pulse. Such a continuous tunability is also achievable through

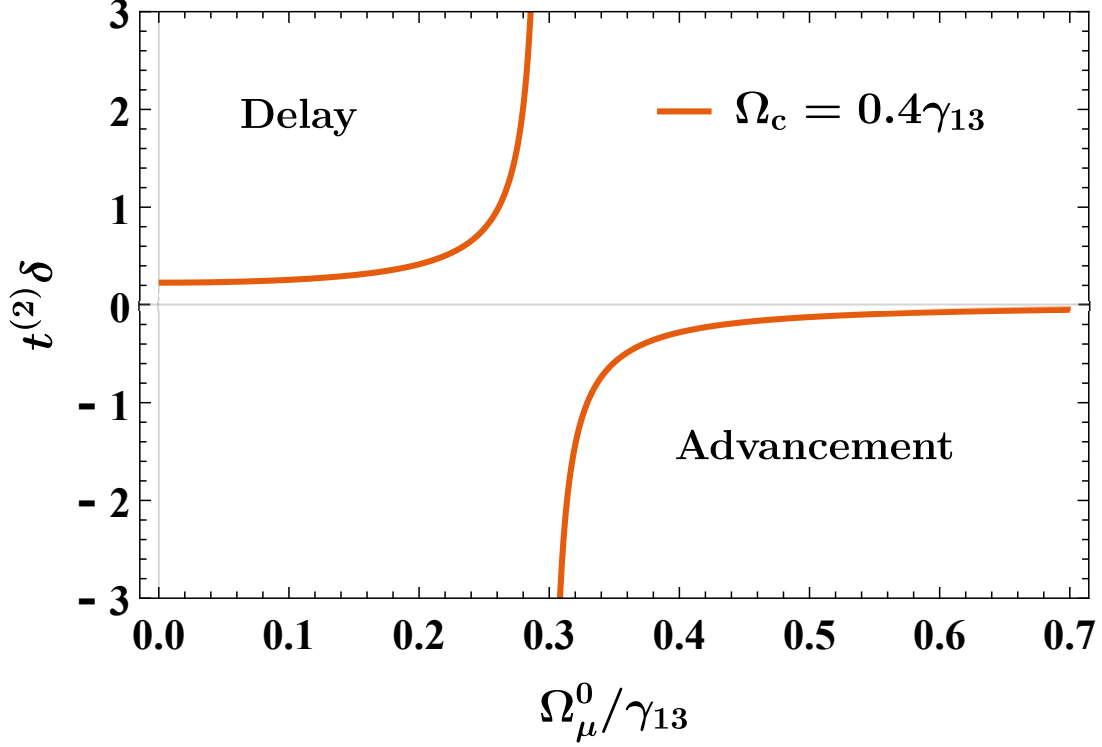


Figure 3.6: Group delay due to generation phase as function of the microwave amplitude, for $\delta = 0.3 \times 10^6$ Hz, $\mathcal{N} = 10^6$ cm $^{-3}$ and $\gamma_{13} = 5\pi \times 10^6$ rad/s.

changes in Ω_c .

The temporal shape of the generated pulse is governed by $|\chi_p^{(2)}|$ as seen in Eq. (3.6.0.10). Our calculations show that the delayed and advanced probe pulse temporal width (standard deviation) in Figs. 3.7 and 3.8 is contracted by 79 % and 91 % respectively, with respect to the input pulsed microwave. As can be seen, the higher value of Ω_μ^0 required for advancement of the generated pulse shown in Fig. 3.8 results in increased temporal distortion of the generated pulse. For higher microwave intensities the levels connected by the microwave field suffer AC stark shift resulting in increased $|\chi_p^{(2)}|$ values away from resonance. This effect in the frequency domain translates to the ringing effect in the time domain.

The temporal profile of the generated probe pulse is obtained by numerical integration of Eq. (3.6.0.10) and is shown in Figs. 3.7 and 3.8. These plots show a pulse delay of $0.5 \mu\text{s}$ and a pulse advancement of $0.3 \mu\text{s}$ respectively, for a change of $\Omega_\mu^0 = 0.004\gamma_{13}$ to $\Omega_\mu^0 = 2.4\gamma_{13}$ at $\mathcal{N} = 10^6$ cm $^{-3}$. To observe the same order of delay or advancement in an input probe pulse through propagation effects, one requires a number density of the order 10^{11} cm $^{-3}$. Thus, by manipulating the generated phase profile even with very dilute samples of atoms, comparable changes in group delay can be achieved.

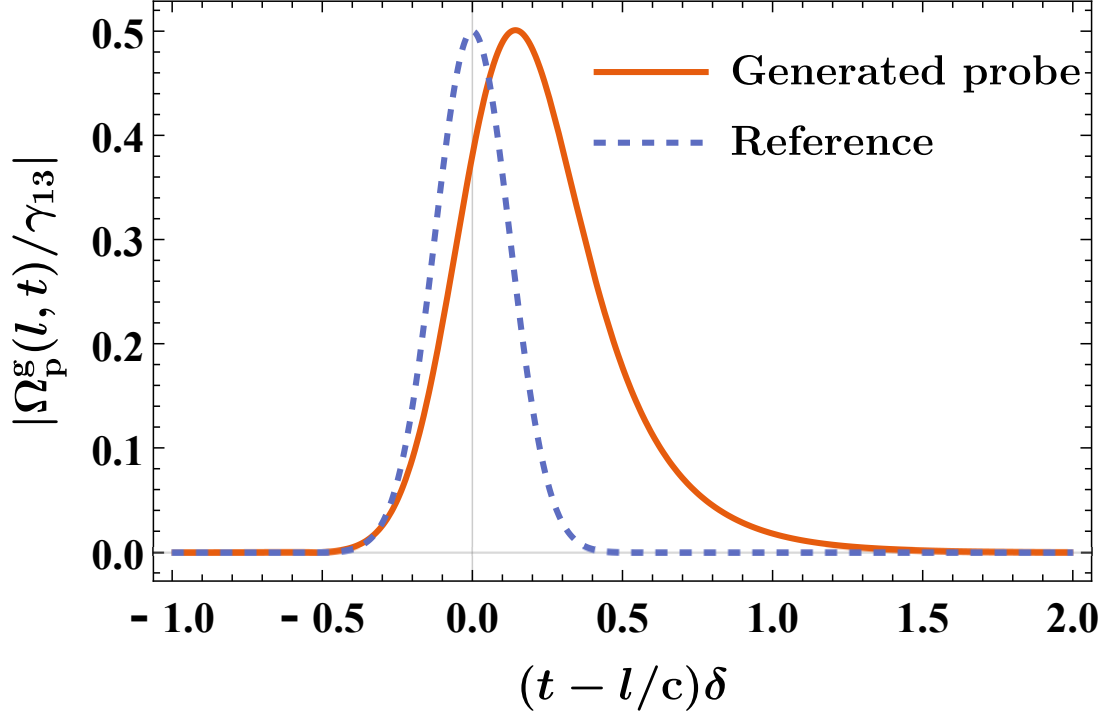


Figure 3.7: Delay of $0.5 \mu\text{s}$ observed in the generated probe with respect to the reference, for $\Omega_\mu^0 = 0.004\gamma_{13}$, $\Omega_c = 0.4\gamma_{13}$, $\delta = 0.3 \times 10^6 \text{ Hz}$, $\gamma_{13} = 5\pi \times 10^6 \text{ rad/s}$, $\mathcal{N} = 10^6 \text{ cm}^{-3}$ and $l = 1 \text{ cm}$. The mean number of photon in the generated probe is 1.24. The generated probe has been scaled by a factor of 10416. The parameters of the reference are $\Omega_r^0 = 0.5\gamma_{13}$ and $\sigma = 2.4 \times 10^6 \text{ Hz}$.

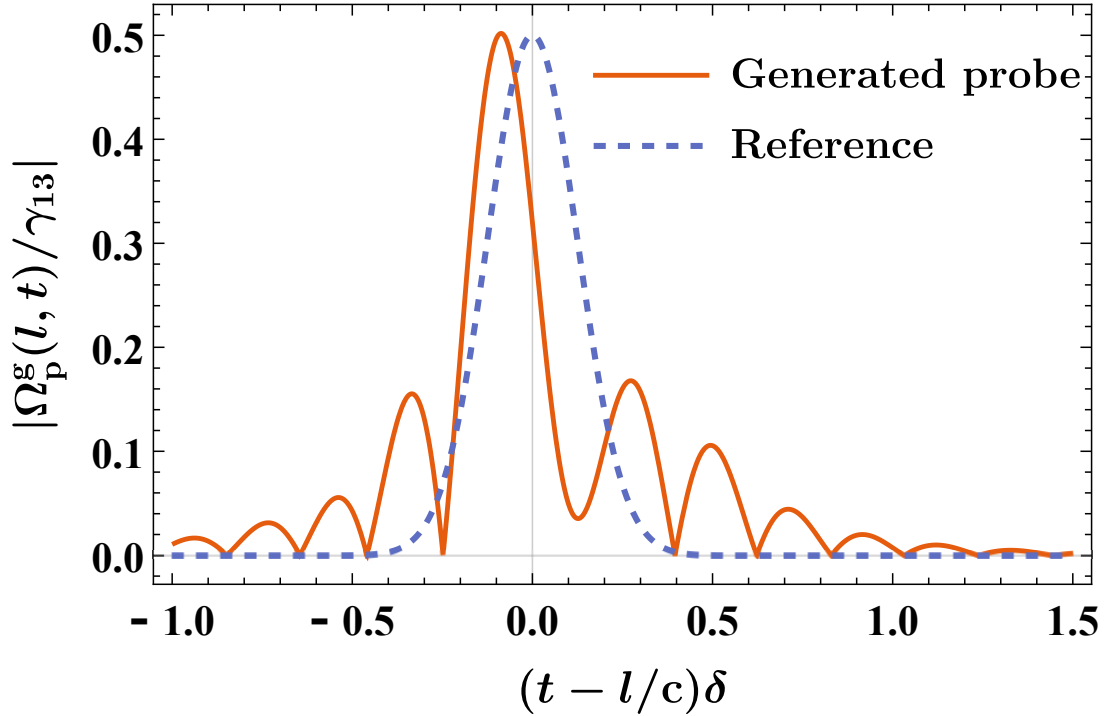


Figure 3.8: Advancement of $0.3 \mu\text{s}$ observed in the generated probe with respect to the reference, for $\Omega_\mu^0 = 2.4\gamma_{13}$, $\Omega_c = 0.4\gamma_{13}$, $\delta = 0.3 \times 10^6 \text{ Hz}$, $\gamma_{13} = 5\pi \times 10^6 \text{ rad/s}$, $\mathcal{N} = 10^6 \text{ cm}^{-3}$ and $l = 100 \mu\text{m}$. The mean number of photon in the generated probe is 1.29. The generated probe has been scaled by a factor of 75757. The parameters of the reference are $\Omega_r^0 = 0.5\gamma_{13}$ and $\sigma = 2.4 \times 10^6 \text{ Hz}$.

To see the effect of Doppler broadening on the group delay of the generated probe pulse we have done a Doppler averaging of the $\chi_p^{(2)}$ susceptibility profile. We incorporate Doppler broadening by defining the Doppler shifted detunings as

$$\Delta_c(v) = \Delta_c - \omega_{32}v/c, \quad \Delta_p(v) = \Delta_p - \omega_{31}v/c, \quad (3.6.0.12)$$

where v is the velocity of atom. We average $\chi_p^{(2)}$ using the Maxwell-Boltzmann velocity distribution for room temperature $T=300$ K. This procedure leads to the Doppler averaged $t^{(2)}$ using Eqs. (3.6.0.3) and (3.6.0.7) and is plotted in Fig. 3.9. It is clear from this figure that even with room temperature atoms a microwave-intensity change acts as an effective knob to convert a delayed pulse to an advanced pulse. However, this change happens at a higher microwave intensity. For a temperature of 100°C the microwave amplitude required to change from delay to advancement is about 6 % more than what is required at room temperature.

