

A P P E N D I X

Poincare Sphere :

The Poincare sphere, conceived by Henri Poincare in about 1892 provides a convenient way of representing polarized light and predicting how any given retarder will change the polarization form. The method is essentially *ma* of mappings: each point on the sphere represents a different polarization form. The mapping may be carried out with the aid of a three-dimensional model, a two-dimensional projection, trigonometry, or analytic geometry (The method is applicable only if the incident beam is completely polarized.)

(In addition to its mapping function, the sphere leads to short-cut solutions of problems involving retarders, or combinations of retarders and ideal homogeneous polarizers. With the sphere in mind, *ma* may often see at a glance the solution of a problem that would be difficult to solve by conventional methods. This is true, for example, of problems encountered by Pancharatnam and Koester in searching for a design of an achromatic, multilayer retarder.

Despite its great usefulness, the Poincare sphere has been given little attention in the literature. The writer has been unable to find any substantial account

of it in any text book. However, brief descriptions have appeared in a number of technical journals; see Ramachandran and Ramaseshan Jerrad, Koester, McMaster, Perrin, Skinner, Walker and Wright.

Figure 1 indicates the significance of the various parts of the sphere. The upper and lower poles represent left and right-circularly polarized light. Points on the equator indicate linear polarization. Other points indicate elliptical polarization. An arbitrarily chosen point H on the equator designates horizontal polarization and the diametrically opposite point V designates vertical polarization. Beams whose polarization forms are orthogonal are represented by points at opposite ends of a diameter. Usually, the radius of the sphere is assumed to be unity; sometimes, it is regarded as proportional to the intensity of the beam.

A general point P on the surface of the (Unit-radius) Poincaré sphere is specified in terms of the longitude (2ψ) and the latitude (2θ), where $-180^\circ \leq 2\psi \leq 180^\circ$ and $-90^\circ \leq 2\theta \leq 90^\circ$. The longitude is positive when measured clockwise from point H; the latitude is positive when measured downward from the equator, that is, toward the pole representing right circularly polarized light. Thus the coordinates of the point P in Fig. 1 are positive.

The significance of the general point 'P' is easily stated. It represents a completely polarized beam whose ellipse has azimuth α ellipticity $\tan|w|$ and a handedness that is left or right according to whether P lies in the upper or lower hemisphere. In summary P represents an elliptical polarized beam whose section pattern is described thus. (

$$\alpha = \lambda, \quad b/a = \tan |w|$$

handedness: left or right for w negative or positive respectively.

Obviously, each point on the sphere represents a different polarization form. Conversely, each polarization form is represented by a different point in the sphere.

Cartesian Coordinates : Alternatively, P may be specified by means of the right-handed cartesian coordinates X, Y, and Z indicated in Fig.2. When the sphere has unit radius, the coordinates of P are (2)

$$\begin{aligned} X &= \cos 2w \cos 2\lambda \\ Y &= \cos 2w \sin 2\lambda \\ Z &= \sin 2w \end{aligned}$$

When the light is horizontally polarized, $2v = \delta = 0$ and the full expressions reduce to $X = 1, Y = 0, Z = 0$. When the light is vertically polarized ($2v = 0, \delta = \pi$) and we find that $X = -1, Y = 0, Z = 0$. (We shall see that, in general, any completely polarized beam has X, Y , and Z values that are identical to the normalised stokes parameters, I, C , and S respectively.

From the equations presented above, one sees that if the X, Y , and Z values of a beam are given, the azimuth, ellipticity and handedness may be computed from the formulas

$$\alpha = 1/2 \arctan Y/X = \lambda$$

$$b/a = \tan |w| = \tan |1/2 \arcsin Z|$$

handedness: left or right for Z negative or positive respectively.

The main use of the Poincare is in finding the effect of inserting a retarder in a polarized beam. The effect may be found merely by constructing an arc of a circle on the sphere. The method is described in detail latter, and only a brief summary

is given here. The choice of point (on the sphere) to serve as a centre of the arc depends solely on the type of retarder (its fast eigenvector's azimuth), ellipticity, and handedness); the choice of arc length depends solely on the retardance. One end of the arc is defined by the point that location describes the emerging beam. Latter besides describing the procedure in detail, presents specific application.

In general, the Poincare-Sphere representation is useful not merely in solving problems but also in formulating them - it clarifies one's thinking and talking. It is a kind of blackboard on which to indicate the starting point, the procedure, and the outcome, all with maximum economy of effort. The principal step obviously, is drawing an arc, or, to use a more common expression, rotating the sphere.

