

Chapter 2

Observational strategy and the RRL modelling procedure

In this chapter, we discuss the observations carried out for this thesis. The telescope used is described in some detail, followed by a discussion of the strategy of observation, the observed sample and the data analysis procedure. We also describe in detail the modelling scheme along with the approximations used and their limitations.

2.1 Description of telescope used : Very Large Array (VLA)

We have used the Very Large Array¹ (VLA) for the projects described in this thesis. The VLA is a 27-antenna connected element interferometer, arranged in the shape of a Y. The individual antennas are movable along tracks which are the arms of the Y. Due to this mobility, the VLA is available in four different configurations, called A, B, C and D, corresponding to a maximum baseline of 36, 11, 3 and 1 km respectively. The shortest spacing (and hence the maximum angular scale to which the array is sensitive) decreases (increases) from A to D configurations. In order to obtain a near circular synthesized beam for observations of southern sources, hybrid configurations are also available in which the antenna positions in only the southern arm correspond to the next largest configuration. For example, the DnC configuration has the southern arm antennas in the C configuration positions whereas the other two arms are in the D configuration. The antenna positions are along a ‘log spiral’ and hence different configurations used at suitable frequencies will act as scaled arrays and yield similar uv -coverage. The frequency bands at which the VLA operates are centred around 74 MHz, 330 MHz, 1.4, 4.8, 8.3, 15, 23 and 43 GHz. The maximum available bandwidth is 50 MHz. Given the large number of antennas, the VLA is the most sensitive interferometer in the world at $\nu > 1.5$ GHz. Since the line emission from nearby starburst galaxies is expected to be ~ 1 mJy or less, we choose to use the VLA for our observations.

In figures 2.1 and 2.2 are shown the sensitivity attainable with the VLA for observations of thermal ionized gas. The minimum emission measure ($EM \sim n_e^2 l$) of ionized gas detectable through its bremsstrahlung continuum emission is shown in figure 2.1. Similarly, the minimum EM of ionized gas detectable through its RRL emission is shown in figure 2.2. The assumed observational parameters are mentioned in the figure captions. The RRL strengths were calcu-

¹The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.

lated for internal emission only. Any additional stimulated emission will reduce the minimum emission measure, and this reduction will be a non-linear function of the size of the HII region.

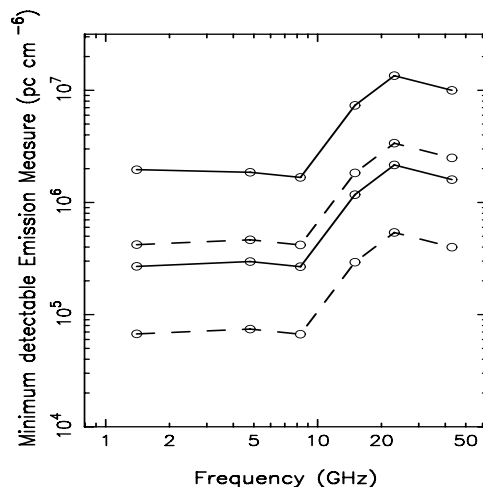


Figure 2.1: The minimum detectable emission measure of the photo-ionized gas through its free-free continuum emission, using the VLA. An on-source integration time of 6 hours, a bandwidth of 25 MHz with one IF and 26 usable antennas were assumed. The solid and dashed lines indicate the limits for a 5 pc and a 10 pc sized HII region respectively, in an external galaxy. The distance to the galaxy is assumed to be 5 Mpc and 10 Mpc for the lower and upper pair of curves respectively. The frequencies available at the VLA are marked as open circles.

2.2 Observations

This thesis deals with Hydrogen radio recombination line observations of nearby starburst galaxies. The motivation for observing these lines from starburst regions has been discussed in section 1.3.1. The observational details, the strategy employed, and the limitations of our observing technique are described here.

In principle, we would like to study ionized gas over a wide range of densities; from diffuse gas at $\sim 1 \text{ cm}^{-3}$ (Extended Low Density Warm Ionized Medium or ELDWIM; Roshi & Anantharamaiah 2001) to proto-stellar condensations at $\sim 10^7 \text{ cm}^{-3}$ (Super UltraCompact HII regions or SUCHII regions; Franco et al. 2000). Gas at different density ranges will best be probed by RRLs at specific frequencies, but in practice, the density range accessible to us is limited by the instrumental sensitivities at various frequencies. Detection of low density diffuse gas does not seem possible with current instruments (for e.g., see chapter 5) since this component of ionized gas usually lacks enough emission measure for detectable internal emission at low frequencies. Also, almost all starburst galaxies lack a background nuclear source bright enough to produce detectable stimulated line emission from this low density gas. Observations of very high density gas through cm-wave RRLs are hampered both by a large continuum optical depth and also by the relatively small number of such objects. A possible way of detecting such HII regions is through mm-wave RRLs, and this has been attempted for a few nearby starburst galaxies (Puxley et al. 1989, Puxley et al. 1991, Puxley et al. 1997, and Viallefond, private communication). Therefore, by using cm-wave RRLs, we are sensitive only to ionized gas with sufficiently large emission measures (see figures 2.1 and 2.2 for emission measure limits corresponding

