

CHAPTER 1

INTRODUCTION

The different states of condensed matter are classified on the basis of the inherent positional and orientational order in the arrangement of their constituent molecules. The all-familiar liquids, also called isotropic liquids, do not have either positional or orientational order while the equally familiar crystals exhibit both positional and orientational order. A wide variety of materials exhibit a mesophase or a state that is intermediate between that of a crystal and a liquid. If the constituent molecules are highly anisotropic in shape, then they can exist in a condensed state with a preferential orientation of their molecules. In addition, these phases may possess positional order in one or two dimensions. Such mesophases of condensed matter, also called *liquid* crystals, may appear in single component or multicomponent systems. In the case of single component systems where phases change in response to changes in temperature, they are called thermotropic liquid crystals. And, in the case of multicomponent systems, they are referred to as lyotropic liquid crystals. In the second case, the existence of the liquid crystalline phase is also sensitive to the relative proportions of the different components. If the molecules are nearly spher-

ical in shape, then there can be another kind of mesophase, called a plastic *crystal*, which is endowed with only positional order and no orientational order. This thesis deals with only liquid crystals.

Liquid crystals can be broadly classified into two depending upon their orientational order, *viz.* *uniaxial* and *biaxial* liquid crystals. Uniaxial liquid crystals possess an axis of rotational symmetry. Since this is a direction of optical isotropy, it is also the direction of its optic axis. In the language of electromagnetism, we can say that the electric or magnetic susceptibility tensor is an ellipsoid of rotation about this direction. Two well-known structures in this category are:

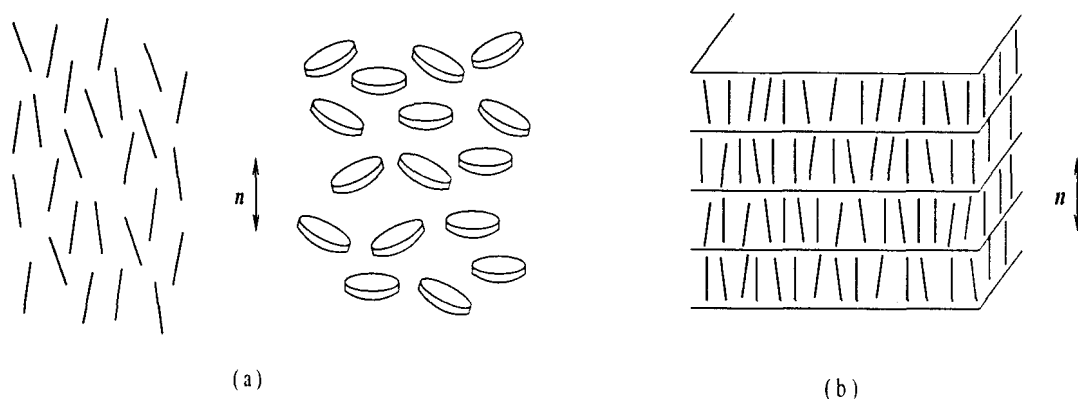


Figure 1.1: (a) Schematic representation of two types of uniaxial nematic phases: orientationally ordered rod-like and disc-like molecules. The direction of preferred alignment is represented by the unit vector $\mathbf{n}$ called the director. (b) Smectic-A is shown with layering along $\mathbf{n}$ and fluid-like arrangement in the other two dimensions. The director $\mathbf{n}$ is represented by a 'double-headed' arrow.

Uniaxial nematics (N_u): This phase has no positional order. The constituent molecules in a pure component system are either rod-like or disc-like. In the first case, the molecules are preferentially aligned about a direction, and in the second case, perpendicular to a direction. A schematic representation of this phase is shown in Figure 1.1(a). In either case, this direction about which cylindrical symmetry exists is represented by a unit vector, $\mathbf{n}$, called

the director. The molecular arrangement in both the cases is such that $\mathbf{n}$ and $-\mathbf{n}$ are indistinguishable states. This is one reason why $\mathbf{n}$ is often represented by a 'double-headed' arrow.

Smectic-A (Sm-A): This phase has not only the orientational order as in uniaxial nematics, but also has the constituent molecules arranged in a one-dimensional lattice. Here, the molecules are stacked in layers along the director $\mathbf{n}$, and without positional order in the layers. Hence, in any direction perpendicular to $\mathbf{n}$, the medium has fluidity. Figure 1.1(b) shows the essential features of such a phase.

On the other hand, biaxial liquid crystals possess two optic axes since the electric or magnetic susceptibility tensor is a triaxial ellipsoid having two directions of optical isotropy, i.e., two optic axes. Two well-known examples are:

Biaxial nematics (N_b): This phase has orientational ordering in all three directions described by a triad of directors, $(\mathbf{a}, \mathbf{b}, \mathbf{c})$, each of which has a two-fold symmetry. Further, the medium does not possess positional ordering. In Figure 1.2(a) is shown such a phase with orthorhombic symmetry.

Smectic-C (Sm-C): This phase exhibits layering like Sm-A, but the director, $\mathbf{n}$, is tilted at an angle with respect to the layer normal, $\mathbf{N}$. As a result of molecular confinement in layers, the structure is not uniaxial about $\mathbf{n}$. In fact, it is biaxial with monoclinic symmetry with a two-fold axis perpendicular to $\mathbf{n}$ and a mirror plane along $\mathbf{n}$ and the layer normal $\mathbf{N}$. The projection of $\mathbf{n}$ onto the layers, represented by $\mathbf{c}$, is often useful in the description of the phase. Sm-C is schematically represented in Figure 1.2(b). We have used the accepted notation in which the nail heads indicate the end of the director coming out of the plane.