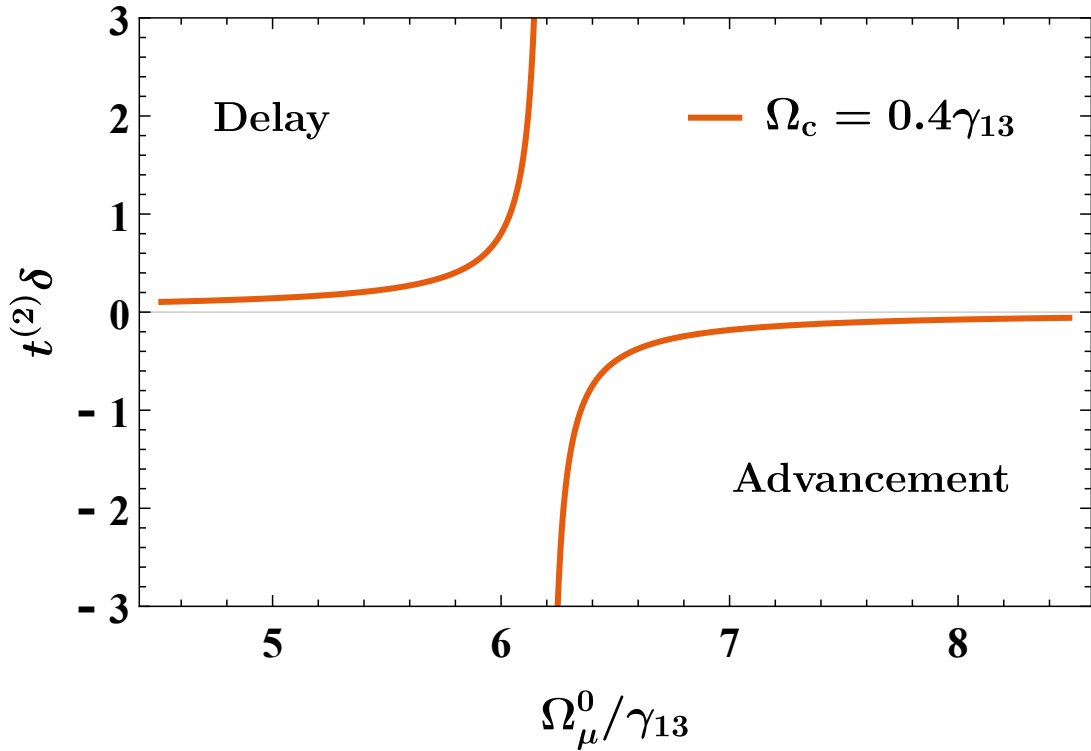


Figure 3.9: Doppler averaged group delay due to generation phase as a function of microwave amplitude, for $\gamma_{23} = 0.4\gamma_{13}$, $T=300$ K, $\delta = 0.3 \times 10^6$ Hz and $\gamma_{13} = 5\pi \times 10^6$ rad/s.

The repetition rate of the on-demand pulse with the ability to delay or advance the peak depends on the bandwidth of the near-linear region around $\Delta_p = 0$ in Fig. 3.5. From this figure it is clear that the bandwidth is of the order of a few MHz for our parameters

and so the repetition rate of our generation also would be in the order of a few MHz.

Experimental realization of generation and group delay control of a few photon pulse described above can be achieved. Generation of probe optical field with ^{85}Rb atoms from a sum frequency process between the coupling and microwave fields has already been realized [34]

In order to show that the delay or advancement of the generated pulse is independent of number density or length of the cell, the number density in a vapour cell could be varied by changing the temperature or by using identical cells of varying vapour density or cells with varying physical dimensions.

3.7 Conclusion

A scheme for on-demand generation of optical pulses and control of their group delay using a hybrid second order-nonlinearity in an atomic Δ system is presented. An unexplored regime of manipulating group delay arising from the initial phase of the generated pulse is presented. We rigorously establish that this leads to density-independent group delays. We show that the generated pulse can be delayed or advanced through change in intensity of the input microwave field and discuss in detail the average photon number and temporal shape of the generated pulse. Thus, our scheme can be exploited to generate and control group delay of optical pulses with few photons using ultra-dilute samples with small spatial dimensions. These results open a way for producing and manipulating group delays with microscopic atomic systems that can be integrated in scalable platforms. All the above features in combination with our hybrid nonlinearity will have a definitive impact in introducing controllable delays in both classical and quantum information processing channels which use both microwave and optical fields.

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Chapter 4

Simultaneous observation of six-wave and three-wave mixing processes

4.1 Introduction

Nonlinear processes in a two-level atomic system, excited by a weak probe and a strong pump, have been exhaustively studied for inducing parametric amplification and oscillation in the probe waves [1, 2, 3, 4, 5, 6]. These processes capitalize on the existence of four-wave parametric interactions, three-photon gain, and coherent population oscillations (CPO) in the presence of a transparency window in the probe's absorption profile. A large class of these experiments were performed at high-light intensities and were limited to the observation of third-order nonlinear phenomena.

The emergence of electromagnetically induced transparency (EIT) has enabled the experimental realization of higher-order nonlinear processes with low-light intensities in atomic systems. By reducing linear absorption and simultaneously enhancing nonlinear susceptibilities at atomic resonances, EIT has been responsible for numerous studies of optical nonlinear processes in multi-level atomic systems [7, 8, 9, 10, 11]. The studies primarily constitute observation of four-wave mixing (FWM) and six-wave mixing (SWM) processes. Simultaneous observation of the FWM and SWM processes have also been reported via dual EIT windows by spatially aligning the input beams in a square-box pattern in a Y-type atomic system [12].

Nonlinear processes in atomic systems have also been utilized to interface microwave and optical frequencies, a requirement for realizing quantum hybrid systems to trans-

mit quantum signals. Assisted by EIT, coherent microwave-to-optical conversion has been realized via an SWM process in cold Rydberg atoms [13] and via a three-wave mixing (TWM) process in room-temperature three-level atoms [14]. Recently, a coherent microwave-to-optical conversion of efficiency $82\pm 2\%$ has been achieved in Rydberg atoms [15] via off-resonant scattering. Opening up nonlinear channels in EIT systems has also aided in achieving phase-insensitive amplification (PIA) [16] and phase-sensitive amplification (PSA) [17, 18] of optical signals. Particularly, the realization of a microwave-controlled phase-dependent amplifier [19], driven by a TWM process, has been utilized to perform digital communication from the microwave to optical spectrum [20].

In this study, we experimentally investigate and theoretically analyze co-existing TWM and SWM processes in room-temperature ^{85}Rb atoms. The scheme utilizes three hyper-fine levels of the D1 manifold, cyclically connected by two co-propagating optical fields, the probe and coupling field, and one microwave field as shown in Fig. 4.1. The pump coupling field dresses the excited state ($|3\rangle$) and the meta-stable ($|2\rangle$) state causing EIT for the weak probe. The microwave field connects the ground ($|1\rangle$) and metastable levels. By suitable detuning of the microwave or probe field from its resonance, the three-photon resonance condition is broken, and CPO is induced in the system. This causes the TWM and SWM processes to appear in distinct frequency channels simultaneously. The SWM process grows exponentially with propagation distance due to its parametric coupling to the input probe while the TWM process grows linearly due to the absence of parametric coupling to the input probe field. Nevertheless, the simultaneous presence of EIT and CPO greatly enhances the TWM and SWM processes by reducing the linear absorption and enhancing the non-linear susceptibilities.

To the best of our knowledge, this is the first observation of the simultaneous generation of a near-infrared optical field through a TWM and SWM process in an atomic system. The TWM process combines one electric dipole and one magnetic dipole transition while the SWM process combines three electric dipole and two magnetic dipole transitions. The novel magneto-optical interaction between the electric and magnetic dipoles, enhanced through a microwave cavity, breaks the centro-symmetry in our thermally broadened atomic system and opens up the TWM and SWM channels. We have theoretically analyzed the TWM and SWM phenomena and identified the requirement of a broken three-photon resonance and the resulting CPO as being responsible for the

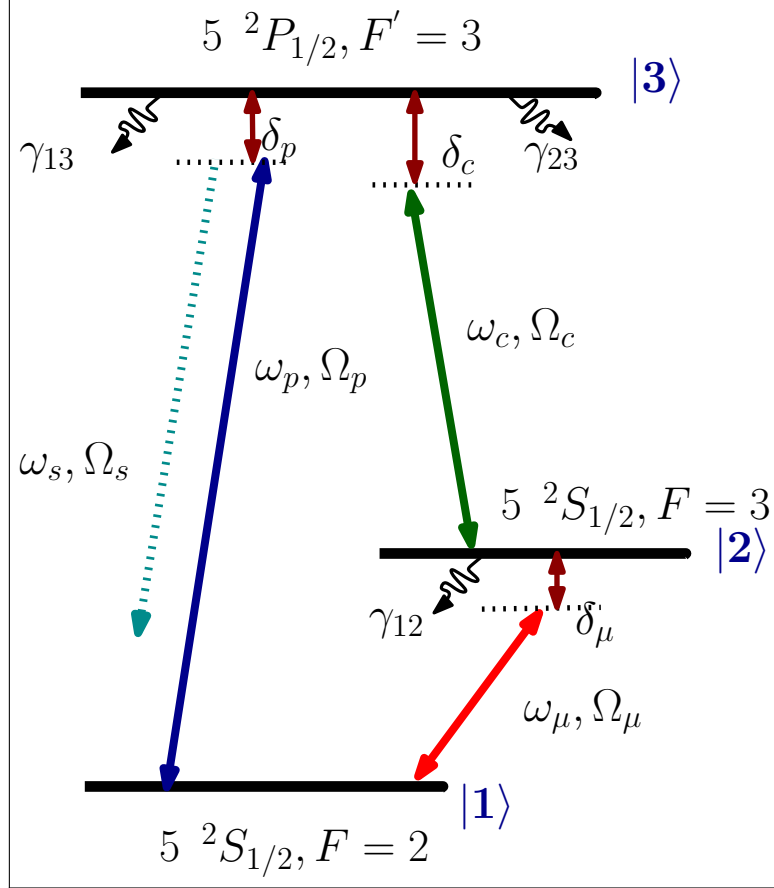


Figure 4.1: Hyperfine energy levels of ^{85}Rb in the D1 manifold used in the experiment. The detunings of probe field from $|1\rangle \rightarrow |3\rangle$ transition is denoted by δ_p . The detunings δ_c and δ_μ represent coupling and microwave field detunings from $|2\rangle \rightarrow |3\rangle$ and $|1\rangle \rightarrow |2\rangle$ respectively. Ω_p , Ω_c and Ω_μ are the Rabi frequencies of the probe, coupling, and microwave fields, respectively. γ_{ij} denote the natural decay widths connecting $|i\rangle \rightarrow |j\rangle$ transition. The dotted green arrow denotes the generated optical signal field.

SWM process. The CPO oscillations are observed in real time to validate our theory.

Moreover, the nonlinear channels also offer the possibility to realize an amplifier operating simultaneously in different regimes of linear amplification in distinct frequency channels. The phase-dependent amplification (PDA) resulting from the TWM process has been thoroughly analyzed in our previous manuscript [19]. Opening up the SWM channel offers the possibility of achieving PIA and PSA simultaneously. Thus, the results reported in this chapter substantially strengthen the versatility of the neutral atom-based transducer in not only converting signals from the microwave to the optical domain but also in aiding various types of amplification to be enabled on a single physical platform.

