Effects of temperature and ground-state coherence decay on enhancement and amplification in a Δ atomic system

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We study phase-sensitive amplification of electromagnetically induced transparency in a warm ^{85}Rb vapor wherein a microwave driving field couples the two lower-energy states of a Λ energy-level system thereby transforming into a Δ system. Our theoretical description includes effects of ground-state coherence decay and temperature effects. In particular, we demonstrate that driving-field-enhanced electromagnetically induced transparency is robust against significant loss of coherence between ground states. We also show that for specific field intensities, a threshold rate of ground-state coherence decay exists at every temperature. This threshold separates the probe-transmittance behavior into two regimes: probe amplification vs probe attenuation. Thus, electromagnetically induced transparency plus amplification is possible at any temperature in a Δ system.

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I. INTRODUCTION

Electromagnetically induced transparency (EIT) [1] has become foundational for creating, storing, and transferring quantum features between interacting systems. EIT has its origins in atom-light interaction phenomena wherein an atom with a three-level Λ configuration of atomic levels interacts with two coherent electromagnetic fields ($|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow 3\rangle$ of Fig. 1). Under a two-photon resonance condition for these two transitions, absorption for both these fields is eliminated due to establishment of quantum coherence between levels $|1\rangle$ and $|2\rangle$. Several other nonintuitive physical phenomena arise due to this quantum coherence. These include lasing without inversion (LWI) [2], the realization of slow and stopped light [3], ultralow light level optical switches [4], and single-photon quantum nonlinearities [5].

For a long time [6,7], it was realized that connecting the unconnected two levels of a traditional EIT system by a drive field makes the absorption and dispersion properties dependent on the relative phase between all three fields. For a Λ EIT system, this would result in connecting the lower two levels $|1\rangle$ and $|2\rangle$, resulting in a Δ system (Fig. 1). From an analysis of dark states in a Λ system controlled by a microwave field, the sensitivity of the dark state to the relative phase between the interacting fields was established [8]. Spatially separated interaction with the Raman optical fields and the microwave field translated the optical dark state to either one of the microwave-dressed spin states or vice versa [9]. A study of slow and fast light propagation in a Δ system [10] explicitly brought out the control on dispersive properties of such systems by the drive field. Experimental demonstration of both

EIT [11,12] and associated nonlinear effects [13] in Δ systems of ^{85}Rb vapor has opened the possibility of demonstrating phase-sensitive coherence-related effects in these systems.

Phase-dependent amplification for microwave fields in a fluxonium superconducting circuit with an artificial atom featuring a Δ configuration of energy levels reveals simultaneous existence of LWI and EIT phenomena resulting in probe amplification [14]. As the propagation phases of electromagnetic (EM) fields governed by Maxwell equations depend on the polarization of the medium, the EM waves participating in a closed-loop interaction scheme thus have their absorptive and dispersive properties determined not only by Kramers-Kronig relations but also by the refraction experienced in the medium [15]. Hence, for appropriate phases, when the medium as a whole exhibits transparency for all three interacting fields in a Δ system, the total EM energy can oscillate between any of these fields.

This oscillation can give rise to large lossless amplification in any one or two of the three fields [16]. An analysis of phase-dependent amplification of a probe pulse controlled by the drive field [17] used amplification to compensate for losses experienced inside the medium. Fourier decomposition of pulse propagation in a Δ system [18] showed the simultaneous existence of absorption and gain channels due to multiphoton processes.

Despite such strong reasons to observe amplification in Δ systems, lossless amplification of the probe field has not been experimentally verified so far, even though enhancement of EIT has been seen [12]. Factors that affect population and coherence between the ground states of a Δ system play a major role in determining such experimental outcomes. For room-temperature experiments, the presence of an associated thermal bath determines steady-state population between the ground levels. In addition, various factors affect the coherence between the ground states of a Δ system.

