Gravitational waves from inspiralling compact binaries: 2.5PN waveform, 3PN angular momentum flux and 3.5PN parameter estimation

by

K G Arun

Thesis submitted to the
Jawaharlal Nehru University
for the award of the degree of
Doctor of Philosophy

2006

Raman Research Institute
Bangalore 560 080
India

Certificate

This is to certify that the thesis entitled

Gravitational Waves from Inspiralling Compact Binaries:

2.5PN Waveform, 3PN Angular Momentum Flux and 3.5PN Parameter Estimation

submitted by

K G Arun

for the award of the degree of

Doctor of Philosophy

of

Jawaharlal Nehru University

is his original work. This has not been published or submitted to any other University for any other Degree or Diploma.

Prof. Ravi Subrahmanyan
(Center Chairperson)
Director
Raman Research Institute
Bangalore 560 080 INDIA

Prof. Bala R Iyer
(Thesis Supervisor)
Raman Research Institute
Bangalore 560 080 INDIA

DECLARATION

I hereby declare that this thesis is composed independently by me at the Raman Research Institute, Bangalore, under the supervision of Prof. Bala R Iyer. The subject matter presented in this thesis has not previously formed the basis for the award of any degree, diploma, membership, associateship, fellowship or any other similar title of any University or Institution.

Prof. Bala R Iyer
Theoretical Physics Group
Raman Research Institute
Bangalore, INDIA

K G Arun
Theoretical Physics group
Raman Research Institute
Bangalore, INDIA

Dedicated

To my (late) grandmother

Acknowledgement

- Having **Bala Iyer** as my thesis advisor was one of the greatest fortunes I had in my life. More than an advisor he had to play the role of a mentor and a friend many a time. I thank him for all his invaluable advise and constructive criticisms at every juncture of my Ph. D, all the enthusiastic discussions we had almost daily, all his support during times when I felt a bit low, the freedom he gave me in my work, making me pursue problems on my own and last but not the least for his immense patience in guiding me through. It was a great experience working with you Bala. Hope we will continue collaborating in the future.

- It was a great experience to collaborate with **Luc Blanchet** on projects related to the computation of 2.5PN polarization and later on eccentric orbit binaries. I thank him for the collaboration and stimulating discussions, which were always a great learning experience. Finally his comments on one of the parameter estimation projects is also gratefully acknowledged. Merci beaucoup Luc!

- I gratefully acknowledge **B S Sathyaprakash** for collaboration on the parameter estimation project, insightful discussions on various aspects of data analysis of gravitational waves, informative e-mail exchanges (which drive our collaboration) and his encouragement to pursue problems on my own. Thanks a lot Sathya!

- I thank **Moh'd S S Qusailah** for the collaborations which we had, all the discussions on projects in which we both were involved and for a close friendship which we shared. Thanks a lot Moh'd!

- I got interested and involved in theoretical issues concerning gravitational wave data analysis, which forms almost half of my thesis, through my interactions with **Ajith Parameswaran**. I thank him for all the lively discussions we had both in person and through mails and always keeping me posted about the latest developments in the field. I also thank Ajith for intimate friendship.

- It was with **Pranesh Sundararajan** that I took up the first problem related to data analysis. We had great time discussing and figuring out things together. I thank him for the collaboration and friendship.

- I had the great privilege to have **Shukre**, **Madhavan**, **Sam** and **Vishu** as part of my students advisory committee, at different points of time. I thank them for constantly monitoring my academic progress, their critical comments on my performance and suggestions about the possible improvements. I specially thank Shukre for his support whenever I approached him with any problem, academic or non-academic.

- At different stages of my Ph. D, I have benefitted from useful discussions and/or comments from **Alessandra Buonanno**, **R Balasubramanian** and **Sanjeev Dhurandhar**. I thank all of them.

- I thank Siddhartha for all the questions he raises every now and then which made me think more deeply about the subject and also for all useful discussions.

- At different stages during my Ph. D, I had the chance to interact and work with many summer students which was always a learning experience. I specially thank Nithyanandan and Anosh with whom I had a great time figuring out many things.

- It is a memorable experience to be part of the theory physics group in RRI. I thank every member of the group. My special thanks to Abhishek Dhar for the reading course I had with him and for all the chats we used to have about academics, computers,....His presence on the floor always lifts up our spirit! I also thank Sumati and Damien for their active presence in GRIM and useful conversations.

- I thank Prof. Ravi Subrahmanian (Director) and Prof. N. Kumar (former Director) for their interest in my work and useful comments.

