Implementing the Aharon-Vaidman quantum game with a Young type photonic qutrit

Piotr Kolenderski[1,2], **Urbasi Sinha**[1], **Li Youning**[3], **Tong Zhao**[1], **Matthew Volpini**[1], **Adán Cabello**[4,5], **Raymond Laflamme**[1] **and Thomas Jennewein**[1]

[1] *Institute for Quantum Computing, University of Waterloo, 200 University Ave. West, Waterloo, Ontario, CA ,*
[2] *Institute of Physics, Nicolaus Copernicus University, Grudziadzka 5, 87-100 Toruń, Poland,*
[3] *Department of Physics, Tsinghua University, Beijing 100084, P. R. China,*
[4] *Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain,*
[5] *Department of Physics, Stockholm University, S-10691 Stockholm, Sweden*

Abstract: The Aharon-Vaidman (AV) game exemplifies the advantage of using simple quantum systems to outperform classical strategies. We present an experimental test of this quantum advantage by using a three-state quantum system (qutrit) encoded in a spatial mode of a single photon passing through a system of three slits. The preparation of a particular state is controlled as the photon propagates through the slits by varying the number of open slits and their respective phases. The measurements are achieved by placing detectors in the specific positions in the near and far-field after the slits. This set of tools allowed us to perform tomographic reconstructions of generalized qutrit states, and implement the quantum version of the AV game with compelling evidence of the quantum advantage.

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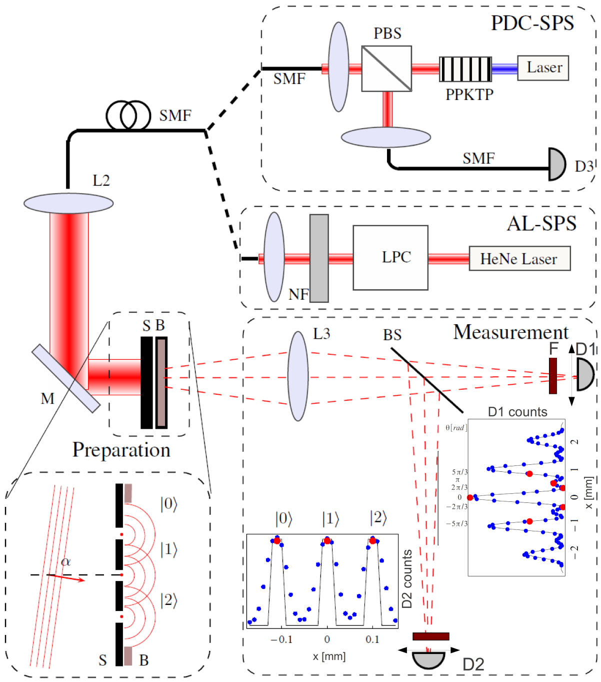
1. Introduction

We present here for the first time the experimental realization of the AV quantum game using single photon as the incident particle and a system of three slits in lieu of the boxes. The setup comprising of a single photon source (SPS), triple slit [1] and single photon detectors allowed us to perform optimized quantum tomography to characterize the qutrit states and to play the game correctly in the next step.

1.1. Aharon-Vaidman game

The Aharon-Vaidman game [2] is a conceptually simple example of how quantum mechanics can be both beneficial and counter-intuitive. In the classical analogue, Alice puts a particle in one of three boxes such that when Bob, who in next turn is allowed to check only two of them, is most likely to find it. Alice wins whenever Bob discovers the particle. Hence, it is obvious that Alice will not use the box that Bob does not have access to and therefore her chance to win is 50%. On the other hand, when she uses quantum particles her chance rises above this limit and ideally reaches 100%, when she chooses her particle to be in the state $|\psi_A\rangle = \frac{1}{\sqrt{3}}(|0\rangle+|1\rangle+|2\rangle)$ in the first turn of the game. In second turn, assuming Bob has access to slits (boxes) 0 and 2, if he decides to check if the photon is passing though slit number 0, he does a projective measurement on the state $|0\rangle$. If he finds the particle, then his state becomes $|\psi_B^{(p)}\rangle = |0\rangle$, otherwise $|\psi_B^{(n)}\rangle = \frac{1}{\sqrt{2}}(|1\rangle+|2\rangle)$. The photon detection results in losing the photon from the system, otherwise the photon goes through opened slits. Next, in the third turn of the game Alice makes a projective measurement on $|\psi_{Am}\rangle = \frac{1}{\sqrt{3}}(|0\rangle-|1\rangle+|2\rangle)$. If Alice detects a particle, she accepts the game trial, and if she does not, she cancels it. Now it is clear that Alice cannot lose as whenever Bob does not detect a photon, the state after the second turn is $|\psi_B^{(n)}\rangle$ and Alice's detector never clicks as $|\langle\psi_B^{(n)}|\psi_{Am}\rangle|^2 = 0$. If Bob found the particle in a slit 0 and tried to leave no trace of that, Alice has $|\langle\psi_B^{(p)}|\psi_{Am}\rangle|^2 = 1/3$ chance to detect it after that. The same reasoning holds if Bob chooses the slit number 2.