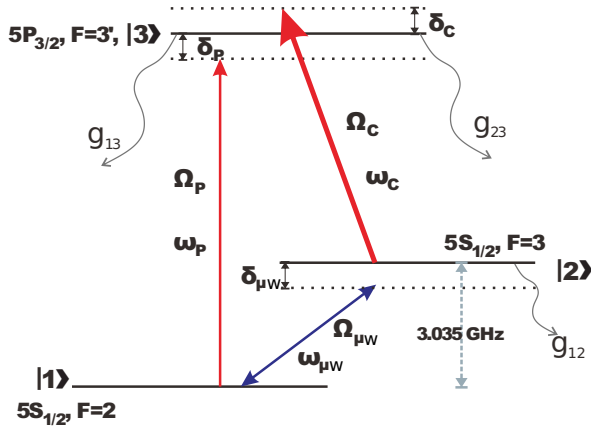*andal@rri.res.in

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FIG. 1. (Color online) Level scheme showing Δ atomic level configuration in ^{85}Rb formed by two optical fields and a microwave field. The coupling field at frequency ω_c, with strength Ω_c (thick red), connects the $5S_{1/2}, F = 3(|2\rangle) \leftrightarrow 5P_{3/2}, F = 3'(|3\rangle)$ transition. The probe field at frequency ω_p (thin red), with strength Ω_p, connects the $5S_{1/2}, F = 2(|1\rangle) \leftrightarrow 5P_{3/2}, F = 3'(|3\rangle)$ transition. The microwave drive field at frequency $\omega_{\mu w}$, with strength $\Omega_{\mu w}$ connects levels $5S_{1/2}F = 2(|1\rangle) \leftrightarrow 5S_{1/2}, F = 3(|2\rangle)$. The detunings of the coupling and probe fields from $|3\rangle$ are denoted by δ_c and δ_p respectively. The decay rate γ_{ij} from level $|j\rangle$ to level $|i\rangle$ assumes values $\gamma_{12} = 0.001$ MHz and $\gamma_{23} = 5$ MHz.

One such factor is the finite bandwidth of the optical and microwave drive fields. For finite bandwidth electromagnetic fields, the phase diffuses over time across their bandwidths. For a Λ system, with critically cross-correlated probe and coupling fields, such phase diffusion does not destroy ground-state atomic coherence and preserves the dark state [19]. In a Δ system, the issue of phase diffusion on coherence between ground states has not been addressed so far. However, Agarwal's treatment of δ-correlated phase fluctuating fields interacting with two-level atoms [20] suggests that any formal treatment of phase fluctuations in a Δ configuration will lead to ground-state coherence decay. In addition to phase fluctuations, collisions between Rb atoms and between the Rb atoms and the walls of the cell also contribute to decay of ground-state coherence.

Here we undertake a comprehensive analysis of the consequences of thermal bath and the effect of decay rate of ground-state coherence on a Δ atomic system. For the purpose of this study, we have used the natural decay parameters pertaining to a realistic Δ system in a warm ^{85}Rb vapor. In addition, we have treated the rate of ground-state coherence decay as a phenomenological constant. This constant represents the various physical effects that give rise to a decay of ground-state coherence in a Δ system. Collisional factors that give rise to ground-state coherence decay can be mitigated by using buffer gas additions to rubidium vapors and by using paraffin-coated vapor cells. Therefore, our present study is most relevant to ground-state coherence decay given rise by the ever-present finite bandwidth of electromagnetic fields used in the experiment. In this context, our study helps to quantify the robustness of phase-sensitive induced transparency effects, with phase diffusing fields.

For given intensities of coupling, probe, and microwave fields, we establish the existence of a threshold rate of ground-state coherence decay at every temperature. This threshold separates the behavior into regimes. Below this threshold, probe enhancement and amplification are possible for a wide range of coherence decay values. Importantly, our analysis establishes that enhancement of the probe field in the presence of a drive field is a precursor to probe amplification. Furthermore, transparency and amplification are possible even for warm Δ systems, provided the rate of ground-state coherence decay is below a certain limit.

The outline of our paper is as follows. Section II details our theory of Δ systems, which includes thermal bath and ground-state coherence decay effects. Section III presents results of our theoretical model. Section IV discusses the results and gives predictions for future experiments. We summarize our conclusions in Sec. V.

II. THEORY

The theory of our atomic Δ system differs from other theories of closed-loop systems [21] in two respects. In contrast to previous analyses, we have included the consequences of decoherence between the lower two levels $|1\rangle$ and $|2\rangle$ by making the ground-state coherence decay rate as a variable in our calculations. In addition, we have included thermal bath effects for finite temperature Δ systems.

We consider the atomic levels of our Δ system as shown in Fig. 1, which also presents interactions with the probe (p), coupling (c), and microwave (μw) fields. The Rabi frequency Ω for dipole interaction of a pair of levels $|i\rangle$ and $|j\rangle$, with an applied field $\boldsymbol{E}$, is given by $\Omega = \boldsymbol{d}_{ij} \cdot \boldsymbol{E}$, with $\hbar$ taken to be equal to 1, and $\boldsymbol{d}_{ij} \equiv \langle i|\boldsymbol{d}|j\rangle$ with $\boldsymbol{d}$ the dipole moment vector.