Poincare - Sphere Method. The preferred method of predicting the effect of retarder on a beam of polarized light is to employ the Poincare sphere;. The method is simplicity itself. One marks the point P that describe the polarization form Sf the incident beam and rotates the sphere about the appropriate axis and through the appropriate angle. The new location of point P (relative to the original sphere) characterizes the polarisation of the emerging beam.

As axis of rotation we employ the radius vector from the center of the sphere to the point R that describes the fast eigenvector of the retarder. One rotates the sphere through an angle equal to the retardance

deg (which has been defined so as to be always positive). The sense of rotation is always the sense that appears clockwise to an observer situated far out on the (extended) radius vector. The rotation may be carried out on an actual sphere, or on graph paper suited to the stereographic projection. Suitable spheres, provided with degree scales, have been described by Jerrad and Koester; special graph paper of "Wulff-net" type has been described by Hatshorne and Stuart (1916) and employed successfully by Koester (such paper is obtainable from the University of Toronto Press). A free hand perspective sketch of the sphere (fig. 3) will often suffice for qualitative purposes. As an illustration of the method, consider a beam of light that is linearly polarized at 45° as suggested by point P in Fig. 3 and suppose that the beam encounters a linear retarder whose retardance is 90° and whose fast axis is at 22.5° as indicated by R. Then the pertinent rotation indicated by the righthanded quadrantal arc that starts at P and is generated by a rotation about the radius vector through R. The end point P' is the answer: it indicates the polarization from the emerging beam. Clearly the beam is left-

elliptically polarized (because 'P' lies in the upper hemisphere); the major axis is at 22.5° (because 'P' lies on the same meridian as R, namely meridian for which $2\psi = 45^\circ$); the ellipticity is $\tan 22.5^\circ = 0.4142$ (because $2\omega = -45^\circ$ and hence $|\omega| = 22.5^\circ$)

If the linearly polarized incident beam were to have vibration direction that differs by 45° from the 90° retarder's fast axis, the end point of the arc would necessarily be at a pole; hence the use of 90° retarders in converting linearly polarized light to circularly polarized light.

If, in Fig. 3 the retardance were 180° the arc would be a semicircle and the end point would be found at H, representing Horizontally polarized light. If the arc, were extended to form a complete circle, the various points on the arc would represent every possible outcome from inserting (in the given beam) azimuth of fast axis. a linear retarder having every possible value of retardance (and having the given azimuth of the fast axis).

If the retarder is of right-circular type, the point R coincides with the lower pole of the sphere and the rotation is performed about the radius vector passing through that pole. The resulting arcs are all parallel to the equator, showing that if the incident beam is right elliptically polarized the emerging beam will also be right-elliptically polarized and with the same ellipticity. Changing the retardance of a circular

retarder changes only the azimuth of the major semiaxis of the emerging beam.

In general, the Poincare-sphere representation is useful not merely in carrying out a problem involving polarized light and retarders but also in formulating the problem - thinking about it and talking about it. Various puzzling problems become clear once the investigator translates the problem into the graphic, maximum-brevity language of the Poincare sphere.

In general any polarization form P_2 can be converted to any other form P_1 by finding (on the sphere) a point R midway between them, then inserting in the beam P_1 a 180° retarder whose fast eigenvector corresponds to R . If both P_1 and P_2 indicate linear (and nonorthogonal) forms, the required linear retarder is a 180° retarder and its fast axis must bisect the angle between the vibration directions implied by P_1 and P_2 (Whether the fast axis or the slow axis bisects the angle is immaterial; any two points P_1 and P_2 are; of course, connected by two arcs of a great circle - a short arc and a long arc; the midpoint of either represents a solution for R , and these two solutions correspond to have the fast axis or the slow axis bisect the angle in question. Stated differently the fast axis of the retarder may bisect either the acute

