Light transmission through and its complete stoppage in an ultra slow wave optical medium

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Light wave transmission – its compression, amplification, and the optical energy storage – in an ultra slow wave medium (USWM) is studied analytically. Our phenomenological treatment is based entirely on the continuity equation for the optical energy flux, and the well-known distribution-product property of Dirac delta-function. The results so obtained provide a clear understanding of some recent experiments on light transmission and its complete stoppage in an USWM.

Keywords: Continuity equation, light transmission, slow wave medium, stopped light, ultra slow light.

ULTRA slowing down of light and its complete stoppage in certain highly dispersive optical media is now an established experimental fact[1–5]. The extreme degree of dispersion required for this had been realized by the use of electromagnetically induced transparency (EIT)[6]. In the present work we have studied analytically the case for a light wave traversing such an ultra slow wave medium (USWM). Based on the continuity equation for the optical energy flux, we derive a number of results such as wave compression, intensity growth (i.e. amplification), and the wave energy storage peaking at the point of accumulation in the dispersive medium where the group velocity is at a minimum, even tending to zero, i.e. complete stoppage. While these analytical results are interesting in their own right, they also seem to provide a clear phenomenological understanding of the experimental observations of light wave stoppage and the associated energy storage[2–5], especially for the EIT-slowed light.

Consider the case of a light wave propagating through a 1D USWM of length L along the x-axis $((0 \leq x \leq L)$. The strongly dispersive USWM is modelled here by a spatial profile for the group velocity $v_g(x) = c_0 \eta(x)$, where the slowness function $\eta(x) \to 1$ as $x \to 0$, and $\eta(x) \to 0$ as $x \to \Delta < L$ (Figure 1). Here c_0 is the speed of light outside the USWM. Thus, $x = \Delta$ is the point of accumulation for the wave energy in the USWM. (For a tunable USWM, the slowness parameter $\eta(x)$ can be switched on ($\eta(x) = 1$), or off ($\eta(x) = 0$) in time, and can thus allow even gating of the light wave.)

The energy flow through the USWM obeys the continuity equation

$$\frac{\partial I(x,t)}{\partial t} = -\frac{\partial}{\partial x}(v_g(x)I(x,t)) - \frac{I(x,t)}{\tau} \qquad 0 \leq x \leq \Delta, \quad (1)$$

where $1/\tau$ is the energy dissipation rate (as the medium may be lossy) giving an exponential attenuation of the wave intensity $I(x, t)$. (Note that in the case of non-zero loss, the group velocity v_g should, strictly speaking, be replaced by the energy flow velocity v_E (ref. 7). We will, however, ignore this finer point here, and continue to use the group velocity v_g for the purpose of our phenomenology.)

Now, consider the light wave of intensity $I(0)$ from a continuous wave (cw) laser, say, to be incident on the USWM at $x = 0$. As the light cannot escape beyond the point of accumulation $x = \Delta$, we expect a steady state to reach when the incident energy flux is balanced by the dissipation in the USWM because of the loss rate $(1/\tau)$. The steady-state energy stored in the USWM can be readily calculated. Let $I_{ss}(x)$ be the steady-state intensity in the USWM. The total wave energy stored in the steady state U_{ss} is obtained by equating the rate of the influx $c_0 I(0)$ with the loss rate U_{ss}/τ, giving

$$U_{ss} = c_0 I(0)\tau. \qquad (2)$$

This is, of course, independent of the detailed shape of $\eta(x)$ for the USWM.

Next we consider the intensity profile $I_{ss}(x)$ in the steady state.

Setting $(\partial I/\partial t)(x, t) = 0$ in eq. (1), we get the equation for the steady-state intensity

$$0 = -\frac{\partial}{\partial x}(c_0 \eta(x)I_{ss}(x)) - \frac{I_{ss}(x)}{\tau} \qquad 0 \leq x \leq \Delta, \qquad (3)$$

giving

$$\frac{\partial I_{ss}(x)}{\partial x} = -\frac{1}{c_0\eta(x)}\left(c_0\frac{\partial \eta(x)}{\partial x} + \frac{1}{\tau_0}\right)I_{ss}(x) + A\delta\left(1 - \frac{x}{\Delta}\right)\ldots$$
$$(4)$$