For our Δ system, we take the optical probe and coupling fields to be propagating through the cell along the z axis. They are represented by

$$\boldsymbol{E}_{\perp p}(\boldsymbol{r}_\perp) \cos(\omega_p t - k_p z + \phi_p) \qquad (1)$$

and

$$\boldsymbol{E}_{\perp c}(\boldsymbol{r}_\perp) \cos(\omega_c t - k_c z + \phi_c), \qquad (2)$$

with angular frequencies $\omega_{p,c}$, wave numbers $k_{p,c}$ and initial phases $\phi_{p,c}$ respectively. We have taken the microwave field to be a standing wave inside a microwave cavity; therefore there is no propagation phase associated with it. The microwave field is thus represented by

$$\boldsymbol{E}_{\mu w}(\boldsymbol{r}) \cos(\omega_{\mu w} t + \phi_{\mu w}). \qquad (3)$$

We start with the Hamiltonian of the Δ system in the interaction and rotating wave picture, taking the reference energy level as the energy of level $|3\rangle$:

$$\hat{H}(\boldsymbol{r}, \boldsymbol{v}) = \delta_p'(\boldsymbol{v})|1\rangle\langle 1| + \delta_c'(\boldsymbol{v})|2\rangle\langle 2| + \Omega_{\mu w}(\boldsymbol{r})|1\rangle\langle 2| $$
$$+ \Omega_p(\boldsymbol{r}_\perp)|1\rangle\langle 3| + \Omega_c(\boldsymbol{r}_\perp)|2\rangle\langle 3| + \text{H.c.} \qquad (4)$$

for H.c. denoting Hermitian conjugate. Here

$$\delta_p'(\boldsymbol{v}) \equiv \delta_p - \boldsymbol{k}_p \cdot \boldsymbol{v}, \ \delta_c'(\boldsymbol{v}) \equiv \delta_c - \boldsymbol{k}_c \cdot \boldsymbol{v}, \qquad (5)$$

are Doppler-shifted detunings of the coupling and probe fields (see Fig. 1) seen by an atom moving with velocity $\boldsymbol{v}$. The terms

Ω_j's are complex Rabi frequencies of the fields are

$$\Omega_c(\boldsymbol{r}_\perp) := \boldsymbol{d}_{23} \cdot \boldsymbol{E}_{\perp c}(\boldsymbol{r}_\perp)e^{i\phi_c},$$

$$\Omega_p(\boldsymbol{r}_\perp) := \boldsymbol{d}_{13} \cdot \boldsymbol{E}_{\perp p}(\boldsymbol{r}_\perp)e^{i\phi_p},$$

$$\Omega_{\mu w}(\boldsymbol{r}) := \boldsymbol{d}_{12} \cdot \boldsymbol{E}_{\mu w}(\boldsymbol{r})e^{i(\omega_p-\omega_c-\omega_{\mu w})t+i(k_p-k_c)z+i\phi_{\mu w}}. \quad (6)$$

Using the dipole approximation, the Rabi frequencies of the probe, coupling, and microwave fields are taken to be spatially uniform, yielding the constants $\Omega_c(\boldsymbol{r}_\perp) \equiv \Omega_c$, $\Omega_p(\boldsymbol{r}_\perp) \equiv \Omega_p$ and $\Omega_{\mu w}(\boldsymbol{r}) \equiv \Omega_{\mu w}$.

Unlike a Λ system, the propagation and temporal phases of the EM fields in closed-loop systems do not vanish in the interaction picture. Choosing $\delta_c = 0$, and maintaining $\delta_p = \delta_{\mu w}$ in all our calculations, makes the temporal phase factor

$$\omega_p - \omega_c - \omega_{\mu w} = 0, \quad (7)$$

thus ensuring time-independent Rabi frequencies. However, the propagation phases give rise to an effective position-dependent microwave Rabi frequency, which is given by $\Omega_{\mu w}(z) = \Omega_{\mu w}e^{i\phi_{\mu w}}e^{i(k_p-k_c)z}$.