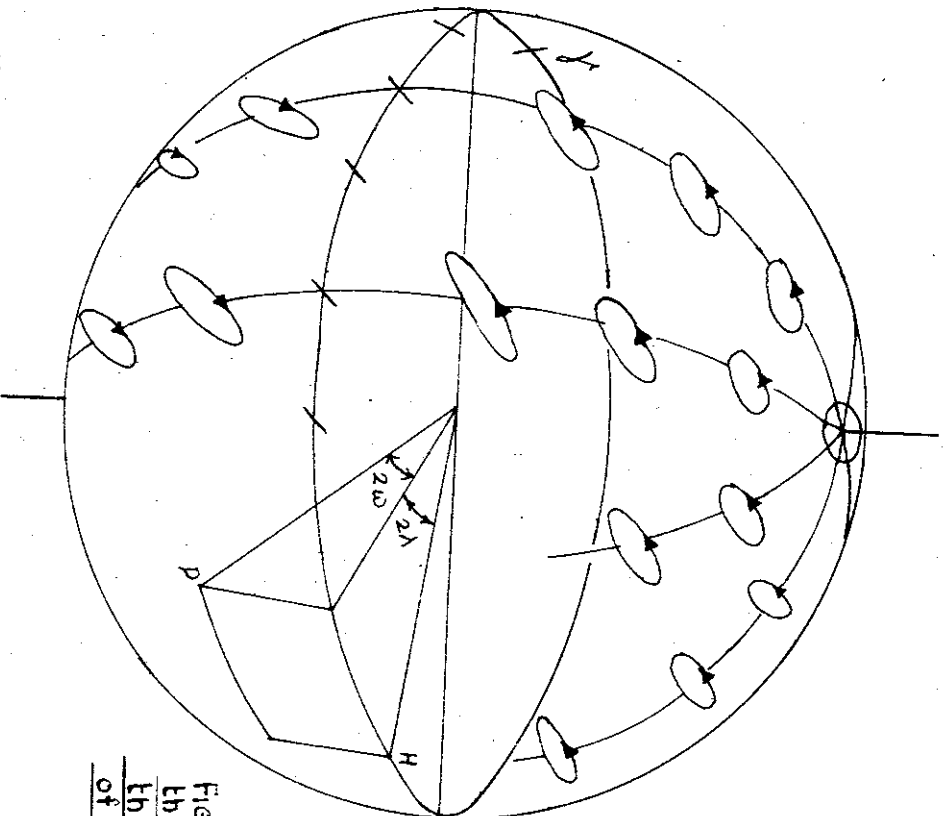


Fig. 4. Use of Poincaré sphere in finding the result of interposing a train of three elliptical retarders in a beam of elliptically polarised light.

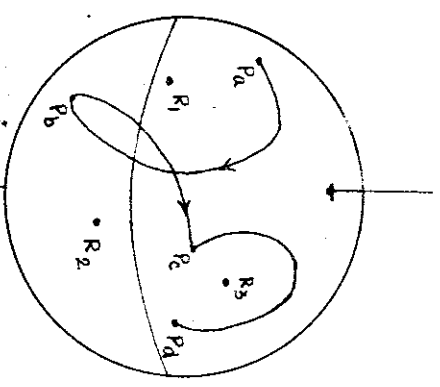


Fig. 1. The Poincaré sphere showing the general significance of the different areas and the specification of the general Point 'P' in terms of the angles 2λ , measured clockwise from H and 2ω measured downward from the equation.

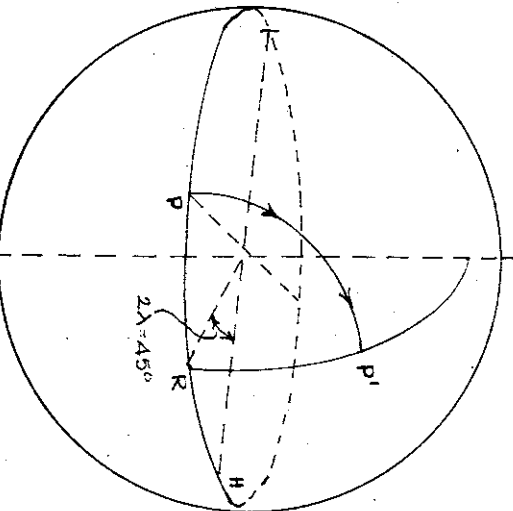


Fig. 3. Use of Poincaré sphere in predicting the effect of a 90° linear retarder R (with first axis at 22.5°) on a linearly polarized beam P. (With vibration direction at 45°)

