Wide spectrum H₂O sources—Astrophysical Raman masers

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Abstract. The extraordinarily wide 22 GHz emission from W49A and two other galactic HII regions is discussed, and arguments presented why the entire width of these spectra cannot be attributed to the $6_{16} \rightarrow 5_{23}$ transition of H_2O . It is suggested that most of the weak emission with large frequency shifts is due to stimulated Raman scattering of the strong central features appearing at the expected frequency of the water vapour line. It has not been possible to identify the scattering agent, but it is predicted that the spectra of millimetre—wave maser sources should also show such frequency-shifted features.

Keywords. Masers; Interstellar Molecules; Raman Scattering; HII regions; Radio Lines.

1. Introduction

In searching for water vapour emission at 22 GHz from various galactic HII regions, it was discovered by Knowles et al (1969), and Meeks et al (1969), that the spectrum obtained in the direction of W49A was anomalously wide. So remarkable was the width that several authors have discussed the nature of this extraordinary spectrum, e.g. Sullivan (1973). In all of these discussions, the various authors have in conformity with standard practice, interpreted the observed frequency spread of the profile in terms of Doppler shifts (of the $6_{16} \rightarrow 5_{23}$ transition of H_2O) due to radial velocities of the emitting regions. The velocities thus obtained have posed a serious problem in producing any satisfactory model to account for the motions, the life-time, and other characteristics of this unusual source—see e.g. Strel'nitskii and Syunyaev (1973). Observations by Baudry et al (1974), also provide evidence on the relative positions and sizes of many of the individual point sources, all of which information has in no way helped to produce an acceptable model.

For many years the spectrum of W49A was unique in that its anomalously large width ($\sim 360 \text{ km/s}$) was irreconcilable with all other measurements on velocities associated with interstellar gas concentrations, and suggested strongly that an interpretation of this entire spectrum in terms of Doppler shifted $6_{16} \rightarrow 5_{23}$ transitions was incorrect. More recently however, some other measurements of HII regions have become available also showing large widths for their 22 GHz spectra. Waak and Mayer (1973) observed Sgr. B2 and found features which were interpreted as indicating a velocity spread of over 100 km/s in this source, and obser-

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vations in December 1973 at the Haystack Observatory (Radhakrishnan and Johnston, to be published) showed that the spectrum of W51 at the water vapour frequency spanned an apparent velocity range of at least 90 km/s.

In this paper we conclude after consideration of all of the evidence available to date on these three anomalously wide spectra, that an interpretation of their entire widths in terms of Doppler shifted $6_{16} \rightarrow 5_{23}$ H₂O transitions is still unacceptable; instead, we suggest an alternative explanation involving stimulated Raman scattering. In putting forward this alternative interpretation we are naturally aware that an identification of the molecules and levels involved in such a process would have been desirable to add strength to the interpretation. This has not yet been possible, but we nevertheless advance phenomenological arguments strongly favouring the suggested interpretation.

2. Interstellar gas velocities

In attempting to judge the likelihood of any given spectrum resulting solely from Doppler shifts, it is ironical that the bulk of the information on velocities we bring to bear on the problem is in fact based on a Doppler interpretation of observed spectra—proper motion measurements are possible only on nearby stars and in a few cases on supernova shells and planetary nebulae. The information we have on interstellar gas velocities comes from studies on (a) optical interstellar lines, (b) the 21 cm line of neutral hydrogen, (c) recombination lines from HII regions, and (d) molecular spectra. We shall discuss each of them in turn.

The 21 cm line of HI arises from gas distributed throughout the galactic plane and the spectra generally refer therefore to long path lengths. In HI and optical interstellar absorption lines, the effective path length extends only to the source or star whose continuum emission shows evidence of absorption, whereas in the case of HI emission all the gas along the line of sight contributes to the profile. In both cases, the pattern of differential galactic rotation reflected in the observed radial velocities has only small variations due to: (i) systematic deviations, (ii) random motions in the central velocities of the gas concentrations, and (iii) internal motions within the concentrations due to thermal and turbulent broadening.

