Srinivasa Ramanujan

S. Ramaseshan

When I think of Indian science at its best, two names spring foremost to my mind—the towering figures of Raman and Ramanujan. Both were born a hundred years ago on the banks of the river Cauvery, and both were from poor middle-class stock. But when they grew up, they did things which made the world sit up and take note. The former was an experimental physicist *par excellence*, who won for India the Nobel Prize; the latter was one of the greatest mathematicians of the twentieth century. When the mathematicians of the world made a bust of Ramanujan, one copy of which was presented by the Nobel Prize-winning astrophysicist Prof. S. Chandrasekhar and Mrs Lalitha Chandrasekhar to the Indian Academy of Sciences, S. Chandrasekhar wrote:

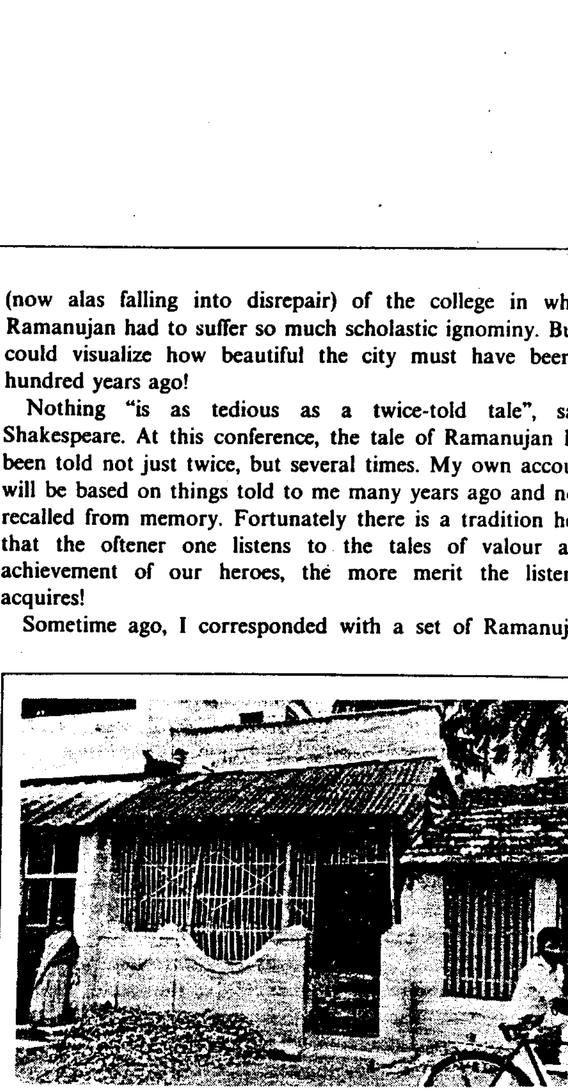
> ... as a companion to the bust of Raman so that the bust of the greatest physicist of India could be along with that of the greatest mathematical genius of our times who happened to be an Indian.

I do feel greatly honoured that I have been invited to speak about Srinivasa Ramanujan here. It would have been more appropriate for this talk to have been given by a mathematician, but it is too late for that now.

I feel very happy today, for I have seen the house in which Ramanujan lived (it is a pity that Ramanujan's house is still not a national monument; but this is not surprising in India); I visited the school which introduced him to mathematics, and mingled with the children there. I saw the river Cauvery, the life stream of the south flowing through the town (but sadly polluted). I admired the exquisite 19th century buildings (now alas falling into disrepair) of the college in which Ramanujan had to suffer so much scholastic ignominy. But I could visualize how beautiful the city must have been a hundred years ago!

Nothing "is as tedious as a twice-told tale", says Shakespeare. At this conference, the tale of Ramanujan has been told not just twice, but several times. My own account will be based on things told to me many years ago and now recalled from memory. Fortunately there is a tradition here that the oftener one listens to the tales of valour and achievement of our heroes, the more merit the listener acquires!

Sometime ago, I corresponded with a set of Ramanujan



Ramanujan's house in Kumbakonam

Lecture delivered at the Ramanujan Centennial International Conference (15–18 December 1987) at Kumbakonam. Reprinted with permission from the *Proceedings*, © Ramanujan Mathematical Society.

scholars, who gave me a fleeting vision of the greatness of Ramanujan and the quality of his mind. All that I shall try to do today is to convey this to you.

The story really began with a letter from S. Chandrasekhar:

I should not be writing this letter if I were not sure of your long-standing friendship and understanding....