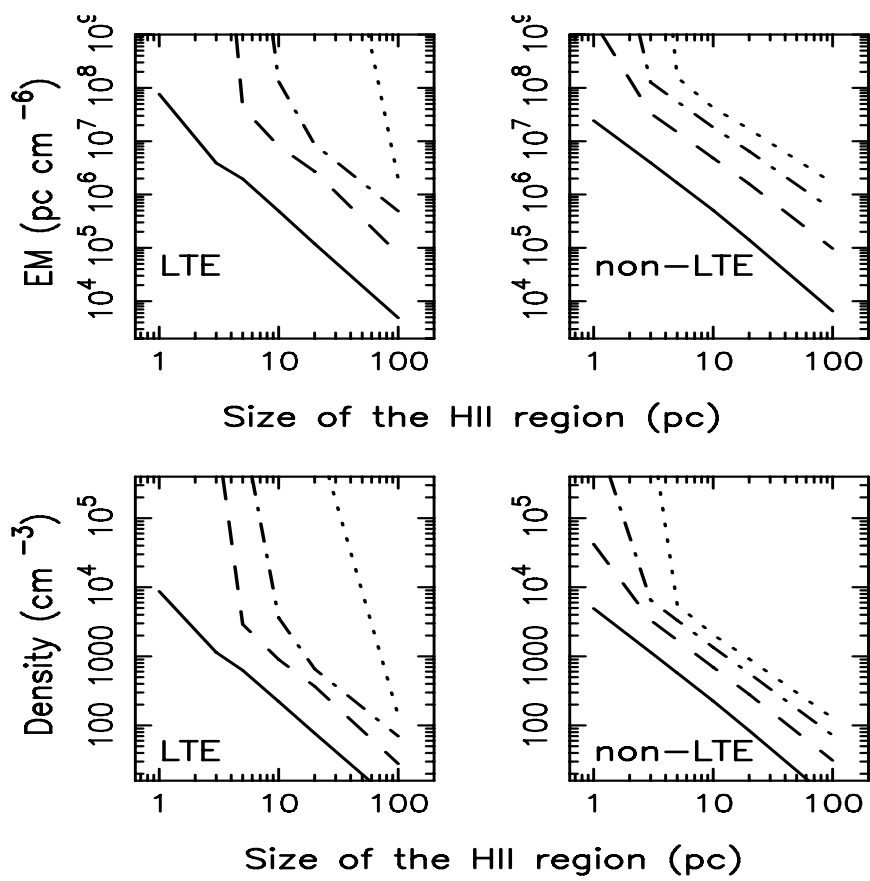


Figure 2.2: The minimum detectable emission measure (*top*) and density (*bottom*) of a HII region in an external galaxy through its 8.3 GHz H92 α RRL emission, using the VLA in the D configuration. The assumed parameters are: on-source integration time of 6 hours, 26 available antennas, one IF and the FWHM of the line of 200 km s $^{-1}$. The values were calculated assuming that the line formation is in *LTE* (*left*) and is not in *LTE* (*right*). The solid, dashed, dash-dot and dotted curves are for a galaxy at a distance of 0.5, 2, 5 and 10 Mpc respectively.

to the VLA), and to densities between few 10^2 cm^{-3} and few 10^5 cm^{-3} . This density range corresponds, in our galaxy, to classical HII regions up to compact HII regions.

We have used the VLA for all our observations, mainly because of its superior sensitivity and its flexible range of spatial resolution and frequency coverage. Nevertheless, it has a few limitations which constrain its use. One of the main drawbacks in using the VLA for this project is its bandwidth limitation. The widest available bandwidth is 50 MHz. Most extragalactic RRLs are wider than 50 MHz at frequencies higher than 15 GHz and are hence unobservable without using fairly complicated multi-bandwidth techniques. Second, the well-known 3 MHz ripple in the bandshape limits the stability in the spectral domain to about 1 in 100-500. In our observations, the spectral dynamic range is usually limited by this phenomenon rather than by the thermal noise. Our observations using the VLA are of two kinds –

1. Search for RRL emission from a new sample of nearby galaxies, and
2. Multi-line follow-up of a sub-sample of RRL-detected galaxies.

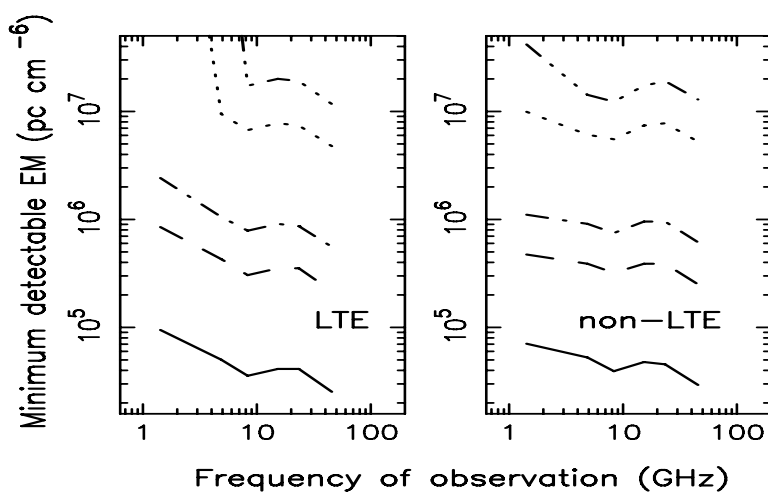


Figure 2.3: The minimum detectable emission measure of a HII galaxy in a galaxy 5 Mpc away, through RRLs, using the VLA in the D configuration. The assumed parameters are: on-source integration time of 6 hours, 26 available antennas, one IF and the FWHM of the line of 200 km s^{-1} . The values were calculated assuming that the line formation is in *LTE* (left) and that it is not in *LTE* (right). The curves from bottom to top are for densities of 10, 50, 100, 500 and 1000 cm^{-3} .

2.2.1 Search for RRLs from nearby starburst galaxies

Our search for recombination lines was carried out at 8.3 GHz with the VLA. The sensitivity of the VLA is maximum from 1.4 GHz up to 8.3 GHz, is much worse at 15 and 23 GHz and becomes significantly better at 45 GHz. Figure 2.3 shows the minimum detectable emission measure in RRLs as a function of observing frequency. This minimum value is least at 8.3 GHz and then at 45 GHz. We have chosen 8.3 GHz as the search frequency, as opposed to 45 GHz, due to (i) the bandwidth limitations at higher frequencies mentioned earlier, and (ii) since the resolution of the VLA at 45 GHz is far too high, as explained below. The VLA was used in the C and D configurations in these observations. This was to maximise the line flux per synthesized beam, this being a detection experiment. The D configuration synthesized beam at 8.3 and 45 GHz, using natural weighting, corresponds to a linear size of 120 pc and 20 pc for a galaxy 2 Mpc away. Since the line emitting region could consist of a collection of HII regions extended over the entire starburst region (typically less than one kpc), we chose to observe RRLs at 8.3 GHz using the VLA in the C or D configuration as the optimum combination of observing parameters.

The sample of galaxies were selected based on the following criteria :

- Zhao et al. (1996) showed that, for the sample of starbursts with detected RRLs, the $\text{H}92\alpha$ line strength is weakly correlated with the $\text{Br}\alpha$ luminosity (the scatter being a factor of a few). A sample of nearby starburst galaxies with published $\text{Br}\alpha$ fluxes were chosen and the expected $\text{H}92\alpha$ line strengths were computed. Galaxies with the latter value more than $\sim 0.3 \text{ mJy}$ were selected, based on the VLA sensitivity.
- Phookun, Anantharamaiah, & Goss (1998) showed that the nuclear radio luminosity is correlated with the $\text{H}92\alpha$ luminosity. Based on their work, we select those galaxies whose radio continuum flux densities in the starburst region are greater than about 40 mJy at 8.3 GHz.