- I thank Mr. Krishnamaraju, Administrative officer, for all his helps on numerous occassions.

- I thank the computer division of RRI, especially Jacob Rajan, Nandakumar, Vivek, Sunil, Dhanesh, for timely assistance whenever I approached them with problems.

- I am grateful to all the staff of the RRI library for their co-operation throughout my Ph. D.

- My first interaction with Shrirang was when he was my GR course grader. That later on developed into a close friendship. Words are less to thank him for his advice throughout my Ph. D days on both academic as well as personal issues, his company during visits and conferences.I am grateful to him for his support at the darkest hour of my Ph. D life. Thanks a lot Shrirang!

- Remaining sane at the end of a Ph. D is a big achievement by itself. If I could remain more or less stable throughout my Ph. D days, the credit goes to a nice bunch of friends I had.

  - Amit has been a constant companion to me from the day we both joined RRI. He is the first one whom I would look for when I encountered any problem in my day-to-day matters and he has never let me down. I thank him for his jovial company and support at different stages of my RRI life and all the gossips we shared.

  - Anija is someone who makes her presence felt with her silence. I gladly acknowledge her for pleasant company and all the discussions we used to have.
  - Atish and I have been together since the days of our course work. He is some one who has the immense patience to answer my all stupid queries related to astrophysics *n* number of times. I thank him for the friendship and the nice times we had together and everything I learnt discussing with him.
  - Divya has the rare ability to make others join her laughter which always makes one feel 'lighter' in her company. I thank her for the friendship and the nice time we had doing the course work together.
  - My friendship with Ganesh is something which I will always cherish. We had a great time both creating and solving problems together. I thank him for an intimate friendship we shared.
  - Most of the astrophysics I picked up after my course work, which helped my research, has been initiated by my discussions with Resmi. I also thank her for constantly monitoring my progress as a researcher and constructive criticisms, suggestions about possible improvements, discussion on various issues related to physics, philosophy and others, lending support at difficult times and for her cheerful company throughout. Thanks a lot Resmi!

- It was a nice experience to do my course work with Dipanjan and Chandreyee. I thank them for all the discussions and their company.

- It is a great pleasure for me to thank my LC lab seniors, Viswanath (for all his useful tips on computers, especially LaTeX) , Panikumar (for all the 'student's talks' he used to organize), Shrinivas Murthy, Rajkumar, Sanat for their company and tips on various of issues.

- I thank my LC colleagues, Bharat Kumar, Brindaban Kundu, Govind, Nagaraju, Rakesh Pandey, Suresh Kumar....for their company in the hostel and the discussions/arguments during breakfast and dinner times.

- It was a wonderful experience to share the office with Sudipto, Kripa, Rahul, Dibyendu at different times. I thank them for all the discussions and assistance I got from them. Also I thank my other theory colleagues Sutirtha, Abhijit, Javed and Abhishek for their pleasant company.

- I thank Manju, secretary of TP group, who always responded positively with whatever help I needed which made my academic hours at RRI trouble-free. I also thank Vidya, secretary of Astrophysics group and Radhakrishna, secretary of Liquid Crystals Group, Marisa and Radha of the administrative department for helping out on various occassions.

- Yoga has always helped me to relax and put me in a better state of mind. My sincere thanks to Ms Karpagam, my Yoga instructor for guiding me through.

- My sister **Jyothi** has always been a joyful company to me whether or not I was at home. I thank her for all the telephonic conversations whenever I was stressed (and her patience to listen to whatever I say) which was always an effective medicine.

- I am grateful to my **Achhan** and **Amma** for all their support till now. They have always given me the freedom to pursue a career on my own. I thank them for all their trust and faith.

- My special thanks to Bhabhi, Sunilettan and Meghna for all the weekends I spent with them which always made me feel at home in Bangalore.

Contents

Preface

A compact binary system consisting of neutron stars (NS) and/or black holes (BH) loses energy and angular momentum via gravitational radiation as it orbits about its common centre of mass causing the orbit to shrink. Gravitational waves (GWs) emitted during this inspiral phase (when the change in orbital frequency due to GW emission is smaller than the orbital frequency itself) constitute one of the most important sources for the ground-based interferometers such as LIGO, VIRGO and the proposed space-based experiment LISA. The reason why these sources are prime targets is that using different approximation schemes to general relativity (GR), one can compute the gravitational waveforms accurately for the inspiral phase. This allows experimenters to use data analysis techniques such as matched filtering for detecting the signal and estimating the parameters of the signal.