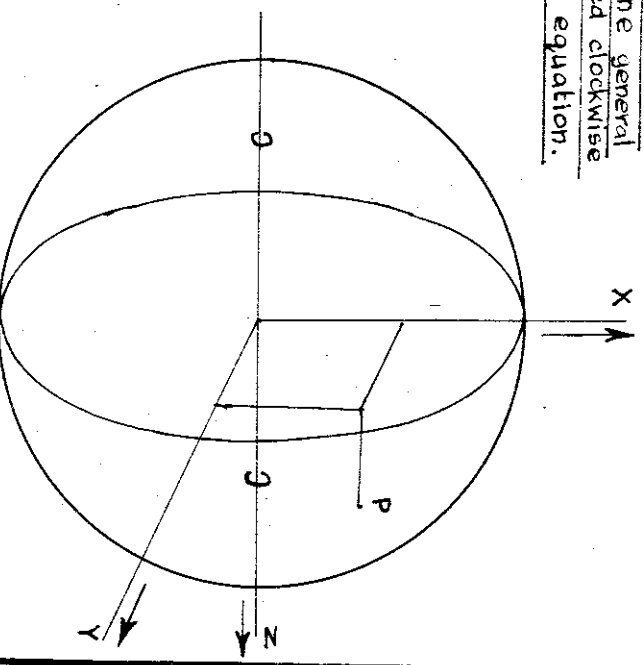


Fig. 2. Axes of the cartesian co-ordinate system used in specifying Point P on the unit radius Poincaré sphere. Note that positive z axis extends downwards.

or the obtuse angle between P_1 and P_2) in succession. If the train of , say, three retarders is used, three rotations must be performed in succession about the radius vectors through the three points R_1 , R_2 and R_3 (Fig. 4) The rotation are described by three arcs, each connected to the next. The properties of the finally emerging beam P_4 are determined without need for pausing and interpreting the intermediate outcomes P_1 and P_2 .

JONES VECTOR

The Jones vector, introduced in 1941, by R.Clark Jones, describes a polarized beam with the maximum algebraic brevity, and is eminently suited to the solving of problems involving beams whose phase relations are important. The present section defines the Jones vector, lists its commonest forms and relates it to the more familiar ways of describing polarized light. ~~Also~~ Also it shows how the vector is used in problems involving the combining of polarized beams of any polarization form and any phase relationship. (The use of the vector in problems involving polarizers and retarders is treated latter).

's polarization form and amplitude
The Jones vector is a two-element column vector which describes a beam

components at some given the vector has the general form :

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} \text{ or } \begin{bmatrix} E_x \\ E_y \end{bmatrix} \text{ or } \begin{bmatrix} A_x \exp(i(\phi_x + 2\pi f t)) \\ A_y \exp(i(\phi_y + 2\pi f t)) \end{bmatrix}$$

Where E_x and E_y are the scalar components (of the instantaneous electric vector) along the X - and Y. axes; A_x is the maximum value of E_x is the phase of the component E_x at time $t = 0$ and at the given location; A_y is the maximum value of E_y . The quantity of E_y is the phase of the component E_y at time $t = 0$ and at the given location; ϕ_y is the phase of the component E_y the letter R may be used to represent $\tan A_y/A_x$. In general, each element of the column vector is a complex quantity.

We may convert the vector to the following equivalent form :

$$\exp(i2\pi f t) \begin{bmatrix} A_x \exp(i\phi_x) \\ A_y \exp(i\phi_y) \end{bmatrix}$$

Since the magnitude of any quantity of the form $\exp(im)$ is a unity, the magnitude of $\exp(i2\pi f t)$ is unity. Thus this latter quantity may be dropped entirely in those problems in which details of variation with time are not of Interest. Most problems are of this type, form, and accordingly, the Jones vector is often written in the following form, called the full Jones vector.

$$\begin{bmatrix} A_x \exp(i\epsilon_x) \\ A_y \exp(i\epsilon_y) \end{bmatrix}$$

Obviously, if it turns out that the time variation is of interest, the factor $\exp(i2\pi r t)$ can be restored at any desired stage of the calculation.

In certain cases, the full vector can be simplified further. Consider horizontally polarized light : here $\epsilon_y = 0$, so the vector reduces to

$$\begin{bmatrix} A_x \exp(i\epsilon_x) \\ 0 \end{bmatrix}$$

For light that is linearly polarized at 45° , the two quantities A_x and A_y are equal, the quantities ϵ_x and ϵ_y are equal and the vector reduces to

$$\begin{bmatrix} A_x \exp(i\epsilon_x) \\ A_x \exp(i\epsilon_x) \end{bmatrix} \quad \text{or} \quad A_x \exp(i\epsilon_x) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For right-circularly polarized light, $A_x = A_y$ and $r = \epsilon_y - \epsilon_x = 1/2 \pi$. Thus the vector is

$$\begin{bmatrix} A_x \exp(i\epsilon_x) \\ A_x \exp(i\epsilon_x + 1/2 \pi) \end{bmatrix}$$

Table 1 lists the full Jones vectors for various forms of polarization.