The dynamical evolution of density matrix elements, in the interaction picture is given by the master equation

$$\dot{\rho}(\boldsymbol{v},z) = -i[\hat{H}(\boldsymbol{v},z),\rho(\boldsymbol{v},z)] + \sum_{k=1}^{5} \mathcal{L}(\hat{c}_k)\rho(\boldsymbol{v},z) \quad (8)$$

with $\mathcal{L}(\hat{c}_k)$ being the Lindblad superoperator

$$\mathcal{L}(\hat{c})\rho := \hat{c}\rho\hat{c}^\dagger - \tfrac{1}{2}\{\rho,\hat{c}^\dagger\hat{c}\} \quad (9)$$

acting on operators

$$\hat{c}_1 = \sqrt{(\bar{n}+1)\gamma_{12}}|1\rangle\langle 2|,$$

$$\hat{c}_2 = \sqrt{\bar{n}\gamma_{12}}|2\rangle\langle 1|, \quad \hat{c}_3 = \sqrt{\gamma_{13}}|1\rangle\langle 3|,$$

$$\hat{c}_4 = \sqrt{\gamma_{23}}|2\rangle\langle 3|, \quad \hat{c}_5 = \sqrt{\gamma_c}(|1\rangle\langle 1| - |2\rangle\langle 2|) \quad (10)$$

with γ_{12} and $\gamma_{23} = \gamma_{13}$ representing the natural linewidth of levels $|2\rangle$ and $|3\rangle$ respectively.

The symbol $\bar{n}$ is the average number of thermal photons in the bath at temperature T, and γ_c is the rate of ground-state coherence decay. As our optical fields are copropagating along the z direction, we henceforth denote the z component of the velocity vector by v. We solve Eqs. (4)–(10) for steady-state values of $\rho(v)$, which is then averaged over the Maxwell-Boltzmann velocity profile at some temperature T

$$\bar{\rho} = \frac{\int_{-\infty}^{\infty} \rho(v)e^{-\left(\frac{v}{v_{mp}}\right)^2}dv}{\int_{-\infty}^{\infty} e^{-\left(\frac{v}{v_{mp}}\right)^2}dv} \quad (11)$$

with $v_{mp} = \sqrt{\frac{\pi}{4}} \; \bar{v}$ the most probable speed of atoms at temperature T and $\bar{v}$ the average speed of the atoms.

As is well known in a Δ system, the steady-state $\bar{\rho}$ matrix values depend on the relative phase between all fields [6]. As the coupling and probe fields differ in wavelength, they have differing phase values during propagation. We take both these optical fields to be derived from the same source thereby making their initial phases identical. Thus, the relative phase between all three fields is

$$\phi(z) = z(k_p - k_c) + \phi_{\mu w}. \quad (12)$$

Therefore, we can vary the value of ϕ by controlling z. As experiments with Δ systems typically employ a Rb vapor cell of a finite length L, z-dependent phase variations of density-matrix elements need to be calculated, taking into account the phase changes experienced over the entire length L. In addition, we have assumed that the probe and coupling fields are right- and left-circularly polarized, reflecting experimental demonstration of probe transmission sensitivity to polarizations [12].

In order to simulate the changes in the probe field as it passes through a cell of length L, we treat the Rb cell as a sequence of small cells along the propagation direction. The propagation equation for the probe field is then calculated using the slowly varying envelope approximation in each cell, which is given by [12]

$$\frac{\partial \Omega_p}{\partial z} = -i\eta\bar{\rho}_{31}. \quad (13)$$

Here $\bar{\rho}_{31}$ represents the phase-dependent steady-state density matrix element corresponding to probe absorption, obtained using Eqs. (4)–(10), and η is the coupling constant taken to be close to 1.

In the following section, we present results for probe transmission from our Δ system using our theoretical model. The absorption experienced by the probe field during interaction with an ^{85}Rb atom at a position z, are calculated using the imaginary parts of the density matrix element $\bar{\rho}_{31}$. Using realistic parameters, we present results of change in probe transmission for a system of ^{85}Rb atoms contained in a vapor cell of length L, for various values of T and for various rates of ground-state coherence decay (γ_c) using Eq. (13).

It is well known that ground-state coherence decay rate in a Λ system affects the contrast of EIT transmission resonance. In subsequent sections, we explore the transmission loss in the probe beam of our Δ system as a function of T and γ_c, both of which affect the coherence between the ground states.