- The widths of the Br α lines, published in the literature, should be less than $\sim 600 \text{ km s}^{-1}$, due to the bandwidth limitations of the VLA.
- The size of the starburst region, seen in the optical and NIR, should be between 2-20". The upper limit to the size arises from the need for the line emission to be spread over not more than 2-3 synthesized beam areas so that the line flux per beam is detectable. The lower limit is to avoid beam dilution.

Table 2.1: Parameters of galaxies

Galaxy	Heliocentric vel. (km/s)	S _{8 GHz} (mJy) ^a	Br α ($\times 10^{13}$ erg/s/cm ²)	Area of Br α emission	FWHM of Br α (km/s)
NGC 4038	1640	170	2.1	$\sim 12''$	350
NGC 1808	1000	160	5.3	6''	600
VV 114	6016	57	1.1	6'' ^b	450
NGC 7552	1585	50	18	6'' ^c	600
NGC 5253	404	47	7.0	4''	250
He 2-10	873	45	2.6	4''	300
II Zw 40	789	40	2.5	5''	220

^aFrom the starburst region except for NGC 4038, which is the total emission.

^b $\sim 60\%$ from a 6'' clump and the rest is extended.

^cThe emission is from a ring of 6'' diameter and $\sim 3''$ thickness.

The final sample consists of seven galaxies, whose properties are listed in table 2.1, along with their properties. Detailed notes on each galaxy, along with the data, models and interpretation of each of these galaxies are presented in chapters 4 and 6.

2.2.2 Multi-line observations of selected galaxies

Our objective in observing RRLs is to derive the local electron density and the size of the ionized gas responsible for the line emission. This can be achieved by measuring a single RRL, if multi-frequency radio continuum observations are available over the region of interest. The reasons for this are discussed in detail in section 2.3. On the other hand, there is a need for multi-frequency RRL observations as well. As explained previously, gas in a particular range of densities is expected to emit RRLs preferentially in a particular range of frequencies. Hence, to probe gas at widely different densities needs measurement of RRLs at different frequencies. This might not be feasible in practice due to instrumental limitations, summarised in section 2.2. Nevertheless, previous studies have shown (Anantharamaiah et al., 1993; Zhao et al., 1996, 1997) that models based on single-RRLs usually allow two classes of models : (a) a high density model, with $n_e \gtrsim 10^4$ and size of a few pc, and (b) a lower density model, with $n_e \sim 10^2$ - 10^3 and size of few tens of pc. Both these models are consistent with observations. A possible way to resolve this degeneracy is to observe either higher or lower frequency RRLs (see Zhao et al. 1997, fig 11). An additional reason to observe multiple lines is to attempt to build a multi-density model of the ionized gas, which is described in section 2.3.2. Therefore we have observed a few galaxies which have been detected in the H92 α line emission, at 15 GHz as well as 1.4 GHz in order to constrain the properties of high density and low density gas respectively. These observations will be dealt with in chapters 3 and 5.

2.2.3 Data analysis

We now briefly describe the data analysis procedure common to all our observations. The data obtained at the VLA uses the instrument in the standard interferometer mode, i.e., we obtain the complex visibilities (as a function of frequency) corresponding to the sky, as a function of time, from each possible pair of antennas. The theoretical framework which deals with obtaining images and spectra from the observed visibilities is well known and will not be discussed here (the book Thompson, Moran & Swenson 1986 is an excellent reference). All data analysis is carried out using standard algorithms available in the software package AIPS. Practical aspects of data analysis are dealt with in the collection of lectures from the NRAO Synthesis Imaging Workshop (Taylor et al., 1999). A pseudo-continuum dataset is obtained by averaging the central 75 % of the channels of the line dataset. This is a faithful representation of the continuum except that this data is not corrected for the various bandshapes of each antenna. Nevertheless, it is known that for the VLA, this averaged dataset is a fairly accurate representation of the true continuum, since the phase of the bandshapes does not have a significant variation across the band. Flux calibration is performed by observing a standard primary calibrator like 3C 48 or 3C 286 and bootstrapping the flux densities of all other sources to the known flux densities of these calibrators. Initial phase calibration of the continuum data is carried out using a nearby point source chosen from the VLA calibrator manual. This phase calibrator is observed every ~ 30 -45 minutes, for a duration of ~ 2 minutes. If the phase calibrator or the flux calibrator is not a point source up to the maximum uv distance achieved in the observation, i.e., if the image of the calibrator, made using all the visibilities, is even marginally resolved with the synthesized beam obtained, then only those inner visibilities are used for the bootstrapping procedure for which the calibrator remains a point source. The complex gains obtained from this phase calibrator are used to derive approximate time dependent antenna-based phases. These phases are then interpolated in time to obtain the correction terms for the on-source scans which are temporally located in between two phase calibrator scans. The calibrator and source visibilities are ‘flagged’; visibilities which have amplitudes higher than, say, 10σ are discarded. So are antennas with significantly lower gains than the rest of the array, and also domains in the baseline-time-frequency space which show regular patterns, for e.g., ripples, phase jumps or significantly offset gains. The time dependence of the amplitudes of the complex gains is not calibrated using the phase calibrator and a single time-averaged value for the amplitude of each antenna is transferred to the source visibilities. These visibilities are then imaged and deconvolved using the CLEAN algorithm, which was originally devised by Hogbom (1974). The Cotton-Schwab (Schwab, 1984) variant of the Clark FFT-based (Clark, 1980) CLEAN algorithm has been used, wherein a major cycle subtraction of CLEAN components is performed on the ungridded visibilities, thus increasing accuracy. Initially, a low resolution image is created by tapering the visibilities with a gaussian in order to identify possible strong sources within about two primary beam diameters of the phase centre. While imaging the galaxy of interest, these sources are also imaged separately and simultaneously deconvolved. This needs to be done primarily at 1.4 GHz and 0.3 GHz for the more compact configurations of the VLA. During the CLEANing process, tight fitting boxes are defined around the known strong sources, atleast for the initial cycles of iteration. All our target sources have strong enough flux densities that we can perform self-calibration with a solution interval as small as 2-5 minutes. Hence, after imaging the source data using the initial phase calibration, the source visibilities are iter-