It is hardly necessary for me to tell you that Ramanujan is the greatest mathematician of India; and that in some sense the quality of his mathematical genius has not been equalled anywhere in the world during this century. You must also be aware that the discovery a few years ago by George Andrews of Ramanujan's lost (and his last) notebook has created a renewed interest in his mathematical discoveries during the last years of his life. In this renewed interest, two other American mathematicians, besides George Andrews, have played an important role—Richard Askey and Bruce Berndt.*

The letter continues to say that Richard Askey took the initiative to have a bust of Ramanujan made by the distinguished sculptor Paul Granlund. (Paul Granlund, Sculptor in Residence, Gustavus Adolphus College, St. Peters, Minnesota, USA, who successfully transformed the two-dimensional passport photograph of Ramanujan into a three-dimensional work of art.) That is how I became involved in the project to get two of these busts into India and to present one of them to Janaki Ammal, Ramanujan's widow.

Ramanujan's life—A melodrama

The life of Ramanujan reads like a fairy tale, as melodramatic as a bad Indian film. Born on 22 December 1887 he could not pass his F.A. examination (pre-university class examination of today). He failed many times but secured 100% marks in mathematics. He lived in obscurity for 25 years, all the time working on mathematics, which none of his contemporaries in India really understood.

Then his luck, and that of mathematics, changed. He was invited to Cambridge where his mathematics was appreciated and understood. Says Hardy, his mentor:

One gift he had which no one can deny ... invincible originality ... on this side I have never met his equal, and I can compare him only with Euler and Jacobi.

He achieved fame. He was elected Fellow of Trinity College, and Fellow of the Royal Society at the age of 30—amongst the youngest ever to be so honoured. Then, as in a bad movie, fate struck again. In 1917 he fell seriously ill and spent the next three years in hospitals and nursing homes. He returned to India in 1919, and died on 26 April 1920 at the age of 32 years and 4 months and 4 days. As the Greek poet said:

Thus died Lycidas who left no peer.

Of his last days his wife Janaki Ammal said:

He was only skin and bone and often complained of severe pain. Yet he continued doing his mathematics filling sheet after sheet with numbers. It possibly helped him to forget his pain.

This was presumably the way *'the lost notebook'* was written, to be discovered later and leave the world stunned. We can only compare his life to one of the great tragedies of Sophocles or Euripides.

*It was embarrassing for me that this triumvirate, whom I quote often here, were present listening to my talk.

There was something magical about Ramanujan's thinking. At 12, he understood trigonometric functions—not as ratios, but as expansions of series. At 15, a book by G. S. Carr* fell into his hands and many say it became famous because Ramanujan used it. After reading Carr he was certain that the things throbbing in his head were not scary nightmares but real mathematics. Then the floodgates opened, and the deluge followed. He re-discovered many things that generations of mathematicians before him had found, but he also discovered many new formulae, never previously thought of. His pre-occupation with mathematics became total. No wonder he failed in his examinations and lost his scholarship. But one thing must have become quite clear to him, that he was a man with a strange and incredible power, a power over numbers, and over formulae.

Then followed a phase that is so sad to recall and relate. The great Ramanujan, who had become aware of his inordinate power, went round like a street hawker all over South India trying to display his wares, and seeking patronage. And this in a country where we pride ourselves as having invented the zero and the decimal system, we brag of the *Aryabhatiya* and even relate to the world the prowess of our Kerala mathematicians who invented many series expansions 300 years before the Western masters Newton, Leibnitz or Gregory thought of them. (M. S. Rangachari, 'The Indian Tradition in Mathematics', *J. Indian Inst. Sci.*, 1987, pp. 3–9.)

Said Ramachandra Rao to whom Ramanujan went as a mendicant:

He came with a *frayed notebook* under his arm. He was miserably poor. He just wanted a pittance to live on, to get simple food and leisure to pursue his studies.

He wrote to Sir Francis Spring, Chairman of the Madras Port Trust where Ramanujan was a clerk earning Rs 25 per month. Francis Spring sent Ramanujan's papers to Col. Gilbert Walker, Director of Meteorology (who was later to become a great friend of C. V. Raman and of Indian Science), who in turn recommended Ramanujan's case to the Registrar of Madras University, and the Syndicate of the Madras University gave him a scholarship of Rs 75 per month.

I want you all to sit back and think about this. How many Registrars in this country today, or for that matter how many Vice-Chancellors of today, 100 years after Ramanujan was born, would give a failed Pre-University student a research scholarship of what is now equivalent of Rs 2000 or Rs 2500 today. This is after 40 years of independence, when we can no longer blame a colonial power for not encouraging Indian talent.