Intensity of the Beam a The intensity of the beam is

proportional to the sum of the squares of the magnitudes of the individual elements. If the units of intensity or amplitude are chosen so that the proportionally constant is unity, the relation becomes

$$I = A_x^2 + A_y^2$$

Standard Normalized Jones Vector : Any full vector may be converted to a standard normalized form by multiplying it by whatever scalar (Ordinarily a complex scalar) reduces, the intensity to unity and also reduces the vector to simplest form. The process is called normalizing. Examples of nonnormalized and normalized vectors are as follows :

Nonnormalized form

Standard normalized form

$$\begin{bmatrix} 3 \exp(i\epsilon_x) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 + i \\ 1 - i \end{bmatrix}$$

$$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

Table 1 lists the standard normalized vectors for various forms of polarized light.

If a normalized Jones vector is denoted by $\begin{bmatrix} m \\ n \end{bmatrix}$ the vector of an orthogonally polarized beam is $\begin{bmatrix} -n^* \\ m^* \end{bmatrix}$ where the asterisk denotes the complex conjugate. Thus, for example, the following two forms are orthogonal :

$$\frac{\sqrt{2}}{2} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ and } \frac{\sqrt{2}}{2} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Interpretation of the Vector. The polarization form implied by a given Jones Vector may be expressed in conventional terms with the aid of the following 'tabulation of inferences.' :

Status of E_x and E_y	Inference as to polarization form
$E_x \neq 0, E_y = 0$	Linear; $\alpha = 0^\circ$
$E_x = 0, E_y \neq 0$	Linear; $\alpha = 90^\circ$
$E_x \neq 0, E_y \neq 0, E_x = E_y$	Linear; $\alpha = 45^\circ$
$E_x \neq 0, E_y \neq 0, E_x = -E_y$	Linear; $\alpha = -45^\circ$
$E_y/E_x = 1$	Right circular
$E_y/E_x = -1$	Left circular
General case	Elliptical

When a vector not falling one of the simple cases is encountered a three-step procedure is followed. Step 1 consists in converting the vector to the form of the full Jones vector :

$$\begin{bmatrix} A_x \exp(i\epsilon_x) \\ A_y \exp(i\epsilon_y) \end{bmatrix}$$

Thus if an investigator encounters the vector

$$\begin{bmatrix} 0.31 \\ 0.5 \exp(i(-0.2)) \end{bmatrix}$$

he alters the first element by replacing 1 by its equivalent $\exp(i\pi/2)$ and obtains the

$$\begin{bmatrix} 0.3 \exp(i\pi/2) \\ 0.5 \exp(i(-0.2)) \end{bmatrix}$$

Step 2 consists in computing the angles R and r defined as $|\arctan(A_y/A_x)|$ and $(\epsilon_y - \epsilon_x)$. In the step 3 consists in employing the present example

$$R = |\arctan(0.5/0.3)| = 59.0^\circ$$

$$r = (-0.2 - 1/2 \pi) = -1.77 \text{ rad or } -101^\circ$$

Step 3 consists in employing the following rules :

If $\sin R > 0$, the ellipse is right-handed; if $\sin R < 0$, the ellipse is left handed.

: 15 :

The azimuth α of the major axis is given by

$$\alpha = 1/2 \text{ arc tan } \left[(\tan 2 R) \cos r \right]$$

The ellipticity b/a is given by

$$b/a = \tan \beta$$

where $\beta = 1/2 \text{ arc sin } (\sin 2R |\sin r|)$

Applying these rules to the present example, one finds that :

$\sin V < 0$, and consequently the ellipse is left handed.

$$\begin{aligned} \alpha &= 1/2 \text{ arc tan } \tan 118^\circ \cos (-101^\circ) \\ &= 1/2 \text{ arc tan } 0.37 = 10^\circ \end{aligned}$$

$$\begin{aligned} \beta &= 1/2 \text{ arc sin } |\sin 118^\circ \sin (-101^\circ)| \\ &= 1/2 \text{ arc sin } 0.86 = 30^\circ \end{aligned}$$

$$b/a = \tan 30^\circ = 0.58.$$

An investigator who is familiar with the Jones vector can easily compute the result of combining two coherent beams of completely polarized light. He writes down the two full vectors appropriate to the given polarization forms, given intensities and given phases. He then adds the vectors.