ated upon, using the technique of self-calibration (Readhead & Wilkinson, 1978; Cornwell & Wilkinson, 1981), in order to obtain the final images. The source visibilities, being complex, have a complex error, or gain. However, it is known that for the VLA, the effects due to the phase of this correction term is much more than that of the amplitude. Whether the amplitudes or the phases are more in error can be gauged from the initial images themselves, since phase errors lead to features which are antisymmetric about the phase centre and amplitude errors lead to symmetric features. Also, it is usually found that the timescale of variation of the amplitude is longer than that of the phase. Hence, initial rounds of self-calibration are performed only for the phases. The model used for subsequent self-calibration cycles is the CLEANed image of the source, obtained during the previous cycle. When the change in the phase corrections from one self-calibration cycle to the next becomes comparable to the expected noise, then the final phase corrected visibilities are self-calibrated for both amplitudes and phases. Typical amplitude corrections are $\lesssim 10\text{-}15\%$. With every cycle of self-calibration, the dynamic range of the image is expected to increase, and the rms of the noise is expected to decrease towards the theoretical value. However, the rms of the noise in the final image is often limited by missing short spacings, low elevation due to southern declination of the sources, poor uv coverage and dynamic range. A more important diagnostic of the fidelity of the image is the intensity distribution of the noise, away from the source. These regions are examined for large scale correlated patterns which usually take the form of either ripples or scaled versions of the dirty beam (which is the fourier transform of the sampling function in the uv plane, otherwise known as the transfer function). Self-calibration should, for the most part, remove these artifacts. Exceptions arise due to two main reasons: inadequate uv coverage and non-closure errors. The former cannot be completely remedied, but its effect can be mitigated by tapering and weighting the visibilities in order to obtain a more suitable dirty beam, and also by a judicious choice of spatial constraints (in the form of boxed areas) while CLEANing. The various weighting schemes, though not entirely adequate, have been augmented recently based on the work of Briggs (1999) which has been used extensively. The latter problem, that of non-closure errors, cannot be corrected easily and we do not attempt a baseline-based calibration in order to mitigate the effects of this error. Hence, individual antennas or baselines identified as causing significant non-closure errors are discarded.

The frequency dependence of the antennas-based complex gains of the instrument is determined by bandpass calibration. This is performed by observing a strong source once every hour or so. This bandpass calibrator is selected such that it is a point source or atleast that its structure does not vary appreciably across the observed bandwidth. Since the VLA correlator is an XF or a lag correlator, the response of a δ function or a sharp feature in the time domain is a *sinc* function in the frequency domain. This causes what is termed as Gibbs ringing, which can be dealt with by hanning smoothing the data online, inside the correlator. Since we require large bandwidths for high frequency RRL observations and since the number of channels available per IF is less for larger bandwidths, we opt for off-line hanning smoothing instead. Hence, both the bandpass calibrator data and the source data are hanning smoothed using a triangular function covering five channels with weights 0, 0.5, 1.0, 0.5, 0. The initial calibration as well as the self-calibration solutions are applied to the bandpass calibrated line data. The continuum emission is subtracted from the line data by fitting linear functions to the real and imaginary parts of the visibility spectra (Cornwell, Uson & Haddad, 1992). Only channels identified as

free from line emission are used in fitting the complex baseline. The continuum-subtracted dataset is imaged to produce a 3-dimensional image with RA and Dec as two coordinates and frequency as the third. The process of continuum subtraction ‘redistributes’ the noise in the line cube, i.e., the rms of the noise in the channels used in fitting the baseline is suppressed and that of the remaining channels is enhanced (Cornwell, Uson & Haddad, 1992). This enhancement and suppression is closer to unity for a sufficiently large number of channels. The bandwidths used in our observations are at least 25 MHz in order to cover the entire line emission at high frequencies and hence the number of channels available is limited. Therefore, the non-uniformity of the noise across the band is readily apparent in the line cubes. Additionally, it is not necessary that the rms of the noise in the R.A.-Dec plane for any given channel is the same as the rms of the noise for a single pixel as a function of frequency. Since most of the line emission we detect is spatially unresolved, we use the rms of the noise in the entire cube as a conservative estimate. All further analysis of the line emission is carried out on the line cube.

2.3 RRL modelling

The observed line strengths depend on the electronic level populations and the subsequent radiative transfer through the medium. The level populations are determined by the local electron density (n_e), electron temperature (T_e) and the ambient radiation field. Given the typical radiation intensities and electron densities in the ISM, collisions are expected to dominate the determination of level populations and therefore we ignore the effects of the ambient radiation field in our analysis. Hence, using an observed RRL, the properties of the ionized gas like n_e , T_e and size of the region can be derived if the line formation process can be modelled. However, the determination of these parameters is neither analytical nor is it unique. The former problem arises since the formation of RRLs is almost always not an *LTE* process and hence the level populations themselves depend on electron density. Nevertheless, the values of density, temperature and size can be constrained due to several factors. The observed line strength can be used to derive the required emission measure (and hence the line-of-sight path length l) as a function of assumed density, for a given transverse size L . Imaging observations, like ours, yield an upper limit to the transverse size, which in turn limit the range of solutions in the EM- n_e space. Even for a range of filling factors, the value of l cannot be orders of magnitude different from L for reasonable geometries. However, the main constraint on the upper bound on l arises due to continuum optical depth (τ_c) effects; increasing l increases τ_c , and the line strength decreases for sufficiently optically thick gas (even for non-*LTE* conditions). The lower envelope of the EM (and hence l) vs. n_e is determined by the assumption that the ionized gas produces all of the observed line emission.

The two main factors though, apart from those mentioned above, which constrain the solutions for n_e , T_e , l and L are the non-*LTE* nature of the line formation process and the free-free emission spectrum. The efficiency of line emission at a particular frequency for a given mass and EM of the gas depends sensitively on the local density due to non-*LTE* effects. Additionally, since the free-free emission of the line emitting gas can be calculated at various frequencies and compared with observations, multi-frequency continuum data also strongly constrained the solution space: for example, very high emission measures can be ruled out by using continuum measurements at frequencies high enough that $\tau_c < 1$. Measurements of two different recombi-

nation lines provides a stronger constraint on density, since the line ratio in non-*LTE* conditions is a sensitive function of the electron density. Figure 2.4 shows the dependence of the flux ratio of the 8.3 GHz H92 α line to the 15 GHz H75 α line as a function of density. The curves plotted in the figure are essentially flat ($\sim n_e^0$) for *LTE* conditions.