Systematic deviations from the velocities predicted by simple theories of galactic rotation have been found in some regions of the galaxy, e.g. the Perseus spiral arm and the Carina spiral arm (Kerr 1969). These deviations, generally accepted as a conequence of the density wave theory of spiral arms, are of the order of 10 km/s.

The random motions have been shown from optical studies (Miller 1968) to be ≤ 10 km/s in general, and ≈20 km/s in some extreme cases from HI studies (Radha-krishnan et al 1972 a). For (iii) the estimates from optical interstellar lines are ~3 km/s (Spitzer 1968) and from HI measurements, it is found that the widths (full width to half power) for individual concentrations are typically 3 km/s, and that the maximum such widths observed are only about 10 km/s (Radhakrishnan and Goss 1972). Much wider lines are produced by the diffuse intercloud medium and typical values for the widths are 20–30 km/s obtained at intermediate and higher latitudes where this component can be separated from cloud emission (Radhakrishnan et al 1972 b).

As we are interested here only in velocities encountered in the galactic disk, we ignore the so-called high-velocity HI clouds; these peculiar objects found at higher latitudes all have negative velocities and belong in a category by themselves. Very high velocities ($\sim 100-150 \text{ km/s}$) are also found near the galactic centre but are known to be peculiar to the nuclear region of the galaxy.

The thermal broadening in recombination lines from HII regions is greater than in the case of HI because of the higher temperature ($\sim 10^4$ K) but when allowance is made for this component of the broadening, the remainder due to random motions is again only of the order of 20 km/s (Wilson *et al* 1970).

Coming to radio emission from interstellar molecules, we have two general classes. Several molecules are observed to be widely distributed in the galactic plane and produce spectra reminiscent of HI spectra. OH and H₂CO are seen in absorption against continuum sources and CO and CH are observed in emission somewhat less widespread than that of HI. The velocities in these cases also reflect the effects of galactic rotation and the random motions are no greater than those observed for the HI concentrations.

Most of the other molecules that have been detected show spectra indicating a very high degree of localisation in both velocity and position. Although excitation conditions must govern to some extent the radiative evidence for their existence, it seems unlikely that these molecules are distributed as uniformly as HI (Zuckerman and Palmer 1974). The most localised sources are of course of the OH, H₂O and SiO masers and they are invariably associated with HII regions or infrared stars. The evidence for their localisation is their concentration in the plane of the sky which makes it extremely unlikely that they are distributed along the line of sight. Further, in H₂O and type I OH sources, the radial velocities of the recombination lines from the corresponding HII regions are within 5-10 km/s of the molecular velocities (Robinson et al 1974) thus strongly supporting the hypothesis of a physical association between them. Habing et al (1974) have recently shown that many type I OH sources are closely associated with high density compact HII regions of sizes approximately 0·1 parsec or smaller. VLB observations have shown that both H₀O and type I OH sources are clusters of very small diameter sources grouped together within a solid angle of a few seconds of arc.

In general, the total spread of velocities in the spectra of such sources is only $10-15~\rm km/s$. This is illustrated in figure 1 where we present the data from a sample of $\rm H_2O$ and type I OH sources. We have taken the maximum velocity spread (the velocity difference between extreme features in the spectrum) and plotted a histogram of the number in each velocity spread interval. For comparison, we have included in the figure a curve of the expected distribution of velocity separations between pairs of points located at random within an HI concentration. This was derived from the distribution of observed velocity widths of absorbing HI concentrations shown in Figure 2b of Radhakrishnan and Goss (1972). It might be pointed out here that 180 km/s is the full velocity width of the HI emission spectrum in the direction of W49 A; this defines the range of all velocities associated with neutral atomic hydrogen over a path length of \approx 20 kpc, along the line of sight to and well beyond W49 A.