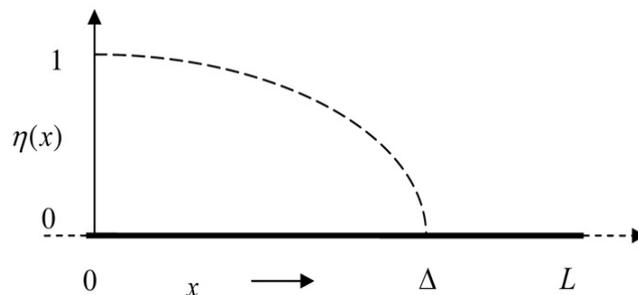**Figure 1.** Group velocity profile $\eta(x)$ for the model ultra slow wave medium (USWM) ($0 \leq x \leq L$), with the point of accumulation Δ, where the group velocity vanishes (schematic).
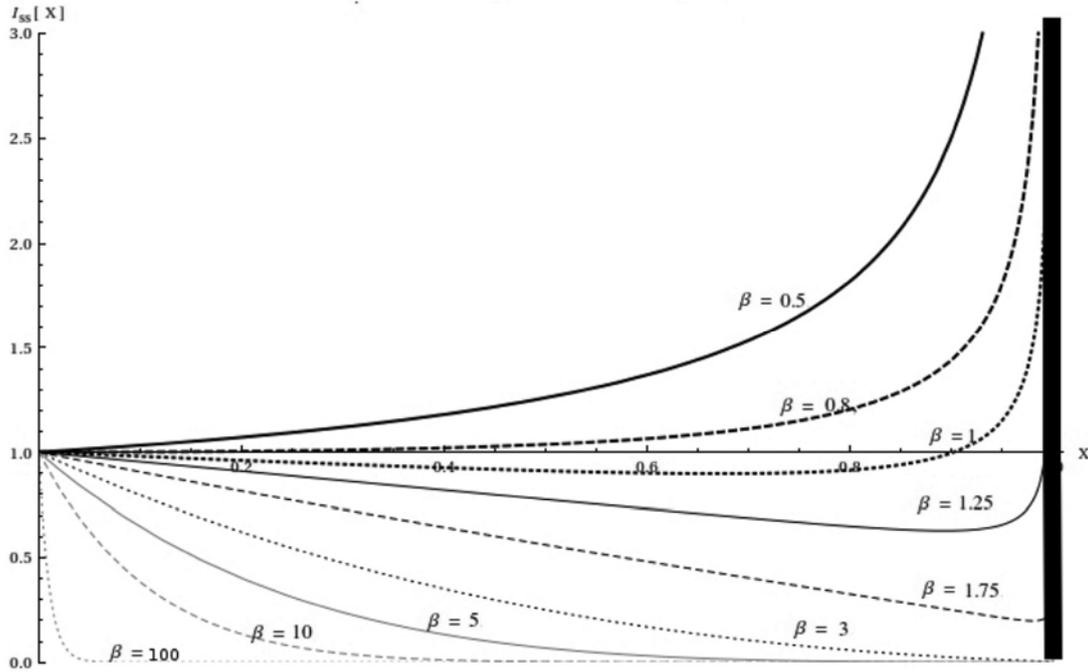
Figure 2. Plot of steady-state intensity $I_{ss}(X)$ against the normalized distance $x/\Delta \equiv X$ in a USWM for the choice of parameters: $\alpha = 0.8$ and $\Delta/c_0\tau \equiv \beta$. Here, β ranges from 0.5 to 100. Note the thick vertical line at $x/\Delta = 1$ representing the delta-function peaks at the point of accumulation (the bright spot).

The last term $A\delta(1 - (x/\Delta))$ in eq. (4) arises because of the division by $\eta(x)$, which vanishes at the accumulation point $x = \Delta$. (Such a term follows from the Dirac distribution-product identity, viz. for any $f(x)$ with $xf(x) = xg(x)$, we have $f(x) = g(x) + k\delta(x)$, where the constant k must be determined from, for example, a normalization condition. Recall that $\eta(x)\delta(x - \Delta) = 0$ for our model of the slowness function $\eta(x)$ as above, and the light wave is confined to $x \leq \Delta$.)

Equation (4) is readily solved to give for $x < \Delta$,

$$I_{ss}(x < \Delta) = I(0)\,\frac{1}{\eta(x)}\exp\left(-\frac{1}{c_0\tau}\int_0^x \frac{1}{\eta(x')}\mathrm{d}x'\right), \quad \text{for } x < \Delta.$$

(5)

Thus, the point $x = \Delta$ will contribute a delta-function at $x = \Delta$ so as to give the total steady-state energy stored in the USWM equal to U_{ss}, consistent with eq. (2). For the purpose of explicit illustration now, the USWM will be modelled as $\eta(x) = (1 - (x/\Delta))^\alpha$, with the slowness exponent $0 < \alpha < 1$.

The condition $0 < \alpha < 1$ ensures a finite time

$$\int_0^\Delta \frac{\mathrm{d}x}{c_0\eta(x)} = \frac{\Delta}{c_0(1-\alpha)}$$

for the wave to reach the point of accumulation Δ. (Of course, $\alpha > 1$ is not ruled out, and can be treated in the

same way.) With this choice, we have the steady-state solution

$$I_{ss}(x < \Delta) = \frac{I(0)}{(1-x/\Delta)^\alpha}\cdot\exp\left(-\frac{\Delta}{c_0\tau}\frac{1-(1-x/\Delta)^{1-\alpha}}{1-\alpha}\right)$$

$$+ \text{ a boundary term,}$$

(6)

where the boundary term is localized at $x = \Delta$. It is a delta-function that will be fixed now as follows.