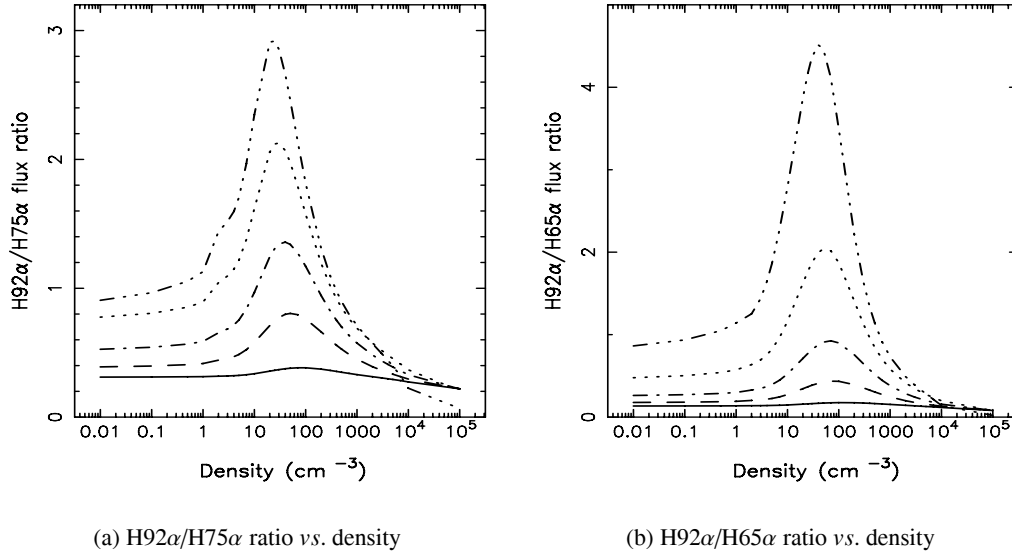


Figure 2.4: The RRL flux ratios H92 α /H75 α (fig 3a; left) and H92 α /H65 α (fig 3b; right) as a function of density for various values of EM, for non-*LTE* conditions and $T_e=7500$ K. The curves from bottom to top are for an EM of $10^7 \times (0.1, 1, 3, 10, 100)$ pc cm⁻⁶, i.e., τ_c at 8.3 GHz of $0.056 \times (0.1, 1, 3, 10$ and 100) respectively.

The radiative transfer of the line emission is carried out assuming a static medium. Though the line optical depth does indeed assume values higher than unity in some cases, the line intensity always remains much smaller than the continuum intensity. Hence we do not expect the velocity field to affect the radiative transfer of the line emission significantly. The optical depth is calculated for a gaussian line profile. The width of this profile can either be calculated using the assumed electron temperature, density and a rms turbulent velocity or can be assumed to be equal to the observed value, which is larger than the intrinsic width by almost an order of magnitude in all the observed galaxies. Whether the former or the latter assumption is used in the model depends on whether the observed line width is assumed to arise from relative motions predominantly in the lateral direction or in the direction of the line of sight. The models considered are of two classes : single-density models and multi-density models. In the former, all of the ionized gas is assumed to have a constant density n_e , to be constrained. In the latter, a multitude of densities is assumed to co-exist and contribute to the continuum and line radiation. Multi-density models can be constructed either in terms of a minimum number of distinct different density components or in terms of a continuous range of densities with some distribution function. In most cases, results from single-density models have been quoted and used. The justification for this single-density approximation is discussed in section 2.3.2. The various geometries considered in the modelling of the ionized gas are a spherical HII region, a collection of spherical HII regions and a rectangular slab. The solution space for the various geometries are not very different from each other; the range of derived values obtained is dominated by the degeneracy in model parameters and errors in the measurements rather than geometry-dependent

effects. Yet another parameter in the model is the intensity of the background radiation which gives rise to stimulated line emission. Though the total continuum flux density from the line emitting region can be measured, the exact geometry of this radiation and the ionized gas along the line of sight is unknown. Therefore, the fraction of continuum intensity which arises from behind the HII region is varied in the models, thereby changing the fraction of the total line emission which is due to stimulated emission. A higher fraction of stimulated emission will naturally reduce the emission measure and the mass of the ionized gas required to explain the observations. What is observed is the total continuum emission which includes both thermal and non-thermal emission, wherein the latter component includes the effects of free-free absorption by thermal gas. For a given model, the expected free-free emission can be calculated. For the assumed geometry of the ionized gas and the relative locations of the non-thermal and thermal emission along the line of sight, the amount of free-free absorption of the ‘true’ non-thermal emission can be calculated (which is simply $e^{-\tau_c}$ for a plane parallel slab of thermal gas of unit filling factor in front of the non-thermal emission). Hence, correcting for this absorption, the ‘deabsorbed’ continuum can be calculated. This deabsorbed non-thermal continuum emission could consist of multiple components of non-thermal emission as well as thermal emission from gas which does not emit observable RRLs. Hence this quantity is estimated by two different methods, which are equivalent in that one attributes the uncertainties to the spectral index of the emission and the other attributes the uncertainties to the optical depth of a possible non-RRL emitting thermal component. In the first method, the deabsorbed continuum is calculated self consistently from the total observed continuum, the calculated free-free emission and also the free-free optical depth at all frequencies at which continuum measurements exist. Any discrepancy is then attributed to a varying spectral index of the non-thermal continuum, constrained to be not steeper than -0.9 . In the second method, the deabsorbed continuum at the highest observed frequency is assumed to be the value of the true non-thermal emission and a spectral index is assumed a priori (and is a free parameter to be varied in the model). The stimulated line and free-free continuum emissions are calculated using these two values and any discrepancy is attributed to the optical depth of an unknown thermal component which emits negligible RRLs at the observed frequencies. This optical depth of the unknown component is constrained to be positive for physical solutions.

