Gravitational waves from inspiralling compact binaries: 3PN polarisations, angular momentum flux and applications to Astrophysics and Cosmology

by

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DECLARATION

I hereby declare that the thesis titled "**Gravitational waves from inspiralling compact binaries: 3PN polarisations, angular momentum flux and applications to Astrophysics and Cosmology**" is composed independently by me at the Department of Physics, Indian Institute of Science, Bangalore, under the supervision of Prof. Bala R. Iyer (Theoretical Physics Group, Raman Research Institute, Bangalore) and Prof. Arnab Rai Choudhuri (Dept. of Physics, IISc). The subject matter presented in this thesis has not previously formed the basis for the award of any degree, diploma, membership, associateship, fellowship or any other similar title of any University or Institution.

Siddhartha Sinha

Dedicated

To dear Javed

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Synopsis

Binary systems comprising of compact objects like neutron stars (NS) and/or black holes (BH) lose their energy and angular momentum via gravitational waves (GW). Radiation reaction due to the emission of GW results in a gradual shrinking of the binary orbit and an accompanying gradual increase in the orbital frequency. The preliminary phase of the binary evolution when the radiation-reaction time-scale is much larger than the orbital time-scale is called the *inspiral* phase. GW emitted during the final stages of the inspiral phase constitute one of the most important sources for the ground-based laser interferometric GW detectors like LIGO, VIRGO and the proposed space-based detector LISA. For the ground-based detectors, NS and/or stellar mass BH binaries are primary sources, while for LISA supermassive BH (SMBH) binaries are potential targets. Inspiralling compact binaries (ICB) are among the prime targets for interferometric detectors because using approximation schemes in general relativity (GR) like the post-Minkowskian (PM) and the post-Newtonian (PN) approximations one can compute the GW emitted by them with sufficient accuracy both for their detection and parameter estimation leading to GW astronomy.

The extreme weakness of gravitational interactions implies that if a GW signal from an ICB is incident on a detector, it will be buried in the noisy detector output. Therefore, sophisticated data analysis techniques are required for detecting the signal in presence of the dominant noise and also estimating the parameters of the signal. From the pre-calculated theoretical waveforms called *templates*, one already knows the structure of the waveform from an ICB. The technique for detecting signals which are of *known* form in a noisy detector is matched filtering. This technique consists of cross-correlating the output of a noisy detector assumed to contain the signal of known form with a set of templates. It then finds an 'optimal' template that would produce, on average, the highest signal-to-noise ratio (SNR). The efficient performance of matched filtering as a data-analysis strategy for GW signals from ICB presupposes very accurate theoretical templates. Slight mismatches between the signal and the template will result in a loss of signal to noise ratio. Computing very accurate theoretical templates and including effects such as eccentricity are challenging tasks for the theoreticians. This thesis addresses some of the issues related to the waveform modelling of the ICB and their implications for GW data analysis.

It is known theoretically that compact binaries reduce their eccentricity through the emission of GW. When GW signals from prototype ICB reach the GW detector bandwidth, their orbits are almost circular. Hence one usually models the binary orbit to be circular for computation of the search templates. The waveform from an ICB in a circular orbit is, at any given PN order of approximation, a linear combination of a finite number of harmonics of the orbital frequency. At the lowest order of approximation, called the Newtonian order, the waveform comprises a single harmonic at twice the orbital frequency. Inclusion of higher order PN corrections lead to the appearance of higher harmonics of the orbital frequency. Since the amplitudes of the higher harmonics contain higher powers of the PN expansion parameter, relative to the Newtonian order, they are referred to as amplitude corrections. The phase of each harmonic, determined by the orbital phase, is known upto 3.5PN order (nPN is the order of approximation equivalent to terms $\sim (v/c)^{2n}$ beyond the Newtonian order, where v denotes the binary's orbital velocity and c is the speed of light). Matched filtering is more sensitive to the phase of the signal rather than its amplitude, since the correlation builds up as long as the signal and the template remain *in phase*. Motivated by this fact, search templates so far have been a waveform model involving only the dominant harmonic (at twice the orbital frequency), although the phase evolution itself is included upto the maximum available PN order. Such waveforms, in which all amplitude corrections are neglected, but the phase is treated to the maximum available order, are called *restricted waveforms* (RWF) and these are generally used in the data-analysis of ground-based detectors and also simulated searches for the planned LISA. However, recent studies, in the case of ground-based interferometers, showed that going beyond the RWF approximation could improve the efficiency of detection as well as parameter estimation of the inspiral signal.