4.2 Experimental details

The experiment utilizes Δ configured three hyperfine energy levels in the D1 manifold of the ^{85}Rb atoms. The coupling laser connecting the metastable state ($|2\rangle$) and the ex-

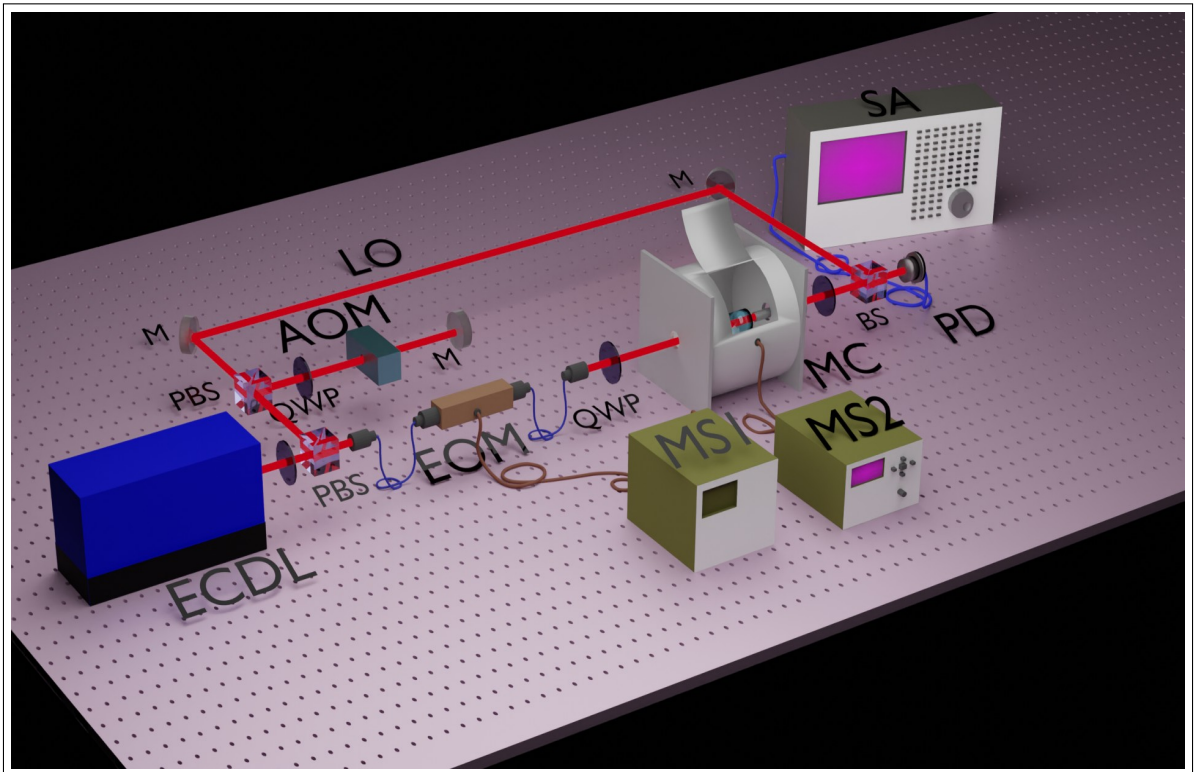


Figure 4.2: Experimental Setup for simultaneous observation of TWM and SWM signals. Here ECDL —External Cavity Diode Laser, MC —Microwave Cavity, MS —Microwave Source, SA —Spectrum Analyzer, PD —Photo-Detector, EOM —Electro-Optic Modulator, AOM —Acousto Optic Modulator, QWP —Quarter Wave Plate, M —Mirror, PBS —Polarising Beam Splitter, BS —Beam Splitter, LO —Local Oscillator.

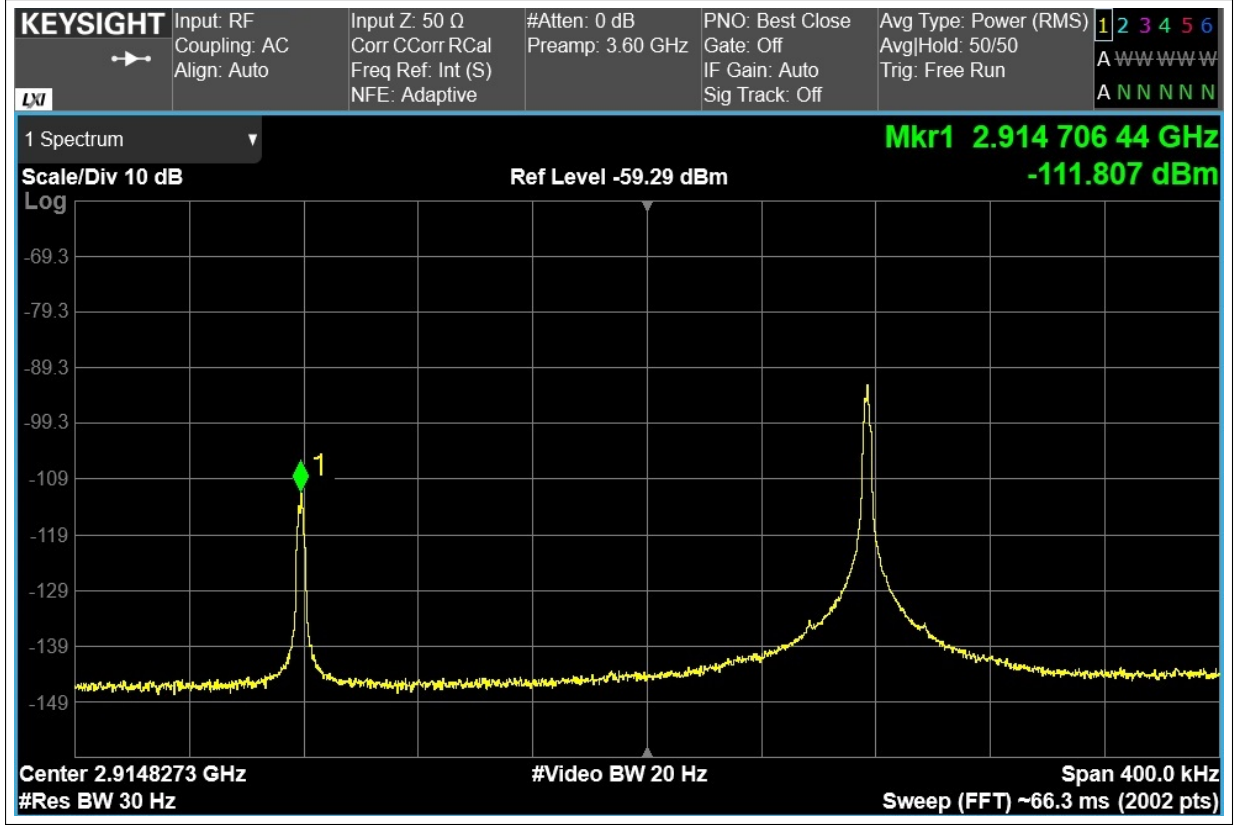


Figure 4.3: A screenshot of the spectrum analyzer showing the simultaneous presence of both SWM(left) and TWM(right) peaks in different frequency bins.

cited state ($|3\rangle$), an electric-dipole transition, is derived from an External Cavity Diode Laser (ECDL), tuned at 795 nm as shown in Fig. 4.2. The ground state ($|1\rangle$) is connected to the excited state, an electric-dipole transition, by a seed probe beam derived from the coupling laser after passing through a phase-modulated Electro-Optic Modulator (EOM). The EOM is driven by the microwave source (MS1). The metastable state and the ground state, a magnetic-dipole transition, are coupled by a 3.035 GHz microwave field derived from another microwave source (MS2). The presence of two optical fields and one microwave field makes the system closed and cyclic (Δ). The Rubidium vapor cell is kept inside a microwave cavity to enhance the magnetic dipole interaction between the atoms and the microwave field. The frequency tunable microwave cavity is designed to support the TE_{011} mode with a quality factor of 9000 and a linewidth of 300 kHz. The microwave is fed to the cavity, after passing through an amplifier providing 30 dB gain, with a small loop antenna placed inside the cavity. By using two different microwave sources to drive the EOM and cavity, we control the three-photon detuning by suitably detuning the microwave or the probe field off-resonant.

The generated TWM and SWM beams, co-propagating along with seed probe and coupling beams, simultaneously exit the microwave cavity and fall on a high bandwidth photo-detector. In order to resolve the generated signals from other co-propagating fields, an optical heterodyning technique is used. The signals exiting the cavity mix with a local oscillator (LO) beam, which is 120 MHz frequency detuned from the coupling beam. The photo-detector output is now recorded in a spectrum analyzer, where the TWM and SWM signals can be frequency resolved. The frequency-resolved TWM and SWM signals appear as shown in Fig. 4.3 in the spectrum analyzer in distinct frequency channels.

4.3 Theoretical model

The experimental observation of the simultaneous TWM and SWM process in distinct frequency channels, as shown in Fig. 4.3, is achieved by breaking the three-photon resonance. The three-photon detuning δ_3 is defined as,

$$\delta_3 = \omega_p - \omega_c - \omega_\mu = \delta_p - \delta_c - \delta_\mu, \quad (4.3.0.1)$$

where $\delta_i, i \in (p, c, \mu)$ represents the single photon detuning from their respective energy levels. $\omega_{(ij)}, ij \in (1, 2, 3), i \neq j$ are the atomic resonances.

The frequencies at which the TWM (ω_3) and SWM (ω_6) signals are generated, can be written as,

$$\omega_3 = \omega_c + \omega_\mu = \omega_p - \delta_3, \quad (4.3.0.2)$$

$$\omega_6 = 2(\omega_c + \omega_\mu) - \omega_p = \omega_p - 2\delta_3, \quad (4.3.0.3)$$

respectively.

For $\delta_c = \delta_\mu = 0$, which is the case in our experiment, the TWM (ω_3) and SWM (ω_6) signals generates at

$$\omega_3 = \omega_{31}, \quad (4.3.0.4)$$

$$\omega_6 = \omega_{31} - \delta_p, \quad (4.3.0.5)$$

and the probe is seeded at $\omega_{31} + \delta_p$.

The TWM process absorbs one microwave photon and one coupling photon to generate a signal photon. The SWM process absorbs two microwave photons and two coupling photons and emits one probe photon to generate a signal photon. Furthermore, as inferred from Eqs. (4.3.0.2) and (4.3.0.3), it is necessary to break the three-photon resonance to distinguish the TWM and SWM processes in the frequency spectrum. In our experiment, by changing δ_p using MS1 in Fig. 4.2, we introduce a non-zero δ_3 .

The TWM and SWM process can be traditionally described as a non-linear interaction between the coupling, microwave, and probe fields, enabled by the hybrid $\chi_p^{(2)}$ and $\chi_p^{(5)}$ non-linearity respectively. In contrast to the TWM process, the SWM process is always accompanied by a probe photon emission, a consequence of the energy and momentum conservation, represented by Eq. (4.3.0.3). As will be shown subsequently, the interplay between the SWM and the probe field will have a significant effect on its propagation.

The SWM process and the probe photon emission can alternatively be viewed as a result of CPO in the system. By breaking three-photon resonance, the medium's response at ω_p (due to the probe field) and at $\omega_c + \omega_\mu$ (due to the coupling and microwave field) is detuned causing CPO at the beat frequency δ_3 . The populations can follow the beat frequency if it is smaller than the decay rates of the transitions. The resulting CPO at δ_3 acts as modulators to the medium's response at $\omega_c + \omega_\mu$ to create the sidebands $\omega_c + \omega_\mu \pm \delta_3$. The sidebands are, respectively, the SWM signal and probe photon generation. In our system, the generated SWM signal takes part in another six-wave mixing process where the probe field also gets generated. The energy conservation for the probe photon generation can be described as

$$\omega_p = 2(\omega_c + \omega_\mu) - \omega_6 \quad (4.3.0.6)$$

4.3.1 Derivation of CPO

From Eq. (3.4.0.4) we know the density matrix element can be written as

$$\sigma_{jk} = \sigma_{jk}^{(0)} \Omega_c \Omega_\mu + \sigma_{jk}^{(1)} \Omega_p e^{-i\Delta_3 \tau} + \sigma_{jk}^{(2)} \Omega_p^* e^{i\Delta_3 \tau}, \quad (4.3.1.1)$$

where $(j, k \in \{1, 2, 3\})$.

The ground state population can be derived from the above equation and can be written as

$$\sigma_{11} = \sigma_{11}^{(0)} \Omega_c \Omega_\mu + \sigma_{11}^{(1)} \Omega_p e^{-i\Delta_3 t} + \sigma_{11}^{(2)} \Omega_p^* e^{i\Delta_3 t}. \quad (4.3.1.2)$$

But, as the population term is a real quantity, the $\sigma_{11}^{(0)}$ should be real. Also

$$\sigma_{11}^{(1)} = \sigma_{11}^{(2)*} \quad (4.3.1.3)$$

as their associated terms are already conjugate to each other. Rewriting Eq. (4.3.1.2) as

$$\begin{aligned} \sigma_{11} &= \sigma_{11}^{(0)} \Omega_c \Omega_\mu + |\sigma_{11}^{(1)}| |\Omega_p| e^{-i(\Delta_3 t + \phi_1 + \phi_2)} + |\sigma_{11}^{(1)}| |\Omega_p| e^{i(\Delta_3 t + \phi_1 + \phi_2)} \\ &= \sigma_{11}^{(0)} \Omega_c \Omega_\mu + 2|\sigma_{11}^{(1)}| |\Omega_p| \cos(\Delta_3 t + \phi_1 + \phi_2) \end{aligned} \quad (4.3.1.4)$$

where, $\phi_1, |\sigma_{11}^{(1)}|$ and $\phi_2, |\Omega_p|$ are the phase and magnitude of $\sigma_{11}^{(1)}$ and Ω_p respectively.