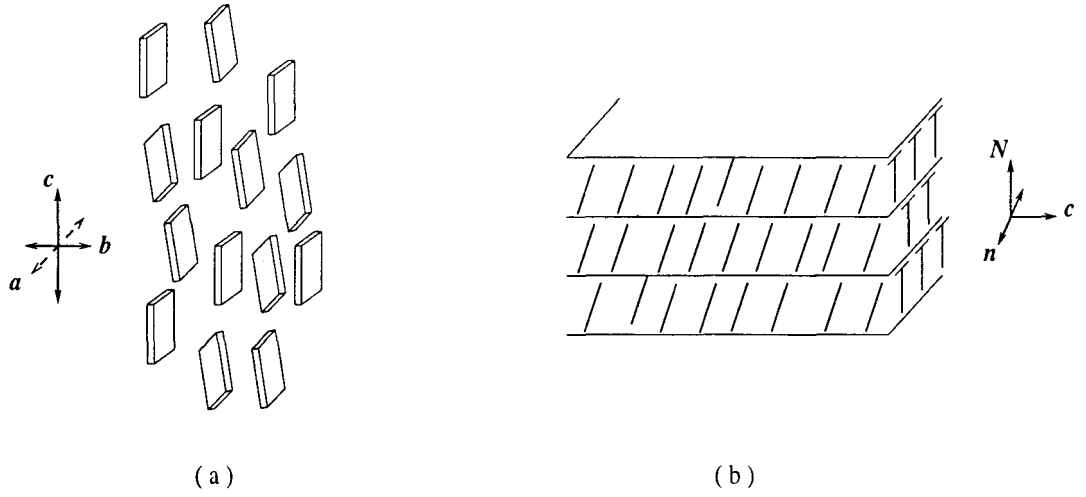


Figure 1.2: (a) Schematic representation of a biaxial nematic of orthorhombic symmetry. The director a is shown to be orthonormal to b and c . (b) Smectic-C is shown with layering along N . The director n is at angle with respect to N . The unit vector c is the projection of the director onto the layers. We use the accepted notation in which the nail heads indicate the end of the director coming out of the plane. This has monoclinic symmetry with a two-fold symmetry about a direction perpendicular to n and a mirror symmetry parallel to the N - c plane.

Liquid crystals are systems with broken continuous symmetry. A necessary consequence of broken continuous symmetry is the existence of topological defects [1, 2]. Topological defects have been studied in systems as diverse as superfluid helium, crystalline solids, liquid crystals, and even in some cosmological models. They have been important in understanding the structure of exotic defect structures such as the Abrikosov flux-lattice phase of type II superconductors, twist grain boundary smectics and blue phases of liquid crystals. In view of their universal nature, topological defects have attracted a lot of attention of experimentalists as well as theorists. In this context, liquid crystals are ideal systems for studying them. This is because the distortions in the director field accompanying these defects are easily generated by boundary conditions on the director, surface geometries, or external fields. Secondly, they can be easily seen and studied under optical microscope in view of their

large scale structures. The first step in any study of topological defects would be to work out the different types of topological defects that are possible in a particular system as dictated by its symmetry. Then, the structure, energetics and interaction of defects are analyzed on the basis of a continuum theory of elasticity and dynamics. This approach also enables us to understand many unique and interesting instabilities seen in liquid crystals in the presence of destabilizing external electric or magnetic fields, or hydrodynamic flows. Such dynamical studies acquire special significance since instabilities in external fields are employed in display devices. Further, the dynamics of defects in liquid crystals is relevant to the study of such phenomena as response to external stresses, or the approach to equilibrium after a quench into an ordered phase. In this context, it is also important to stress that liquid crystals are very useful systems to study front propagation. In recent times, the problem of front propagation has interested mathematicians and physicists in diverse fields such as biological evolution, dynamics of kink states, dendritic growth and instabilities in vesicles.

Much of the existing literature on liquid crystals is on studies of the elastic and dynamical properties of uniaxial liquid crystals [3, 4]. To an equal depth and detail, such studies in biaxial liquid crystals have not been considered. This is one reason that prompted us to undertake investigations in biaxial liquid crystals. This is the theme of this thesis where attention has been addressed to elasticity, dynamics and topological defects in two biaxial liquid crystals, *viz.* biaxial nematics and smectic-*C*. In this introductory chapter, topological defects and hydrodynamics of liquid crystals are briefly reviewed.

1.1 Topological defects in nematics and smectics

Liquid crystals being soft materials allow for a continuous deformation of the director field. Any such deformation can be decomposed into three fundamental

types, *viz.* splay, bend and twist. In Figure 1.3, these deformations have been depicted.

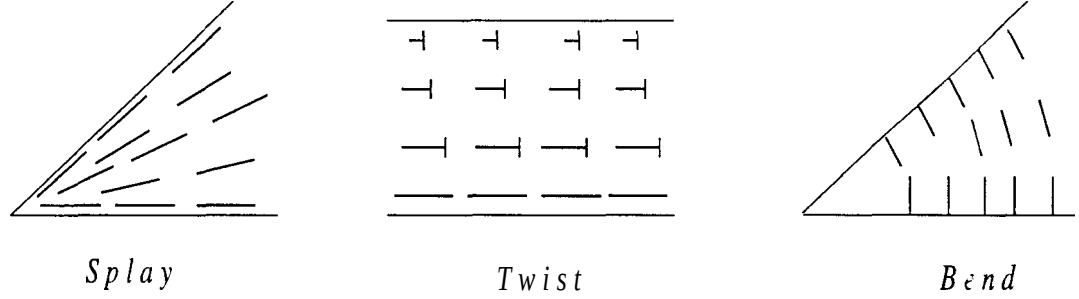


Figure 1.3: Schematic representation of splay ($\nabla \cdot \mathbf{n}$), twist ($\mathbf{n} \cdot \nabla \times \mathbf{n}$) and bend ($\mathbf{n} \times \nabla \times \mathbf{n}$) deformations in the director field. $\mathbf{n}$.

In the case of uniaxial nematics, the elastic energy density, F , of slowly varying spatial distortions in the director $\mathbf{n}(\mathbf{r})$ is given by [5-7]

$$F = \frac{1}{2} \left[K_{11}(\nabla \cdot \mathbf{n})^2 + K_{22}(\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + K_{33}(\mathbf{n} \times \nabla \times \mathbf{n})^2 \right] \quad (1.1)$$

where K_{11} , K_{22} , and K_{33} are respectively, the splay, twist, and bend elastic constants. They are of the order of 10^{-6} dynes. We have ignored surface terms. Topological defects can create physical situations where the director field is not a smooth function of the spatial coordinates at all points. It should be noted over here that apart from a study of the topological aspects of defects [2, 8, 9], investigations on the structure, energetics and interactions of defects have been considered on the basis of continuum theories [1, 3, 4, 10]. In the case of uniaxial nematics, equation (1.1) can be used to study topological defects within the context of a continuum description. Topological defects in uniaxial nematics are now briefly discussed. This will serve as a useful background for topological defects in biaxial nematics and smectic-C.

