Gravitation, Gauge Theories and the Early Universe

Edited by

B. R. Iyer, N. Mukunda and C. V. Vishveshwara

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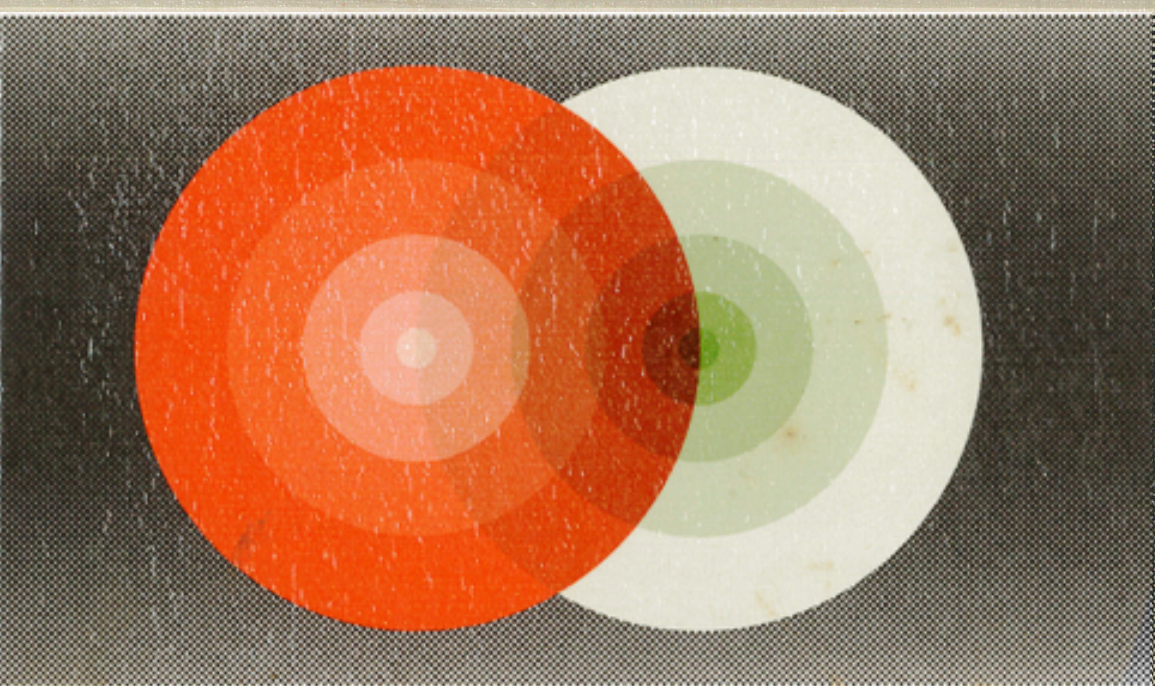

Fundamental Theories of Physics

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Edited by

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This volume contains twenty-six chapters, each written by a leading expert, on various aspects of gravitation, gauge theory and the early universe. The work begins with an introduction to relativity, particle physics, and classical quantum field theory. This is followed by reviews of the latest developments in each of these fields, such as black-hole thermodynamics, electroweak theory, grand unification, and the renormalization group. The concluding part of the work comprises discussions of current research topics, such as problems of big-bang cosmology, quantum fields in curved spacetimes, quantum cosmology, Kaluza – Klein theories, supersymmetry, supergravity and superstrings. In addition, special articles on relevant mathematical topics are included.

The book covers a wide range of topics which are at the forefront of current research interests, and may serve to inspire and guide the student in these different areas, starting from first principles and leading up to the most modern developments.

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