Consider for example, the result of adding two linearly polarized beams, one of which is polarized horizontally and the other vertically. Let each beam

On referring to table 1 one finds that the resulting expression in the Jones Vector of a beam of vertically polarized light. A

As an example of compressing the informations into one matrix, consider the circular polarizer. This consists of a linear polarizer and a 90° linear retarder cemented to it at the azimuth of 45° . If the transmission axis of the polarizer is horizontal and the fast axis of the retarder is at 45° : - one looks up the table the Jones Vector of the incident beam, with which is assumed to ~~the~~ be a unity intensity beam that is left circularly polarized. One looks up the Jones matrix of 4 ideal homogeneous optical devices in question namely 1) a linear polarizer with transmission axis horizontal 2) a linear 90° retarder with fast axis at 45° 3) a linear polarizer with transmission axis at 45° and 4) a 90° right circular retarder. Thus one finds the multiplication called for

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

$$\begin{matrix} \sqrt{2}/8 \\ \sqrt{2}/8 \end{matrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

or

when multiplication is performed using proper sequence the axis

$$\sqrt{2}/8 \begin{bmatrix} 21 & -2 \\ 0 & \end{bmatrix} \text{ or } \sqrt{2}/4 \begin{bmatrix} 1 & -1 \\ 0 & \end{bmatrix} \text{ or } 1/2 \exp(-13\pi/4) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

This corresponds to a horizontally polarized beam whose intensity is $(1/2)^2$ or $1/4$.

N-Matrix The ordinary matrix of the Jones Calculus concerned with an optical device as a whole, and gives NO information as to either the polarization form at the intermediate location within the device or the change of polarization form that is produced per unit path length within the device. However, information of this type is provided by what Jones ~~has~~ has called It-Matrix, a matrix that is obtained from the approximate standard matrix (N-Matrix) by a process involving differentiation. Obviously, the N-Matrix describes the intensity properties of the device, and is indeed a most succinct and informative specification of these properties. Each of the 8 major types of optical behaviour of crystals has its own type of N-matrix. By examining the N-matrix of a given crystal one can determine the average values of the refractive index and the absorption coefficient, the circular birefringence and circular dichroism,

and the linear birefringence and linear dichroism with respect to the coordinate axes and with respect to the bisectors of these axes. The method is explained by Jones and Ramachandran and Ramaseshan.

Outline of the Jones Calculus. This calculus is another treatment in which the incident light is described by a vector the optical device is described by a matrix and the outcome is computed by multiplying the vector by the matrix. It employs a smaller matrix (2×2) and is applicable to problem in which information as to phase must be preserved. But many of its matrix elements are complex and it is inapplicable to optical devices that have depolarizing tendencies. The Jones Matrix of importance polarizers and retarders are given Inter. Each matrix describes a given device on a given orientation and assumes that a given face serves as entrance face. The matrices are derived from the usual mathematical expression for a monochromatic (polarized) wave train and from mathematical analysis of the changes produced by interposing a given polarizer or retarder; The matrices presented in are in the simplest form and are called standard matrices. They are ideally designed for use by an investigator interested in the intensity in polarization form of an emerging beam. However they contain no information as to the change

produced in the absolute phase. An investigator wishing to determine the absolute phase of an emerging beam must use the full matrix. For example, the full matrix of an ideal homogeneous linear polarizer oriented with its transmission axis horizontal is

$$\begin{bmatrix} \exp(-i 2\pi nd/\lambda_v) & 0 \\ 0 & 0 \end{bmatrix} \quad \text{or} \quad \exp(-i 2\pi nd/\lambda_v) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Where n is pertinent refractive index, d is the thickness, and λ_v is the wave length in vacuum. Obviously the term $\exp(-i 2\pi nd/\lambda_v)$ carries the information as to absolute phase. The matrix (P_θ) of a polarizer at some nonprincipal azimuth θ is easily obtained from the Matrix (P_0) of the polarizer in its principle azimuth. As in the course of Mueller Calculus the procedure involves a rotator matrix and a counter rotator matrix. These latter, called ($S(\theta)$) and

$S(-\theta)$, have the forms

$$S(\theta) = \begin{bmatrix} C_1 & S_1 \\ -S_1 & C_1 \end{bmatrix} \quad \text{and} \quad S(-\theta) = \begin{bmatrix} C_1 & -S_1 \\ S_1 & C_1 \end{bmatrix}$$

where C_1 and S_1 are abbreviations for $\cos \theta$ and $\sin \theta$ respectively. The relation between P_θ and P_0 is

$$P_\theta = [S(-\theta)] [P_0] [S(\theta)]$$

A corresponding relation applies to the nonprincipal matrix R_θ of a retarder and the principal matrix R_0

of the retarder. Again, the R_0 matrix describes the retarder in its principal orientation, and hence may be regarded as the simplest, most basic description of the retarder.