In the histogram of OH sources, there is a single anomaly in OH 0739-14. However, as no detectable continuum emission is associated with this

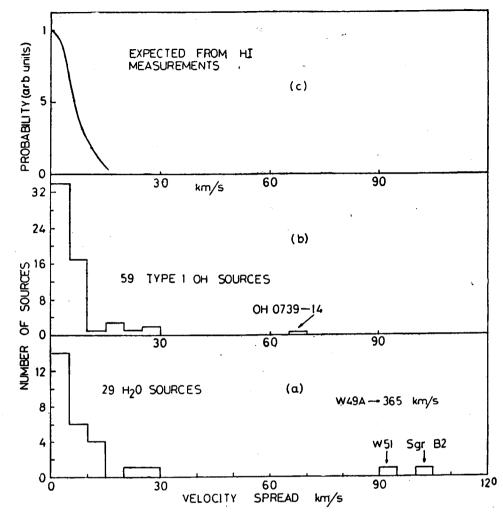


Figure 1. Histogram showing the number of H_2O sources (a) and Type I OH sources (b) whose spectra have a maximum veclocity spread in a given interval. The maximum velocity spread is defined as the velocity difference between extreme features in the spectrum. The curve of figure 1 (c) represents the relative probability of two small regions chosen at random having a given velocity separation in a typical HI cloud. It is derived from a histogram (figure 2b in Radhakrishnan and Goss 1972) of measured velocity widths of a sample of absorbing neutral hydrogen clouds, to illustrate the origin of the widths of molecular spectra. The histogram on water sources was compiled with data taken from Sullivan (1973), Johnston et al (1972) and Yngvesson et al (1975).

source (Goss et al 1973), it is not an HII region and its nature is as yet not understood.

It is evident from the figure that barring certain obvious exceptions, the total OH or H₂O velocity spread is in general just that to be expected from a single HI concentration. The obvious explanation for this is that all the individual components in the OH and H₂O spectra originate in different parts of the same overall physical concentration within which there are large variations in excitation condi-

tions. The quantities that are likely to vary are the pumping rates, the magnetic field, the velocity structure and the relation of one or more of these to the line of sight. The picture that emerges is that inside an HI concentration of which the HII region is a part, there are small regions which are favourable for the production of OH and H₂O masers. Other molecules also exist in these concentrations and display velocities consistent with this picture.

It is also quite evident from figure 1 that the sources with exceptionally large velocity widths clearly cannot be thought of as belonging to the tail of a distribution. Any reasonable distribution law fitted to the rest of the sources would indicate a totally negligible probability of finding spectra with such large widths on a purely statistical basis. We are therefore forced to conclude that there are exceptional reasons for the widths of these few spectra and an explanation is clearly called for. In the following section we advance arguments against a purely Doppler interpretation of these spectra. We emphasise particularly the case of W49 A which although not the only source with an anomalously wide spectrum, is certainly the most outstanding case and in our opinion the archetype of a new kind of source.

3. The Doppler Interpretation

If we assume, as has been done in the literature until now, that the entire widths of all of these spectra are due to Doppler shifts caused by large radial velocities, we run into a number of inconsistencies. We shall list these below and proceed to discuss each of them in turn.

- 1. Such large velocity widths are observed only in the H₂O spectra of these sources, and not in their OH, recombination line or other molecular spectra.
 - 2. There is a tendency for only the strong H₂O sources to show such large widths.
- 3. The sources are concentrated in the plane of the sky inspite of having large velocity differences.
- 4. In the case of W49 A which has been studied interferometrically there is no pattern associated with the radial velocity components of the different sources associated with different frequencies.
- 5. Large velocity differences in such a small region imply large velocity gradients which are unfavourable for maser action.
- 1. If, in fact, the H₂O spectra provide evidence of such high velocities in the molecular clouds in question, it seems extra-ordinary that these motions have left the OH, recombination line, and other spectra observed in these regions quite unaffected. Whether the cause of the apparent motions observed at 22 GHz, is a supernova explosion or any other phenomenon, it seems to us most unlikely that these motions will not be reflected in the other spectra. If there had been only one source such as W49 A, it may have been argued that the 22 GHz radiation originated in a region distinct from the rest of the molecular cloud, and in which some extraordinary acceleration process had taken place. For this to happen in all three sources in question is too unlikely for serious consideration, particularly as there is agreement between the mean velocities of the OH and recombination line spectra and the strong central features in the H₂O spectra.
- 2. There appears to be a correlation of the widths of these anomalous spectra and the intensities in their central regions. The source with the widest spectrum (~400 km/s) is W49 A, which is also the strongest by far. The flux density in the