From Eq. (4.3.1.4), we can see the ground state population oscillates around a steady state value with a frequency equal to the three-photon detuning. This is called coherent population oscillation(CPO), and the same can be derived for the other two levels.

4.3.2 Derivation of propagation equation

The growth of the SWM (A_6) and probe (A_p) amplitudes, as it propagates through the vapor cell, can be analyzed by using wave equation which is shown in Eq. (4.3.2.1).

$$\frac{\partial^2 E(z, t)}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E(z, t)}{\partial t^2} + \mu_0 \frac{\partial^2 P(z, t)}{\partial t^2} \quad (4.3.2.1)$$

where $E(z, t)$ is the electric field propagating in the medium and $P(z, t)$ is the nonlinear source term/polarization of the medium. We represent the source term in Eq. (4.3.2.1) as

$$P(z, t) = P_p(z, t) + P_6(z, t), \quad (4.3.2.2)$$

where

$$P_p(z, t) = \epsilon_0 \chi^{(1)}(\omega + \delta) E_p(z, t) + \epsilon_0 \chi^{(5)}(\omega - \delta) E_c^2 E_\mu^2 E_6^*, \quad (4.3.2.3)$$

and

$$P_6(z, t) = \epsilon_0 \chi^{(1)}(\omega - \delta) E_6(z, t) + \epsilon_0 \chi^{(5)}(\omega + \delta) E_c^2 E_\mu^2 E_p^*. \quad (4.3.2.4)$$

The $P_p(z, t)$ and $P_6(z, t)$ are the nonlinear source terms for the seed probe and six wave mixing process, respectively. E_c, E_μ, E_p and E_6 are the electric field of coupling, microwave, seed probe, and six-wave respectively. Here, $\delta_c = \delta_\mu = 0$ and $\delta_p = \delta$ is the three photon detuning. We have used $\omega_{31} = \omega$ interchangeably.

If the nonlinear source term is not too large (which is indeed the case in our experiment), the solution to Eq. (4.3.2.1) can take the form as described in Eq. (4.3.2.5)

$$\begin{aligned} E_p(z, t) &= A_p(z) e^{i(k(\delta)z - \omega_p t)} \\ E_6(z, t) &= A_6(z) e^{i(k(-\delta)z - \omega_6 t)} \\ E_c(z, t) &= A_c e^{i(k_c z - \omega_c t)} \\ E_\mu(z, t) &= A_\mu e^{i(k_\mu z - \omega_\mu t)} \end{aligned} \quad (4.3.2.5)$$

where A_i , where $i \in (p, 6, c, \mu)$ are the field amplitudes of the respective electric fields. $k(\delta), k(-\delta), k_c$ and k_μ are the propagation vectors for probe, six wave, coupling and microwave field respectively. δ is the three photon detuning. Here $A_p(z)$ and $A_6(z)$ are the slowly varying functions of z .

We now substitute Eq. (4.3.2.5) in Eq. (4.3.2.3). After using Eq. 4.3.0.6, we then obtain

$$\begin{aligned} P_p(z, t) &= \epsilon_0 \chi^{(1)}(\omega + \delta) A_p(z) e^{i(k(\delta)z - \omega_p t)} \\ &\quad + \epsilon_0 \chi^{(5)}(\omega - \delta) A_c^2 A_\mu^2 A_6^*(z) e^{i(2k_c z + 2k_\mu z - k(-\delta)z)} e^{-i\omega_p t} \end{aligned} \quad (4.3.2.6)$$

where $\omega_p = \omega_{31} + \delta$.

As required by the nonlinear wave Eq. (4.3.2.1), the time and spatial derivatives of the required quantity is given as follows:

$$\begin{aligned} \frac{\partial^2 P_p(z, t)}{\partial t^2} &= -\omega_p^2 \epsilon_0 \chi^{(1)}(\omega + \delta) A_p(z) e^{i(k(\delta)z - \omega_p t)} \\ &\quad - \omega_p^2 \epsilon_0 \chi^{(5)}(\omega - \delta) A_c^2 A_\mu^2 A_6^*(z) e^{i(2k_c z + 2k_\mu z - k(-\delta)z)} e^{-i\omega_p t} \end{aligned} \quad (4.3.2.7)$$

similarly,

$$\frac{\partial^2 E_p(z, t)}{\partial t^2} = -\omega_p^2 A_p(z) e^{i(k(\delta)z - \omega_p t)} \quad (4.3.2.8)$$

and

$$\begin{aligned} \frac{\partial^2 E_p(z, t)}{\partial z^2} &= A_p''(z) e^{i(k(\delta)z - \omega_p t)} + 2ik(\delta) A_p'(z) e^{i(k(\delta)z - \omega_p t)} \\ &\quad - k(\delta)^2 A_p(z) e^{i(k(\delta)z - \omega_p t)} \end{aligned} \quad (4.3.2.9)$$

We substitute Eq. (4.3.2.7 - 4.3.2.9) in Eq. (4.3.2.1). we get

$$\begin{aligned} &(A_p''(z) + 2ik(\delta) A_p'(z) - k(\delta)^2 A_p(z)) e^{i(k(\delta)z - \omega_p t)} \\ &= \frac{-\omega_p^2 A_p(z) e^{i(k(\delta)z - \omega_p t)}}{c^2} + \mu_0 (-\omega_p^2 \epsilon_0 \chi^{(1)} (\omega + \delta) A_p(z) e^{i(k(\delta)z - \omega_p t)} \\ &\quad - \omega_p^2 \epsilon_0 \chi^{(5)} (\omega - \delta) A_c^2 A_\mu^2 A_6^*(z) e^{i(2k_c z + 2k_\mu z - k(-\delta)z) e^{-i\omega_p t}}) \end{aligned} \quad (4.3.2.10)$$

now cancelling $e^{-i\omega_p t}$ from both the sides, the Eq. (4.3.2.10) becomes

$$\begin{aligned} A_p''(z) + 2ik(\delta) A_p'(z) - k(\delta)^2 A_p(z) &= \frac{-\omega_p^2 A_p(z)}{c^2} + \mu_0 (-\omega_p^2 \epsilon_0 \chi^{(1)} (\omega + \delta) A_p(z) \\ &\quad - \omega_p^2 \epsilon_0 \chi^{(5)} (\omega - \delta) A_c^2 A_\mu^2 A_6^*(z) e^{i(2k_c z + 2k_\mu z - k(-\delta)z - k(\delta)z)}) \end{aligned} \quad (4.3.2.11)$$

It is quite acceptable to drop the first term on the left-hand side of Eq. (4.3.2.11) as it is very small compared to the second term. This approximation is called slowly varying amplitude approximation and is valid when

$$\left| \frac{\partial^2 A_p(z)}{\partial z^2} \right| \ll \left| k(\delta) \frac{\partial A_p(z)}{\partial z} \right| \quad (4.3.2.12)$$

After approximating and using

$$\frac{\omega_p^2 (1 + \chi^{(1)} (\omega + \delta))}{c^2} = k^2 (\omega + \delta) = k^2 (\delta) \quad (4.3.2.13)$$

the Eq. (4.3.2.11) becomes

$$\frac{\partial A_p(z)}{\partial z} = \frac{i\omega_p^2 \chi^{(5)}(\omega - \delta) A_c^2 A_\mu^2}{2c^2 k(\delta)} A_6^* e^{i\Delta k z} \quad (4.3.2.14)$$

where

$$\Delta k = 2(k_c + k_\mu) - k(-\delta) - k(\delta) \quad (4.3.2.15)$$

is called the phase mismatch.

In a similar way, we can derive the wave equation for the source term $P_6(z, t)$. After substituting Eq. (4.3.2.5) in Eq. (4.3.2.4), we obtain

$$\begin{aligned} P_6(z, t) = & \epsilon_0 \chi^{(1)}(\omega - \delta) A_6(z) e^{i(k(-\delta)z - \omega_6 t)} \\ & + \epsilon_0 \chi^{(5)}(\omega + \delta) A_c^2 A_\mu^2 A_p^*(z) e^{i(2k_c z + 2k_\mu z - k(\delta)z)} e^{-i\omega_6 t} \end{aligned} \quad (4.3.2.16)$$

where $\omega_6 = \omega_{31} - \delta$.

Now taking the double time derivative of the above equation, we get

$$\begin{aligned} \frac{\partial^2 P_6(z, t)}{\partial t^2} = & -\omega_6^2 \epsilon_0 \chi^{(1)}(\omega - \delta) A_6(z) e^{i(k(-\delta)z - \omega_6 t)} \\ & -\omega_6^2 \epsilon_0 \chi^{(5)}(\omega + \delta) A_c^2 A_\mu^2 A_p^*(z) e^{i(2k_c z + 2k_\mu z - k(\delta)z)} e^{-i\omega_6 t} \end{aligned} \quad (4.3.2.17)$$

similarly,

$$\frac{\partial^2 E_6(z, t)}{\partial t^2} = -\omega_6^2 A_6(z) e^{i(k(-\delta)z - \omega_6 t)} \quad (4.3.2.18)$$

and

$$\begin{aligned} \frac{\partial^2 E_6(z, t)}{\partial z^2} = & A_6''(z) e^{i(k(-\delta)z - \omega_6 t)} + 2ik(\delta) A_6'(z) e^{i(k(-\delta)z - \omega_6 t)} \\ & - k(-\delta)^2 A_6(z) e^{i(k(-\delta)z - \omega_6 t)} \end{aligned} \quad (4.3.2.19)$$

Putting Eq. (4.3.2.17 - 4.3.2.19) in Maxwell's Eq. (4.3.2.1), we get

$$\begin{aligned}
& A_6''(z)e^{i(k(-\delta)z-\omega_6t)} + 2ik(-\delta)A_6'e^{i(k(-\delta)z-\omega_6t)} - k(-\delta)^2A_6(z)e^{i(k(-\delta)z-\omega_6t)} \\
& = -\frac{\omega_6^2A_6(z)e^{i(k(-\delta)z-\omega_6t)}}{c^2} + \mu_0(-\omega_6^2\epsilon_0\chi^{(1)}(\omega-\delta)A_6(z)e^{i(k(-\delta)z-\omega_6t)} \\
& - \omega_6^2\epsilon_0\chi^{(5)}(\omega+\delta)A_c^2A_\mu^2A_p^*(z)e^{i(2k_cz+2k_\mu z-k(\delta)z)}e^{-i\omega_6t}
\end{aligned} \tag{4.3.2.20}$$

now cancelling $e^{-i\omega_6t}$ from both the sides, the equation becomes

$$\begin{aligned}
A_6''(z) + 2ik(-\delta)A_6' - k(-\delta)^2A_6(z) & = -\frac{\omega_6^2A_6(z)}{c^2} + \mu_0(-\omega_6^2\epsilon_0\chi^{(1)}(\omega-\delta)A_6(z) \\
& - \omega_6^2\epsilon_0\chi^{(5)}(\omega+\delta)A_c^2A_\mu^2A_p^*(z)e^{i(2k_cz+2k_\mu z-k(\delta)z-k(-\delta)z)})
\end{aligned} \tag{4.3.2.21}$$

By invoking SVAA(Eq. (4.3.2.12)) for $A_6(z)$, the $A_6''(z)$ term can be ignored and using

$$\frac{\omega_6^2(1+\chi^{(1)}(\omega-\delta))}{c^2} = k^2(-\delta) = k^2(\omega-\delta) \tag{4.3.2.22}$$

Eq. (4.3.2.41) can be written as

$$\begin{aligned}
\frac{\partial A_6(Z)}{\partial z} & = \frac{i\omega_6^2\chi^{(5)}(\omega+\delta)A_c^2A_\mu^2}{2c^2k(-\delta)}A_p^*e^{i\Delta kz} \\
& = \frac{i(\omega-\delta)^2\chi^{(5)}(\omega+\delta)A_c^2A_\mu^2}{2c^2k(-\delta)}A_p^*e^{i\Delta kz}
\end{aligned} \tag{4.3.2.23}$$

where Δk is defined as in Eq. (4.3.2.15).