To find the result of interposing several optical devices in a beam of 100 percent polarized light, one writes down (at the right) the Jones Vector of the incidence beam, and then writes down the Jones matrixes of the various devices. Again the sequence of the matrixes important, and must correspond to that of the matrices themselves; the matrix of the least-to-the encountered device must appear the the left.

Example of application : Consider a unit-intensity horizontally polarized beam strikes an ideal homogeneous linear 180° retarder whose fast axis is at 45° what is the character of the emerging beam,

Ans Since the Jones Vector of the incidence beam is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and the Jones matrix of the retarder is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ the axis is found by obtaining the product

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

TABLE 1

Polarization form				Jones Vector	
Sectional pattern	α (deg.)	b/a	$\frac{A_y}{A_x}$	ψ (deg.)	Standard Normalized
0	0	0	0	-	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
90	90	0	∞	-	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
45	45	0	1	0	$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
-45	-45	0	1	± 180	$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
General Linear		0	any +ve number	0 or ± 180	$\begin{bmatrix} \cos R \\ \pm \sin R \end{bmatrix}$
-1	-1	R	1	90	$\frac{\sqrt{2}}{2} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$
-1	-1	L	1	-90	$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
0	0	$1/2, R$	$1/2$	90	$\frac{2\sqrt{2}}{3} \begin{bmatrix} -1 \\ 1/2 \end{bmatrix}$
90	90	$1/2, R$	2	90	$\frac{2\sqrt{2}}{3} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$
22.5, 0, 315	22.5, 0, 315			45	$0.326 \begin{bmatrix} 2.73 \\ 1+1 \end{bmatrix}$
General elliptical					$\begin{bmatrix} \cos R \exp(-i\frac{\psi}{2}) \\ \sin R \exp(+i\frac{\psi}{2}) \end{bmatrix}$
General elliptical - Partially polarized and Unpolarized					None
					Full
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ 0 \end{bmatrix}$
					$\begin{bmatrix} 0 \\ A_y \exp(i\phi_y) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ A_y \exp(i\phi_y) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ \pm A_y \exp(i\phi_y) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ A_x \exp i(\phi_x + \pi/2) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ A_x \exp i(\phi_x - \pi/2) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ 1/2 A_x \exp i(\phi_x + \pi/2) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ 2 A_x \exp i(\phi_x + \pi/2) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ 0.518 A_x \exp i(\phi_x + \pi/4) \end{bmatrix}$
					$\begin{bmatrix} A_x \exp(i\phi_x) \\ A_y \exp(i\phi_y) \end{bmatrix}$

STANDARD MATRICES

Abbreviations: $C_1 = \cos \theta$, $S_1 = \sin \theta$ for polariser and $\cos \rho$ and $\sin \rho$ for retarders. $C_2 = \cos 2\theta$ etc.
 $Y = \cos(2 \arctan b/a)$, $P = \exp(i\pi/4)$,
 $Q = \exp(-i\pi/4)$, $Z = 2 \sin(2 \arctan b/a)$.

Std. Matrix

Jones Matrix

Optical Devices

Ideal plate of isotropic,
absorbing glass where
transmittance is k or P^2

$$\begin{bmatrix} P & 0 \\ 0 & P \end{bmatrix}$$

Region devoid of matter, or
ideal plate of isotropic,
non-absorbing glass

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Totally absorbing plate

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Ideal depolariser (nothing
closely resembles this)

none

Ideal Homogeneous Linear Polarizers

Azimuth θ of transmission axis

0 deg.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

90 deg.

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

45 deg.