central peaks at any phase of their variability is many times greater than in the next strongest source. When the large distance of W49 A relative to those of other HII regions is also taken into account, the luminosity of this source is two orders of magnitude more than the next strongest source. In absolute terms the luminosity of W49 A in just the central peaks of its 22 GHz spectrum is equal (for isotropic radiation) to that of the sun in all wavelengths. The other two sources showing anomalous widths, while much less intense than W49 A, are still among the most luminous. Their maximum observed luminosities are comparable, and an order of magnitude higher than any of the other water sources listed by Sullivan (1973).

On the Doppler explanation, the various regions in a source producing different frequencies must be physically separated, and in fact their separation must be increasing rapidly with time. In such a situation it is hard to imagine that their mean intensities can be correlated, as would be required for weaker "high velocity" features to be seen in only those sources with strong "low velocity" features.

- 3. The close correlation in the plane of the sky observed for the various components of W49 A is extraordinary in terms of the lifetime of the source. The overall angular dimensions of a few arc seconds of the cluster of point sources (Baudry et al 1974) when combined with the velocity spread of the order of 400 km/s, forces one to the conclusion that such velocities have existed only for 10³ years or so; in other words, we are fortunate in observing this expansion in its initial phase. If interferometric observations on the other sources show in like manner that their large "velocities" could also have been operating only for a very short time, this alone would make the Doppler hypothesis suspect.
- 4. Taking the case of W49 A again, if the observed frequency shifts were due to large velocities produced by some common accelerating mechanism, one would expect to find some pattern in the velocities of the various regions in the source. That such a pattern is not found is further evidence casting doubt on the Doppler hypothesis.
- 5. Many of the "high velocity" features observed in these wide spectra must be masing. This is certainly true for W49 A as deduced from the high apparent brightness temperatures obtained for them. In any astrophysical maser, the effective path length will be governed strongly by the velocity gradients present in the region, the maximum obtainable lengths being along directions in which the radial velocities for the molecules are nearly the same. The existence of large velocity differences, as suggested by the Doppler interpretation of the spectra, would lead one to believe that there are tremendous velocity gradients within the molecular cloud making up the region. It is now hard to conceive of a number of sub-regions moving away from each other with enormous velocities, but within each of which the radial velocity components are in sufficient agreement to provide gain for the masers.

Taken together, all of the above arguments indicate strongly that a velocity hypothesis is unacceptable and that most of the "high velocity" features in the 22 GHz spectra of these sources can consequently not be attributed to Doppler shifts of the $6_{16} \rightarrow 5_{23}$ transition of H_2O ; they must originate in other transitions of either the same or other molecules. As we are attempting to explain spectra that are wider than the average by up to an order of magnitude, it is clear that no single additional transition can provide the extra widths necessary. In the case of W49 A the apparent velocity spread is greater than that of the average H_2O source by a

factor of 20 or so, and twice as wide as the full HI spectrum in this direction. To propose that there are many independent transitions fortuitously juxtaposed to give the observed spectra would be absurd. One must therefore look for other explanations such as the one advanced in the next section.

4. An alternative interpretation

It was shown in the preceding section that the features with large frequency shifts in the wide spectrum H₂O sources are unlikely to result from large Doppler shifts of the same transition as that producing the "low velocity" features. Further, we noted that 22 GHz spectra with large frequency widths tend to be associated with strong central features having velocities in agreement with other observations. In particular, the widest spectrum hitherto observed, namely that of W49 A, is associated with the most intense radiation detected so far and lying in the expected velocity range. If the presence of weaker features with large frequency shifts was in fact a direct consequence of the luminosity in the central features, this would be a natural explanation for their presence in the spectra of 'strong' sources only. This will leave untouched the question of why the luminosity of W49 A is so much higher than that of other water sources, but at least it will not require the source to be unique in two quite different and apparently unrelated ways.