1.1.1 Line disclinations

Line disclinations are lines of singularity in the director field. On these lines, the director $\mathbf{n}$ cannot be uniquely defined. In a circuit around a disclination line, the director rotates continuously by $360^\circ \times s$, where s is called the *strength* or winding number of the defect.

Uniaxial nematics

Since $\mathbf{n}$ and $-\mathbf{n}$ are equivalent in this system, s can be integral or half-integral, *i.e.*, $s = \pm m/2$ where m is an integer. Examples of disclinations with $s = \pm \frac{1}{2}$ are shown in Figures 1.4(a) and 1.4(b). The line of singularity is perpendicular to the plane of the figure and goes through the origin. These disclination lines are often called wedge disclinations. Topologically, $s = \frac{1}{2}$ and $s = -\frac{1}{2}$ wedge disclination lines are equivalent since one can be transformed into another by rotating the director locally at every point by 180° about an axis (say, x-axis), *i.e.*, normal to the disclination line. If instead of 180° , the director everywhere is rotated only by 90° , then a wedge disclination gets transformed to a twist disclination. A twist disclination of strength $s = \frac{1}{2}$ is shown in Figure 1.4(c). From the figure, it is clear that as we go in a circuit around a wedge disclination line, $\mathbf{n}$ rotates about an axis that is parallel to the disclination line, while it rotates about a perpendicular direction in the case of a twist disclination line.

In the disclinations depicted in Figures 1.4(a) and 1.4(b), the director configuration is given by $\mathbf{n} = (\cos \phi/2, \pm \sin \phi/2, 0)$, in cartesian coordinate system, with $\phi = \tan^{-1}(y/x)$, the azimuthal angle in the xy-plane. In the case of the twist disclination, the director field is given by $\mathbf{n} = (\cos \phi/2, 0, \pm \sin \phi/2)$. The energy per unit length of a disclination line can be calculated from free energy density (1.1) and it turns out to be

$$E = \pi K s^2 \ln(R/r_c) + \epsilon_c \quad (1.2)$$

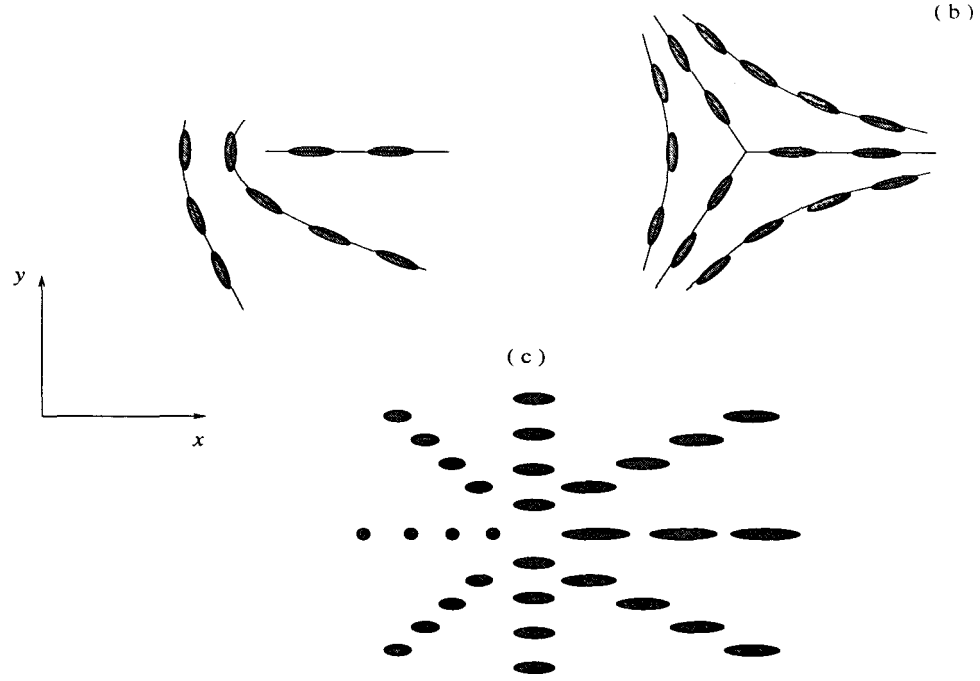


Figure 1.4: Director configurations associated with a half disclination. (a) $s = \frac{1}{2}$ wedge disclination (b) $s = -\frac{1}{2}$ wedge disclination, and (c) $s = \frac{1}{2}$ twist disclination where the director lifts out of the plane of the paper. Since these structures are often studied optically, we show in the figure the projection of the index ellipsoid. In all the three disclinations, the disclination line is along z-axis and it is a line singularity.

where R is the sample radius, r_c is the radius of the disclination core, and ϵ_c is the core energy per unit length. The core radius r_c is of molecular dimensions. A core is necessitated by the fact that there is a singularity in the director field at the origin and at distances comparable to molecular dimensions, continuum description fails. In arriving at (1.2), we have used the "isotropic or one-constant approximation" where $K_{11} = K_{22} = K_{33} = K$. Hence, the energy per unit length of the disclination increases logarithmically with the sample size. Since E varies as s^2 , it is obvious that a disclination of strength, $|s| > \frac{1}{2}$ will split on its own into two half disclinations to lower the energy. It can be shown from energetics that two like disclination lines (i.e., both with same sign of s) repel and two unlike disclination lines (i.e., opposite

signs of s) attract with a force varying inversely with distance.

Yet, in uniaxial nematics, experimentally it is found that disclination lines of strength $|s| = 1$ exist as individual defects. This paradoxical situation was resolved by showing that the planar $|s| = 1$ disclination is topologically unstable [11, 12]. The singularity at the centre can be removed by allowing the director $\mathbf{n}$ to “escape into the third dimension”. As one approaches the centre, the director lifts out of the plane and at the core it is along z -axis, thereby uniquely defining the director at the origin. Such a process is schematically represented in Figure 1.5 for $s = 1$ line defect. The director configuration of such an escaped configuration is given by $\mathbf{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ in cartesian coordinate system. Here, θ is the angle made by the director with the z axis, and ϕ is the azimuthal angle in the xy -plane. From a minimization of total elastic energy, it can be shown that $\theta = 2 \tan^{-1}(r/R)$ where r is the radial distance in the xy -plane and R is the radius of the sample.

