ABOUT THE SPEAKER

Prof Roddam Narasimha, Director, National Aeronautical Laboratory, Professor, Department of Aerospace Engineering and Convener, Centre for Atmospheric Sciences, Indian Institute of Science, is a fluid dynamicist and aerospace engineer.

Prof Narasimha, born in Bangalore in 1933, obtained his Bachelor's Degree from the University of Mysore. He then joined the Indian Institute of Science where he obtained a Diploma and an Associateship in Aeronautical Engineering. He was awarded Ph.D. at California Institute of Technology, Pasadena, in 1961.

Prof Narasimha's major scientific interests relate to aerospace science and engineering, fluid mechanics and atmospheric sciences. He has edited several books and is the author of about 90 papers and 120 reports. His research efforts in fluid mechanics include work on relaminarisation of fluid flows and on the laminar-turbulence transition zone in the boundary layer. In addition to these basic researches in fluid dynamics, Prof Narasimha has been closely involved with developments in aerospace technology in India. His participation in the evaluation of the airworthiness programme with civilian aircraft has led to the concept of "Stochastic corrective processes".

Prof Narasimha is a recipient of several honours and also holds important advisory positions. He is Clark B. Millikan Visiting Professor of Aeronautics in Caltech since 1985, where he was also Sherman Fairchild Distinguished Scholar(1982-83). He is a Fellow of the Indian Academy of Sciences, Indian National Science Academy, New York Academy of Sciences and the Aeronautical Society of India. He has on several occasions been invited to deliver various key-note lectures both in the country and abroad.

His major advisory positions include Member, General Body and Governing Body, Aeronautical Development Agency, Member, Board of Directors, Hindustan Aeronautics Limited. Recently he has been nominated to the Science Advisory Council to the Prime Minister.

4th Raman Memorial Lecture
Indian Institute of Science

ORDER AND CHAOS IN FLUID FLOWS

R NARASIMHA
DIRECTOR
NATIONAL AERONAUTICAL LABORATORY
BANGALORE

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Cover

This is a flow in which there is a "reverse transition" from chaos to order. Flow coming in through the tube at the top is turbulent, as can be inferred from the rapid dispersion of dye. Flow going out of the coils is laminar: a filament of dye injected after a few turns in the coil does not spread. Such reverse transitions are observed in some nonlinear maps.

ORDER AND CHAOS IN FLUID FLOWS

Prof R Narasimha
Director, National Aeronautical Laboratory

I want to speak today about the flow of fluids; to begin with, I should perhaps explain why so many people - engineers, mathematicians and physicists - have spent their whole lives studying the phenomena that we see with our unaided eyes every day around us. Trees sway in the breeze, clouds fill the sky, waves lash the beach, floods sweep rivers: nature therefore is full of a variety of fluid-dynamical phenomena all the way from the very gentle (have you seen small rain drops sliding down a sagging wire?) to the fierce (such as a cyclone). However, fluid flows occur in art as well - or technology - and are crucial in a variety of the devices and appliances that are so commonly used in our daily lives; whether it is fans, water from the taps (when it is available) or, to take more fancy applications, flow past aircraft and turbines which enable us to travel in ways which were unknown a hundred years ago. What adds to the fascination of these problems, in nature and in technology, is that many of them are to this day unsolved as problems in physics. It comes as a great shock to many people to realize that the ancient problem of conveying a fluid like water from point A to point B - a problem essentially solved thousands of years ago by experience, and one solved around the turn of the century in terms of codified information for technology - remains to this day a totally unsolved problem as one in science. By this I mean that it is still not possible to predict, based solely on first principles such as for example Newton's Laws, how much water can be pushed through a given pipe with a given loss of pressure.

The answer is of course known, but cannot yet be predicted without recourse to testing. Some clever analysis reduces the amount of testing required to answer the question, but the need for testing cannot be eliminated yet.

Why is it that, with all the spectacular advances that have taken place in a variety of branches of science, this ancient problem still remains unsolved? There are several reasons for it but the most basic is that the equations governing the flow of such fluids - discovered more than 150 years ago and named after Navier and Stokes - are non-linear. And our understanding of the behaviour of non-linear phenomena is still in many ways rudimentary. One of the characteristics of the particular non-linear system describing the flow of fluids is that under a great variety of situations, and in fact we can say in general, the motion of fluids exhibits a complex combination of order and chaos. What we mean by these terms will I hope become clearer as we go along, but to this day there is no satisfactory method of handling systems with this intimate mixture of order and chaos. Some features of this mixture are indeed visible to all of us. For example we all know how one can argue endlessly about the shape of clouds; are they totally chaotic or is there a hidden order in the shapes we see? This question, which is something that children play with, is actually at the very heart, I believe, of problems in fluid flow. Using the most sophisticated instrumentation available today, and investigating flows far simpler than a cloud in the controlled conditions of a laboratory, we still have to face exactly the same basic question.

Before we proceed to see the implications of this combination let us first look at the simpler limits of the problem and begin with flows which exhibit considerable order.