In a booklet dated 1910, Ramanujan found for the first time formulae like some of his own and acting on the advice

**Synopsis of Elementary Results in Pure Mathematics* by G. S. Carr. The book that gave extensive references which would have been useless to Ramanujan as he had no access to a library that contained them. The lack of proofs did not bother him as he simply worked through the book presumably supplying proofs of his own. Many mathematicians feel that the style of this book encouraged Ramanujan to present mathematical results without proofs. It just so happens that this was also the style of the ancient Indian mathematical works.

of Seshu Iyer, he wrote, on 16 January 1913, a letter to the unknown author of this tract—Hardy*.

Hardy wrote to Ramanujan inviting him to Cambridge† and to the Madras University to give him a Fellowship. The moment he heard from Hardy, Ramanujan knew that his intellectual solitude was at an end. E. H. Neville, Hardy's colleague—a don from Cambridge who visited Madras—was asked by Hardy to persuade Ramanujan to undertake the journey to Cambridge.

My father's description of Ramanujan

On the 50th anniversary of Ramanujan's birth (when I was still in school) I was surprised to learn that my father knew Ramanujan when he was in Madras. According to him:

> Everyone who came across Ramanujam (most of Ramanujan's contemporaries called him Ramanujam) knew that he was gifted, that he was a genius. No one had the least doubt about it. But he was by no means an eccentric, a quality that is often associated with genius. I have often seen him lying on his stomach on a mat with a pillow under his chest writing on a slate. The slate was large but the letters (numbers) were small and the slate pencil invariably squeaked which irritatingly 'set one's teeth on edge'. He had a very peculiar mannerism of rubbing out some of the numbers with his elbow! No one *could* distract him when he was doing his sums (*sic*). When they were done, he would appear to be speaking to himself, smiling, shaking his head, and would enter the results into a *notebook*.
>
> Ramanujam looked like a typical Thengalai Iyengar with a large head with very bushy hair knotted into a big tuft. He also wore a *namam* (the *namam* was the thengalai caste mark). He was dark, not quite five foot six inches, with a chubby, faintly pockmarked faced. He would often forget to shave. He walked in the style we call 'Broad Gauge' in South India. He was so friendly and gregarious; always so full of fun, ever punning on Tamil and English words, telling jokes, sometimes long stories, and going into fits of laughter when relating them. His tuft would come undone and he would try to knot it back as he continued to tell the story. Because of his premature laughter he would often have to be asked to repeat the end of the story.
>
> He was so full of life and his eyes were mischievous and sparkling. He ate with great relish; had a tendency to talk and joke even with his mouth full. He could talk on any subject and it was hard not to like him.

Thirty-five years later I read Neville on Ramanujan:

> His own irrepressible laughter swallowed the climax of his narrative. Success and fame left his natural simplicity untouched. The wonderful mathematician was indeed a loveable man.

Almost the same words as my father's. There was no doubt they were describing the same man. One hears of Ramanujan's jokes and puns. I cannot adequately translate his puckish description of the food dished up at Matlock clinic. And when his English friends asked him about his losing caste on crossing the seas, he is stated to have said that when he goes back he will never be invited to a funeral! (He was referring to the traditional Hindu 13th day ceremony when friends and relatives of the deceased are invited to partake in a feast.)

I have dwelt at some length on this aspect to counteract the impression that many seem to have of him that he was a morose and melancholy character.

Ramanujan's illness

Ten years ago when driving through the Peak District of Derbyshire, I entered a town called Matlock. The name rang a bell, for this was where Ramanujan had spent much time in a nursing home. On enquiring at the post office I found that there was no such nursing home or medical establishment. The kindly postmaster however found an old postman who did remember the place and he showed it to me. The retired postman said that the building continued to be a medical establishment till probably the early 30s, but now it was being used as a municipal office and residences.

It was here that Ramanujan was so unhappy and miserable that on leaving it he attempted suicide. (The story of Ramanujan's attempted suicide became known in India when S. Chandrasekhar mentioned it in a lecture in Delhi, and it created a furore. We in India like our supermen to be without 'blemish'!) According to Janaki Ammal, Ramanujan's depression in 1918 owed not little to the fact that none of the letters she wrote ever reached him (nor did any of his many letters from England ever reach her). It was only on his return to India that it became clear that Ramanujan's mother Komalathammal with whom she lived had intercepted both lots! Clearly another side to the same person that E. H. Neville refers to in his beautiful essay on Ramanujan.

> Death too was a frustration, but the life's purpose of which his mother dreamed was at last in part fulfilled—and it is better to be frustrated by unhappy death than by life.