The Eq. (4.3.2.14) and Eq. (4.3.2.23) in the phase matched($\Delta k = 0$) condition can be simplified as

$$\frac{\partial A_p}{\partial z} = \Theta(-\delta)A_6^* \tag{4.3.2.24}$$

$$\frac{\partial A_6}{\partial z} = \Theta(\delta)A_p^* \tag{4.3.2.25}$$

where,

$$\Theta(\delta) = \frac{i(\omega_{31} - \delta)^2 \chi_p^{(5)}(\delta) A_c^2 A_\mu^2}{2k_p(-\delta)c^2} \quad (4.3.2.26)$$

We differentiate Eq. (4.3.2.24) and Eq. (4.3.2.25) with respect to z and placed the complex conjugate of Eq. (4.3.2.25) and Eq. (4.3.2.24) respectively to eliminate $\frac{\partial A_6^*}{\partial z}$ and $\frac{\partial A_p^*}{\partial z}$ from the right-hand side. We then obtain

$$\frac{\partial^2 A_p}{\partial z^2} = \alpha^2 A_p \quad (4.3.2.27)$$

$$\frac{\partial^2 A_6}{\partial z^2} = \alpha^{*2} A_6 \quad (4.3.2.28)$$

Where,

$$\alpha^2 = \Theta(-\delta)\Theta^*(\delta) \quad (4.3.2.29)$$

Now, the solutions of the above differential Eqs.(4.3.2.27, 4.3.2.28) in the undepleted approximation of the coupling (A_c) and microwave (A_μ) fields is given by

$$A_p(z) = ue^{\alpha z} + ve^{-\alpha z} \quad (4.3.2.30)$$

$$A_6(z) = xe^{\alpha^* z} + ye^{-\alpha^* z} \quad (4.3.2.31)$$

where u, v, x and y are the constants to be identified using boundary conditions. Using boundary conditions as

$$A_p(z=0) = A_p(0) \quad (4.3.2.32)$$

$$\text{and, } A_6(z=0) = 0 \quad (4.3.2.33)$$

We get

$$A_p(0) = u + v \quad (4.3.2.34)$$

$$0 = x + y \quad (4.3.2.35)$$

Also from Eq. (4.3.2.30)

$$\frac{\partial A_p}{\partial z} = u\alpha e^{\alpha z} - v\alpha e^{-\alpha z} \quad (4.3.2.36)$$

Substituting Eq. (4.3.2.36) and Eq. (4.3.2.31) in Eq. (4.3.2.24) and using boundary conditions, we obtain

$$u\alpha e^{\alpha z} - v\alpha e^{-\alpha z} = \Theta(-\delta)x^*(e^{\alpha z} - e^{-\alpha z}) \quad (4.3.2.37)$$

After solving the above equation, we get $u = v = \frac{A_p(0)}{2}$ and $x^* = \frac{A_p(0)\alpha}{2\Theta(-\delta)}$.

But $\delta \ll \omega_{13}$ as frequency of ω_{13} is in THz as compared to that of δ which is in kHz. So, $\Theta(\delta)$ can be written as

$$\Theta(\delta) \approx \frac{i\omega_{13}^2 \chi_p^{(5)}(\delta) A_c^2 A_\mu^2}{2k_p(-\delta)c^2} \quad (4.3.2.38)$$

From Kramers-Kronig relation $\chi^{(5)*}(\delta) = -\chi^{(5)}(-\delta)$. Also, it is assumed that $k_p(\delta)$ is real and is equal to $k_p(-\delta)$ as $|\chi_p^{(1)}|$ is very small.

Using above conditions $\Theta^*(\delta)$ can be written as $\Theta^*(\delta) = \Theta(-\delta)$. Now α can be simplified as

$$\alpha = \Theta(-\delta) = \Theta^*(\delta) \quad (4.3.2.39)$$

The solutions of the coupled differential Eq. (4.3.2.24) and Eq. (4.3.2.25) can be obtained after substituting Eq. (4.3.2.39) in the Eq. (4.3.2.30) which are

$$A_p = A_p(0) \cosh(\Theta(-\delta)z) \quad (4.3.2.40)$$

$$A_6 = A_p^*(0) \sinh(\Theta(\delta)z) \quad (4.3.2.41)$$

respectively.

In contrast to the exponential growth of the SWM with distance (Eq. (4.3.2.41)), the

TWM grows linearly with the above approximations as [19],

$$A_3 = \frac{i\omega_{13}^2 \chi_p^{(2)} z A_c A_\mu}{2k_p c^2}. \quad (4.3.2.42)$$

The exponential growth of A_6 with sample length justifies its efficient generation, despite involving two magnetic-dipole transitions. Physically, the exponential growth can be illustrated by two factors. First, the generated A_6 can reinforce A_p by acting as a seed probe in yet another SWM process. The reinforced A_p , in turn, builds up A_6 at an increased rate than previously. This results in parametric coupling between the input probe field (A_p) and the SWM signal (A_6), represented by Eq. (4.3.2.24) and Eq. (4.3.2.25). Second, the generation of the SWM signal when it acts as a seed to generate the probe field also contributes to the exponential growth. The absence of these parametric interactions in the TWM process leads to its linear growth. Nevertheless, the simultaneous presence of EIT and CPO greatly enhances the TWM and SWM process by reducing the linear absorption and enhancing the non-linear susceptibilities.

4.4 Results and discussions

Several features in this work are different from previous observations. Firstly, the TWM and SWM processes, co-propagating with the input fields, are easily distinguished in the frequency spectrum by breaking the three-photon resonance. This eliminates the need to align the input optical beams in a specific spatial geometry to separate out the nonlinearly generated beams [11, 12]. Additionally, the co-propagation of the input optical beams, which are very close in wavelength, also ensures a simultaneous natural wave-vector phase matching for both the nonlinear processes.

Secondly, the simultaneous generation of the TWM and SWM signals opens up the possibility to convert quantum information from microwave to optical in distinct frequency channels simultaneously. Together, the TWM and SWM channels can simultaneously perform phase-dependent [19], phase-sensitive and phase-insensitive amplification of the probe signal. Thus, the simultaneous presence of these two channels can be used to realize a microwave-controlled optical amplifier operating in different regimes of linear amplification [21]. In the following discussions, we examine the TWM and SWM process's dependence on the pump coupling field and the idler microwave field. In

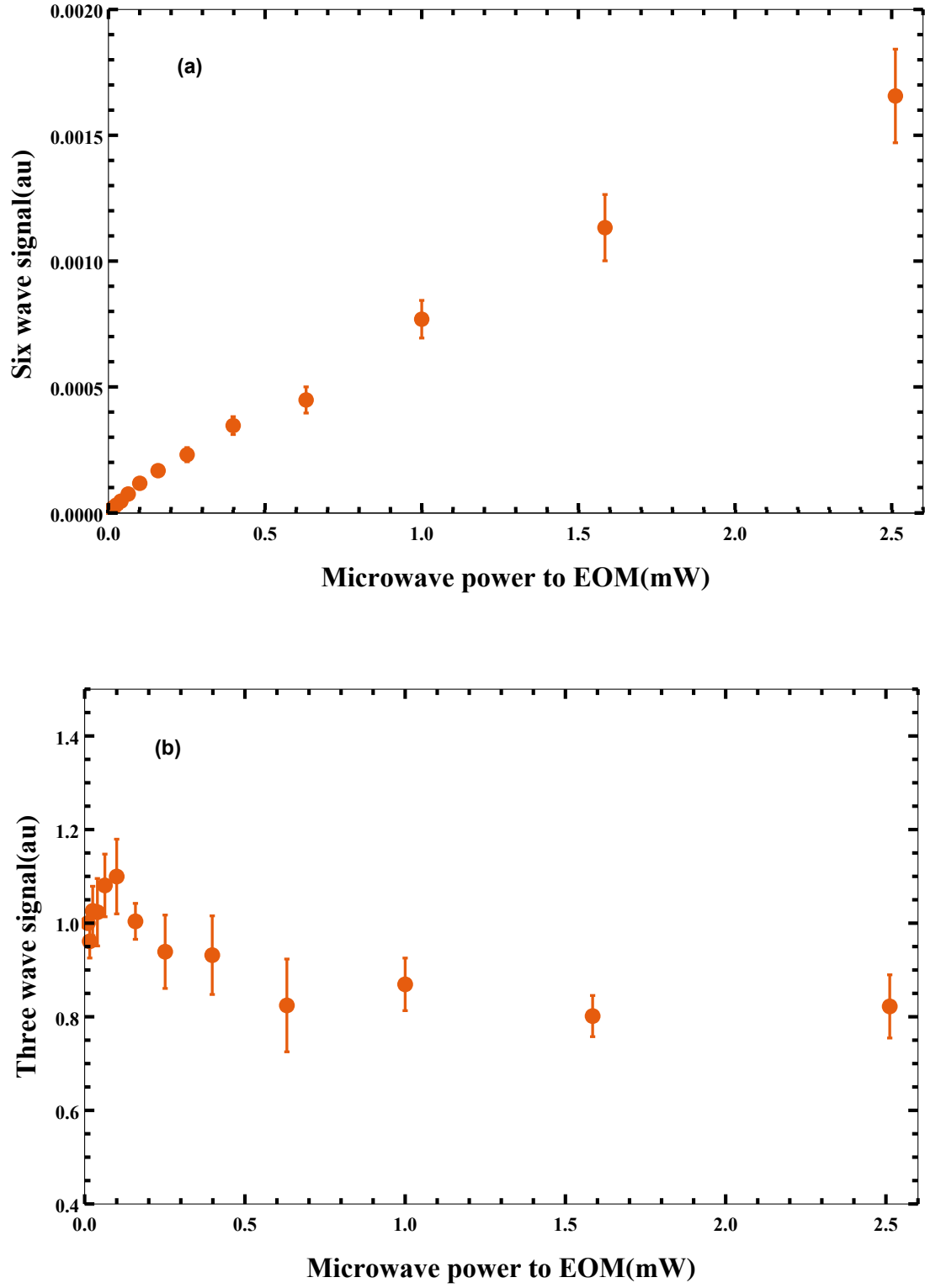


Figure 4.4: Normalized signal of (a)SWM and (b)TWM as a function of EOM source power. With increasing EOM source power the seed probe power increases. The experiment is done for a microwave power of 1 Watt and total optical pump power of $100\mu W$. Here $\delta_3 = 200\text{kHz}$. All data are normalized with respect to the first data point of the generated three-wave signal (TWM).