Figure 1 shows some examples: the degree of order seen in these flows is extraordinary. And the question immediately is why and how such order arises. The question becomes very interesting especially if we start from the molecular point of view. We know that fluids in general, and gases in particular, involve molecules which are in disordered motion; how do all these molecules in such disordered motion suddenly 'cooperate' to produce the kind of patterns that we see in these flows?

The molecular line of enquiry is one that has in fact been pursued by some people, but the fluid dynamicist's viewpoint is rather different. He generally takes the view that while of course it is important to explain everything eventually in terms of molecules in a wide variety of problems including most of the fluid flows that he handles, it is not essential to do so. The basic argument for this viewpoint is that fluids with widely different molecular structure, for example air and water, show essentially the same kind of behaviour under equivalent conditions. What determines the equivalence of different conditions was a question which was answered a little more than a hundred years ago by Reynolds in the case of relatively simple fluids. Investigating the flow of water in a pipe Reynolds demonstrated that there are two kinds of motion possible - he called them direct and sinuous, we now call them laminar and turbulent - a distinction that is very easy to make when you open any water tap. If the opening is very small we know that water usually comes out as a smooth glassy jet. If the opening is increased, the surface loses its smoothness and the water begins to move in a very irregular way, traditionally called turbulent, now more fashionably called chaotic. Reynolds showed that whether the motion is laminar or turbulent does not depend individually on the fluid or on the size of the pipe or on the velocity, but on a combination of properties which has since come to be known as the Reynolds Number, defined as

$$Re = V D / \nu ,$$

where V is the fluid velocity, D is the diameter of the pipe and ν is the kinematic viscosity of the fluid. Reynolds found that if this number exceeded something like 2300 the flow could become turbulent, whereas below this number the flow remained laminar.

Going back to the question of ordered motion, the fluid dynamicist's answer proceeds on the following lines. If the Reynolds Number is extremely low, then the fluid motion is smooth, regular, and steady if boundary conditions are steady; there is no particular associated pattern of motion. As the Reynolds number increases, however, there is a stage at which the flow becomes unstable and spontaneously there is a generation of certain ordered patterns in the flow, some examples of which are shown in Figure 1. As the Reynolds Number increases further, these patterns break down and the flow eventually becomes irregular and turbulent. Figure 2 shows some examples of how disordered the motion can appear to be.

Just exactly how does the kind of highly ordered motion of which we have seen examples break down into chaotic motion? There seems to be no unique way in which this happens. Figure 3 gives some examples of intermediate stages in the transition from order to chaos. In some cases, the basic waves characterising instability seem to produce new instabilities. One conjecture, due originally to Landau, is that there is a succession of instabilities; each time the flow goes unstable it leads to the possibility of new modes of instability and as this keeps on ad infinitum we eventually get choatic motion. No such infinite sequence has actually been observed; however there are cases in which the primary instability leads to a secondary instability, and the secondary to a tertiary. Fluid dynamicists often like to distinguish between two basic modes of transition; one that may be called hard or fast and another which may be called slow or soft. In the latter type the flow goes through a range of instabilities, possibly starting with the onset of a new state of steady motion. In the hard type, at the point of instability the flow is already unsteady, and as it gets more and more unstable, for example as Reynolds Number increases, the flow eventually breaks down rather rapidly into chaotic motion. In the case of the flow past a flat surface, for example, which we can think of as an idealization of an aircraft wing or a fan blade, the question has in fact been of long standing. How rapid is the transition from laminar to turbulent flow in such a case? In 1935, Prandtl, one of the greatest fluid dynamicists of this century, said that transition occupied an appreciable extent but the point of transition on the plate oscillated with time. Four years later Dryden, another well-known scientist, agreed that the transition point oscillated with time but suggested that transition was abrupt. What measurements show is that although the point at which the flow goes chaotic is relatively well defi-ned the time it takes for the flow to become fully turbulent can be very substantial. Figure 4 shows how the fraction of time during which the flow is turbulent, called intermittancy, varies downstream in flow past a flat plate. As one can see, the intermittency goes from zero to one and can often cover a substantial part of the surface.

The equations governing fluid flow are so complicated that there is no case, not even the most idealised one we can think of, where all the stages of this transition process can be quantitatively described. Even computer simulations, using the biggest computers now available, have not been able to achieve this sort of description. There has therefore been an attempt by many scientists to look at the behaviour of equations which mimic fluid flow, although they may not represent it realistically. One such attempt was made by Burgers many years ago but it turned out on closer examination that the equation that was intended to simulate turbulence did not do this at all but rather described shock waves; in other words a model devised for turbulence was one for noise.