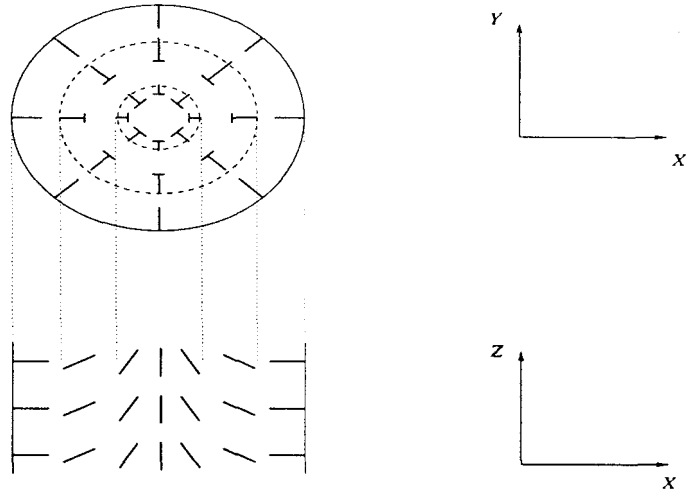


Figure 1.5: The singularity in a $s = 1$ disclination can be removed by an “escape into the third dimension”.

The bulk elastic energy per unit length of this 'escaped' structure is:

$$E = 2\pi K \quad (1.3)$$

Thus, the energy per unit length of such a structure does not depend on the size of the system. Likewise, $s = -1$ is also non-singular. Even though a planar and hence singular $s = 1$ line disclination is energetically unfavourable when compared to a pair of $s = \frac{1}{2}$ disclination lines, the nonplanar nonsingular $s = \pm 1$ line disclinations are favoured when the size of the system is large. This is the reason why such disclination lines are seen in experimental situations. It should be noted that even though topology demands a $s = \pm 1$ disclination line to be nonsingular, energetics and the geometry of confinement can prevent an escape into the third dimension, and thereby a singular integral defect could be stabilized in fine capillaries.

Biaxial nematics

In 1970, Freiser [13] theoretically predicted the existence of biaxial nematics. Immediately thereafter, Toulouse [14] used topological methods to show that in biaxial nematics, there are four stable singular line disclinations, *viz.* three distinct line disclinations of half-integral strength and a line disclination of odd-integral strength.

As already said, biaxial nematics of orthorhombic symmetry can be described by a triad (a, b, c) of mutually perpendicular unit vectors. Just as uniaxial nematics have disclinations in the director field $\mathbf{n}$, here there are such defects in these three director fields. In Figure 1.6(a), a half-strength disclination line in the pair of directors (a, b) is shown. This has no singularity in c. Two other similar structures are possible in (b, c) and (c, a) director fields. Since they cannot be transformed into each other by any continuous transformation of the directors, there are three distinctly different half-disclinations. The fourth class of topologically stable disclination line has odd-integral strength. In Figure 1.6(b) is shown a $s = 1$ line disclination in (a, b) fields. In uniaxial nematics, singular disclination lines of $s = \pm 1$ could be made nonsingular by an escape of the director into the third dimension. But this process

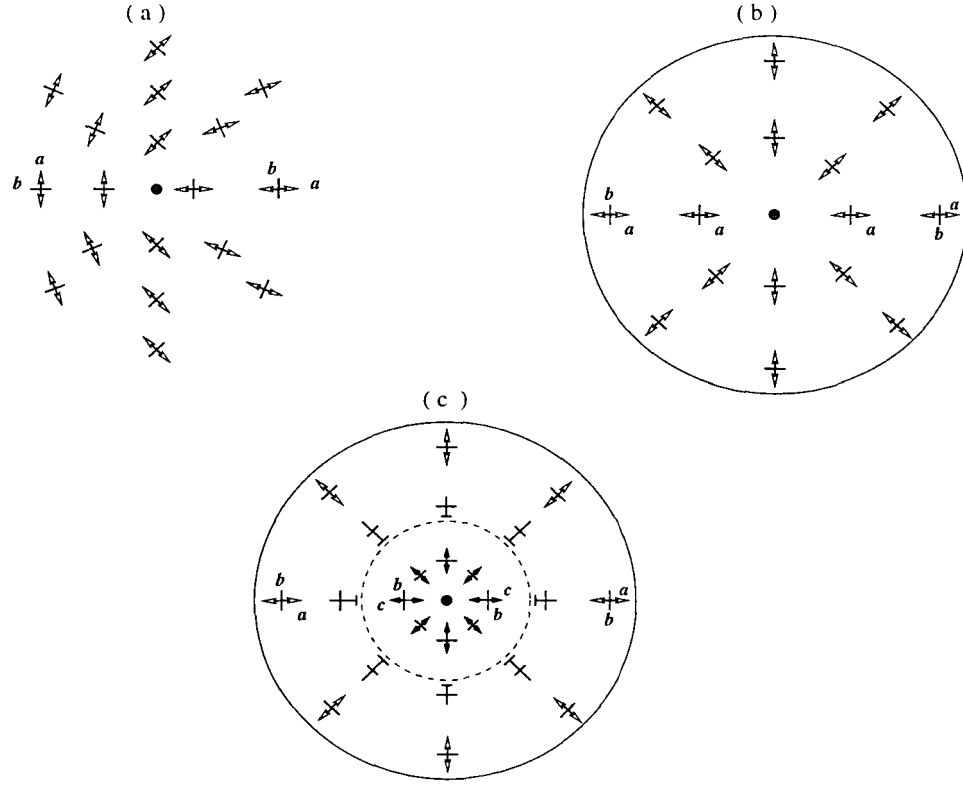


Figure 1.6: Singular line disclinations in biaxial nematics. (a) $s = \frac{1}{2}$ in the **(a, b)** director fields. (b) Disclination line of strength $s = 1$ in **(a, b)**. (c) This shows that an escape into the third dimension as in uniaxial nematic does not remove the singularity in $s = 1$ disclination. In the outer region the configuration is a disclination in **(a, b)**. The director **a** has escaped into the third dimension thereby creating a $s = 1$ strength defect in **(b, c)** near the core.

will not remove singularity of a similar disclination line in biaxial nematics. By an escape into the third dimension, we only transform such a defect in one pair of directors, say **(a, b)**, into a line singularity of $s = 1$ in another pair, say **(b, c)**. This process is shown in Figure 1.6(c). Thus, unlike in uniaxial nematics, $s = 1$ defects are singular in biaxial nematics. This argument is valid for any defect of odd-integral strength. It is only recently that some of these theoretical predictions about defects have been experimentally verified by De'Neve *et al.* [15]. Toulouse speculated that such a phase with many such singular line disclinations may lead to entanglement

between disclination lines thus endowing the medium with a polymer-type structure and 'topological rigidity' in elastic and flow properties.