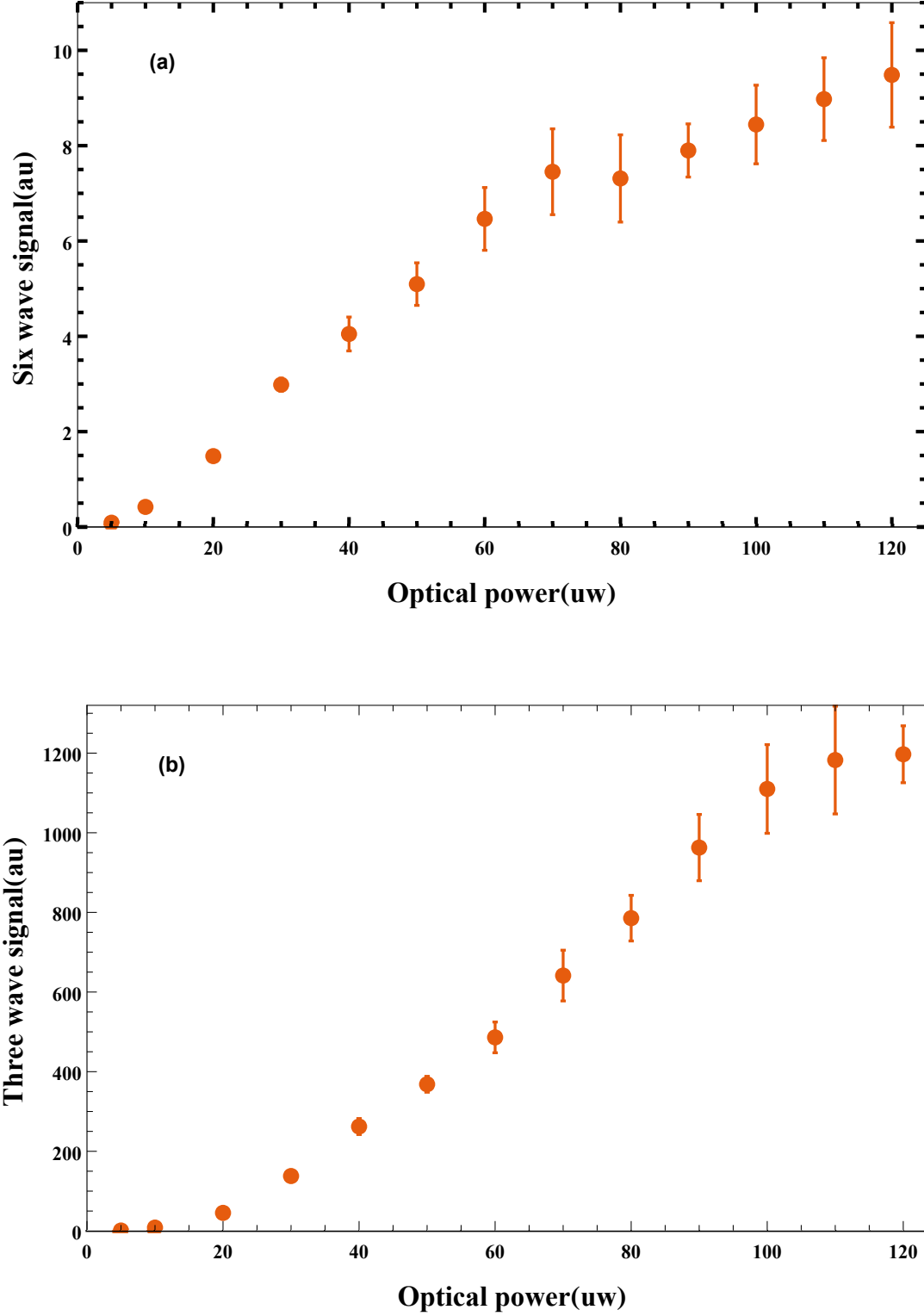


Figure 4.5: Normalized signal (a)SWM and (b)TWM as a function of total optical power entering in to the microwave cavity. Here $\delta_3 = 200\text{kHz}$ and microwave power is 1 watt. All data are normalized with respect to the first data point of the generated three wave signal (TWM).

Fig. 4.4(a), we observe SWM signal power's linear dependence on the input probe power, a consequence of Eq. (4.3.2.41). The probe power in the experiment never reaches the power required to saturate the transition resulting in the absence of saturation region in

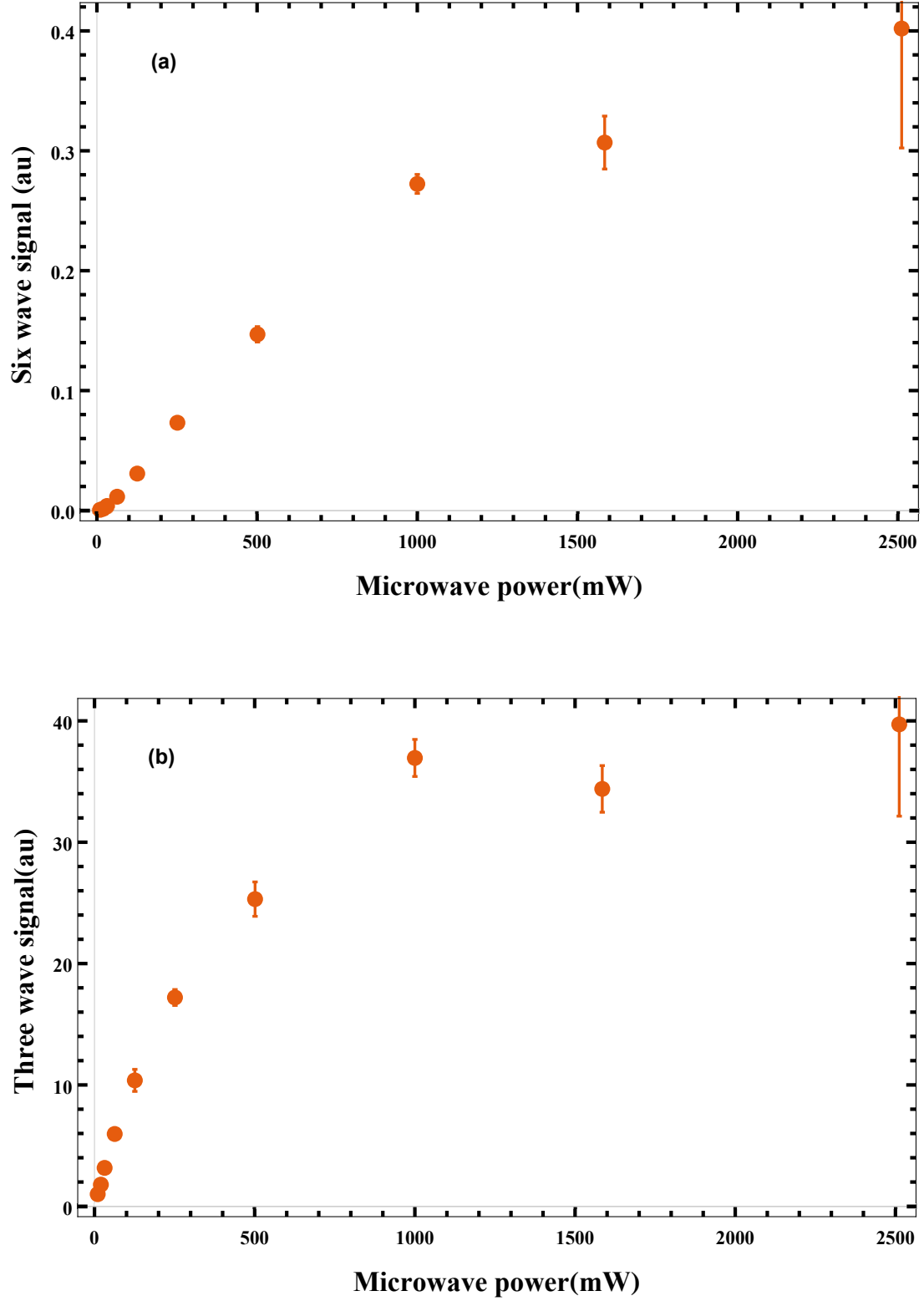


Figure 4.6: Normalized signal of (a)SWM and (b)TWM as a function of input microwave power. Here $\delta_3 = 200\text{kHz}$ and optical power is $100 \mu\text{W}$. All data are normalized with respect to the first data point of the generated three-wave signal (TWM).

Fig. 4.4. In Fig. 4.4(b) we observe a slight decrease in the amplitude of TWM as a function of probe power. Strictly speaking, from Eq. (4.3.2.42), TWM should be independent

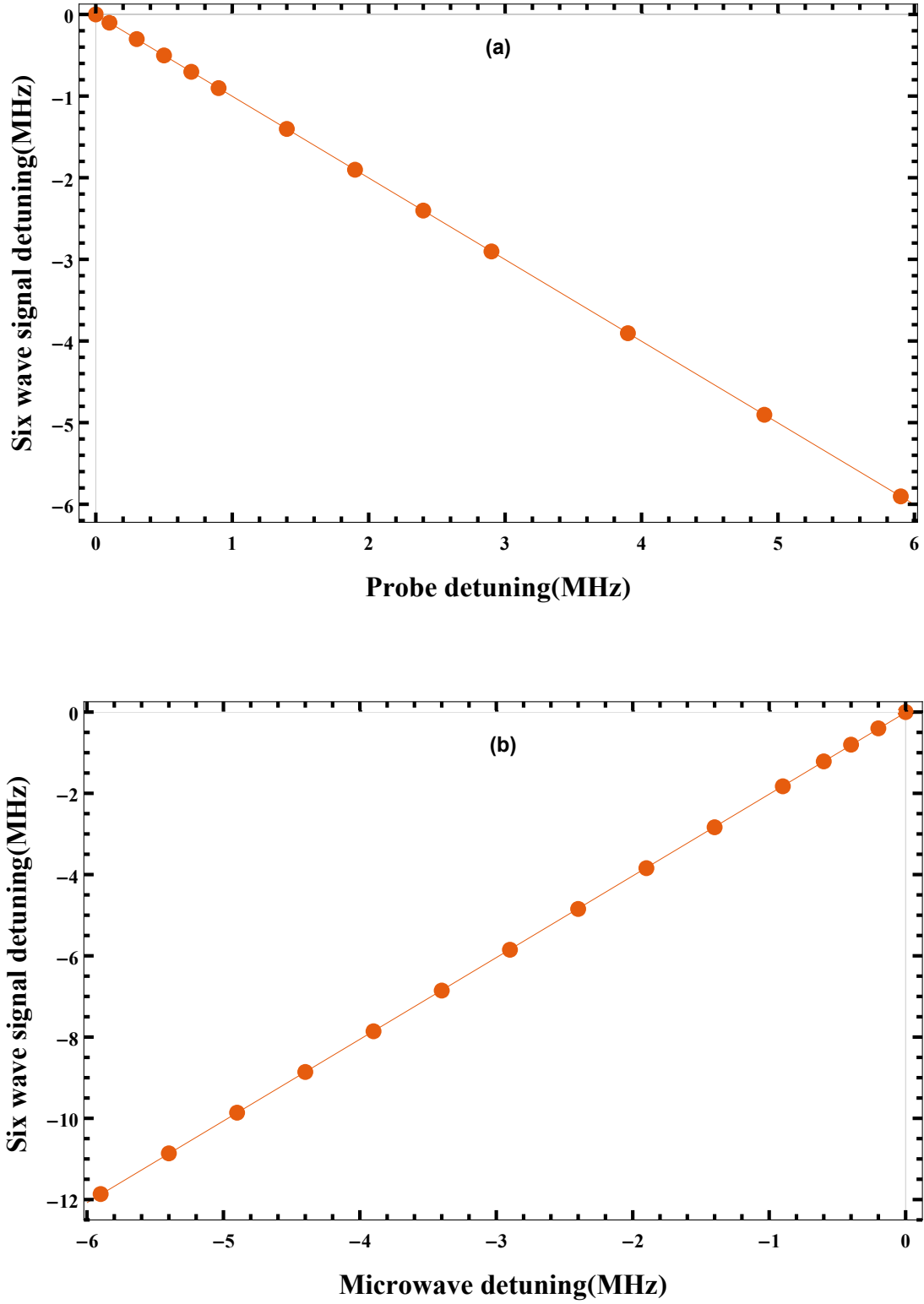
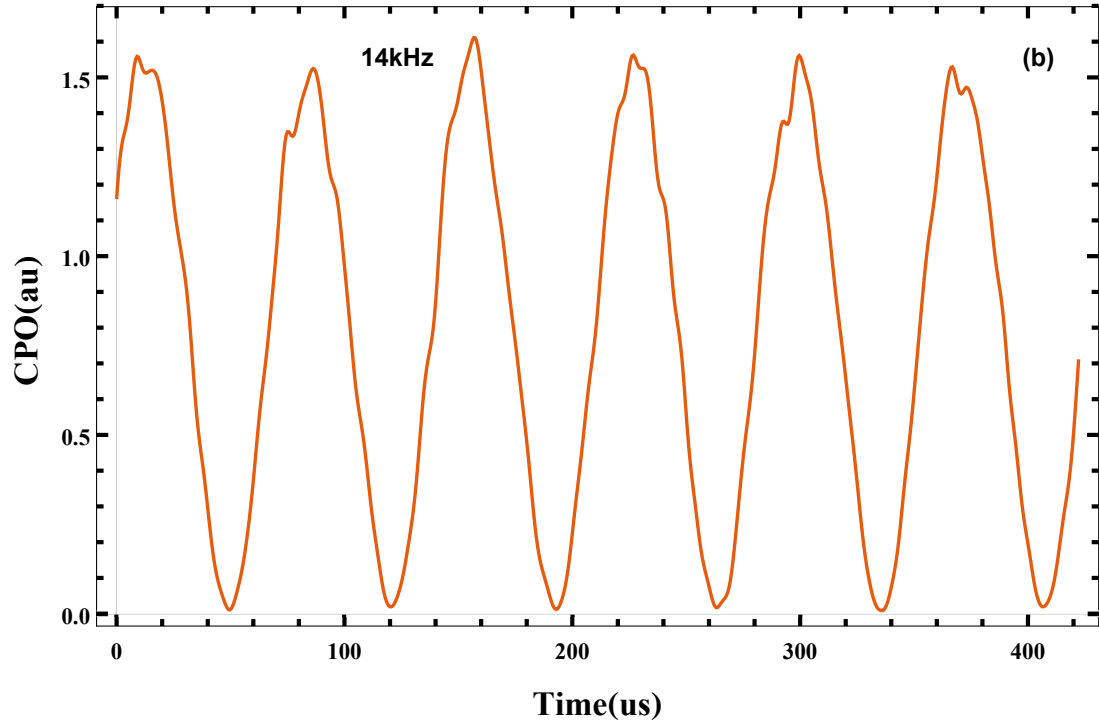
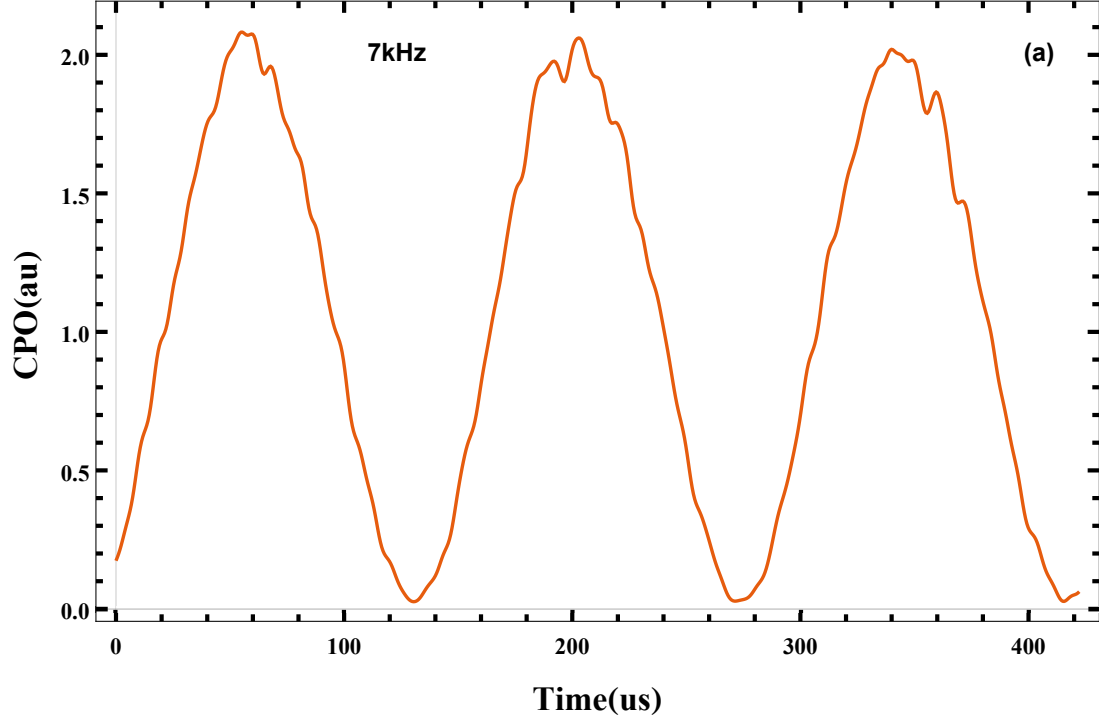