2.3.1 Single-density models

The free-free continuum, the RRL flux density and the deabsorbed background radiation are given by

$$\begin{aligned}
 S_{ff} &= \frac{2kT_e \nu^2}{c^2} \int_{\Omega_{\text{HII}}} (1 - e^{-\tau_c}) d\Omega, \\
 S_l &= S_o e^{-\tau_c} (e^{-\tau_l} - 1) + \frac{2kT_e \nu^2}{c^2} \left(\frac{b_n \tau_l^* + \tau_c}{\tau_l + \tau_c} \right) \int_{\Omega_{\text{HII}}} (1 - e^{-\tau_c - \tau_l}) d\Omega - S_{ff} \\
 \text{and } S_o &= \frac{(S_{\text{obs}} - S_{ff})}{1 - \frac{\Omega_{\text{HII}}}{\Omega_{\text{beam}}} (1 - e^{-\tau_c})}
 \end{aligned} \tag{2.1}$$

which can be solved easily for a rectangular slab of ionized gas of uniform density. The deabsorbed continuum is composed of both non-thermal radiation as well as free-free emission from gas which does not contribute to the observed RRLs.

The expression for the continuum flux density in the case of a spherical HII region was given by Olmon (1975) and Panagia & Felli (1975),

$$S_{ff} = \frac{2kT_e v^2}{c^2} \Omega_{\text{HII}} \left[1 - \frac{2}{p_c^2} \{1 - e^{-p_c} (1 + p_c)\} \right], \quad (2.2)$$

where $p_c = \tau_c(0)$ is the peak continuum optical depth, towards the centre of the spherical HII region. We can derive a similar expression for the spontaneous part of the line emission and also the deabsorbed continuum intensity I_0 . The stimulated emission part, for a background continuum intensity I_0 will then be given by

$$S_l^{\text{st}} = I_0 \Omega_{\text{HII}} \left[\frac{2}{p_{c+l}^2} \{1 - e^{-p_{c+l}} (1 + p_{c+l})\} - \frac{2}{p_c^2} \{1 - e^{-p_c} (1 + p_c)\} \right] \quad (2.3)$$

where $p_{c+l} = \tau_c(0) + \tau_l(0)$. Further, the deabsorbed continuum can also be assumed to be mixed, that is, spatially co-existent, with the RRL emitting gas. In this case,

$$S_o = \frac{(S_{\text{obs}} - S_{ff})}{1 - \frac{\Omega_{\text{HII}}}{\Omega_{\text{beam}}} \left\{ 1 - \frac{2}{p_c} \left(1 - \frac{(1 - e^{-p_c})}{p_c} \right) \right\}} \text{ for a sphere} \quad (2.4)$$

$$\text{and } S_o = \frac{(S_{\text{obs}} - S_{ff})}{1 - \frac{\Omega_{\text{HII}}}{\Omega_{\text{beam}}} \left\{ 1 - \frac{(1 - e^{-\tau_c})}{\tau_c} \right\}} \text{ for a rectangular slab} \quad (2.5)$$

The expressions for stimulated and spontaneous line emission can be similarly derived.

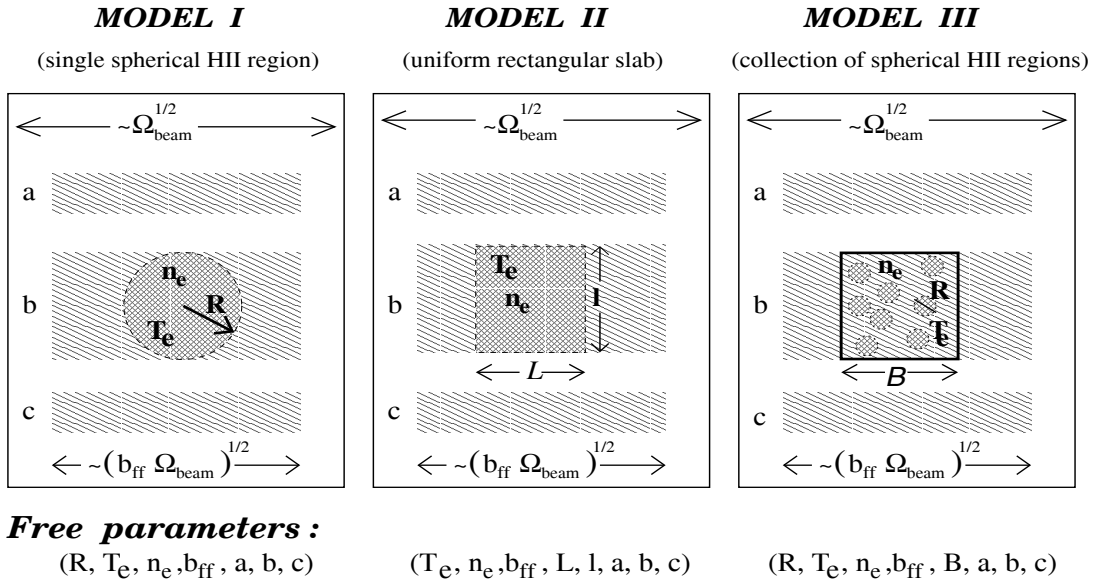


Figure 2.5: A schematic of the three single-density models used to analyse RRL data. (a) Model I consists of a single spherical HII region, (b) Model II consists of a uniform rectangular slab and (c) Model III consists of a collection of spherical HII region. The free parameters in each model are also indicated in the figure. The deabsorbed non-thermal emission is assumed to be behind, mixed with, and also in front of the thermal gas in the ratio of a:b:c, all three of which are free parameters. In practice, solutions are found for extreme values of a, b and c.

