

57. STRANGE BUT TRUE

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We come across once in a way some very unusual and strange facts which seem unbelievable. Such situations exist in science also. The following examples may seem unbelievable but they are true. They give a glimpse of the peculiar world of science.

Toughness of diamonds

It is said that every great 'diamond' has a trail of crimes associated with it. So alluring is the beauty of a diamond that it has enticed man to go to all lengths to possess it through fair or as usually the case, foul means. But an interesting twist to crimes associated with diamonds probably originated here in Karnataka that too under the benevolent kings of Vijayanagar empire. Many were done to death by the Raya rulers of Vijayanagar simply because these people ventured to please the Kings by gifting them diamonds. In the Kingdom of Rayas, the person who presented a diamond had to demonstrate in the court that it is genuine. One particular test that a diamond had to pass was the 'Hammer test'. It was believed that only diamond would withstand a hammer blow in view of its hardness. Whenever a supposed diamond broke into pieces under the blow of a hammer it is said that the person who had brought it as a gift was beheaded, since he was adjudged to be a cheat. We do not know how many lost their lives in this way and for how many years this hammer test was practiced. This particular test evolved out of a misconception that hardness would mean toughness. This is, however, not true. The two are different properties. Hardness refers to resistance of a material to scratch but toughness refers to its resistance to cleavage. Like most crystals diamonds too cleave under a stress. Interestingly this very property of cleavage was discovered in India. Under a hammer blow the diamond cleaves. But unfortunately the Raya Kings of Vijayanagar and most importantly their advisers thought otherwise.

This mistaken notion of hardness crushed to pieces many a good diamond and also resulted in innumerable killings of innocent people.

Bees and the 'golden ratio'

We generally take it for granted that every animal has a pair of parents namely a father and a mother. But this is not always true. Some whiptail lizards are asexual and give birth to only females. Aphids switch from sexual reproduction in spring to asexual reproduction in summer. But in bees we are in for a greater surprise. A male also called a 'Drone' has no father at all. It has only a mother or a female as its parent. On the other hand a female has both a father and a mother. In other words the male needs only one parent while a female needs two parents. This surprising biological fact leads to an interesting mathematical consequence. Let us work out the family tree of a male (M). Its immediate ancestor is a female (F). Its grand parents are 2 in number viz., a male (M) and a female (F). Its great grand parents are 3 in number viz., FMF and its great grand parents are 5 in number viz., FMMFF. Its parents five generations earlier are 8 in number viz., FMFMFFMF. So on the sequence proceeds. The number of parents in the successive ancestors are 1, 2, 3, 5, 8, etc. This is the famous Fibonacci sequence. Every number after the second is the sum of the two immediate previous number. Also we find another surprising fact. The ratio of females to males in the various ancestral generations are $1/1, 2/1, 3/2, 5/3, 8/5$, etc., that is in the ratio of successive Fibonacci numbers. If we go back into the distant past this ratio of females to males will tend to $(1+\sqrt{5})/2$. This number is called the 'golden ratio'. It is an irrational number in the sense that it cannot be expressed as a ratio of integers. It appears in very many places in science. But in art also it has played a great role. Any rectangular frame whose sides are in this ratio appears to be the most proportionate

and beautiful. Thus all paintings were painted inside such rectangular frames. The Golden ratio was not the first irrational number to be discovered. The story of irrational numbers is quite pathetic by itself.

Irrational behaviours

We associate Pythagoras with his famous theorem concerning right angled triangles. Sadly he sacrificed a large number of oxen to propitiate the gods when he discovered this theorem. History also associates him and his followers with many more crimes perpetrated in the name of mathematical truth and beauty. He was a ruthless leader of a cult that did not tolerate any breach of secrecy imposed on its members. One of the greatest discoveries made by this group concerns irrational numbers. This was the first surprising consequence of the Pythagoras theorem. The diagonal of a square would always be $\sqrt{2}$ times its side. And this number $\sqrt{2}$ was strange in that like other familiar numbers it could not be expressed as a ratio of integers. Later on the same Pythagoras discovered that diagonal of a cube is 3 times its side. Even this turned out to be an irrational number. Much later a similar exercise on regular pentagons showed that its diagonal was $(1+\sqrt{5})/2$ times its side. Thus was born the most famous irrational number which later on acquired the name 'Golden ratio'. Unfortunately the secrecy to which the pythagorans were bound prevented them from sharing this discovery with outsiders. It is said that whoever passed this information on to others was done to death by the rest of the clan. How many perished due to this rigid cruel practice is not well recorded. What an irrational way of preventing knowledge from spreading.