Figure 4.7: Spectral detuning of SWM signal as a function of (a) seed probe field detuning(δ_p) (b) input microwave field detuning(δ_μ). Here total optical power in to the cavity is $200\mu\text{W}$ and microwave power is 1 Watt. The values and signs of the linear dependence in both these plots are observed to be the same as predicted by Eq. (4.3.0.3)

of the probe power as it requires only the coupling and microwave fields for its generation. But, since, the probe field is derived from the coupling field through an EOM, an increase



in probe power would result in a decrease in coupling power which in turn decreases the TWM signal as seen in Fig. 4.4(b).

In Figs. (4.5,4.6), we have plotted the dependence of the TWM and SWM processes on the powers of the pump coupling field and idler microwave field respectively. Both these pairs of plots have an initial linear growth of the TWM and SWM signal powers,

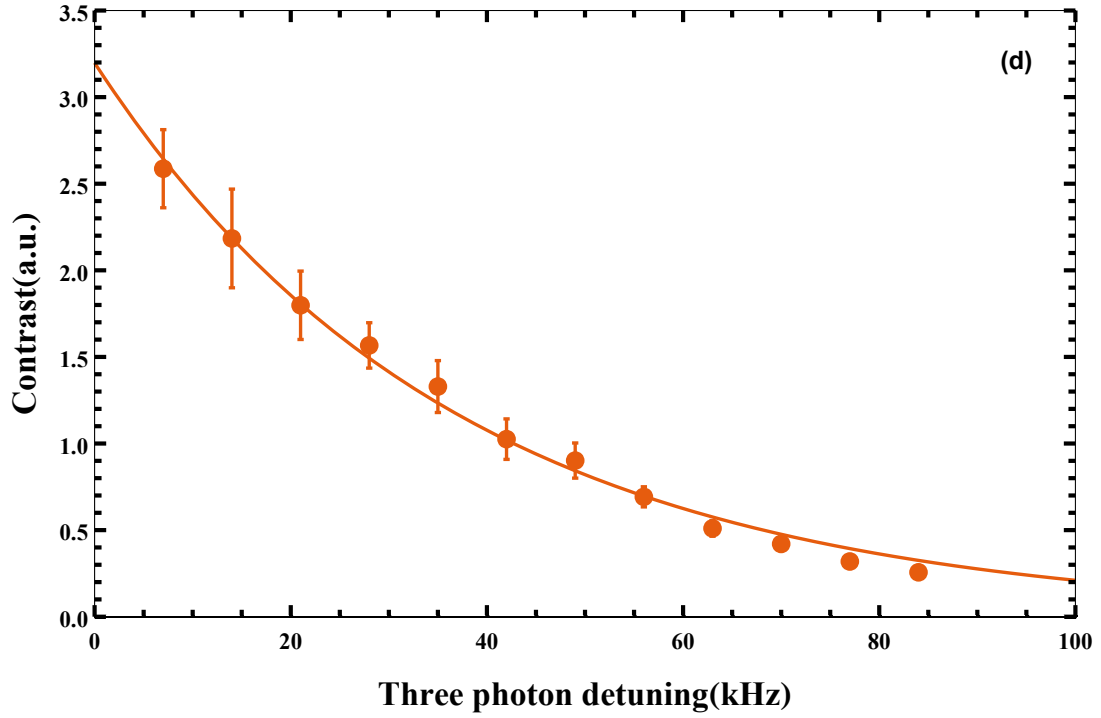
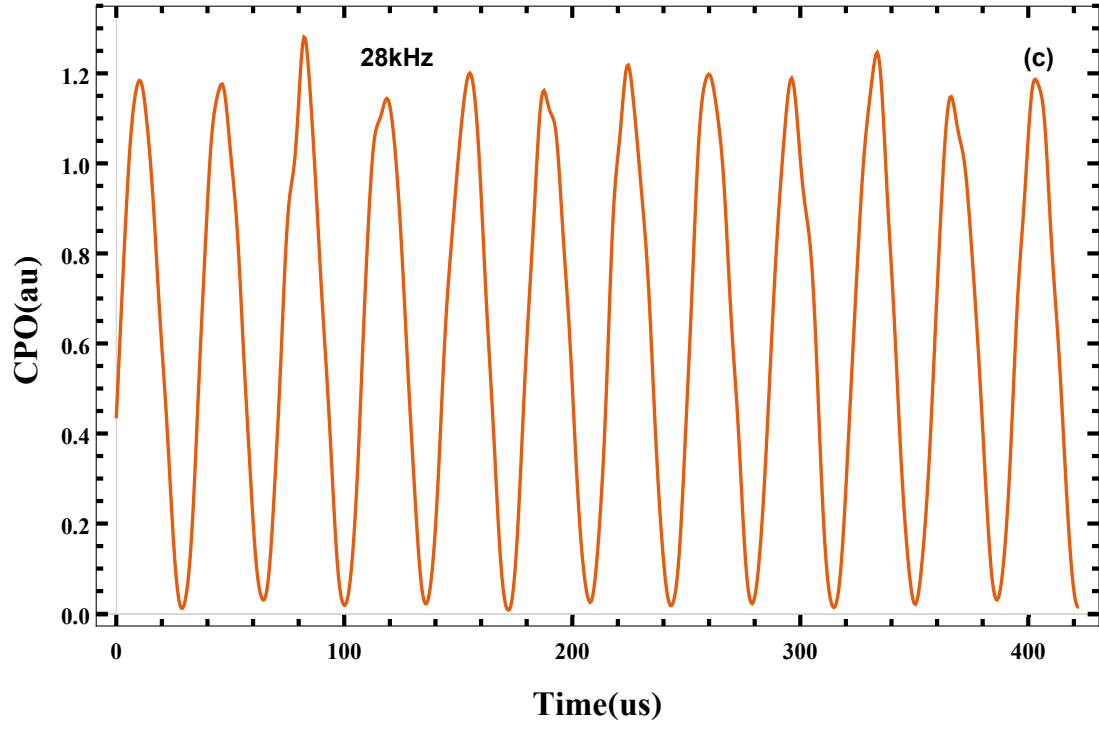


Figure 4.8: CPO oscillations observed in the probe field as a function of time for δ_3 values of (a) 7 kHz (b) 14 kHz and (c) 28 kHz. The sub-plot(d) shows the CPO oscillation contrast as a function of δ_3 . The solid line is a fit to the experimental data points. The contrast is defined as the difference between the maximum and minimum value in the CPO time domain plot. Here total optical power going in to the cavity is $200\mu\text{W}$ and microwave power to the cavity is 1 Watt.

with an increase in pump coupling power and idler microwave power. We also see in both these plots that the signal optical probe generation saturates beyond the threshold power of the pump coupling field and the microwave idler field.

In Fig. 4.7, we observe the SWM signal's perfect linear dependence on probe detuning and microwave detuning with slopes -1 and 2, respectively. This is a consequence of Eq. (4.3.0.3) and it is a signature of coherent generation due to the underlying nonlinear conversion process.

As discussed earlier, the SWM signal generation can be understood as resulting from modulation of the TWM process whenever the three-photon condition is broken. In the absence of three-photon resonance, these modulations result in population modulations in all three levels in the Δ system resulting in CPO. In our experiment, we observe these CPO oscillations, in the time domain of our spectrum analyzer, in populations of the level connected by the probe laser. As predicted from our theoretical model, the frequency of the CPO is equal to the three-photon detuning, as shown in Figs. 4.8(a,b,c). In Fig. 4.8(d), we observe CPO's dependence on three-photon detuning. The contrast of the CPO decreases with increasing three-photon detuning, an indication that the populations are unable to follow the increasing beat frequency.

4.5 Conclusions.

In this work we experimentally observe for the first time, the simultaneous production of a three-wave (TWM) and a six-wave (SWM) mixing signal, from a room temperature collection of ^{85}Rb atoms. Both the TWM and SWM signals generate an optical signal field from a microwave idler field and thus act as a dual-frequency transducer connecting microwave and optical frequencies. The very few number of interacting beams and the simplicity of their geometrical arrangement required to generate these signals make our system a robust platform for microwave-to-optical conversion. Thus our system, which is a traditional quantum memory system [22], can also act as a wavelength division multiplexer (WDM) distributing quantum information in the microwave domain to several frequencies in the optical domain. In addition, we have given a theoretical model for the SWM generation in terms of coherent population oscillations (CPO). These oscillations are experimentally observed in real-time validating the theoretical model. As a future

application, we can show through our theoretical model that the generation of TWM, SWM and probe signal fields make our system an amplifier that can potentially show PDA, PSA [21] and PIA types of amplification in a single physical system.