III. RESULTS

In Fig. 2(a) we show probe absorption as a function of δ_p for an atomic Δ system at position z held at a temperature $T = 0$ K with no coherence decay between the ground states ($\gamma_c = 0$ MHz). Changing z results in modified values of relative phase ϕ. We see from the figure that for $\phi = \pi/2$ there is significant amplification seen in the probe field around two-photon resonance ($\delta_p = \delta_c$) Thus Fig. 2(a) establishes that our Δ-system theory produces phase-sensitive probe amplification at $T = 0$ K in the limit of zero rate of ground-state coherence decay. We also present in Fig. 2(a) the asymmetric and absorptive probe profiles at other ϕ values, which are qualitatively similar to those seen in theoretical calculations of a Δ system in superconducting circuits [14].

The effect increasing the ground-state coherence decay rate and temperature on probe transmission is revealed by the plots in Fig. 2(b), which pertain to $\phi = \pi/2$.

This Δ system is assumed to be in a cell of length L with physical parameters that give rise to a decay rate of ground-state coherence of about 1 MHz. This is a realistic decay rate for experiments conducted in narrow-diameter cells [22].
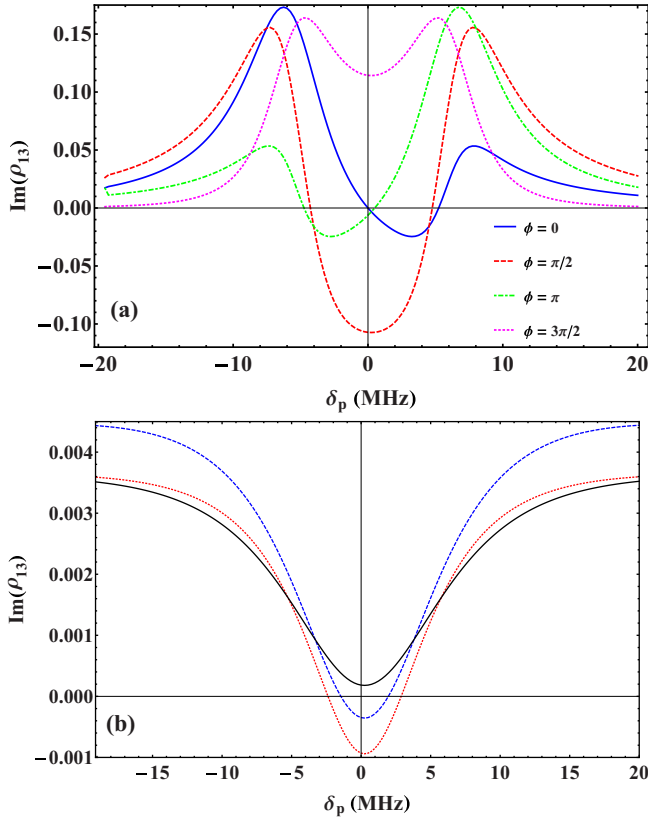
FIG. 2. (Color online) Probe absorption Im(ρ_{13}) vs probe detuning δ_p for a Δ system in ^{85}Rb gas with $\Omega_c = 6.4$ MHz, $\Omega_p = 1.0$ MHz, and $\Omega_{\mu w} = 0.8$ MHz. (a) Probe absorption with no ground-state coherence decay ($\gamma_c = 0$ MHz) at $T = 0$ K is given. The various values of ϕ are indicated in the legend. (b) Probe absorption with finite ground-state coherence decay values and at finite temperatures given for a fixed value of $\phi = \pi/2$. The various cases are: $T = 333$ K and $\gamma_c = 1.0$ MHz (solid black line), $T = 333$ K and $\gamma_c = 0.1$ MHz (dot-dashed red line), and $T = 233$ K and $\gamma_c = 1.0$ MHz (dashed blue line).

With $\gamma_c = 1.0$ MHz, we see that the probe experiences absorption at the two-photon resonance condition for $\phi = \pi/2$. For precisely this ϕ value, significant amplification of the probe transmission was obtained for parameters of Fig. 2(a). Thus, increasing temperatures and rates of ground-state coherence decay contributes towards a loss of probe amplification.

In Fig. 2(b), the transmitted probe field does not endure absorption and is actually amplified at $T = 333$ K for $\gamma_c = 0.1$ MHz. Thus, we observe that nonzero values of γ_c can yield probe-field amplification even for a warm Δ system.