After a brief overview of the properties of GW and their detection strategies in chapter 1, in chapters 2 and 3, we investigate the implications of going beyond the RWF, in the context of the planned space-based Laser Interferometric Space Antenna (LISA). The sensitivity of ground-based detectors is limited by seismic noise below 20Hz. On the other hand, the space-based LISA will be designed to be sensitive to GWs of frequency $(10^{-4} - 1)$Hz. The most important source in this frequency band are supermassive BH (SMBH) binaries. There is strong observational evidence for the existence of SMBH with masses in the range of $10^6 M_\odot$–$10^9 M_\odot$ in most galactic nuclei. Mergers of such galaxies result in SMBH binaries whose evolution is governed by the emission of GW. Observation of the GW from SMBH binaries at high redshifts is one of the major science goals of LISA. These observations will allow us to probe the evolution of SMBHs and structure formation and provide an unique opportunity to test General Relativity (and its alternatives) in the strong field regime of the theory. Observing SMBH coalescences with high (100-1000) SNR is crucial for performing all the aforementioned tests.

The LISA bandwidth $(10^{-4} - 1)$Hz determines the range of masses accessible to LISA because the inspiral signal would end when the system's orbital frequency reaches the mass-dependent last stable orbit (LSO). In the test-mass approximation, the angular velocity $\omega_{\rm LSO}$ at LSO is given by $\omega_{\rm LSO} = 6^{-3/2} M^{-1}$, where M is the total mass of the binary. Search templates using the RWF, which contains only the dominant harmonic at twice the orbital frequency, cannot extract power in the signal beyond $f_{\rm LSO} = 2(\omega_{\rm LSO}/2\pi) \simeq 4.39(M/10^6 M_\odot)^{-1}$mHz. This further implies that the frequency range [0.1, 100] mHz corresponds to the range $\sim 4.39 \times [10^4,\ 10^7] M_\odot$ for the total mass of BH binaries that would be accessible to LISA. In chapter 2, we show that inclusion of higher harmonics will enhance the mass-range of LISA (for the same frequency range) and allow for the detection of SMBH binaries with total masses higher than $4.39 \times 10^7 M_\odot$. The template employed in chapter 2 includes amplitude corrections upto 2.5PN order, while keeping the phase upto 3.5PN order. We call this template the *full waveform* (FWF). The FWF defined above contains higher harmonics of the orbital frequency, the highest of them being 7 times the orbital frequency. For a SMBH binary with total mass $\sim 10^8 M_\odot$, the dominant harmonic at LSO is less than the lower cut-off of the LISA bandwidth. Therefore, if one uses the RWF as a search template, this system is 'invisible' to LISA. However, the seventh harmonic can still enter the LISA bandwidth and produce a significant SNR and thus allow its detection. With the FWF, LISA can observe sources which are favoured by astronomical observations, but not observable with the RWF. More specifically, with the inclusion of all known harmonics LISA will be able to observe SMBH coalescences with total mass $\sim 10^8 M_\odot (10^9 M_\odot)$ (and mass-ratio 0.1) for a low frequency cut-off of 10^{-4}Hz (10^{-5}Hz) with an SNR up to ~ 60 (~ 30) at a distance of 3 Gpc.

The orbital motion of LISA around the Sun induces frequency, phase and amplitude modulations in the observed GW signal. These modulations carry information about both the source's location and orientation. Determination of the angular coordinates of the source also allows determination of the luminosity distance of SMBH binaries. Therefore, SMBH binaries are often referred to as GW "standard sirens" (analogous to the electromagnetic "standard candles"). LISA would also be able to measure the "redshifted" masses of the component black holes with good accuracy for sources up to redshifts of a few. However, GW observations alone cannot provide any information about the redshift of the source. If the host galaxy or galaxy cluster is known one can disentangle the redshift from the masses by optical measurement of the redshift. This would not only allow one to extract the "physical" masses, but also provide an exciting possibility to study the luminosity distance-redshift relation providing a totally independent confirmation of the cosmological parameters. Further, this combined observation can be used to map the distribution of black hole masses as a function of redshift. Another outstanding issue in present day cosmology in which LISA can play a role is the dark energy and its physical origin. Probing the equation-of-state-ratio ($w(z)$) provides an important clue to the question of whether dark energy is truly a cosmological constant (i.e., $w = -1$). Assuming the Universe to be spatially flat, a combination of WMAP and Supernova Legacy Survey (SNLS) data yields significant constraints on $w = -0.967^{+0.073}_{-0.072}$ [1]. Without including the spatial flatness as a prior, WMAP, large-scale structure and supernova data place a stringent constraint on the dark energy equation of state, $w = -1.08 \pm 0.12$. For this to be possible, LISA should (a) measure the luminosity distance to the source with a good accuracy and (b) localize the coalescence event on the sky with good angular resolution so that the host galaxy/galaxy cluster can be uniquely identified. Based on analysis with the RWF, it is found that LISA's angular resolution is not good enough to identify the source galaxy or galaxy cluster, and that other forms of identification would be needed. Secondly, weak lensing effects would corrupt the distance estimation to the same level as LISA's systematic error.