One of the most influential of these models has been the one set up by Lorenz in 1962. Lorenz was concerned with the problem of predicting weather and proceeded to make a highly idealised model for the kind of convective flow that is so common in the atmosphere. He was aware of the radical simplifications that he was making, but nevertheless the behaviour of the solutions of the equations constructed was so strange that he thought, quite rightly, that it would repay attention. His equations are reproduced below.

$$\dot{X} = 10Y - 10X ,$$

$$\dot{Y} = -Y - XZ + rX ,$$

$$\dot{Z} = XY - (8/3)Z ,$$

where

$$(\dot{\ }) \equiv \frac{d}{dt}(\ ) .$$

The three unknowns in these equations in some sense represent the amplitude of the convective motion, of the kind of which an example was given in Figure 1. The numbers in the model represents the conditions of the flow; and the most important of these is the parameter r, which stands for the Rayleigh number that plays the same role in the convection problem as the Reynolds number does in the pipe flow problem. Lorenz integrated the equations of motion on a computer and showed that at r=28, i.e. when the Rayleigh number was 28 times the value at which convection rolls first appeared, the solution appeared erratic as shown in Figure 4. Ironically the original equations to which Lorenz was providing an approximation have been found not to exhibit the kind of chaotic behaviour found in the simplified systems! Nevertheless the results that Lorenz obtained from his system have in recent years profoundly influenced the way we look at the possible mechanisms by which an ordered motion becomes chaotic.

Even the solutions of these simplified equations turned out to be so hard to analyse that a further simplification was thought to be needed. This was obtained by looking at the peaks in the solution; and it led to the discovery that each peak was uniquely determined by the previous peak, in the manner shown in Figure 5. Note however the crucial fact that a given peak does not determine uniquely the previous peak! This has provided the key to an enormous amount of research in recent years. For here we have a connection made between the original partial differential equations governing the problem, through the simplified non-linear ordinary differential equations that were constructed out of them, to a simple kind of mapping that we have discovered between the peaks. Once these connections are seen, it has been realised that it may be worthwhile to look at just the maps themselves. There is the extraordinary possibility that the behaviour of such simple maps hides in it the behaviour of complicated fluid flows.

It is in fact astonishing how such simple maps have been able to teach us about the behaviour of a wide variety of systems and in particular about chaotic behaviour. Physicists and mathematicians have in recent years discovered an enormous variety of results concerning such simple maps. Let us in fact look at a very simple one. Figure 6 shows what is called a 'tent' map. It is symmetric about the midpoint and works in the range 0 to 1, and has a fold at the top. This fold is absolutely crucial. It is very easy to start with a given number and obtain successive numbers in the sequence using this map. The astonishing thing about the map is that any given initial point determines uniquely all future numbers in the sequence; note however that any given number at any given stage does not uniquely determine the previous sequence of numbers leading to it, for the simple reason that the mapping function folds over at the peak.

It is not very difficult to show with this completely deterministic map that the correlation between two successive numbers in the sequence is zero, and that any number is just as probable as any other number in a long sequence of these numbers. What we mean when we say the correlation is zero is that if we look at the sequence of numbers, we would be unable to distinguish them, by any known statistical procedure, from a sequence of random numbers. To make this more specific we can agree to denote all numbers in the sequence which fall between 0 and 0.5 by H, and those that fall between 0.5 and 1.0 by T. And we order the sequence in terms of Hs and Ts. We will then find that the sequence of Hs and Ts from the map cannot be distinguished from the heads and tails obtained by tossing a coin. In other words, we have here a completely deterministic system whose results however are for all practical purposes indistinguishable from what is known as stochastic behaviour, i.e. they appear "random".

This kind of example is, I feel sure, going to affect profoundly our view of statistics, let alone fluid behaviour. Any undergraduate course in engineering, especially on systems theory, starts by making a distinction between deterministic processes and stochastic processes. Examples of the kind that I have just described show that the distinction cannot be sustained; a process can be entirely deterministic and still appear stochastic. I say appear stochastic, but the more basic question can be raised: is it possible that the processes that we consider stochastic are in fact deterministic; but only governed by the kind of non-linear procedure which is caricatured in this simple example?

We will not pursue the implications for statistics here, but come back to turbulence. It is interesting now to speculate that the transition to irregular behaviour that we have noted is in fact nothing other than the kind of stochastic behaviour that the simple model exhibits. Spurred by this possibility, a series of results have been proved in recent years about such maps, and in fact it has become a small field on its own. The advantage of maps is that, unlike that the Navier-Stokes equations, theorems can be stated and proved; and the validity of these theorems can be checked on the computer; furthermore many theorems that cannot be proved can be suggested by computer experiments. One such result that has attracted considerable attention everywhere is due to Feigenbaum, who noticed that such maps had certain universal characteristics. These characteristics are summarised in two numbers which basically describe the relation between the parameter values at successive bifurcations of the solutions of the map. For the maps themselves, these numbers can be computed to extraordinary accuracy, and are in fact now widely known. The fascinating question is whether we can now make the connection backwards from the map to the fluid flow, and expect to find the same numbers operating in the fluid flow problems. For example can the Rayleigh number at which the flow becomes chaotic in the convection problem be related to the Rayleigh number at which the first instability appears in the same way that the value of our parameter in the map at which chaotic solutions appear is related to the primary instability revealed in the map?

The experimental evidence here is still somewhat ambiguous. It is true that the sequence of numbers observed in certain

experiments closely mimic the Feigenbaum sequence. It is still however not possible to state categorically that that is in fact the way that chaos appears in either the convection flow or any other problem.