We compare absorption at $T = 333$ K vs $T = 233$ K in Fig. 2(b), which shows that the transmitted probe in the cooler case is amplified. As the drive-field intensities of all three graphs in Fig. 2(b) are identical, these plots illustrate that probe-field amplification can be obtained for suitable values of the temperature T and the ground-state coherence decay rate γ_c.

In Fig. 3 we give a contour plot of change in transmitted probe intensity, as it emerges from a cell of length $L = 5$ cm, for wide ranges of ground-state coherence decay rate values and temperatures. The decay rate ranges from the kHz to the MHz

domain, which incorporates regimes where collisional decay is the dominant decohering mechanism as well as regimes where the finite linewidths of the electromagnetic fields contribute dominantly to the decay.

IV. DISCUSSION

We see from the contour plot of Fig. 3 that, for given intensities of the coupling, probe, and microwave fields at every temperature T, there exists a threshold rate of ground-state decoherence γ_{th}. This threshold at every temperature lies along the contour denoted by zero in Fig. 3. For γ_c values below the threshold value, we obtain amplification in the optical probe field indicated by positive-valued contours. For γ_c values above this threshold the probe exhibits absorption signified by negative-valued contours.

Experimentally an increase of the transmitted optical probe intensity has been observed when the microwave field was switched on compared to the value when the microwave field was switched off [12,13]. The increased transmitted intensity was still lower than the input probe intensity, hence no amplification.

To understand this enhanced transmission of the probe field, we show in the same contour plot of Fig. 3 the special contour labeled Reference Line. Along this contour, the transmitted intensity of the probe field is the same, with and without the microwave drive field on. With the help of the Reference Line contour and the zero contour line, the contour plot can be divided into three regions.
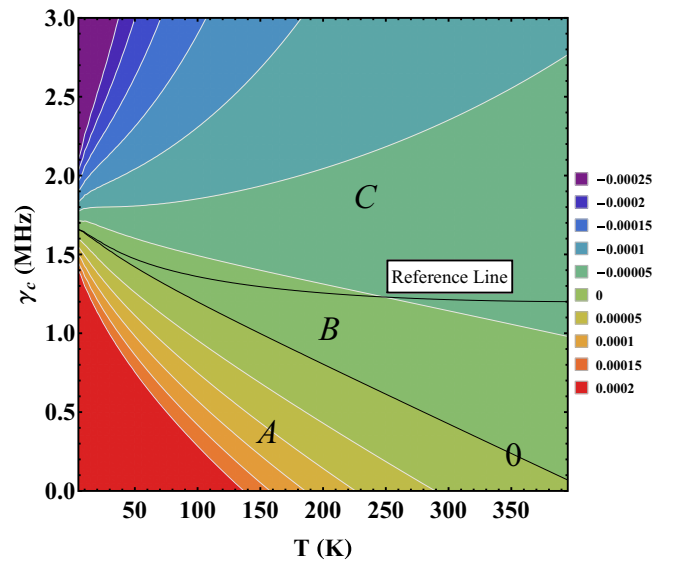

FIG. 3. (Color online) Contour plot showing the change from probe-field input intensity to transmitted intensity for temperatures $T = 0$ K to $T = 400$ K and for γ_c ranging from 0.001 MHz to 3.000 MHz. The Rabi frequencies are fixed at $\Omega_c = 6.4$ MHz, $\Omega_p = 1.0$ MHz, and $\Omega_{\mu w} = 0.8$ MHz. "Reference Line" denotes a reference contour along which the probe-field transmitted intensity is identical whether the microwave drive is on or off. The symbols A, B, and C refer to regions of amplification, enhancement and absorption respectively.

Region A of the contour plot is below the zero contour line, which is the region for which the probe field experiences amplification. Region B, which is sandwiched between the zero contour and the Reference Line, is where, despite absorption of the probe field, increased transmittance occurs compared to the absence of the microwave drive field. Region B is the region of enhancement.

Both of the A and B regions are below the Reference Line. Region C is above the Reference Line and gives the regime of probe absorption where the probe experiences greater absorption than it did in the absence of the microwave drive field.

Comparing γ_c values along the Reference Line and the threshold γ_{th} value for amplification along the zero-valued contour, it is clear that $\gamma_{th} < \gamma_c$ at all values of temperature T. This clearly indicates that a loss in ground-state coherence is the main reason for absence of probe amplification in a Δ system.