The input parameter space consists of the density n_e , which is varied between 10^{-2} cm^{-3} to 10^6 cm^{-3} , temperature T_e , which is varied between 2500 K and 12500 K, and the size and geometry of the ionized gas. The size parameter is the diameter of the HII region for a spherical model (Model I), the line-of-sight path length l and the transverse length L (parametrized as the beam filling factor $b_{ff} = L^2/\Omega_{\text{beam}}$) for a rectangular slab model (Model II) and the diameter l and the number of such HII regions N for a collection of spherical HII region model (Model III). The basic features of each model and their free parameters are shown in figure 2.5. The expressions used in models I and II were discussed above and can be extended for the case of Model III. In this model, since a given line of sight could intersect more than one HII region in the case of overlap, the number of HII regions encountered per line of sight will determine the radiative transfer. Hence we model the collection of HII regions to be randomly located in a three dimensional box of side B and this quantity is a free parameter, constrained to be less than L , where L is defined by $\Omega_{\text{beam}} = \pi L^2/(4D^2)$, where D is the distance to the galaxy. The average number of HII regions per line of sight is $N_{\text{los}} = N \frac{l^2}{B^2}$. In an attempt to avoid performing monte-carlo calculations, we calculate the radiative transfer assuming that the actual number of HII region in every line of sight is indeed equal to this average value, N_{los} . The free-free emission, deabsorbed continuum and the total line emission can then be approximated as

$$S_{ff} = \frac{2kT_e v^2}{c^2} \frac{\pi B^2}{4D^2} f_1 \left[(1 - e^{-f_2 \tau'_c}) \right], \quad (2.6)$$

$$S_o = \frac{(S_{\text{obs}} - S_{ff})(1 + X)}{(1 + X) - f_{\text{bck}} f_1 (1 - e^{-f_2 \tau'_c})} \quad (2.7)$$

$$S_l^{\text{st}} = f_{\text{bck}} S_o \left(\frac{f_1}{1 + X} \right) \left(e^{-f_2 \tau'_{c+l}} - e^{-f_2 \tau'_c} \right) + \frac{2kT_e v^2}{c^2} \frac{\pi B^2}{4D^2} f_1 \left[c_2 (1 - e^{-f_2 \tau'_{c+l}}) - (1 - e^{-f_2 \tau'_c}) \right] \quad (2.8)$$

where f_{bck} denotes the covering factor for the deabsorbed continuum, and is assumed to be either L^2 or B^2 in the models, $f_1 = (1 - e^{-N_{\text{los}}})$ and $f_2 = \frac{N_{\text{los}}}{(1 - e^{-N_{\text{los}}})}$. The deabsorbed continuum intensity is assumed to be distributed partly behind and partly in front of the thermal gas. Suitable expressions can also be derived in the case where it is mixed with the thermal gas instead. The primed optical depths, τ'_c and τ'_{c+l} , are equivalent optical depths calculated using eqn.s 2.2 and 2.3, assuming spherical geometry. c_2 is the non-LTE correction factor to the source function. The value of N is derived by iteratively solving the equations for the deabsorbed continuum and the total line emission. We shall now discuss model IV, the multi-density model, and the validity of the single-density approximation.

2.3.2 Multi-density models and the single-density approximation

As was described in section 1.2.2.3, a particular recombination line is emitted most efficiently by gas at a particular density; gas at lower densities preferentially emit lower frequency RRLs and vice versa. This property is due to non-LTE effects and is the main motivation for the single-density approximation, since only gas from a narrow range of densities will contribute towards an observed line. Therefore, if the observed region has gas at a wide range of densities such that the emission measures of gas at different densities are the same, then almost all of the observed line emission at a particular frequency will arise from that part of the gas which is at

the corresponding density where the line intensity peaks. This is illustrated in figure 2.5a for the H92 α line, which is most efficiently emitted by gas at $\sim 10^2 \text{ cm}^{-3}$. Therefore, if multi-line data is available at various frequencies, which cannot be explained by a single-density, then a simple multi-density model can be constructed. A minimum number of distinct density components are invoked so as to explain the available observations (for e.g., Anantharamaiah et al. 2000).

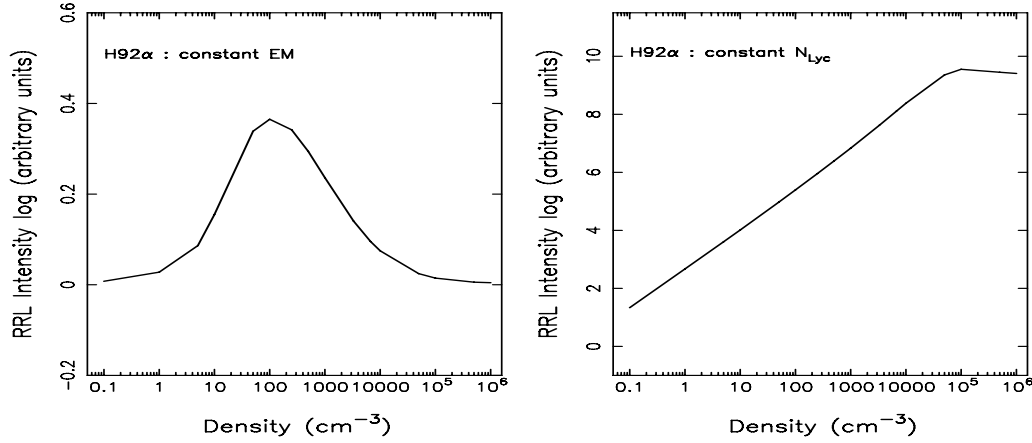


Figure 2.6: H92 α intensity as a function of density for a constant EM = 10^7 pc cm^{-6} (fig 4a; *left*) and for a constant $N_{\text{Lyc}} = 3 \times 10^{43} \text{ s}^{-1}$ (fig 4b; *right*). Pressure broadening is neglected.

However, the peak in the line intensity as a function of density, as shown in figure 2.5a, exists only for a certain dependence of emission measure on density (i.e., on the relative values of emission measures for gas at different densities). Since the peak is due to non-*LTE* effects, the peak is most pronounced for $\text{EM} \propto n_e^0$, in which case, the line intensity depends on density only through the factors b_n and β_n . Shown in figure 2.5b is the dependence of the H92 α line strength on the gas density for a constant N_{Lyc} , i.e., $n_e^2 l^3 \sim \text{constant}$ or $\text{EM} \propto n_e^{4/3}$. In this case, the line intensity has no maximum as a function of density. Hence, we attempt to construct a multi-density model (Model IV) to investigate the nature of the line emission.