Matlock House was also the nursing home about which his friend Ramalingam wrote to Hardy complaining about the food and the treatment given to Ramanujan. It was also the place where he received the news that he had been elected to the Royal Society and from which he wrote the very charming letter to Hardy.

I also happened to visit the other nursing home in Putney (London) with which is associated the most quoted story about Ramanujan; 'Hardy and the taxi cab with number 1729'. This too was the place where Ramanujan, too ill to write, dictated a letter (which was taken down by Hardy himself) requesting the Madras University to use his surplus income for helping poor students and orphans.

There is one more story that is imprinted in my mind.

In the thirties we met Dr P. S. Chandrasekhara Iyer, the tuberculosis expert, who treated Ramanujan when he returned to India. To an enquiry from my father as to whether Ramanujan's life could have been saved, he replied, almost livid with rage, that Ramanujan's life could have and should have been saved. According to him even the diagnosis of Ramanujan's illness was initially wrong and he had been neglected in the early stages of his illness due to the ignorance of his contemporaries! (R. A. Rankin. 'Ramanujan as a Patient', *Proc. Indian Acad. Sci.* (*Math. Sci.*), 1984, 93, 79.)

The entry in his diary on 27 April 1920, a day after

*In the exercise of displaying his wares Ramanujan also wrote to several others abroad, e.g. H. F. Baker and E. W. Hobson, mathematicians with some reputation in England. Both returned his letters without comment, and one wonders what they felt after Ramanujan was 'discovered' by Hardy.

†Hardy's comment on Ramanujan's formulae though oftrepeated is worthy of mention again. "They defeated me completely; I had never seen anything in the least like them before. A single look at them is enough to show they could only be written down by a mathematician of the highest class. They must be true because if they were not true no one would have had the imagination to invent them". It also illustrates the honesty, generosity and discernment of Hardy.

Ramanujan's death, says:

> His death is the saddest event in my professional career. It is not for me to assess Ramanujan's mathematical genius. But at the human level, he was one of the noblest men I have met in my life—shy, reserved and endowed with an infinite capacity to bear the agonies of the mind and spirit with fortitude.

I include this as a contrast to what has been written about him as a very difficult patient in Matlock clinic.

Ramanujan's notebooks

Once while visiting two of my friends (S. Ramu and his son the late R. Subbu, two outstanding master printers of India of the Commercial Printing Press, Bombay) in their printing press, I was taken to an inner room and shown a stack of browned old paper with magic squares and beautiful mathematical formulae systematically written in an elegant hand. I could not believe that I was face to face with Ramanujan's notebook, the one I had heard about first from my father in 1937, the famous *'frayed notebook'* of Ramachandra Rao. I ran the tips of my fingers gently over the old paper—to feel the sheets which Ramanujan himself had filled with a smile on his face when he was without a job and everything else in his life seemed so bleak.

It was realized that if Ramanujan's notebooks were published in *facsimile* form even if they be without notes, commentary, or proofs, it would be a great service to the mathematical community. My friends considered the printing of these fast deteriorating *notebooks* as one of the most exciting jobs they had ever undertaken.

The story of the notebooks is a bit confusing even when one reads the detailed accounts given by G. N. Watson and R. A. Rankin (G. N. Watson, 'Ramanujan's Notebooks', *J. London Math. Soc.*, 1931, **6**, 137; R. A. Rankin, 'Ramanujan's Manuscripts and Notebooks', *Bull. London Math. Soc.*, 1982, **14**, 81).

We heard the story of Ramanujan's lost notebook ('Ramanujan's lost notebooks (with an introduction by George Andrews)', Narosa Publishing House, New Delhi, 1988), at this conference. About it George Andrews said:

> It is my contention that this manuscript or notebook was written during the last year of Ramanujan's life after his return to India from England. My evidence for this assertion is all indirect; in the words of Stephen Leacock, 'It is what we call circumstantial evidence—something people are hanged for'!