A different suggestion has been made by Ruelle & Takens. They propose that in a variety of systems chaos does not appear after the infinite sequence of bifurcations that Feigenbaum has described, or the infinite sequence of instabilities that Landau originally proposed. They show that in a wide variety of systems, chaos can appear at the end of three bifurcations. This they do by proving that in certain systems what is known as a strange attractor appears at the end of three bifurcations. The nature of such attractors is illustrated in <u>Figure 7</u>, in particular for the Lorenz system. The idea of a strange attractor can be described in simplified terms as follows. Suppose we take a cup of coffee, stir it and let go, then usually the flow comes to rest after a while. That is, the stable state of motion for the conditions of this familiar fluid-dynamical problem is one of rest. To put this result in a little bit of jargon the state of rest is an attractor for this problem, that is to say all states tend towards the state of rest eventually. There are other systems in which the solution eventually is not one of rest but rather one of oscillation. A wire galloping in wind or a pipe carrying water that some times produces surges or hunting are examples of such phenomena; these are known as limit cycles, and have the property that no matter where you start your motion, eventually the system settles down to one of steady oscillation. Both of these are attractors; the state of rest is an attractor of zero dimension and the limit cycles are attractors of dimension 1. We can also have an attractor of dimension 2. In state space, this would correspond to a torus or an object shaped like a donut. Here the state of the system, if it can be thought of as an ant crawling on the surface, is described by trajectories which could go either around the donut on its major circumference, so to speak, or around the smaller circumference. What Ruelle & Takens showed was that beyond these three possibilities there was a fourth one and that is that the point denoting the state of the system wanders for ever without lying on any particular surface, possibly getting quite close to previous positions at various times but never staying close to any position or cycle. It is suggested that the kind of turbulent motion that we often see in fact represents such a strange attractor of the Navier-Stokes equations. The Lorenz system showed one of the first such strange attractors, although it was not called that at the time that Lorenz made his studies; the nature of the strange attractor in the space of the variables X, Y and Z in the equations is shown in Figure 8.

Although these new view points are exciting and promise fresh insights into the problem of the development of chaos in fluid flows, we must remember that there are many difficulties - some of them not even faced yet by the new approaches. Let me just give two examples of the inadequacy of current models. The first is that in all of these models chaos develops at low frequencies, i.e. by the appearance of amplitudes at sub-harmonics of a basic fundamental frequency. But in all turbulent shear flows, a characteristic feature is the appearance of high frequency chaos. The filling of the spectrum at the low wave number end does not explain how the high frequency oscillations that characterise turbulent flow arise. A second problem is that in most of these theories the critical value of the parameter, like the Reynolds or Rayleigh number, at which the onset of chaos is predicted, does not depend on external disturbances. On the other hand we know from observations extending over many decades now that there is no unique critical value for the parameter, but rather that the value of this parameter depends strongly on the external disturbances in the flow. For example although it is difficult to maintain turbulent flow below a Reynolds number of about 2300 in a flight, laboratory studies have shown that if due care is taken, the critical Reynolds number at onset of chaos can be increased to something like 10^5. That is a factor of 50. A similar factor operates in transition on a flat plate, where the Reynolds number can vary from less than 10^5 to something like $< 5 \times 10^6$.

We are right now in the process of formulating models which we hope will include these basic features of chaos in fluid flows. How far such models can go and what light they will eventually throw on the problem is something which is still a very open question, but it is certainly something which should be very exciting to pursue.

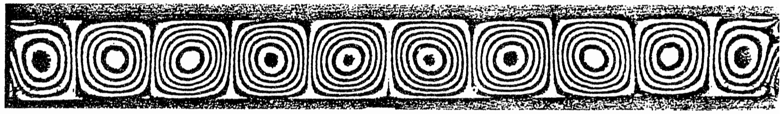


Figure 1: Convection rolls in a horizontal layer of fluid when lower plate is hotter than the upper plate. The curves in the diagram are density contours: in the absence of convection they would have been a series of parallel lines.

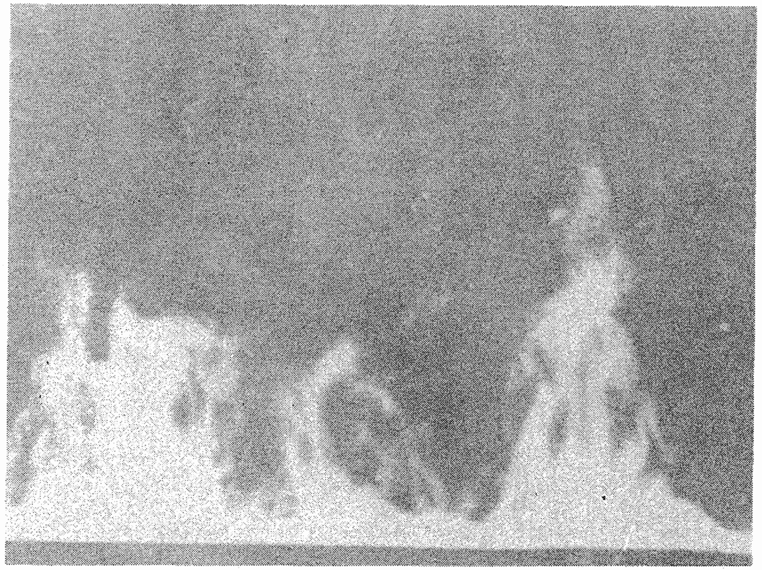


Figure 2A: Flow over a very hot surface, showing convective motion that is highly turbulent or chaotic.