The steady-state intensity $I_{ss}(x < \Delta)$ together with the delta-function term located at the boundary $x = \Delta$ must give the total integrated stored energy $\int_0^\Delta I_{ss}(x)\,\mathrm{d}x = U_{ss}$ as obtained in eq. (2). This at once fixes the boundary term uniquely to be

$$2c_0\,\tau I(0)\exp\left(\frac{-\Delta}{c_0\tau(1-\alpha)}\right)\delta(x-\Delta).$$

With this, we finally have the complete steady-state solution

$$I_{ss}(x) = \frac{I(0)}{(1-x/\Delta)^\alpha}\cdot\exp\left(-\frac{\Delta}{c_0\tau}\frac{1-(1-x/\Delta)^{1-\alpha}}{1-\alpha}\right)$$

$$+2c_0\,\tau I(0)\exp\left(\frac{-\Delta}{c_0\tau(1-\alpha)}\right)\delta(x-\Delta).$$

(7)

This is our main result for the model and is plotted in Figure 2. It has in it the essential features of wave compression, wave amplification and energy storage in the USWM, as stated above. It clearly shows the competition between the loss ($1/\tau$) and the group velocity dispersion (η). Note the power-law divergence co-existing with the delta-function condensation (pile-up) of the wave intensity at the point of accumulation $x = \Delta$.

The intensity peaking (which in fact happens to be an integrable singularity for the above model choice of $\eta(x)$) is important to our phenomenological interpretation of any experiment on light storage in a USWM.

It is to be noted, however, that in the limit of $\tau \to \infty$ (that is, for no dissipation) and with a finite incoming optical pulse, the light wave simply piles up to a complete stop at $x = \Delta$. This should be the case for a USWM as realized in a Bose–Einstein Condensation (BEC)[1], and will manifest as a bright spot appearing at the point of stoppage $x = \Delta$.

It is important to emphasize here that the above phenomena of wave compression, intensity growth and energy storage are generic to any USWM. One may note in passing that this phenomenon of wave compression and amplification is, of course, analogous to that of breaking of waves at the seashore: as the wave train approaches the shore, the leading edge (being in shallower waters) advances slower relative to its trailing edge, resulting in wave compression. This in turn leads to amplitude growth or a pile-up of the compressed wave, and then to its eventual breaking close to the shoreline. The seashore acts as the slow wave medium here.

The above analytical treatment is, of course, readily generalized to the case of an arbitrary group velocity profile other than the one shown in Figure 1. This may include, in particular, the case where the point of accumulation (complete stoppage) gets replaced by a broad maximum, allowing the transmission of light beyond the peak.

In conclusion, we have proposed and solved analytically a model for light propagation in USWM having a point of accumulation where the group velocity becomes extremely low, or even vanishes. Our solution explicitly displays wave compression (pile-up), optical energy storage and a point of accumulation that characterize the USWM.

1. Dutton, Z., Ginsberg, N. S., Slowe, C. and Hau, L. V., The art of taming light: ultra-slow and stopped light. *Europhys. News*, 2004, **35**, 33–39.
2. Hau, L. V., Harris, S. E., Dutton, Z. and Behroozi, C. H., Light speed reduction to 17 metres per second in an ultracold atomic gas. *Nature*, 1999, **397**, 594–598.
3. Bigelow, M. S., Lepeshkin, N. N. and Boyd, R. W., Superluminal and slow light propagation in a room-temperature solid. *Science*, 2003, **301**, 200–202.
4. Liu, C., Dutton, Z., Berhoozi, C. H. and Hau, L. V., Observation of coherent optical information storage in an atomic medium using halted light pulses. *Nature*, 2001, **409**, 490–493.
5. Philips, D. F., Fleischhauer, A., Mair, A., Walsworth, R. L. and Lukin, M. D., Storage of light in atomic vapor. *Phys. Rev. Lett.*, 2001, **86**, 783–786.
6. Kasapi, A., Jain, M., Yin, G. Y. and Harris, S. E., Electromagnetically induced transparency: propagation dynamics. *Phys. Rev. Lett.*, 1995, **74**, 2447–2450.
7. Milonni, P. W., Fast light, slow light and left-handed light. In *IOP Series Optics and Optoelectronics* (eds Brown, R. G. W. and Pike, E. R.), Taylor & Francis, New York, 2005.

Prediction of Indian summer monsoon rainfall using surface temperature and sea-level pressure cluster parameters

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The scientific community has been putting in continuous efforts to improve long-range forecast of Indian summer monsoon rainfall (ISMR). In this study we try to search for new predictors which may improve the prediction of ISMR. The shared nearest neighbour technique has been applied to surface temperature (ST) and sea-level pressure (SLP) to obtain the clusters in pre-monsoon months (January through May) and seasons (winter, spring). The powers of time series averaged over the clusters are used as parameters for predicting ISMR. Instead of a single prediction equation, two separate equations are developed based on the positive and negative phase of effective strength index (ESI) tendency. Simple multiple regression equations are developed using these cluster parameters for predicting ISMR during the contrasting phases of ESI tendency. During positive (negative) phase of ESI tendency, the SLP (ST) cluster parameters can predict ISMR. The prediction of ISMR is improved if we use the prediction equation depending upon the phase of ESI tendency.

Keywords: Cluster parameters, effective strength index, rainfall prediction, sea-level pressure, surface temperature.

INDIA being an agrarian country, the survival of its people mainly depends on the monsoon rains received during the

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