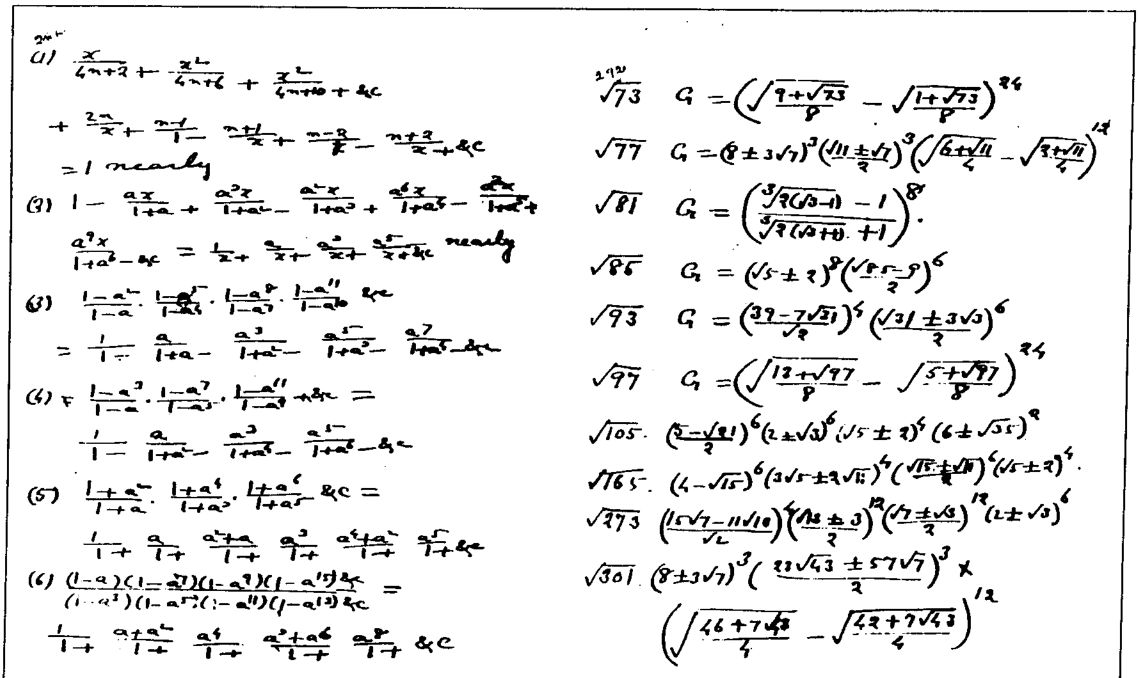
According to Bruce Berndt and others, a large number of papers of Ramanujan are still missing and it is of the utmost importance that a concerted effort be made to trace them.

Ramanujan's style and influence

Hardy says that he tried to educate Ramanujan in mathematics. The implication is that he was not very successful. From all that I have read, Ramanujan was perhaps too engrossed in his own work to 'learn' much.

Says Richard Askey:

> I still do not understand how he could have educated himself and done all the mathematics he did. I cannot think of any other person who accomplished so much with so little aid from others.



Pages from Srinivasa Ramanujan's Notebook

And George Andrews:

> It would be nice to say that if Ramanujan had had more education he would have done more. It is also plausible to say that more education might have ruined Ramanujan.

Many views have been expressed about Ramanujan's methods. That his style was different, that he was a child of intuition, and that if a mixture of intuition and some evidence gave him a *feeling* of certainty, he looked no farther, that he was like an artist who was convinced if he felt satisfied with his work. It was also said that he was not interested in rigour and had no strict logical justification for many of his operations. But no one is certain that he did not have proofs for his formulae. In fact his letters show that he was sometimes quite unhappy when he did not have a good proof.

Yet it is acknowledged that Ramanujan's collaboration with Hardy produced some of his best mathematics. The contribution that Hardy made to Ramanujan and to his growth as a mathematician is clearly substantial but it is not for me to estimate it. But as for the effect on Hardy of their collaboration, we have his own words:

> I still say to myself when I am depressed and find myself forced to listen to pompous tiresome people, 'Well, I have done one thing you could never have done, to collaborate with Littlewood and Ramanujan on something like equal terms'.

On the aesthetic appeal of Ramanujan's work Freeman Dyson says:

> Such mathematics as Ramanujan's has helped to derive new concepts in theoretical physics like superstring theory. As pure mathematics it is as beautiful as any of the flowers that ripened in Ramanujan's garden.

And on its value to researches today, says R. W. Gosper another mathematician:

> Ramanujan is for ever reaching out from his grave and snatching away our latest results.

Hardy thought it a shame that Ramanujan was not born a 100 years earlier, during the great age of Euler, Gauss and Jacobi, the period of discovery of relationships and formulae in mathematics. But the conclusion of many modern mathematicians appears to be otherwise.

> It is a shame that Ramanujan was not born a 100 years later for we are now trying to do problems in several variables. These problems are much harder. It would be marvellous to have somebody with his insight to help us get started. [Askey]

At the conference. *From left,* N. Renganathan (Annamalai Univ.), S. Ramaseshan, A. R. Meenakshi (Annamalai Univ.), R. Balakrishnan (Annamalai Univ.), escort of George Andrews, Seetha Ramaseshan, George Andrews, Bruce Berndt, Basil Gordon, Richard Askey, Mrs. Askey.