As mentioned earlier in this paper, a large number of types of molecules are found in interstellar space and there is a strong suggestion that the surroundings of HII regions are particularly favourable sites for their formation. All of the evidence seems to indicate that there must be many more species of molecules to be found in the neighbourhood of HII regions than have been identified thus far, and there are naturally many more transitions even in the known molecules than have been observed to date. It seems not unreasonable to us to suppose that there must be many possible transitions with energies in the range of a few Megahertz, which have not been observed because of their low transition probabilities among other reasons. Some of these transitions could however be Raman active and produce frequency shifts corresponding to their energies if incident radiation of a definite frequency is scattered by the appropriate molecules.

The intensity of spontaneous Raman scattered radiation is generally many orders of magnitude below that of the incident radiation and therefore unlikely to be observable. But if the incident radiation is itself from a maser and sufficiently intense, it can give rise to stimulated Raman scattering which will be very much stronger than ordinary Raman scattering; in fact, it has been shown that under suitable circumstances it can even have an intensity approaching that of the incident radiation (Woodward 1967). At very high incident intensities, higher harmonics of the frequency shift can also be produced giving rise to a number of lines with progressively decreasing intensity for each incident frequency and operative pair of Raman levels.

A well known but important point to note here about any form of scattered radiation is that no Doppler shift is observed in the forward direction due to the velocity of the scattering agent in the frame of the observer. For the same reason it is easily seen that the frequency of Raman scattered radiation—for shifts small compared to the exciting frequency—is also unaffected in the forward direction.

This becomes of enormous significance in an astrophysical context because the path length now available for stimulated Raman scattering in the forward direction is not limited by velocity gradients in the distribution of the scattering molecules; this is in contrast to ordinary maser emission where even small radial velocity differences causing Doppler shifts of the order of the natural width of the line will reduce the gain of the maser. The implication of the above in the present context is that the likelihood of stimulated Raman emission is greatest in the general direction of the exciting radiation, along which direction it can build up unaffected by the thermal and turbulent motions in the scattering medium. No separate pumping mechanism is required for this kind of maser as the incident radiation is itself the pump. We have here the possibility of a very natural explanation for the presence of weak features with large frequency shifts in the spectra of sources with central features of high luminosity.

This hypothesis also provides a natural explanation for the occurrence of this phenomenon at 22 GHz in preference to 1.6 GHz where we also have strong OH maser sources. The general formula for the build-up of intensity I_a in the stimulated Raman scattered beam is (Lallemand 1971),

$$I. \propto e^{I_i q_L} \tag{1}$$

where I_i is the intensity of the incident exciting radiation. L is the effective path length of the maser and g is the gain per unit length. The gain constant g is given by

$$g \propto \frac{A}{hv_i} \times N$$
 (2)

where N is the number of scattering molecules per unit volume and A is the spontaneous scattering probability per unit time for a single photon of frequency v_i to be scattered into a Raman mode of frequency v_i . It is known (Berestetskii et al 1971) that

$$A \propto \left| \sum_{n=1}^{\infty} \left\{ \frac{\vec{d}_{2n} \cdot \hat{e}^{\prime *}) (\vec{d}_{n1} \cdot \hat{e})}{\nu_{n1} - \nu_{i}} + \frac{(\vec{d}_{2n} \cdot \hat{e}) (\vec{d}_{n1} \cdot \hat{e}^{\prime *})}{\nu_{n2} + \nu_{i}} \right\} \right|^{2} \nu_{i} \nu_{s}$$
 (3)

where $hv_{n1} = E_n - E_1$, $hv_{n2} = E_n - E_2$; E_1 , E_2 are the energies of the two Raman levels and E_n that of an intermediate level; d's are the dipole matrix elements. We see from the preceding two relations, therefore, that the gain constant g has a frequency dependence due to two factors; (i) it is proportional to v_s and (ii) it depends on v_s through the energy denominators. Even if we ignore the factor (ii), which is unimportant when the incident frequency v_s is much smaller than any allowed resonance frequency v_{n1} , the intensity in the Raman scattered beam builds up exponentially with a gain that is proportional to v_s . This would favour the occurrence of stimulated Raman lines from water vapour radiation over hydroxyl radiation by a factor of approximately 14 in the gain, 14 being the ratio of their frequencies (note $v_s \simeq v_s$). This would imply a much greater enhancement in the scattered intensity for a given intensity of the incident beam.