(a)



(b)

(c)

Bob's query		Alice's photon counts	
		AL-SPS	PDC-SPS
	□■■	24400	142
	■□□	6189	25
	■■□	31780	138
	□□■	6692	18

Fig. 1. (a)Experimental setup. The AL-SPS comprises of HeNe laser ($\lambda = 632$ nm), laser power controller (LPC) and neutral filter (NF). The PDC-SPS ($\lambda = 810$ nm) is based on PPKTP crystal pumped by blue continuous wave laser. Heralding photon is detected by detector D3. The single photons from both sources are coupled to single mode fibers (SMF). A qutrit is prepared using the blocking mask and three slits. Next the measurement part of the setup comprises of a 2 inch diameter $f = 150$mm lens (L3), 2 inch diameter pellicle beamsplitter (BS), colour filters (F) and two detection systems (D1, D2), each comprised of multimode fiber mounted on a precise motorized stage (Thorlabs ZST13) and a Perkin Elemer avalanche photodiode.
(b) Experimental and theoretical, best classical and best quantum winning trials in the quantum game. The simulation shows the quantum advantage over classical limit of 50% as Alice wins in 82% (87%) of accepted game trials when using AL-SPS (PDC-SPS).
(c) Table shows the number of photon counts measured by Alice for each of possible actions by Bob and his measurement outcome. For AL-SPS (PDC-SPS) photons were collected for 2 s (1 min, coincidence window 1 ns).

1.2. Encoding and measuring a qutrit

Let us move now to the implementation. The Young type qutrit is realized using triple slits and a single photon source. The spatial wave function of the photon passing through the nth slit can be written as [3,4]:

$$|n\rangle = \int \mathrm{d}k_x \sqrt{\frac{a}{2\pi}} \mathrm{sinc}\left(\frac{k_x a}{2}\right) e^{-ink_x d} |k_x\rangle, \tag{1}$$

where a is the slit width, d is the distance between the slits and k_x represents the transverse component of wave vector. These definition allow one to write the state of the transmitted photon as:

$$|\psi\rangle = \frac{1}{\sqrt{3}} \left(s_1 |0\rangle + s_2 |1\rangle + s_3 |2\rangle\right), \tag{2}$$

which accounts for the basic definition of a Young type qutrit. Here amplitudes s_1, s_2 and s_3 depend on the transmission amplitude though the slit.

The projective measurements are determined by the laws of propagation and the geometry of the setup. For simplicity, we chose to detect in the positions corresponding to near and far field. This can be done using a lens and placing a detector in the focal plane (far field) and in the plane where the image of the slits is formed (near field). In near filed there are three positions which can be associated with the measurement operator defined as:

$$M_{\mathrm{nf}}(n) = \mu_{\mathrm{nf}} |n\rangle \langle n|, \tag{3}$$

where μ_{nf} is the normalization factor to be specified later and subscript nf stands for near field. Similarly, one can define the measurement operator in the far field as:

$$M_{\mathrm{ff}}(\theta) = \mu_{\mathrm{ff}}(\theta) |\phi(\theta)\rangle \langle \phi(\theta)|, \tag{4}$$

where we introduced $|\phi(\theta)\rangle = |0\rangle + \exp(i\theta)|1\rangle + \exp(i2\theta)|2\rangle$, $\mu_{\mathrm{ff}}(\theta)$ is the normalization factor, the phase parameter reads $\theta = 2\pi xd/f$ and the subscript ff stands for far field. Next, by taking three near field measurements $M_{\mathrm{nf}}(n)$, $n = 0,1,2$ and six far field operators $M_{\mathrm{ff}}(\theta)$ corresponding to $\theta = \{0, \pi, 2\pi/3, -2\pi/3, 5\pi/3, -5\pi/3\}$ one can construct POVM set allowing one for reconstruction of arbitrary pure state.

2. Experiment

The experimental setup is depicted in Fig. 1(a). For the quantum game the qutrit was prepared in $|\psi_A\rangle$, which was characterized before by quantum tomographic method. We simulated all possible scenarios of Bob's measurement using PDC-SPS and AL-SPS. The measured photon counts are presented in table 1(c). In the perfect case one expects no counts when two slits are open. Here, it is attributed to finite size of the multimode fiber core, dark counts and background noise. We estimate that the former two contribute approximately 3 coincidence counts. Despite the practical limitations of our experimental setup, Alice had a 87% chance to win using PDC-SPS and 82% using AL-SPS, see Fig. 1(b). This is much better than classical strategy and close to the ideal quantum limit.

In conclusion, we experimentally presented a simple way to implement a qutrit system into a single photon's spatial degree of freedom, which allowed us to perform state tomography and simulate the AV quantum game. The encoding part resorted to the Young type experiment, where a photon passes through 3 slits, which defined its state. Controlling an initial propagation direction of a photon and configuration of the slits it was possible to encode certain class of states. Our state reconstruction technique was based on a small number of measurements over a short period of time that makes our method stable and time efficient in contrast to Ref. [3],

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