I said earlier that Ramanujan's life was like a Greek tragedy, too sad even for tears. But it was also glorious. He was like an exploding star, the brightest in the firmament for the short span of its life. And like the pulsating remnants of many a supernova, his legacy is still with us, after a 100 years revealing with each deep look newer aspects never imagined before.

A flavour of Ramanujan's mathematics

It would be inappropriate for me to talk about Ramanujan's mathematics at this meeting where the experts have discussed it at length. But his magical intuition can be appreciated even by non-mathematicians. Some of these aspects have been brought out in the pamphlet entitled *Ramanujan at Elementary Levels—Glimpses* by V. R. Thiruvenkatachar and K. Venkatachalaiengar, which my young listeners must read. I give below a few examples for which one does not need even college level mathematics to see the reach of Ramanujan's mind and to be awed by it.

a) The genius of the man was to discover relationships that govern the *wilderness of numbers*, as for example when he writes down such a complicated formula as

$$\frac{1}{\pi}=\frac{2\sqrt{2}}{9801}\sum_{n=0}^{\infty}\frac{(4n)!\,[1103+26390n]}{(n!)^4\,396^{4n}},$$

which is one of the fastest converging series for π in which each extra term adds roughly 8 digits to the expansion! Using this, the value of π has now been calculated to 17.5 million digits!! As if this were not enough Ramanujan gave at least 14 other series of π.

b) *Continued fractions*

Continued fractions are fractions of fractions of fractions and are referred to as the "typesetters' nightmares".

$$\pi/4=\cfrac{1}{1+\cfrac{1^2}{2+\cfrac{3^2}{2+\cfrac{5^2}{2+\cfrac{7^2}{2+\cfrac{9^2}{2+\cfrac{11^2}{2+\cfrac{13^2}{2+\cfrac{15^2}{2+\cfrac{17^2}{2+\cfrac{\text{etc}}{}}}}}}}}}}}$$

'In this area Ramanujan is probably unsurpassed in mathematical history ... His most interesting and enigmatic work in continued fractions relates to various products and quotients of gamma functions .... We do not know how he made these discoveries nor do we really understand this particular topic ....' [Bruce Berndt]

c) *Divergent series*

Ramanujan stated much before 1910 that

$$1+2+3+4+\ldots = -1/12$$
$$1^2+2^2+3^2+4^2+\ldots = 0$$
$$1^3+2^3+3^3+4^3+\ldots = +1/240$$

which appear to be 'absurd' even to a schoolboy. They did to M. J. M. Hill, the London mathematician, in 1912. But the considered comment of mathematicians today is that

They are not absurd. When Ramanujan makes a statement in mathematics one must try to understand in what sense he meant it to hold. Ramanujan could not describe analytic continuation as modern mathematicians would, but there is no doubt he understood what he was doing. These functions may be interpreted as the Riemannian zeta functions $\zeta(-1)$; $\zeta(-2)$; $\zeta(-3)$—results which are correct and have, in fact, been given by Euler. They are not only not absurd, the first equation has been used recently to suggest that space has 26 dimensions! and Bruce Berndt's comments 'The French mathematician R. Aprey of the University of Caen solved a famous problem connected with $\zeta(3)$ ... Aprey proved $\zeta(3)$ to be irrational using two 'new' beautiful ideas. One of the ideas had to do with a continued-fraction representation of $\zeta(3)$ *which turns out to be a special case of a very general continued fraction in Ramanujan's Notebook.*'

d) Some of Ramanujan's formulations seem to have been pulled out of the air. Some modern mathematicians feel that these are evidence of theories that are lurking in the future.

In the first letter to Hardy (16 January 1913) Ramanujan stated the coefficient of x^n in $(1-2x+2x^4-2x^9\ldots)^{-1}$ is an integer nearest to

$$\frac{1}{4n}\left(\cos h\pi\sqrt{n}-\frac{\sin h\pi\sqrt{n}}{\pi\sqrt{n}}\right).$$

Of this it was said:

It is inconceivable that Ramanujan could have guessed this extraordinary formula without having some sort of proof ... but it is not as good an approximation as he claimed it to be.

Hence the statement is mathematically 'false' and Hardy says:

Ramanujan's false statement was one of the most fruitful he ever made as it ended by leading us all to our joint work on partitions.

e) *Partitions of integers*: '$p(n)$, the number of partitions of n, is the number of ways in which a whole number or integer n can be written as a sum of integers'. e.g.

