2Physics Quote:
"Needless to say, these advantages would not only arise for antineutrino monitoring of the IR-40 but for any reactor with a power output in the 20-250 MW_{th} range, which are the most likely candidates for being an entry point for a plutonium-based nuclear weapons program. Antineutrino reactor monitoring would not replace other techniques but in combination with those techniques can enhance the overall effectiveness and reliability of non-proliferation safeguards. A practical system appears feasible on a timescale of 1-2 years and the next step would be an actual antineutrino reactor monitoring experiment."
-- Eric Christensen, Patrick Huber, Patrick Jaffke, Thomas E. Shea
(Read Full Article: "Antineutrino Monitoring for the Iranian Heavy Water Reactor")

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Multipolar Post Minkowskian and Post Newtonian Toolkits for Gravitational Radiation

Bala R. Iyer

[This is an invited article reviewing two decades of work by the author and his international collaborators. -- 2Physics.com]

Author: Bala R. Iyer
Affiliation: Raman Research Institute, Bangalore, India

My work on gravitational waves (GW) began during a sabbatical I spent with Thibault Damour at DARC (CNRS-Observatoire de Paris) and Institut des Hautes Etudes Scientifiques (IHES) in France during 1989-90. I was exposed to the powerful Multipolar Post Minkowskian (MPM) formalism that Luc Blanchet and Thibault Damour had set up. Though the MPM formalism then seemed more elaborate than necessary, it is a good example of the advantage that a complete and mathematically rigorous treatment of a problem can eventually bring in the future for more demanding applications.

The wave generation formalism relates the gravitational waves observed by a detector in the far-zone of the source to the stress-energy tensor describing the source. Successful wave-generation formalisms [1] combine post-Minkowskian (PM) methods [expansions in G], post-Newtonian (PN) methods [expansions in $1/c$], multipole (M) expansions [expansions into irreducible representations of the rotation group], and perturbations around curved backgrounds. There are two independent aspects addressing two different problems. The general method (MPM expansion) applicable to extended or fluid sources with compact support, based on the mixed PM and multipole expansion matched to some PN (slowly moving, weakly gravitating, small-retardation) source. And, the particular application to describe inspiralling compact binaries(ICB) by use of point particle models.

Starting from the general solution to the linearized Einstein's equations in the form of a multipolar expansion (valid in the external region), a PM iteration is performed and each multipolar piece is independently treated at each PM order. For the external field, the general method is not *a priori* limited to PN sources. However, closed form expressions for the multipole moments can be only obtained for PN sources because the exterior field may be related to the inner field only if there exists an *overlapping* region where both the MPM and PN expansions are valid and can be matched together. After matching, the multipole moments have a non-compact support since they depend on the gravitational field stress-energy that is distributed everywhere up to spatial infinity. To account for this correctly, the definition of the multipole moments involves a crucial finite part operation based on analytic continuation (equivalent to a Hadamard *partie finie* of the integrals at infinity).

The physical post-Newtonian source, for any PN order, is characterized by six symmetric and trace free (STF) time-varying moments, functionals of a formal PN expansion of the stress-energy pseudo-tensor of the material and gravitational fields. Starting from the six STF *source moments* one can define a different set of two *canonical* source moments, such that the two sets of moments are physically equivalent (i.e. lead to the same metric modulo coordinate transformations). The use of the canonical source moments simplifies the calculation of the external non-linearities and their existence shows that *any* radiating isolated source is characterized by two and only two sets of time-varying multipole moments.

The MPM formalism is valid all over the weak field region outside the source including the wave zone (up to future null infinity). The far zone expansion at Minkowskian future null infinity contains logarithms in the distance which are artefacts of the harmonic coordinates. One can define, order by order in the PM expansion, some *radiative* coordinates such that the log-terms are eliminated. One recovers the standard (Bondi-type) radiative form of the metric from which the *radiative moments* seen by the detector, can be extracted in the usual way.

Nonlinearities in the external field are determined by a post-Minkowskian algorithm and one obtains the radiative multipole moments as some non-linear functional of the canonical moments and thus of the actual source moments. The source moments mix with each other as the waves propagate from the source to the detector and thus the relation between radiative and source moments include many non-linear multipole interactions including *hereditary* (history dependent) effects like tails, memory and tails-of-tails. The radiative moments are also very convenient for the computation of the spin-weighted spherical harmonic decomposition of the gravitational waveform employed to compare analytical PN results to numerical relativity simulations for binary black holes.

The application of the above results to compact binary systems involves a new input. For compact objects the effects of finite size and quadrupole distortion induced by tidal interactions are of order 5PN. Hence, neutron stars and black holes can be modelled as point particles represented by Dirac δ-functions. The general formalism, however, applies only to smooth matter distributions and cannot be directly used for point particles since they lead to divergent integrals at the location of the particles when the energy momentum tensor of a point particle is substituted into the source moments. The calculation needs to be supplemented by a method for self-field regularisation i.e. a prescription for removing this infinite part of the integrals.

Hadamard regularisation, based on Hadamard's notion of partie finie, was employed in all earlier works and led to consistent results in different approaches up to 2.5PN order. Thus, it was surprising to discover that Hadamard regularisation at 3PN order was incomplete as signalled by the presence of four undetermined constants in the final 3PN generation results. The 3PN generation was technically more involved, and only after almost a decade of struggle and by the use of the gauge invariant *dimensional regularisation*, was the problem finally resolved and completed [2]. For non-spinning ICB on quasi-circular orbits, the equation of motion and the gravitational wave polarisations [3] are now known to 3PN accuracy. The radiation field determining GW phasing is known to 3.5PN order beyond the leading Einstein quadrupole formula [2]. The 3PN results for non-spinning ICB on quasi-elliptical orbits [4] and 2.5PN results for spinning binaries [5] have recently been completed. In the test particle limit results are known to order 5.5PN [1].

The PN results for ICB are the basis for the construction of *all* templates employed by the LIGO and VIRGO detectors [6]. Resummation methods like Pade approximants and Effective One Body models [7] going beyond the adiabatic inspiral phase to include plunge, merger and quasi-normal-mode ringing improve the convergence and extend the domain of validity of the PN approximants. In the context of the recent exciting numerical relativity simulations [8] of GW from plunge, merger and ringdown of binary black holes the best analytical PN results for inspiral (3.5PN in phase, 3PN in amplitude) are crucial for calibration and interpretation.

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