If on the other hand, the incident frequency increases to a value close to that of one of allowed resonance frequencies v_{n1} the intensity of Raman scattering would be further enhanced. The intermediate levels governing the intensity of

the Raman scattering are most likely to be those corresponding to pure rotational excitation and could easily lie around 100 GHz for several molecules. An incident radiation of frequency higher than 22 GHz being nearer to the resonance frequency, should therefore produce greatly enhanced scattering by virtue of both factors (i) and (ii). If the present hypothesis is correct, namely that the observed large frequency shifts in 22 GHz spectra are due to stimulated Raman scattering, then there should definitely be evidence to support this in mm-wave spectra from maser sources. It is important to note, however, that if the scattering molecules are the same, these mm-wave spectra will not show the same dramatic increase of apparent velocity widths as in the case of water radiation; it is the *frequency shifts* and not the fractional frequency shifts (apparent velocities) that would be the same.

5. Further implications of the hypothesis

In the case of Raman scattering by a single pair of levels, one would expect to find Stokes and anti-Stokes components symmetrically placed about the frequency of the exciting radiation, and having an intensity ratio given by the relative populations of the two levels. In more complicated cases the pattern will consist of many Raman lines and their intensity ratios would be governed by many factors. However, in the case of the wide spectra discussed in this paper it has not been possible as yet to detect any meaningful pattern in the spectrum of frequencies observed.

The complexity of the observed spectra and the absence of any clearly discernible symmetry in them implies that if Raman scattering is taking place, then (a) there must be many pairs of Raman levels contributing, and (b) that the relative intensities of the Stokes and anti-Stokes lines be highly unequal. Since even for massive molecules the lowest rotational levels would have separations well in excess of the observed frequency shifts, it would appear that such small energy differences could only be due to hyperfine (or similar) transitions of which there could easily be many. Under equilibrium conditions, the populations in such closely spaced levels will be almost equal; but because of the exponential in the stimulated Raman scattering formula, the intensities can be unequal in favour of the Stokes or down-shifted components. Under non-equilibrium conditions as found frequently in molecular clouds, the relative populations can differ appreciably either way, and this can lead to enormous differences between the Stokes and anti-Stokes intensities favouring one or the other as the case may be Further, for the same reasons, the apparent Doppler width of the Raman shifted features will always tend to be less than that of the exciting features.

The requirement of high gain for the production of stimulated Raman scattering implies that the exponent of e in equation (1) is large compared to unity, and consequently that the intensity of the Raman lines will be sensitively dependent on the intensity of the exciting radiation. One might therefore expect that the large time-variations seen in the intensity of the central features in these sources would be accompanied by greatly amplified and easily detectable changes in the intensities of the weaker lines with large frequency shifts. As the available observations have not indicated such a correlation, we consider below two possible reasons for it.

- (a) The observed intensity variations in the central features are really due to changes in the direction of beaming of the maser radiation, and not due to an actual change in the strength of the radiation. Since small departures from the precise forward direction do not introduce appreciable Doppler shifts due to the motions in the gas, it is conceivable that small changes in the direction of the exciting radiation need not be immediately reflected in the strength of the scattered radiation.
- (b) The intensity changes observed in the central features could be due to a variation in the efficiency of frequency conversion through stimulated Raman scattering if a significant fraction is indeed being converted. Since it is the unconverted exciting radiation that we receive, one would now expect an inverse correlation of intensity variations between the central features and the frequency shifted ones if both happened to be precisely beamed in our direction. As we now have two masers in series, and the possibility that the beaming and/or pumping of either or both can change due to other factors not yet understood, it may not be unreasonable to have observed no intensity correlations over the width of these spectra.

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