The fact that $s = \pm 1$ line disclination is singular implies that it will split into a pair of $s = \pm \frac{1}{2}$ disclination lines. Thus, there have been claims that a sample of biaxial nematic would only have half-strength disclination lines. But, nonsingular disclinations are also permitted in biaxial nematics [10]. They are of two types. The first type is a wedge or twist disclination of even integral strength which can also become nonsingular through a director escape in three dimensions. And, the second is a hybrid disclination - with both wedge and twist components - of total even integral strength which can become nonsingular again through a director escape in three dimensions. There has not been much study on the structure and energetics of these defects. In a later chapter of this thesis, results on the structure, energetics and interaction of such disclinations in biaxial nematics are presented.

Smectic- C

The c vector in Sm- C is such that c and $-c$ do not represent one and the same state. Therefore, half-integral strength line disclinations are not permitted in this vector field whereas integral strength defects are permitted. But, unlike in uniaxial nematics, $s = \pm 1$ line disclinations are topologically stable because the layering will not allow a director escape of c into the third dimension. The only way singularity gets removed is by a gradual tilt of n towards the disclination line. This however changes the order parameter and is akin to a quantum fluid vortex.

1.1.2 Point disclinations

While line disclinations are characterized by their winding number s , point disclinations are characterized by an integral topological charge q specifying the number of times the unit sphere is wrapped by the director on any surface enclosing

the defect core. These topological defects, also called point defects, have been of much interest in a wide variety of soft-condensed matter systems such as superfluid helium, vesicles: polymer dispersed liquid crystals, free standing smectic films, nematic emulsions and Langmuir monolayers.

Uniaxial nematics

These systems can have a wide variety of point disclinations or point defects and they are collectively called *hedgehogs*.

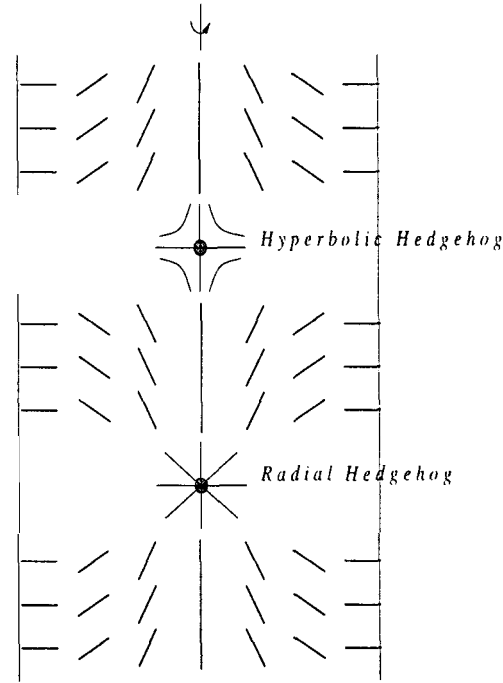


Figure 1.7: Creation inside a capillary of a hyperbolic and a radial hedgehog when the director "escapes into the third dimension". The director n is perpendicular to the walls of the capillary.

Consider a spherical droplet of nematic. If the director n is under the constraint that n is normal to the surface of the drop, then elementary geometry indicates a stable point defect in n at the centre of the droplet. This structure is called a *radial* hedgehog. If the director is not anchored at the surface of the drop, other point defect configurations can be obtained from the radial hedgehog via rotations

through arbitrary angles about any axis. For example, when the director of a radial hedgehog is rotated locally about a fixed axis through π , a hyperbolic hedgehog is produced. Thus, they are topologically equivalent even though they can be energetically different. In Figure 1.7, an experimental situation is depicted where such defects have been seen in a capillary. When a $s = 1$ defect escapes into the third dimension in a capillary, it could do so by lifting the director upward or downward along the axis. Here, we have a situation where a region in which the director that has escaped downwards is sandwiched between two regions where the director escape is upwards. A necessary consequence of this situation is that a radial and a hyperbolic hedgehog are created as shown. Their respective director configurations are $\mathbf{n} = (x, y, z)/r$ (for the radial hedgehog), and $\mathbf{n} = (-x, -y, z)/r$ (for the hyperbolic hedgehog), where $\mathbf{r} = (x, y, z)$ and $r = |\mathbf{r}|$. Further, in a spherical region of radius R , both these have the same energy given by

$$E = 4\pi KR$$

in the one-constant approximation and neglecting surface contributions. However, when $K_{11} \neq K_{22} \neq K_{33}$, these point defects have different energies.

If the boundary condition on $\mathbf{n}$ is such that $\mathbf{n}$ is tangential to the surface of the drop, then elementary considerations will show that there can exist a point defect of strength, $s = 2$ on the surface of the droplet or a collection of defects of total strength $s = 2$. For example, if the director $\mathbf{n}$ is along the longitudes (or latitudes), there will be two all radial (or all circular) defects, each of strength $s = 1$, at the poles. This structure is called a *bipolar defect*. A schematic representation of one such is shown in Figure 1.8(a).