Troublesome traffic

Movement of traffic in a network of roads has an interesting puzzle associated with it. In fact a traffic network is truly paradoxical. The total cost of travel in such a network and the congestion that goes with a particular route are two important features that any one using network would be concerned with. The total cost of a vehicular movement is the sum of the costs over the various branches covered during the transit through the network. Each driver seeks a route from the beginning to the end so that the total travel cost is a minimum. In other words any change in the path would lead to an increase

in the cost of travel. This means that in an equilibrium flow of traffic all vehicles travel by paths that have the same total cost.

When the network is not congested the change of the route by any one driver will not affect others. However, in a congested network change in the path of even one driver will not only change the cost of his or her travel but will also lead to changes in the cost of other travellers.

Again when the traffic is not congested adding new branches to the network will not affect the traffic flow at all. In other words the equilibrium remains undisturbed. Adding more branches only provides more choices to a driver. But this principle is inapplicable to congested networks. In 1968 Braess showed that such networks are paradoxical in behaviour. Adding more branches will neither save money nor time for anybody. In fact it even becomes counter productive. Braess showed that the cost of the travel actually increases for everyone. That is, after an equilibrium is attained by inclusion of new branches or channels is the wrong thing to do!

Interestingly, in recent times, in many other branches of science similar paradoxical behaviours have been observed. In electrical networks, hydraulic networks and networks with springs and strings an equivalent puzzle is seen. Adding more branches to such networks in no way adds to the performance of the network. Surprisingly, the overall performance may actually come down.

The moral of the story is that congestion in networks cannot be relieved by adding more links to the network.

Story of a flag

Soon after the French revolution a republic was established. Along with this France acquired a new national flag: This was a tricolour consisting of three equally wide coloured bands. It had a white band interposed between a red band and a blue band. Strangely the people complained that the three bands in the flag had unequal widths. The blue band appeared to be the widest and the red band the narrowest with the central white band having an intermediate width. This public reaction really shocked the founding fathers of the

republic. All accurate measurements indicated that the three bands had equal widths. In sheer desperation they appointed a committee to look into the public complaint and suggest remedial procedures. Fortunately the committee had some scientists on it. Probably one of them was even a physicist, because the primary cause for this asymmetrical appearance was traced to an optical illusion resulting from the chromatic aberration in the human eye. Due to this inevitable defect, which every ordinary lens, has the blue band's image will be cast farther from the retina and nearer the eye lens than the red band's image, with the white band imaged in between. Therefore to see the blue band the lens muscles will stretch the lens to a thinner shape and to see the red band they will stretch less resulting in a lens of a bigger curvature. Interestingly this is precisely the mechanism used by the eye to see far off and nearby objects respectively. The brain therefore interprets that the blue band is farther away from the observer than the red band is. As the bands are of equal widths this means the blue band will appear wider than the red band. After pinpointing the cause for this, the committee suggested the widths of the blue, white and red bands to be in the ratio of 30:33:37. Due to the optical illusion the flag appears to have bands of equal width though in reality the bands on the flag are of unequal widths.

Wars due to the length of a line

It is an elementary knowledge that any line connecting two given points must have a definite length. If a straight

line connects them it is quite straight forward to measure this length. Even if different people use different length units in the end the net length will be the same. Extending this equivalence of lengths, as measured on different scales, to international boundaries lead to wars between Portugal and Spain. Though the two countries agreed upon the exact location of their common boundary, yet they could never agree on the total length of the boundary. As elsewhere the boundary between these two countries passed through such natural barriers as river banks, mountain ridges and so on. In other words it was not straight but irregular. The point of dispute that led to these wars between them was that according to Spanish surveyors the length of the common boundary was 987 Km but according to the Portuguese surveyors it was 1214 Km. Now we know the root cause of this dispute. The Spanish and the Portuguese surveyors had used scales of different lengths to measure the distance. The smaller scale had covered deeper kinks in the boundary which the bigger scale could not reach. This had led to a greater length being covered by the Portuguese surveyors.

The most important lesson to be learnt is that no irregular curve has a definite length. The net length depends upon the scale used to measure it. Such curves are said to be fractal by nature.