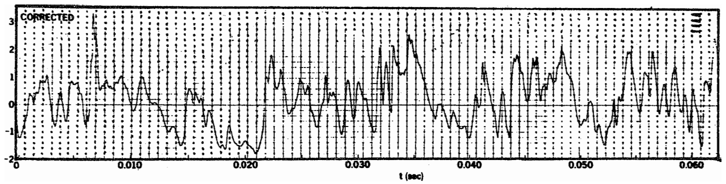
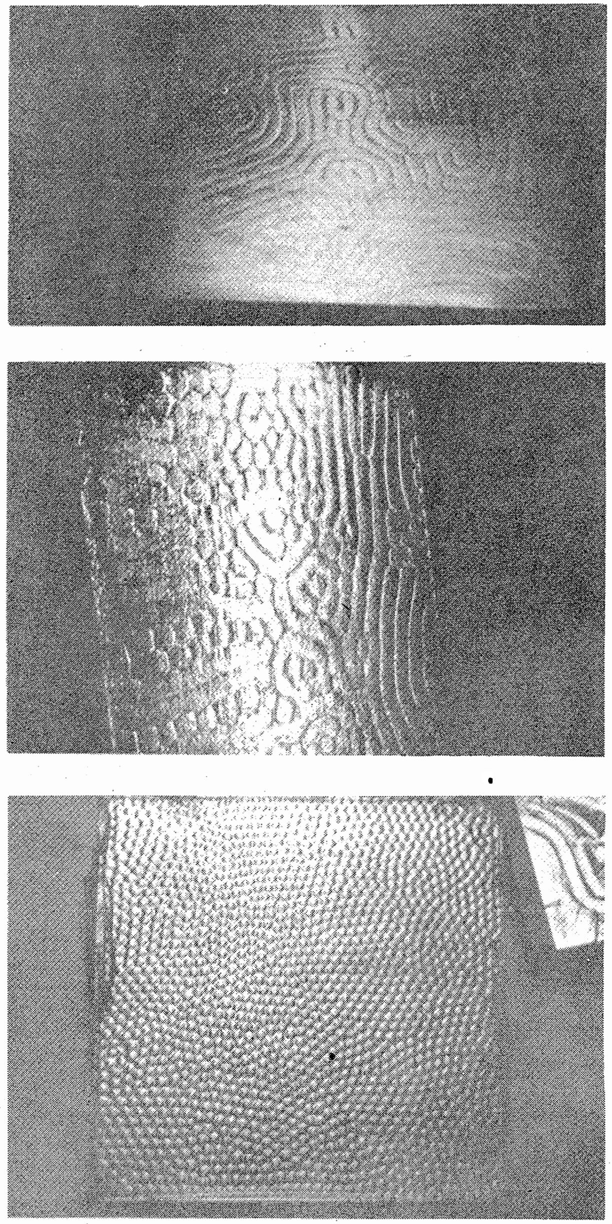


Figure 2B: Typical velocity fluctuation in a turbulent flow.



a)
b)
c)

***Figure 3A:* Patterns produced on frozen cyclohexane when a layer of the substance is hot enough at the bottom to liquefy it. There is considerable order in all these flows, but crystallographers will note various types of dislocations marring the symmetry of the patterns.**