There is another interesting defect configuration in which the director is again everywhere normal to the spherical surface of a drop. In this structure, the director

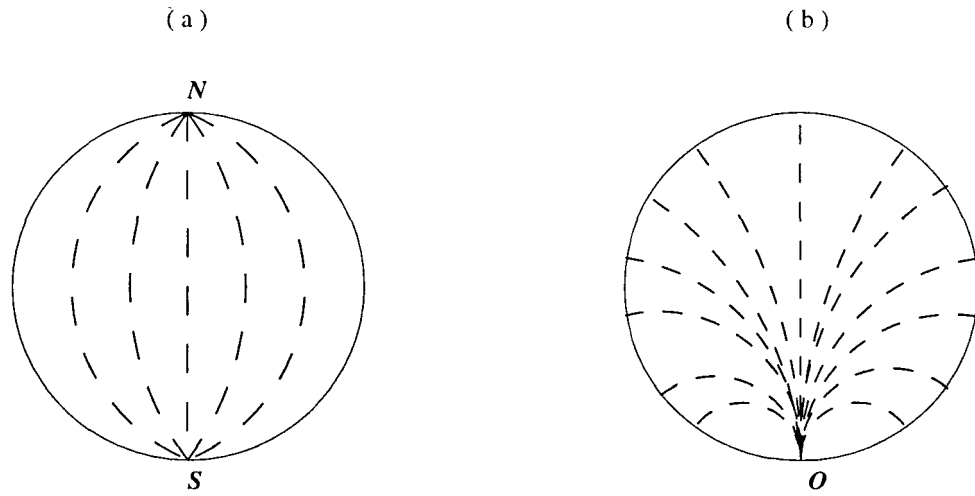


Figure 1.8: (a) The director is along the longitude in this example of a bipolar defect. The two singularities are at N and S. (b) A boojum with singularity at O and a "fountain-like" configuration of the director.

emerges out like a fountain from a point on the surface of the drop. Such a defect was called a *boojum* by Mermin [16]. It is a point singularity on the surface of the drop. This defect is schematically depicted in Figure 1.8(b). Since there does not appear to exist in literature a proper theoretical description of boojums in uniaxial nematics, it will be discussed in a later chapter.

Biaxial nematics

In the case of biaxial nematics, point defects do not exist. The structures depicted by the radial hedgehog and the bipolar defect would also have line disclinations attached to them from the centre to the surface. This is because there are three director fields $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in a biaxial nematic. For example, $\mathbf{c}$ can be as in a radial hedgehog. Then $\mathbf{a}$ can be along longitudes and $\mathbf{b}$ along latitudes. This happens on every spherical surface. Thus, there will be a pair of $s = 1$ disclination lines from the poles to the centre of the defect. This is depicted in Figure 1.9. In literature such a structure, with line disclinations attached to a point defect, has been called a **Dirac** monopole in honour of Dirac who predicted a magnetic monopole

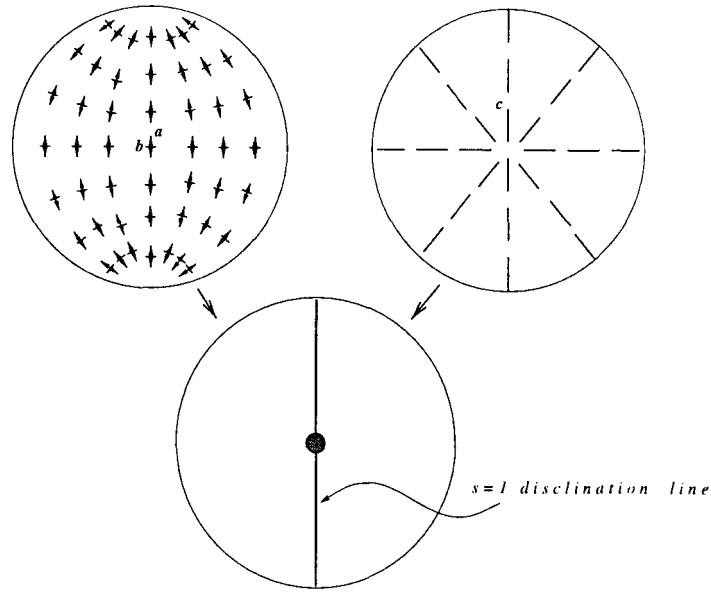


Figure 1.9: The directors **a**, **b** and **c** are along the longitudes, latitudes and radial direction respectively. The resultant Dirac monopole consists of two singular lines of strength $s = 1$ connected to the singularity in **c** at the centre.

with the magnetic field, **B** in the radial direction, and a singular line defect in the magnetic vector potential, **A** given by $\mathbf{B} = \nabla \times \mathbf{A}$.

A boojum is also an allowed defect in biaxial nematics. Interestingly, this does not have singular disclination lines connected to the point singularity. A boojum can be created by (i) bringing the two $s = 1$ line disclinations of a bipolar defect together to form a $s = 2$ disclination line, and then (ii) contracting the $s = 2$ line disclination by shifting the point singularity in **c**, along the disclination line, from the centre to the surface of the drop. This leads to a point singularity on the surface with the disappearance of the $s = 2$ disclination line. Figure 1.10 shows the structure of the end state. Thus, in this process a Dirac monopole has been transformed into a boojum.

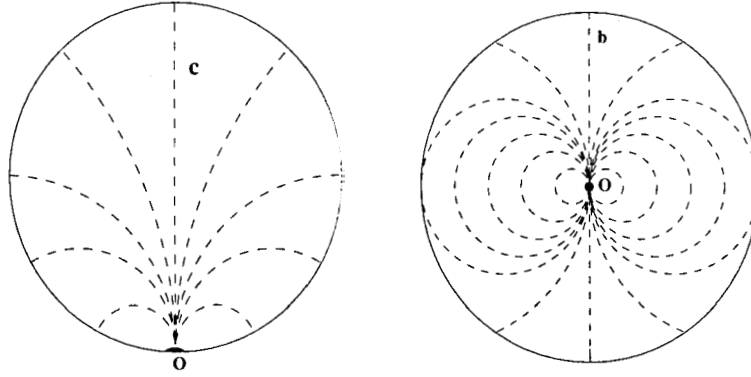


Figure 1.10: Boojum in biaxial nematics. In the meridional plane, the c director has a "fountain-like" structure. The a and b directors get mapped onto the surface in a $s = 2$ director configuration. The singularity of the b director configuration on the surface of the sphere is shown. The point singularity is at O which is a point on the surface.