$1=1$	$p(1)=1$
$2=2,\ 1+1$	$p(2)=2$
$3=3,\ 2+1,\ 1+1+1$	$p(3)=3$
$4=4,\ 3+1,\ 2+2,\ 2+1+1,\ 1+1+1+1$	$p(4)=5$
$5=5,\ 4+1,\ 3+2,\ 3+1+1,\ 2+2+1,\ 2+1+1+1,\ 1+1+1+1+1$	$p(5)=7$
$200=200,\ 199+1,\ 198+2,\ 198+1+1,\ldots$	$p(200)=397299029388.$

Ramanujan was interested in the arithmetical properties of $p(n)$. For example, when $p(n)$ is odd or even. He showed that $p(5n+4)$ [i.e. $p(9)$, $p(14)$, $p(19)$, ...] is divisible by 5, and $p(7n+5)$ [i.e. $p(12)$, $p(19)$, $p(26)$, ...] is divisible by 7. Ramanujan proved many identities like

$$p(4)+p(9)x+p(14)x^2\ldots = \frac{5\{(1-x^5)(1-x^{10})(1-x^{15})\ldots\}^5}{\{(1-x)(1-x^2)(1-x^3)\ldots\}^6}$$

If I had to select one formula (to be representative) for all Ramanujan's work I would agree with Major Macmahon in selecting this. [Hardy]

Ramanujan was interested in the approximate or asymptotic formula for $p(n)$. The one he and Hardy gave is so good that $p(200)$ comes out to be 397299029388.004! Just 0.004 greater than the correct number, i.e. the error is 1 part in 10^{14}.

f) Entry (29), Chapter V, of Ramanujan's notebook, e.g.

$$\frac{1}{(1-x^2)(1-x^3)(1-x^5)(1-x^7)(1-x^{11})(1-x^{13})\&C}$$
$$=1+\frac{x^2}{1-x}+\frac{x^{2+3}}{(1-x)(1-x^2)}+\frac{x^{2+3+5}}{(1-x)(1-x^2)(1-x^3)}+\&C$$

has been cancelled. Why?

Suppose both the expressions are expanded in powers of x:

$$1+c_2x^2+c_3x^3+c_4x^4+\ \ldots = 1+d_2x^2+d_3x^3+d_4x^4+\ldots$$
$$c_n=d_n \text{ for } n=1 \text{ to } 20,$$

whereas $c_{21}=30$, $d_{21}=31$, so that $c_{21}\neq d_{21}$.

Did Ramanujan have an intuitive feeling that the expression was wrong after $n=20$; Or did he actually calculate these terms and find the discrepancy? What is significant is that even this cancelled entry has generated much research on what are now called 'Ramanujan Pairs'.

g) It is clear from Ramanujan's lost notebook that he was interested in the congruences of $p(n)$ and $\tau(n)$.

Ramanujan and the physical sciences

It would never have occurred to Ramanujan to enquire whether his formulae or his theorems would ever be put to 'use'.

I understand that lesser mathematicians who invent new formulations, feel and sometimes even take pride in that their mathematics will never be useful.

George Gamov says:

Pure mathematics tries to avoid morganatic relations with other sciences. Unfortunately, this haughty queen has not been successful in standing apart.... Even number theory, the purest of pure mathematics, has lost its crown of purity.

Let me give you an example (from Ramanujan's work) of the view that mathematics is a tool by which physical phenomena are quantified. Plastics that are so commonly used today consist of polymers. A polymer is a long-chain molecule with n identical basic molecular units, where n is very large. In dealing with the chemistry of plastics one is interested in how such a polymer would break up. Let us for simplicity assume that the polymer has only 5 basic units in it, i.e. $n=5$. We can then count the number of ways in which the

breakdown takes place. It does not break at all (case 1) or breaks into different pieces:
i.e. case (1), $5+0$; case (2), $4+1$; case (3), $3+2$; case (4), $3+1+1$; case (5), $2+2+1$; case (6), $2+1+1+1$; case (7), $1+1+1+1+1$.

We have written what Ramanujan calls the partition function $p(5)$. Ramanujan's work on partition of numbers becomes useful in understanding the break-up of these long-chain molecules in polymers. In the same manner it has become invaluable in two-dimensional statistical mechanics. In fact it has literally reached the streets and is used in the splicing of telephone cables. Some of Ramanujan's work on zeta functions has found application in the theory of pyrometry.

With the progress in modern physics the attitude of physicists towards mathematics has also taken a radical change. They believe that mathematics is really a source for concepts and principles by means of which new theories in physics can be created. Says one mathematician:

Some of Ramanujan's work is what has spurred on the solution of the hard hexagon model in statistical mechanics.