In chapter 3, we study the problem of parameter estimation in the context of LISA, but using the FWF. We investigate systematically the variation in parameter estimation with PN orders by critically examining the role of higher harmonics in the fast GW phasing and their interplay with the slow modulations induced due to LISA's motion. More importantly, we explore the improvement in the estimation of the luminosity distance and the angular parameters due to the inclusion of higher harmonics in the waveform. We translate the error in the angular resolution to obtain the number of galaxies (or galaxy clusters) within the error box on the sky. We find that independent of the angular position of the source on the sky, higher harmonics improve LISA's performance on both counts raised in earlier works based on the RWF. We show that the angular resolution enhances typically by a factor of $\sim$ 2-500 (greater at higher masses) and the error on the estimation of the luminosity distance goes down by a factor of $\sim$ 2-100 (again, larger at higher masses). For many possible sky positions and orientations of the source, the inaccuracy in our measurement of the dark energy would be at the level of a few percent, so that it would only be limited by weak lensing. We conclude that LISA could provide interesting constraints on cosmological parameters, especially the dark energy equation-of-state, and yet circumvent all the lower rungs of the cosmic distance ladder.

Having emphasized the need to consider the FWF as a more powerful template, in chapter 4 we calculate a higher order term in the amplitude corrections of the waveform. In chapters 2 & 3, the FWF incorporated amplitude corrections upto 2.5PN order. In chapter 4 the waveform is calculated upto 3PN order. Recent progress in Numerical Relativity (NR) has resulted in computation of the late inspiral and subsequent merger and ringdown phases of the binary evolution (where PN theory does not hold good) by a full-fledged numerical integration of the Einstein field equations. A new field has emerged recently consisting of high-accuracy comparisons between the PN predictions and the numerically-generated waveforms. Such comparisons and matching to the PN results have proved currently to be very successful. They clearly show the need to include high PN corrections not only for the evolution of the binary's orbital phase but also for the modulation of the gravitational amplitude. This leads to one more motivation for the work in this chapter: providing the associated spin-weighted spherical harmonic decomposition to facilitate comparison and match of the high PN prediction for the inspiral waveform to the numerically-generated waveforms for the merger and ringdown.

For the computation of waveforms from the inspiralling compact binaries one needs to solve the two-body problem in general relativity. The nonlinear structure of general relativity prevents one from obtaining a general solution to this problem. The two-body problem is tackled using the multipolar post-Minkowskian (MPM) wave generation formalism. The MPM formalism describes the radiation field of any isolated post-Newtonian source. The radiation field is first of all parametrized by means of two sets of radiative multipole moments. These moments are then related (by means of an algorithm for solving the non-linearities of the field equations) to the so-called canonical moments which constitute some useful intermediaries for describing the external field of the source. The canonical moments are then expressed in terms of the operational source moments obtained by matching to a PN source and are given by explicit integrals extending over the matter source and gravitational field. The extension of the waveform by half a PN order requires as inputs the relations between the radiative, canonical and source multipole moments for general sources at 3PN order. We also require the 3PN extension of the source multipole moments in the case of compact binaries. The waveform in the far-zone consists of two types of terms, *instantaneous* and *hereditary*. The instantaneous terms are determined by the dynamical state of the binary at the retarded time. The hereditary terms, on the other hand, depend on the entire past history of the source. These terms originate from the nonlinear interactions between the various multipole moments and also from backscattering off the curved spacetime generated by the waves themselves. In this chapter, we compute the contributions of all the instantaneous and hereditary terms (which include tails, tails-of-tails and memory integrals) up to 3PN order. The end results of this chapter are given in terms of both the 3PN plus and cross polarizations and the separate spin-weighted spherical harmonic modes.

Though most of the sources will be in circular orbits by the time the GWs emitted by the system enter the sensitivity band of the laser interferometers, astrophysical scenarios such as Kozai mechanism could produce binaries which have nonzero eccentricity. Studies have shown that filtering the signal from an eccentric binary with circular orbit templates could significantly degrade the SNR. For constructing a phasing formula for eccentric binaries one has to compute the energy and angular momentum fluxes carried away by the GWs and then compute how the orbital elements evolve with time under gravitational radiation reaction.

The far-zone energy and angular momentum fluxes, like the waveform, contain both instantaneous and hereditary contributions. The complete 3PN energy flux and instantaneous terms in the 3PN angular momentum flux are already known. In chapter 5, the hereditary terms in the 3PN angular momentum flux from an ICB moving in quasi-elliptical orbits are computed. A semi-analytic method in the frequency domain is used to compute the hereditary contributions. At 3PN order, the quasi-Keplerian representation of elliptical orbits at 1PN order is required. To calculate the tail contributions we exploit the doubly periodic nature of the motion to average the 3PN fluxes over the binary's orbit. The hereditary part of the angular momentum flux provided here has to be supplemented with the instantaneous part to obtain the final input needed for the construction of templates for binaries moving in elliptical orbits, a class of sources for both the space based detectors and the ground based ones. Using the hereditary contributions in the 3PN energy flux, we also compute the 3PN accurate hereditary contributions to the secular evolution of the orbital elements of the quasi-Keplerian orbit description.

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