Significantly, our result shows that enhancement is a precursor to amplification even for hot Δ systems. This conclusion predicts that, in experiments with hot Δ systems, if enhancement but not amplification is observed, then reducing factors, which contribute to ground-state decoherence will enable to obtain amplification. We have not considered intensity-dependent variations of susceptibility in this study, since the probe field amplification is small.

We give below an analytical derivation of γ_{th} at $T = 0$ K to show its dependence on the intensities of coupling, probe and microwave fields. At $T = 0$ K, all the atoms are initially in the ground state $|1\rangle$ ($\rho_{11} = 1$ and $\rho_{22} = 0$) and we assume $\rho_{23} \approx 0$ holds.

Using Eqs. (4)–(10) we can solve for probe absorption in the medium, using the steady-state ρ_{31} expression given by

$$\rho_{31} = \frac{i\Gamma_{12}^0 \Omega_p - \Omega_c \Omega_{\mu w} e^{i\Delta k z}}{\Gamma_{12}^0 \Gamma_{13}^0 + |\Omega_c|^2} \tag{14}$$

with

$$\Gamma_{12}^0 = \frac{1}{2}\gamma_{12} + 2\gamma_c - i\Delta_{\mu w}, \quad \Gamma_{13}^0 = \frac{\gamma_c}{2} + \gamma_{13} - i\delta_p \tag{15}$$

and

$$\Delta k = k_p - k_c. \tag{16}$$

Using this expression for ρ_{31}, we apply the slowly varying envelope approximation for the probe field entering the vapor cell of length L at z_0 and exiting at $z_0 + L$.

Denoting the initial intensity of the probe field as Ω_{p0}, we derive an expression for the intensity of the probe at the exit of the cell to be

$$\Omega_p(z_0 + L)$$
$$= e^{-\alpha L}\left(\Omega_{p0} - i\frac{\alpha\Omega_c\Omega_{\mu w}e^{i\Delta k z_0}}{\Gamma_{12}^0}\frac{e^{(\alpha + i\Delta k)L} - 1}{\alpha + i\Delta k}\right) \tag{17}$$

with

$$\alpha := \frac{\eta\Gamma_{12}^0}{\Gamma_{12}^0 \Gamma_{13}^0 + |\Omega_c|^2}. \tag{18}$$

By considering experimentally realistic parameters with $L = 0.05$ m and by incorporating ^{85}Rb hyperfine ground-state separation of 3.035 GHz, we obtain $\Delta k = 63.624$ m^{-1} and $\alpha L \ll 1$.

At resonance with $\delta_p = \delta_{\mu w}$ and $\delta_c = 0$, we obtain the threshold γ_c (γ_{th}) by constraining the input and output intensities at the beginning and at the end of the vapor cell to be equal: $\Omega_p(z_0 + L) = \Omega_{p0}$. with $\Delta k L \sim \pi$, this gives us

$$\gamma_{th} = \frac{\Omega_{\mu w}\Omega_c}{\Omega_{p0}\pi}. \tag{19}$$

The analytically estimated value of γ_{th} at $T = 0$ K, for our intensities of probe, coupling, and microwave field is around 1.62 MHz, which is in quite good agreement with the full numerically simulated value seen in our contour plot of Fig. 3 at $T = 0$ K.

From Eq. (19), we see that γ_{th} can be modified by altering the intensities of coupling, probe, and microwave fields as long as the population remains predominantly in the ground state. With increasing temperature T, the thermal redistribution of ground-state population undermines the assumptions $\rho_{11} = 1$ and $\rho_{22} = 0$, thereby making the dependence of γ_{th} on intensities complicated. In such regimes, the threshold has to be obtained from a numerical plot as given in Fig. 3.

V. CONCLUSIONS

We study an atomic Δ system interacting with optical probe and coupling fields and a microwave drive field. Our analysis incorporates effects of ground-state coherence decay rate and effects of thermal bath associated with finite-temperature systems with a view to understanding regimes of probe amplification.

Our numerical results predict the existence of a threshold value for rate of ground-state coherence decay at every temperature, below which the probe field experiences amplification and above which it experiences absorption. We find that experimental observation of enhancement and not amplification in such atomic Δ systems is mainly due to the presence of ground-state decohering factors.

We predict that enhancement is actually a precursor to probe amplification, and that amplification can be obtained if suitable reduction in ground-state decoherence can be achieved. Our theory thus indicates that it is possible to obtain probe amplification even for warm Δ systems. We believe that this is an important step in experimentally obtaining phase-sensitive room-temperature amplification effects in equivalent Δ system architectures.

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