1.1.3 Kinks or walls

Apart from the topologically singular and nonsingular line defects described so far, the symmetry of nematics and smectic-C also allow for very interesting nonsingular planar topological defects in an external electric or magnetic field. The field contribution to the free energy density is

$$F_{elec} = -\frac{\epsilon_a}{2}(\mathbf{n} \cdot \mathbf{E})^2$$

for electric fields. Here, $\epsilon_a = \epsilon_{||} - \epsilon_{\perp}$ where $\epsilon_{||}$ and $\epsilon_{\perp}$ are the dielectric constants parallel and perpendicular to $\mathbf{n}$. For magnetic fields, the contribution to energy density is

$$F_{magn} = -\frac{\chi_a}{2}(\mathbf{n} \cdot \mathbf{H})^2$$

where $\chi_a = \chi_{||} - \chi_{\perp}$ and, $\chi_{||}$ and $\chi_{\perp}$ are the diamagnetic susceptibilities parallel and perpendicular to $\mathbf{n}$. In Figure 1.11, one such planar defect is shown in a nematic with $\chi_a > 0$ placed in a magnetic field. When the field is along the x -axis, then the states at $z = +\infty$, and $z = -\infty$ have $\mathbf{n}$ parallel to $\mathbf{H}$ and they are equivalent in their dielectric or diamagnetic energies. Thus a distorted structure shown in Figure 1.11, can connect these two states, called base states, through a nonsingular continuous deformation of the director through 180° along the z -axis. This is the director configuration in every plane parallel to the xz -plane. Thus, it is a bend wall parallel to xz -plane.

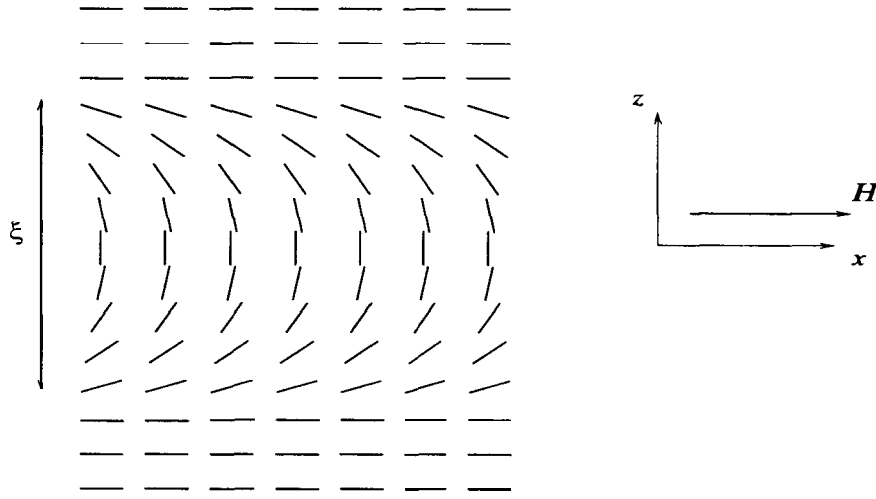


Figure 1.11: An external field, $\mathbf{H}$, is along the x -axis. A static π kink is a permitted topological defect. The director turns through an angle of π as we go from $z = -\infty$ to $z = \infty$.

These were called walls by Helfrich who was the first to discuss them. These days, they are called kinks. In the present case, we have a π kink. And, the kink shown is rich in the bend distortion of the director. It is easy to construct in a similar manner, kinks which are splay-rich or have only twist distortions. The bend-rich kink director configuration, as shown in Figure 1.11, is described by $\mathbf{n} = (\cos \theta, 0, \sin \theta)$ with $\theta = \theta(z)$. Then, from the equation of elastic and magnetic torque balance, we

can show that

$$\theta = 2 \tan^{-1} \exp(z/\xi) \quad (1.4)$$

Here, ξ is the electric (or magnetic) coherence length defined by $\sqrt{K/\epsilon_a E^2}$ (or $\sqrt{K/\chi_a H^2}$). The energy per unit area of the wall or kink is $E = 2 K/\xi$. Unlike the case of disclinations, the energy of a wall is finite in spite of its infinite extent along z -axis. Similar kink states are possible in c vector field of smectic-C in an external field.

Apart from such static kinks, there are also propagating kink states. It is easy to see that a static kink connects two base states that are energetically equivalent in the presence of an external field. And, any two states at the same distance from the centre of the kink are also equivalent energetically. If any of these two conditions are violated, then the kink would start moving.

The violation of the first condition is met if a kink connects a stable state at $z = -\infty$ to a metastable or an unstable state at $z = \infty$. An example of this possibility in smectic-C is shown in Figure 1.12(a). When the magnetic field is applied at an angle with respect to the layers, it is immediately apparent that in a π -twist wall connecting c and $-c$, the base states are not energetically equivalent. The favoured state with n at the smaller angle with respect to H will invade into the unfavoured state of n which is at a larger angle with respect to H . The stable base state will invade into the unstable or metastable base state as a uniformly moving front. In this case, the kink is supposed to move due to broken base state symmetry. Similar situations can be realized in other liquid crystalline systems, and in a later chapter we shall study the related problem of front evolution into a kink and its propagation. This is a topic of current research interest.

Even if the base states are energetically equivalent, the kink could move because the second condition is violated, *viz.* two states at the same distance from the

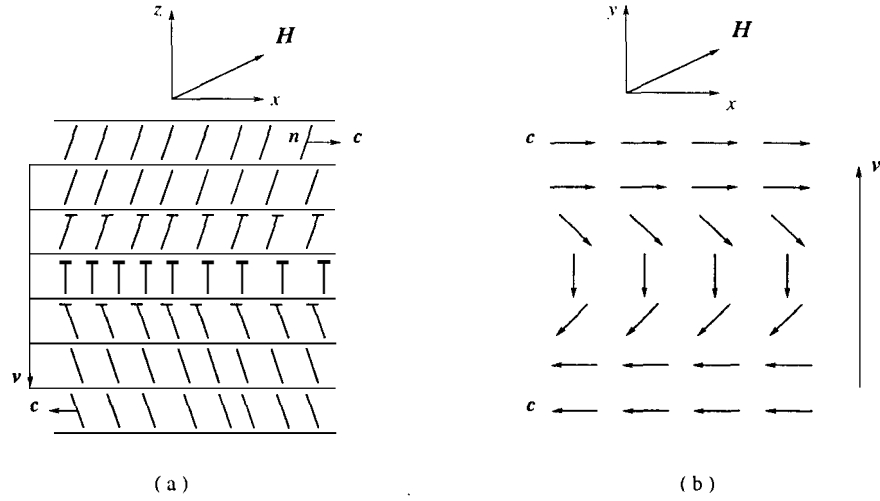


Figure 1.12: (a) A magnetic field is applied at an angle with respect to the layers of smectic-C. This is a case of broken base state symmetry. The π twist wall in c propagates uniformly with a velocity, v , as shown by the arrow. The nail heads indicate that the director comes out of the plane. (b) A bend-rich π wall in c in a rotating magnetic field. The field rotates in the plane of the smectic layers, *i.e.*, xy -plane. The base states follow the field at a constant angle. The direction of propagation of the uniformly moving wall is shown. This is due to broken core symmetry.