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

- 45 deg.

$$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

General

$$\begin{bmatrix} C_1^2 & C_1 S_1 \\ C_1 S_1 & S_1^2 \end{bmatrix}$$

Ideal Homogeneous Non-linear Polarizers

Right Circular

$$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Left Circular

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Right Elliptical

$$\theta = 0, b/a = 0.5$$

$$\frac{2}{5} \begin{bmatrix} 2 & -1 \\ 1 & 0.5 \end{bmatrix}$$

Right Elliptical

$$\theta = 22.5 \text{ deg.}$$

$$b/a = 0.318$$

$$0.288 \begin{bmatrix} 2.73 & 1-1 \\ 1+1 & 0.733 \end{bmatrix}$$

Elliptical General Case

$$\begin{bmatrix} |n|^2 & mn^* \\ m^*n & |n|^2 \end{bmatrix}$$

where Jones major and minor eigen vectors are

$$\begin{bmatrix} m \\ n \end{bmatrix} \text{ AND } \begin{bmatrix} -n^* \\ -m^* \end{bmatrix}$$

Other Polarizers

Homogeneous, non-scattering, non-depolarizing nonbirefringent polarizer with principle transmittance $k_1 = P_1^2$ and $k_2 = P_2^2$ and with the transmission axis horizontal ($\theta = 0$ deg.)

$$\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

Same but with $\theta = 90$ deg.

$$\begin{bmatrix} P_2 & 0 \\ 0 & P_1 \end{bmatrix}$$

In homogeneous, right circular polarizer consisting of two ideal homogeneous layers: linear polarizer with horizontal axis and linear retarder with retardance of 90 deg. and fast axis at 45 deg.; light assumed to be incident on linearly polarizing layer

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Inhomogeneous Linear Retarder with Retardance

$\delta = 90 \text{ deg.}$ A

Asimuth of fast axis

0 deg.

$$\begin{bmatrix} \exp(i\pi/4) & 0 \\ 0 & \exp(-i\pi/4) \end{bmatrix}$$

90 deg.

$$\begin{bmatrix} \exp(-i\pi/4) & 0 \\ 0 & \exp(i\pi/4) \end{bmatrix}$$

45 deg.

$$1/\sqrt{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

-45 deg.

$$1/\sqrt{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

General

$$\begin{bmatrix} C_1^2 P + S_1^2 Q & \sqrt{2} i C_1 S_1 \\ \sqrt{2} i C_1 S_1 & C_1^2 Q + S_1^2 P \end{bmatrix}$$

With retardance $\delta = 180 \text{ deg.}$

0 or 90 deg.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\pm 45 \text{ deg.}$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

General

$$\begin{bmatrix} C_2 & S_2 \\ S_2 & -C_2 \end{bmatrix}$$

Other Ideal Homogeneous Linear Retarders:

$\delta = 360 \text{ deg., any}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

any δ , $= 0 \text{ deg.}$

$$\begin{bmatrix} \exp(i\delta/2) & 0 \\ 0 & \exp(-i\delta/2) \end{bmatrix}$$

any δ , any

$$\begin{bmatrix} C_1^2 \exp(i\delta/2) + S_1^2 \exp(-i\delta/2) & C_1 S_1 2i \sin \delta/2 \\ C_1 S_1 2i \sin \delta/2 & C_1^2 \exp(-i\delta/2) + S_1^2 \exp(i\delta/2) \end{bmatrix}$$

where: $m_{11} = \cos^2 \delta + \exp(-i\delta) \sin^2 \delta$
 $m_{22} = \sin^2 \delta + \exp(-i\delta) \cos^2 \delta$
 $m_{12} = m_{21} = [1 - \exp(i\delta)] \cos \delta \sin \delta$

(or)

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Ideal Homogeneous Non-linear Retarders

Right circular, $\delta = 90$ deg.
(hence $= 45$ deg.)

$$1/\sqrt{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Left circular, $\delta = 90$ deg.
(hence $= -45$ deg.)

$$1/\sqrt{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Right or Left circular, $\delta = 180$ deg.
(hence $= \pm 90$ deg.)

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Right circular, any δ

$$\begin{bmatrix} \cos \delta/2 & \sin \delta/2 \\ -\sin \delta/2 & \cos \delta/2 \end{bmatrix}$$

Left circular, any δ

$$\begin{bmatrix} \cos \delta/2 & -\sin \delta/2 \\ \sin \delta/2 & \cos \delta/2 \end{bmatrix}$$

Elliptical, any δ , any

$$\begin{bmatrix} C_P^2 \exp(i\delta/2) & C_P S_P (2i \sin \delta/2) \\ + S_P^2 \exp(-i\delta/2) & \\ C_P S_P (2i \sin \delta/2) & C_P^2 \exp(-i\delta/2) \\ & + S_P^2 \exp(i\delta/2) \end{bmatrix}$$

where:

$C_P = \cos R$ and $S_P = \sin R$, the quantities R and δ having the significance indicated before.

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