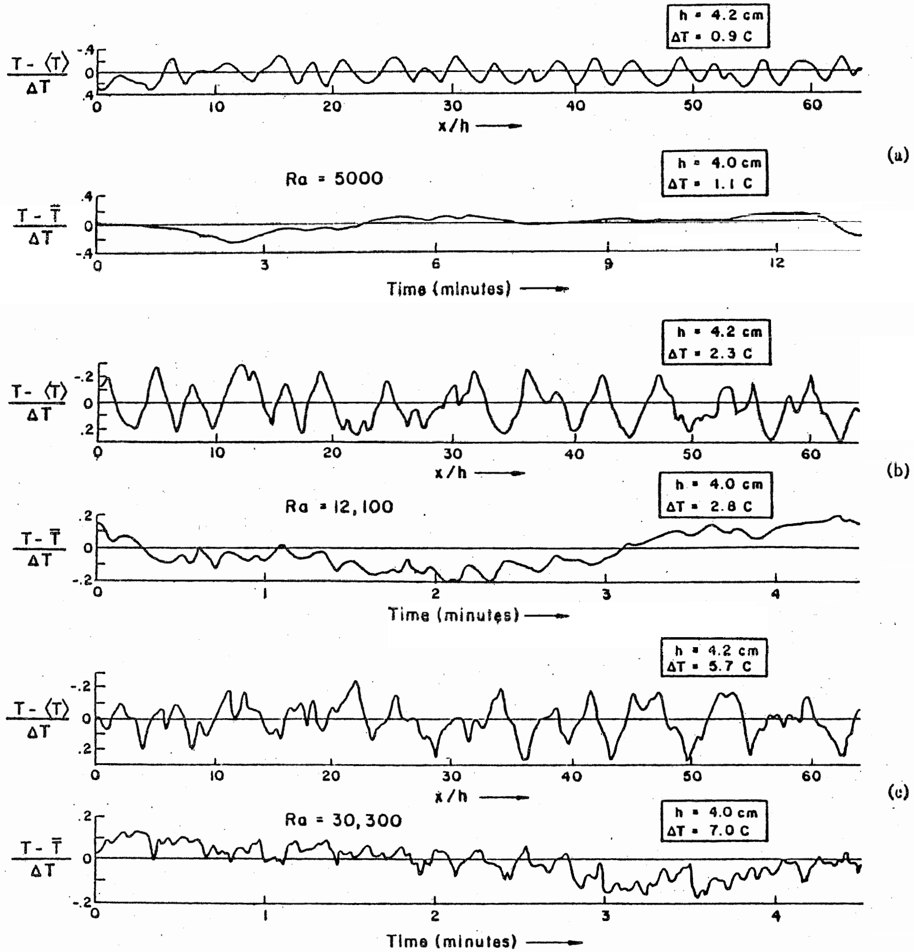


Figure 3B: *How much order and how much chaos? These diagrams present three sets of graphs showing the variation of temperature in space and time in convective flow between two horizontal plates, as the bottom plate is made hotter. The top traces, at the lowest temperature differential, exhibit considerable order. How much order is there in the bottom ones?*

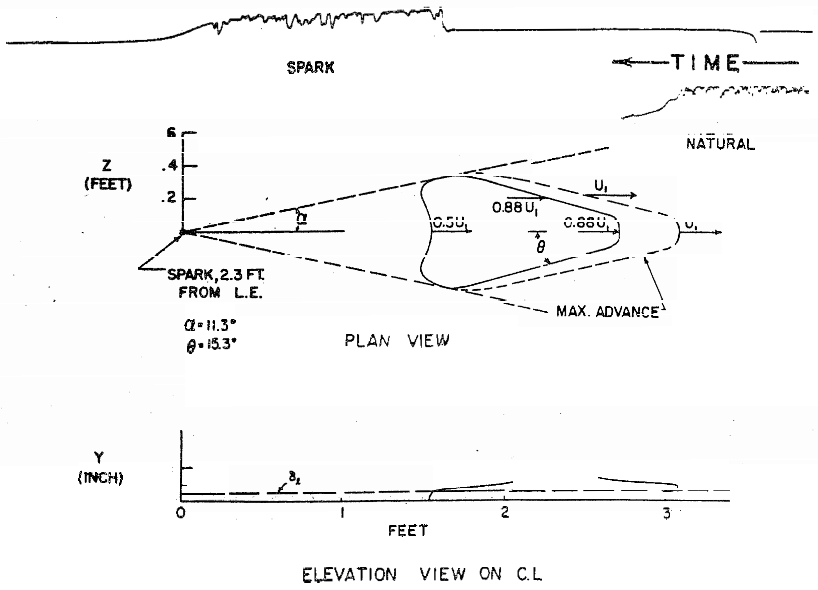


Figure 4A: *One route from order to chaos is through the formation of 'spots' of turbulence, which are islands of chaos in a laminar sea, moving with the flow and generally growing in size. In flow past a flat plate such turbulent spots are heart-shaped, as shown in this figure, and move within the confines of a fairly well-defined cone lying on the surface. Spots tend to be born at some critical location on the surface, making the appearance of chaos rather sudden, but take appreciable distances to grow to such sizes that the laminar sea is completely covered by them.*

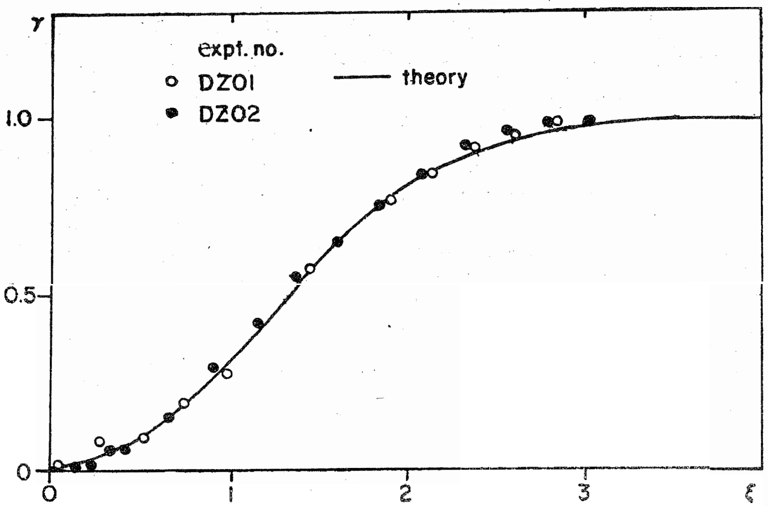


Figure 4B: *The 'intermittency', which is the fraction of time that the flow is turbulent, increases gradually from zero to unity. The extent of the transition zone is generally comparable to that of laminar flow, but is relatively larger at lower Reynolds numbers.*