While another remarks:

Ramanujan's work in the area of number theory known as modular forms is what physicists have long been looking for, when they work on the mathematics of cosmic strings.

To many, it is a wonder how mathematics can mirror the physical universe. Indeed, men like Einstein and Planck have speculated about the very process by which mathematics is created by the human mind and have concluded, surprisingly, that it may be closely related to the process by which the physical world is understood by the human mind.

Kepler said:

Therefore, I chance to think that all nature and the graceful sky are symbolized in the art of geometry.

Physics (and in fact the physical sciences) is intimately connected with nature. Experimental observation followed by theoretical speculation are the essential steps which lead to the discovery of the laws of Nature. It may seem strange that many discoveries, theoretical and experimental, are made in different parts of the world almost simultaneously; but this may be an indication that progress in physics is made by the accumulation of knowledge and by the advances made in experimental methods. Many scientists often feel that when time is ripe it will lead to a discovery in a field. It is said that had Einstein not formulated the special theory of relativity, it would have been discovered within a year or two, but one might have had to wait for decades in the case of the general theory of relativity. It is a salutary thought for the ego of a scientist to realize that discoveries in the physical sciences are inevitable, that the existence of any particular scientist is not so important—that the role of an outstanding man of science is just to accelerate the process of what is inevitable.

What then is the position in mathematics? One view is that the mathematician after propounding a few postulates creates a world of his own, following logically the rules of the game he has laid down. Since human imagination is limitless mathematics too could be unbounded. Unfortunately this romantic view is being abandoned by mathematicians themselves.

They too have become Platonists and believe that mathematics is discovered rather than invented. Therefore, according to them, in the long run it does not matter what the individual mathematician does. If someone does not do it now, someone else will surely discover it later.

Says Richard Askey:

One counter example to this I know in this century is Ramanujan. I am tempted to write that Ramanujan did create some mathematics. At least, without Ramanujan, some mathematics we know would have never been discovered in my life-time.

Motivations?

What motivates a man like Ramanujan to do mathematics? There is no shortage of essays and books written explaining the motivations for pursuing art, science or mathematics—from the 'search for truth' to the 'quest for beauty'.

C. V. Raman, whom I mentioned in the beginning, was one obsessed and pre-occupied with the beauty of Nature. The blue of the sky, the speck of dust in a sunbeam, the glory of a rainbow, the twinkling of the stars, the shimmer of a butterfly's wings all fascinated him and motivated much of his science. He was the perfect example of Poincare's dictum that a scientist studies Nature because he takes pleasure in it, and he takes pleasure in it because it is beautiful.

It has often been asked whether this probing into the working of Nature distorts one's vision of its beauty. Raman's answer is a definite no.

The man of science observes what Nature offers with the eye of understanding but her beauties are not lost on him for that reason. More truly, it can be said that understanding refines our vision and heightens our appreciation of what is striking and beautiful. [C. V. Raman]

The revelation of the underlying order in Nature also has an aesthetic appeal of its own. As an example let me take a crystal with its resplendent faces, whose external beauty anyone can see—a masterpiece of Nature's art. The scientist delves deeper with his X-rays and electron microscopes and unravels an incredible atomic order. From it emerge Nature's laws of the architecture of crystals which again is a thing of beauty in a different way. When mathematics abstracts the very essence of a crystal something extraordinary occurs and we are transported to a different world. The external form of the crystal vanishes. In this strange world objects no longer exist, nor matter, and a new beauty emerges. This must surely be the kind of beauty which excites mathematicians.

When Littlewood came back from the war and saw Ramanujan's work, he exclaimed:

The beauty and singularity of his results is entirely uncanny!

To explain this beauty to one who has no inkling of the aesthetics of mathematics is like explaining the beauty of music to one who is tone-deaf.

Says Richard Askey:

I think of Ramanujan and Mozart as analogous in their fields.

G. N. Watson:

The thrill of a mathematical formula is indistinguishable from the thrill of seeing the incomparable sculptures of Michael Angelo.

And Emma Lehmer:

The discovery of *the lost notebook* of Ramanujan is comparable to the discovery of a complete sketch of the tenth symphony of Beethoven.

In 1920, when he was nearing his end, Ramanujan was working on a family of identities which had profound implications to mathematics. He called these the Mock Theta Functions, and in his last letter to Hardy, Ramanujan writes:

These enter into mathematics beautifully.

Was his motivating force a craving to find beauty? Perhaps. But to me all attempts to explain the motivations of genius are never meaningful. If one must have an explanation the simplest might be that Ramanujan did what he could not help doing.

As in yonder valley
The myrtle breathes its fragrance into space
Through such as these God speaks.