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Chapter 5

Summary and future work

5.1 Summary

The problems discussed in this thesis concern the creation of a neutral atom-based platform that not only up-converts microwave signal into optical signal but also acts as an antenna to send and receive information signal. We have chosen three hyperfine atomic level strategies in the D1 manifold of ^{85}Rb atoms at room temperature that are connected with microwave and optical fields to accomplish this. With the help of this approach, the atomic system becomes a microwave-to-optical transducer, a radio-over-fibre device with atoms as an antenna. Additionally, this atomic Δ system has the ability to delay or advance an optical pulse, the latency of which can be controlled by the intensity of a microwave field.

The first experimental work in this thesis reports a dilute sample of ^{85}Rb atoms that can be used as an antenna for signal communication. The connection of two optical fields along with one microwave field with the appropriate atomic energy levels makes the system closed. This cyclic closed interaction enables the system phase-dependent and as a result, the intensity of the generated optical field can be controlled by the phase of the microwave field. The information is encoded in the phase of the microwave field, which then up-converts to an optical field by the process of three-wave mixing through hybrid second-order nonlinearity. The demodulation of information happens by detecting the generated optical field with a photodetector. We explored both BPSK and C-4PSK protocols for the communication of information signals. We have measured the correlation coefficient between the demodulated data with an ideal template and established a viable

symbol bandwidth up to 1 MHz. We have also shown that communication can happen faithfully even by detuning the microwave field up to 15 MHz, which shows the tunability of the carrier wave. This phase-dependent atomic platform has inherent features of signal demodulation, radio-to-optical conversion, and amplification. The ground states used in the scheme are spin states, which makes our scheme a strong candidate for applications involving communication and memory states of quantum signals.

In the second chapter we have explored a theoretical model that explains the nonlinear interaction between a CW laser field with a microwave pulse in the presence of dilute atomic vapor. By using the hybrid second-order nonlinearity possessed by the atom, the microwave pulse gets up-converted to an optical pulse through sum frequency generation in the presence of a coupling optical field. The prime result of this study is, the absolute phase of the nonlinearity can be controlled by the intensity of the microwave field or the coupling field. This feature gives a handle to control the delay or advancement of the generated optical pulse inside the atomic medium. Moreover, this second-order nonlinearity-assisted delay is atom number density-independent. As the optical pulse is generated through a sum frequency process, it can be made on-demand by switching any one of the input pump fields (coupling optical field or microwave pulse). All These features make the system a platform for a pulse on demand whose arrival time can be tuned.

In the next study, we have both experimentally and theoretically investigated the simultaneous generation of both TWM and SWM processes in the same atomic system. The breaking of three-photon resonance plays a major role in order to separate and observe both the generation process distinctly. It also aids in CPO in all three energy levels. The generation of two optical fields from a microwave field makes the system an atomic multiplexer as it opens dual channels for microwave-to-optical conversion.

5.2 Future work

Various future possibilities can be made feasible by the work provided in this thesis. This section addresses a few of them.

The implementation of BPSK and C-4PSK is shown in Chapter 2 using room temperature atoms. BPSK uses one bit, whereas C-4PSK uses 2 bits to encode data. By using better contrast in the optical intensity, one can use 3 bits or higher to encode information.

This will result in a higher data rate. Also, by quantifying the number of thermal photons that get up-converted to optical photons in this room-temperature-based radio-over-fiber device, the noise figure can be measured, which will decide the performance of this atomic receiver.

The atomic Δ system discussed in this thesis can be used as a bidirectional converter. By using the optical probe and coupling field, the microwave field at their difference frequency can be generated. This will help in manipulating optically transmitted data at microwave frequencies after down-conversion. A similar approach of microwave emission [1] from the atomic ground states is also reported.

The three-level atomic system addressed in the thesis can be a platform for phase-dependent amplification [2], phase-insensitive amplification, and phase-sensitive amplification processes. The presence of another optical probe field exactly at the generated six-wave signal frequency will create phase-sensitive amplification [3] of the probe field. Since the ground states in this atomic system are spin states, quantum information can be stored in them.

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Chapter 6

Appendix

By using the perturbative expansion given by Eq. (3.4.0.4), we can expand all the density matrix elements as given below. By looking at the coefficients of $\Omega_c\Omega_\mu$ (three-wave mixing term) and Ω_p (linear propagation term) we can infer that our quantity of interest are $\sigma_{31}^{(0)}$ and $\sigma_{31}^{(1)}$ as they are proportional to $\chi_p^{(2)}$ and $\chi_p^{(1)}$. The $\sigma_{31}^{(2)}$ represents a higher order wave mixing term. we can also see this by back converting σ matrix into ρ matrix and comparing it with Eq. (3.4.0.2). We have eliminated the ϕ_{rel} term in the equations as it will not play any role in solving them.

$$\sigma_{33} = \sigma_{33}^{(0)} \Omega_c \Omega_\mu + \sigma_{33}^{(1)} \Omega_p e^{-i\Delta_3 t} + \sigma_{33}^{(2)} \Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.1a)$$

$$\sigma_{22} = \sigma_{22}^{(0)} \Omega_c \Omega_\mu + \sigma_{22}^{(1)} \Omega_p e^{-i\Delta_3 t} + \sigma_{22}^{(2)} \Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.1b)$$

$$\sigma_{32} = \sigma_{32}^{(0)} \Omega_c \Omega_\mu + \sigma_{32}^{(1)} \Omega_p e^{-i\Delta_3 t} + \sigma_{32}^{(2)} \Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.1c)$$

$$\sigma_{31} = \sigma_{31}^{(0)} \Omega_c \Omega_\mu + \sigma_{31}^{(1)} \Omega_p e^{-i\Delta_3 t} + \sigma_{31}^{(2)} \Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.1d)$$

$$\sigma_{21} = \sigma_{21}^{(0)} \Omega_c \Omega_\mu + \sigma_{21}^{(1)} \Omega_p e^{-i\Delta_3 t} + \sigma_{21}^{(2)} \Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.1e)$$

The five density matrix equations are

$$\dot{\sigma}_{33} = i\Omega_c\sigma_{23} + i\Omega_p e^{-i\Delta_3 t}\sigma_{13} - i\Omega_c^*\sigma_{32} - i\Omega_p^* e^{i\Delta_3 t}\sigma_{31} - 2(\gamma_{13} + \gamma_{23})\sigma_{33} \quad (6.0.0.2a)$$

$$\dot{\sigma}_{22} = i\Omega_c^*\sigma_{32} + i\Omega_\mu\sigma_{12} - i\Omega_c\sigma_{23} - i\Omega_\mu^*\sigma_{21} + 2\gamma_{23}\sigma_{33} - 2\gamma_{12}\sigma_{22} \quad (6.0.0.2b)$$

$$\dot{\sigma}_{32} = -\gamma\sigma_{32} + i\Omega_p e^{-i\Delta_3 t}\sigma_{12} + i\Omega_c(\sigma_{22} - \sigma_{33}) - i\Omega_\mu^*\sigma_{31} \quad (6.0.0.2c)$$

$$\dot{\sigma}_{31} = -\alpha\sigma_{31} + i\Omega_p e^{-i\Delta_3 t}\sigma_{11} + i\Omega_c\sigma_{21} - i\Omega_p e^{-i\Delta_3 t}\sigma_{33} - i\Omega_\mu\sigma_{32} \quad (6.0.0.2d)$$

$$\dot{\sigma}_{21} = -\beta\sigma_{21} - i\Omega_p e^{-i\Delta_3 t}\sigma_{23} + i\Omega_c^*\sigma_{31} + i\Omega_\mu\sigma_{11} - i\Omega_\mu\sigma_{22} \quad (6.0.0.2e)$$

To make it simpler, we have rewritten the Eq. (6.0.0.1) as

$$\sigma_{31} = a\Omega_c\Delta_3 + b\Omega_p e^{-i\Delta_3 t} + c\Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.3a)$$

$$\sigma_{21} = d\Omega_c\Delta_3 + n\Omega_p e^{-i\Delta_3 t} + f\Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.3b)$$

$$\sigma_{32} = h\Omega_c\Delta_3 + I\Omega_p e^{-i\Delta_3 t} + x\Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.3c)$$

$$\sigma_{33} = j\Omega_c\Delta_3 + k\Omega_p e^{-i\Delta_3 t} + y\Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.3d)$$

$$\sigma_{22} = l\Omega_c\Delta_3 + m\Omega_p e^{-i\Delta_3 t} + z\Omega_p^* e^{i\Delta_3 t} \quad (6.0.0.3e)$$

As we are looking for steady-state solutions, all these 15 variables are constant in time.

Now substituting Eq. (6.0.0.3a) in Eq. (6.0.0.2d) and eliminating Ω_p^2 terms we get

$$\begin{aligned} & ic\Delta_3\Omega_p^* e^{i\Delta_3} - ib\Delta_3\Omega_p e^{-i\Delta_3} \\ &= -a\alpha - \alpha b\Omega_p e^{-i\Delta_3} + id\Omega_c - 2ij\Omega_p e^{-i\Delta_3} \\ &\quad - il\Omega_p e^{-i\Delta_3} + in\Omega_c\Omega_p e^{-i\Delta_3} + i\Omega_p e^{-i\Delta_3} \\ &\quad - \alpha c\Omega_p^* e^{i\Delta_3} + if\Omega_c\Omega_p^* e^{i\Delta_3} + (-i)h\Omega_\mu - iI\Omega_\mu\Omega_p e^{-i\Delta_3} \\ &\quad - ix\Omega_\mu\Omega_p^* e^{i\Delta_3} \end{aligned} \quad (6.0.0.4)$$

In a similar way, we will get the other four equations for the remaining four density matrix equations.

Now, comparing the coefficients of $1, \Omega_p$, and Ω_p^* from both the side of the above Eq. (6.0.0.4), we obtain

$$0 = -a\alpha + id\Omega_c - ih\Omega_\mu \quad (6.0.0.5)$$

$$-ib\Delta_3 = -\alpha b - iI\Omega_\mu - 2ij - il + in\Omega_c + i \quad (6.0.0.6)$$

$$ic\Delta_3 = -\alpha c + if\Omega_c - ix\Omega_\mu \quad (6.0.0.7)$$

Following the steps shown above, we will get 15 such equations (three equations for each density matrix equation). Since all these 15 variables are complex quantities, it is better to split the variables into real and imaginary parts as

$$a = a_1 + ia_2 \quad (6.0.0.8a)$$

$$b = b_1 + ib_2 \quad (6.0.0.8b)$$

$$c = c_1 + ic_2 \quad (6.0.0.8c)$$

$$d = d_1 + id_2 \quad (6.0.0.8d)$$

$$f = f_1 + if_2 \quad (6.0.0.8e)$$

$$h = h_1 + ih_2 \quad (6.0.0.8f)$$

$$I = I_1 + iI_2 \quad (6.0.0.8g)$$

$$j = j_1 + ij_2 \quad (6.0.0.8h)$$

$$k = k_1 + ik_2 \quad (6.0.0.8i)$$

$$l = l_1 + il_2 \quad (6.0.0.8j)$$

$$m = m_1 + im_2 \quad (6.0.0.8k)$$

$$n = n_1 + in_2 \quad (6.0.0.8l)$$

$$x = x_1 + ix_2 \quad (6.0.0.8m)$$

$$y = y_1 + iy_2 \quad (6.0.0.8n)$$

$$z = z_1 + iz_2 \quad (6.0.0.8o)$$

After substituting the fifteen variables with their real and imaginary parts, we will get thirty linear equations. Now, we can equate the real and imaginary parts from both sides of each equation. After solving these thirty equations, we will get the value of “ a ” ($\sigma_{31}^{(0)}$) and “ b ” ($\sigma_{31}^{(1)}$) which will give us the susceptibility after multiplying the appropriate

prefactor as shown in Eq. (3.4.0.5) and in Eq. (3.4.0.6). In solving equations we have assumed Rabi frequencies of all the fields are real without any loss of generality.