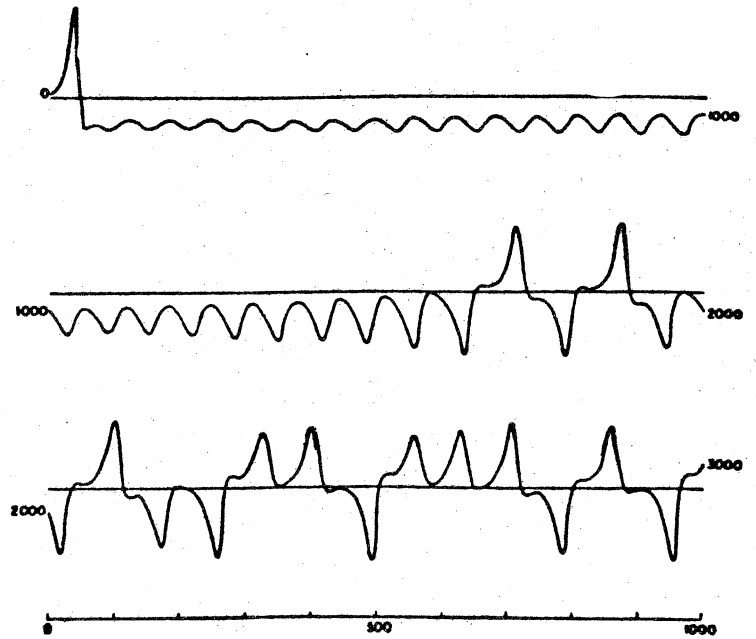


Figure 5: Typical solution of the Lorenz equations, showing "chaotic" behaviour..

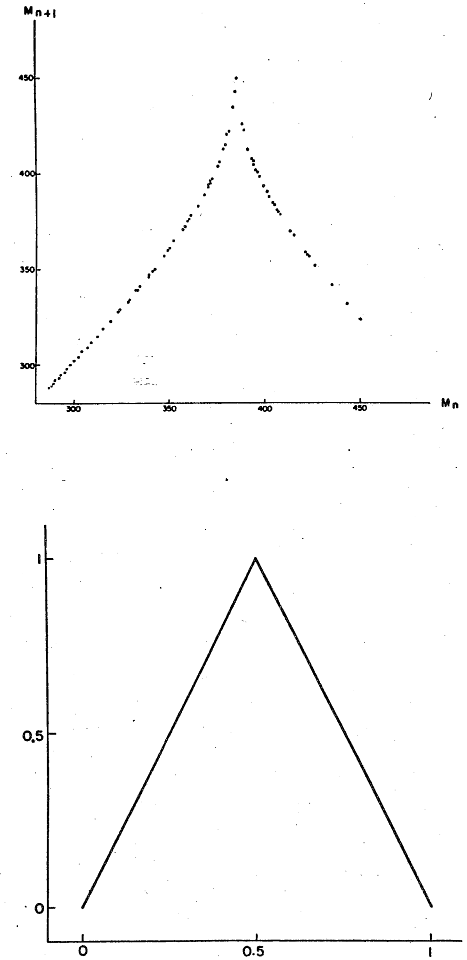
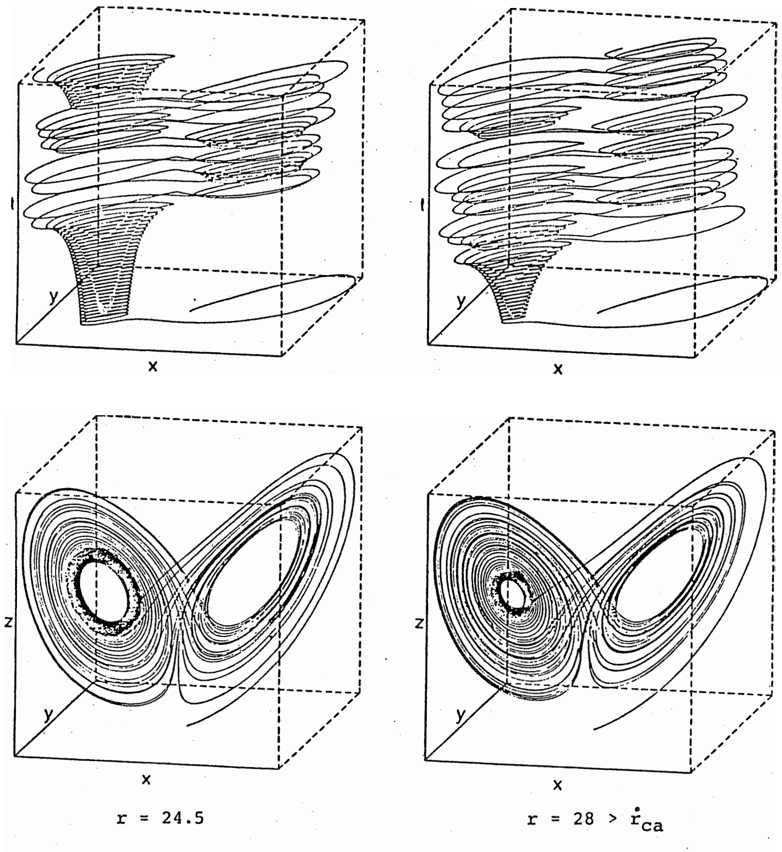


