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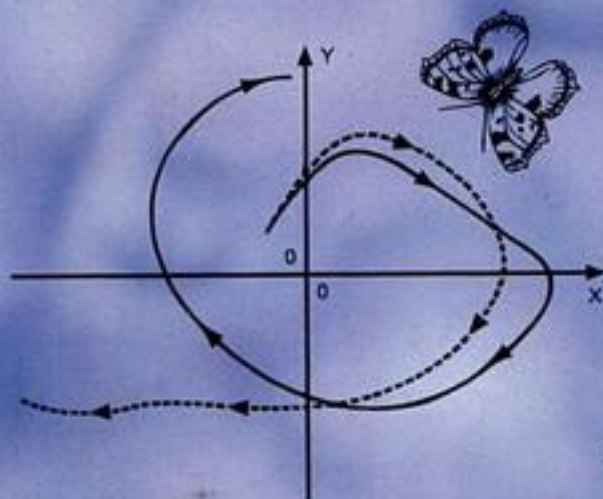
Educational Monographs



Jawaharlal Nehru Centre for  
Advanced Scientific Research

# DETERMINISTIC CHAOS

Complex Chance Out Of Simple Necessity



**N KUMAR**

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*Deterministic Chaos* addresses the newly emerged paradigm of complexity of change. It describes how a simple system ruled by a deterministic law can evolve in a manner too complex to predict in the long run, even in principle, in an operationally qualitative, well-defined sense. With brevity and without oversimplification, it presents the concepts of deterministic chaos and powerful qualitative techniques to explore it. The book is within the grasp of any curious reader with a science background.

*Dr N Kumar* is the Director of the Raman Research Institute, Bangalore. In a career spanning over thirty years, he has been Fellow of a number of prestigious institutions including the Indian Academy of Sciences, the Indian National Science Academy, the American Physical Society, and the Third World Academy of Sciences (TWAS), to name a few. He has received a number of awards including the S S Bhatnagar Award in 1985 and the TWAS Award in Physics in 1992. Dr Kumar is an accomplished author and has a number of research papers, articles, and books to his credit.

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Cover: A schematic representation of the butterfly effect.



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