centre of the kink are not energetically equivalent. Such a situation can be easily realized in smectic-C when a magnetic field, H , is rotated about the layer normal in the xy -plane. From the dynamics of c , it can be shown that when the frequency of rotation is below a certain critical value, c follows the field H at a constant angle. A π kink is considered in this uniform state. In Figure 1.12(b), this situation is shown. Even though the base states are at the same angle with respect to H , *i.e.* they are energetically equivalent, it can be seen that there exists no symmetry within the kink. It is apparent that the c on one side of the centre is inclined closer to the field than c at an equal distance on the other side of the centre. Thereby, the favourable side will invade into the unfavourable side leading to kink movement. Here again, we have an example of a uniformly moving front. Very similar arguments are valid in a nematic for a π kink in a rotating field. In this geometry, the kink is supposed

to move due to broken core symmetry.

In a later chapter, new results on the motion of kinks will be presented, and it will be shown that the kink dynamics can be extremely complex leading to many new and interesting features. In this context, it may be remarked that in certain cases, it would be easier to realize certain features of kink dynamics in smectic-C than in nematics.

1.2 Hydrodynamics of liquid crystals

Study of the propagation of kinks in nematics and smectic- C naturally relies on the hydrodynamic equations that govern these systems. In a chapter of this thesis, a theory of hydrodynamics of smectic-C is formulated, and it has the features of both nematics and smectic-A. In this section, the basic features of the hydrodynamics of nematics and smectics is discussed briefly.

1.2.1 Nematodynamics

The hydrodynamic equations of any phase of matter are always in terms of variables that are characterized by slow decay times proportional to some power of the wavelength of the fluctuations of these variables. The number of such variables is determined by the sum of the number of conservation laws and the number of "continuous broken symmetries". As in ordinary fluids, here also we have the density, the three components of the velocity and the energy density as five hydrodynamic variables associated respectively with the conservation of the mass, the three components of momentum and the energy. In addition to these, we have variables describing dynamics of the director. Since there is the constraint $|\mathbf{n}| = 1$, there are two dynamical equations that describe the dynamics of the director, $\mathbf{n}$. Thus, there are seven hydrodynamic equations in uniaxial nematics.

A nematic flows with translational motions of the fluid particles coupled to

orientational motions of the molecules. The nematodynamics as developed by Ericksen, Leslie and Parodi [17-19] is mainly used in analysing flow behaviour of nematics. According to this description, in a uniaxial nematic, the stress tensor is asymmetric. The dissipative part of the stress tensor has symmetric terms that describes viscous dissipation due to shear flows as in ordinary fluids but appropriately generalized to accomodate the inherent anisotropy of nematics. The asymmetric part describes dissipative angular motions. This also allows for viscous dissipation due to rotations of the director alone, *i.e.*, without any macroscopic flow. The equations of nematodynamics imply that viscous torques on the director are balanced by torques due to a variation of the elastic free energy density which in turn results from spatial variations in the director, or torques resulting from external electric or magnetic fields.

1.2.2 Dynamics of smectics

In certain situations, the general principles underlying nematodynamics of $\mathbf{n}$ can be applied to the $\mathbf{c}$ vector of Sm-C. But, in general, there are important additional features. Even in some recent investigations, the $\mathbf{c}$ vector dynamics has been considered to be nematic-like. Such simplified theories have been applied to the dynamics of switching in chiral and achiral Sm-C, and in various flow geometries. In these studies, certain important consequences of smectic being layered structures have not been incorporated. The hydrodynamic theory of smectic-C has been reformulated and this is the subject matter of a chapter of this thesis. Here, the essential aspects of the hydrodynamics of smectic-A are briefly presented which acts as a good background to the hydrodynamics of all layered systems including that of smectic-C.

The first continuum theory for the Sm-A phase was proposed by de Gennes [20]. Here, the director $\mathbf{n}$ is normal to the layers. The layer displacement $\mathbf{u}$ is the hy-

hydrodynamic variable linked to the broken translational symmetry. Along the layer, there is fluid-like flow. But, what makes the dynamics of the Sm-A phase unique is that the velocity of the molecules and the velocity of the layers at the same spatial location and along the layer normal may not be the same. This process is called permeation. As pointed out first by Helfrich [21], molecules may permeate through the layers and the permeation flow is like that through porous media. It should be noted that in the case of smectics, the porous media and the permeation flow are both provided by layers and the constituting molecules. In any flow of smectics involving permeation, it is found that the apparent viscosity is usually very large, a million times more than that found in nematics or in smectics with layers slipping past one another. Another feature of smectics is that the layers could experience compressive or dilatative deformation in addition to the curvature of the layers. These additional features are essential to a proper continuum theory of smectics.

There are six hydrodynamic equations in Sm-A. Four of these are similar to that of a simple fluid, and they result from the conservation of mass, energy and the two components of momentum along the layers. The fifth equation resulting from conservation of momentum along the layer normal has an extra 'force' term due to the elastic deformation of the layers. The sixth equation encompasses permeation process. It implies that any relative velocity between the molecules and the layers at the same location and along the layer normal results in a permeative force which is proportional to the force necessary to compress, dilate or curve the smectic layered structure.

In a later chapter of this thesis dealing with the hydrodynamics of smectic-C, the nature of permeation and its consequences are explored in more detail. Interestingly, this system combines the hydrodynamics and elasticity of uniaxial nematics and smectic-A apart from additional features due to its inherent biaxiality.

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