Figure 6: Relation between successive maxima in the variable Z in the Lorenz equations. If you enter on the horizontal axis with any particular maximum, the value of the next maximum in the solution is given by the folded curve. An idealization of this folded curve is the triangular 'tent' map, also shown. Such 'maps' are defined by the relation $X_{n+1} = f\ (X_n)$, $n = 0,1,2...$ For the tent map,

$$f\ (X) = 2X \text{ if } 0 \leq X \leq 1/2,$$

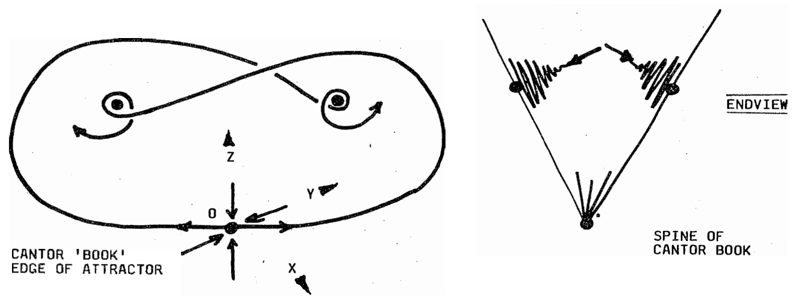
$$= 2 - 2X \text{ if } 1/2 < X \leq 1.$$



Intermittent Periodical and Aperiodical Oscillations

Chaotic Attractor

Figure 7: The nature of the solutions of the Lorenz equation, showing behaviour characteristic of strange attractors..



O : STATE OF REST

⊗ : FIXED POINTS

Figure 8: A sketch of the Lorenz strange attractor. The state of the system wanders endlessly on the attractor, getting arbitrarily close to previous positions but never staying close. Once the state moves away from rest, it is sucked towards one of the fixed points but is eventually flung away from it, to be sucked by the other fixed point, in turn flung away from it, and so on. Near the origin the attractor resembles a book-like object with infinitely many sheets of zero thickness, constituting a Cantor set. If the attractor is chopped into bits of given size, and the areas of the bits added up, it is found that the sum increases more rapidly than the square of the (linear) size of each piece, and (of course) less rapidly than the cube. The attractor therefore is an object with a fractional dimension (approximately 2.06 in the Lorenz case).

ABOUT THE MEMORIAL LECTURE

C.V. Raman Memorial Lectures have been instituted by the Indian Institute of Science Alumini Association (Bangalore) and these lectures are organised in alternate years.

Sir Chandrasekhar Venkata Raman, Nobel Laureate, was born on 7 November 1888 in Thiruvanaikkaval village near Trichinopoly in Tamil Nadu. He matriculated at the age of 11 and won a scholarship for higher studies at the Presidency College, Madras. He passed his B.A. and M.A. examinations in the first class winning gold medals in English and Physics. He appeared for the All India Competitive Examinations which existed in those days for prestigeous positions in the Finance Department and passed the examination at the top of the list. While working in the Finance Department at Calcutta as an Assistant Accountant General, he was attracted by a sign at 210, Bowbazaar Street, Calcutta, which read 'Indian Association for the Cultivation of Science', established by Mr. Amrit Lal Sircar, a man of vision. Dr Raman was given all the facilities of the Association which started off his scientific career. Very soon Dr Raman's outstanding contributions resulted in Sir Asutosh Mukherjee, the Vice-Chancellor of the Calcutta University offering him the Palit Chair of Physics. Dr Raman accepted it readily, relinguising a very lucrative position in the Finance Department. Under his dynamic leadership, the Indian Association became a leading centre for research on scattering of light by liquids,. x-rays by liquids and the viscosity of liquids. The systematic investigations culminated in the discovery of Raman Effect in February/March 1928 for which he was awarded the Nobel Prize in 1930.

In 1933, he joined the Institute as its Director and founded the Department of Physics. Till his retirement in January 1949, his main preoccupation was the physics of diamond and problems in crystal physics and many outstanding contributions were made by him and his students. Besides being an outstanding physicist of the country, he had the innate ability to discover the best in the students who approached him and encouraged them to make significant contributions, and one may even say that that his dynamic leadership was responsible for the creation of a large number of physicists in this country. Till the very end of his life, even after his retirement from the Indian Institute of Science, at the Raman Research Institute, he dedicated himself to work on crystal physics, colour and vision.

The first lecture in this series was delivered by Dr S Ramaseshan on 3 March 1978 on 'C.V.Raman', the second lecture by Dr Sukh Dev on 3 March 1980 on 'Research and Development in chemical industry', and the third lecture by Prof C N R Rao on 'Man, Minerals and Microscopes'.