We consider those densities which correspond to compact ($n_e > 10^3 \text{ cm}^{-3}$ and diameter d between 0.1 and 1 pc) and ultracompact ($n_e > 10^4 \text{ cm}^{-3}$ and $d < 0.1 \text{ pc}$) HII regions. A HII region of density n_e is assigned a size l using the relation $n_e = c_1 l^\alpha$. The parameters α and c_1 depend on the ambient medium and the evolutionary phase of the HII region. In our model, these are free parameters. There is evidence for the existence of such a power law for galactic compact and ultracompact HII regions (see, for example, Habing and Israel 1979 or Garay et al. 1993). The population of HII regions of various densities is assumed to be describable by a power-law distribution function, $N(n_e) dn_e \propto n_e^\beta dn_e$. The distribution function for galactic diffuse HII regions is a power law (Kennicutt, Edgar & Hodge, 1989). Comeron & Torra (1996) found evidence for a power law luminosity function for galactic ultracompact HII regions, although Casassus et al. (2000) claim a peaked distribution function. The actual form of the distribution would depend in detail on the star formation history of the starburst region as well as on the evolution of the HII regions. We use a power-law form of the distribution function in this analysis. In the model, we use β as a free parameter. The fourth free parameter in Model IV is n_e^{min} , the minimum density in a given distribution.

The number distribution of HII regions as a function of density is $N(n_e) dn_e = N_0 n_e^\beta dn_e$,

where the normalisation factor N_0 is fixed by the observed line strength. Since the stimulated component of the computed line strength depends on the deabsorbed continuum which in turn depends on N_0 , its value has to be solved for iteratively. The free-free and spontaneous line flux densities can be computed by summing over the contributions from each density bin from $n_e^{\min}$ up to 10^6 cm^{-3} . The expressions for Model III can be generalised to a case where HII regions of different sizes exist. Since the average optical depth along a line of sight, and hence the continuum and line emissions, cannot be described analytically in Model IV, we assume that all of the deabsorbed continuum lies behind the HII regions. We also calculate the solutions for this model only for those regions of parameter space where the average number of HII regions per line of sight is less than unity, and hence we can derive suitable expressions to be used in the calculations. In this model IV, it can be shown that the condition for a constant emission measure distribution as a function of density is given by $\beta + 2 + 1/\alpha = 0$. Therefore, the line flux density will be strongly peaked as a function of density, similar to the curve in figure 2.5a, if

$$\beta + 2 + 3/\alpha = 0 \quad (2.9)$$

The observed value of α for galactic compact and ultracompact HII regions is ~ -1 . Hence eqn. 2.9 implies that if the number distribution function of HII regions is a power law, then the H92 α line flux density will be a peaked function of density for a power law index of $\beta \sim 1$. Since this is a restrictive condition, it is to be expected that the single-density approximation will not hold for most cases. Nevertheless, the steep dependence of line flux density on n_e for a non-zero value of the left hand side of eqn. 2.9 can itself be used to argue for the validity of the single-density approximation. For such a steep dependence, the line flux density will be dominated by emission from gas at density which is either very close to $n_e^{\min}$ (for a negative slope) or very close to the maximum density considered (for a positive slope). If gas at a wide range of densities were indeed to contribute significantly to the total line emission, then the emission measure required for the gas at density very different from $n_e^{\min}$ (for a negative slope) or from the maximum density considered (for a positive slope) will probably be too large to be physically possible. Since $n_e^{\min}$ and the maximum density are free parameters in the model, this is essentially the same as model III. Only if valid solutions are obtained when the left hand side of eqn. 2.9 is close to zero will the single-density not be applicable. However, as will be discussed in the forthcoming chapters, there are rarely any valid solutions for the latter case and hence the single-density models seem to be valid.

This analysis assumes that the number distribution function with respect to density, $N(n_e) dn_e$, is a power law in n_e . If this assumption is not true, then the preceding analysis is not longer applicable and hence a single-density model cannot be justified. However, in such a case, it is not clear as to what form a possible multi-density model would take. The simplest approach in such a case would be to obtain multi-line data and construct a minimum number of distinct density components. If the radio continuum is known to be fully thermal and is spatially resolved, then the derived rms electron density can be used to constrain the possible multi-density nature of the gas. However, such a measurement is rarely available.

2.3.3 Determining the solutions space of the models

As explained in section 2.3, the expression for the total line strength cannot be analytically inverted to determine the gas properties, due to non-*LTE* effects. Hence we adopt the approach

of a grid search. The input parameters in a model are varied in steps, within an adopted range of values and the continuum and line strengths are calculated as a function of frequency for each combination of input parameters. The following conditions are imposed on the calculated values in order to identify the valid solutions :

1. The total line strength, the sum of stimulated and spontaneous emission, should equal the observed value, within errors, for all frequencies where measurements are available,
2. The computed free-free flux density should be less than the observed total continuum value at all frequencies,
3. The calculated spectral index of the deabsorbed continuum should not be steeper than, say, -0.9 , or, if the spectral index is kept constant, the optical depth of the unknown thermal component should be positive,
4. For Model III, a collection of single-density HII regions, the number of HII regions should be more than one, the volume filling factor should be less than one so as to distinguish it from the slab model, and the number of O stars per single HII region should be greater than one.
5. For Model III, $\left(N \times \Delta\nu_{\text{HII}} \times \frac{\Omega_{\text{beam}}}{\Omega_{\text{HII}}}\right)$ should be greater than the observed FWHM $\Delta\nu_{\text{obs}}$, where $\Delta\nu_{\text{HII}}$ is the FWHM of a single HII region (see Anantharamaiah et al. 1993), These conditions are suitably modified for model IV as well.

The continuum and line strengths predicted by the models are compared with the observed value, within a range determined by the errors in the measured values. Hence the range of input parameters for the valid solutions is indicative of the errors and a separate error analysis for the modelling procedure is not carried out. For the valid solutions identified, the total ionized gas mass and the ionizing photon flux N_{Lyc} are calculated. The solution corresponding to the minimum value of N_{Lyc} is also identified, whose significance is as follows: for an arbitrary continuum optical depth τ_c ,

$$N_{\text{Lyc}} \propto S_{ff} \times \left(\frac{\tau_c}{1 - e^{-\tau_c}} \right), \quad (2.10)$$

where S_{ff} is the free-free radio continuum flux density. Hence, for an observed value of S_{ff} , the calculated value of the required ionizing photon flux N_{Lyc} increases with increasing continuum optical depth, τ_c . A given value of S_{ff} thus implies a lower limit to the value of N_{Lyc} . Hence, since the minimum consistent N_{Lyc} corresponding to the minimum consistent τ_c presupposes the least amount of gas, this solution with this value of N_{Lyc} is considered important. It is observed that the valid solutions obtained does not depend significantly on the value of the electron temperature T_e by more than $\sim 50\%$, in the range considered and hence the solutions discussed are for a temperature whose value is taken from optical line ratio studies in the literature, for a particular galaxy.