For the computation of waveforms from the inspiralling compact binaries one needs to solve the two-body problem in general relativity. No exact solution to this problem is available till today and one resorts to a variety of approximation schemes to tackle it. We approach the problem at hand using the multipolar post-Minkowskian formalism which uses a combination of post-Minkowskian expansion of the gravitational field (expansion in powers of Newton's constant G which is valid at all points outside the source) complemented by a multipole expansion, which makes it easier to handle the technical implementation. This is followed by a post-Newtonian (PN) expansion (expansion in powers of a velocity parameter v/c, which is valid only in the near zone of the source).

The theoretical description of the gravitational waveform needs to be very accurate and as close to reality as possible when one uses matched filtering because mismatches between the signal and the template (which is the pre-calculated waveform) will result in a loss of signal to noise ratio. Computing very accurate theoretical templates and including effects such as spin and orbital eccentricity are challenging tasks for the theoreticians. This thesis addresses some of the issues related to the waveform modelling of the inspiralling compact binaries and their implications for gravitational wave data analysis.

Most of these compact binaries move in circular orbits during the last stages of their inspiral. Hence one usually models the binary orbit to be circular for computation of the waveforms. Usual templates which are used for gravitational wave data analysis are currently the 'restricted waveforms' (RWF). The restricted waveforms ignore the presence of harmonics other than the dominant one at twice the orbital frequency and models the phase of the waveform to the maximum PN accuracy possible, keeping the amplitude at the leading Newtonian order. This is justified because matched filtering is more sensitive to the phase of the wave rather than the amplitude, since the correlation builds up as long as the signal and the template remain *in phase*. Recent studies showed that going beyond the RWF approximation could improve the efficiency of detection as well as parameter estimation of the inspiral signal.

Motivated by the necessity to go beyond the standard RWF approximation we compute, in chapters 2 and 3, the gravitational waveform of inspiralling compact binaries moving in quasi-circular orbits at the second and a half post-Newtonian (2.5PN) approximation to general relativity, using the multipolar post-Minkowskian and matching formalism. The inputs we use include notably the mass-type quadrupole at the 2.5PN order, the mass octupole and current quadrupole at the 2PN order, the mass 2^5-pole and current 2^4-pole at 1PN. The non-linear hereditary terms come from the monopole-quadrupole multipole interactions or tails, present at the 1.5PN, 2PN and 2.5PN orders, and the quadrupole-quadrupole interaction arising at the 2.5PN level. In particular, the specific effect of non-linear memory is computed using a simplified model of binary evolution in the past. The "plus" and "cross" wave polarizations at the 2.5PN order are obtained in ready-to-use form, extending the 2PN results calculated earlier by Blanchet, Iyer, Will and Wiseman. The results we have obtained should be of use for data analysis of the inspiral signal for the ground-based as well as space-based detectors. Recent studies, which actually used the nonrestricted waveform presented in chapters 2 and 3, showed that the higher harmonics in the amplitude could improve the detectability of binaries whose leading harmonic does not enter the detector's sensitivity band or spends too little time to be effectively detected.

As mentioned earlier, higher PN order waveforms should, in principle, improve the efficiency of detection and parameter estimation. Recently, the ambiguity parameters which appeared at 3PN order in the phasing formula for nonspinning compact binaries on circular orbit were computed, and this leads to a complete 3.5PN phasing formula. It is instructive to examine the convergence of parameter estimation as a function of the PN order of the approximation used in modelling the waveform. Chapters 4 and 5 contain an exhaustive analysis of the effect of higher PN order terms in the phasing on parameter estimation in the context of ground-based detectors and space-based detectors, respectively.

We revisit the problem of parameter estimation of the gravitational-wave chirp signals from inspiralling non-spinning compact binaries in the light of the recent extension of the post-Newtonian (PN) phasing formula to order $(v/c)^7$ beyond the leading Newtonian order. We study in detail the implications of higher post-Newtonian orders from 1PN up to 3.5PN in steps of 0.5PN ($\sim v/c$), and examine their convergence. In both initial and advanced detectors the estimation of the chirp mass ($\mathcal{M}$) and symmetric mass ratio (η) improve at higher PN orders but oscillate with every half-a-PN order. In initial LIGO, for a $10M_\odot$-$10M_\odot$ binary at a signal-to-noise ratio (SNR) of 10, the improvement in the estimation of $\mathcal{M}$ (η) at 3.5PN relative to 2PN is $\sim$ 19% (52%). We compare parameter estimation in different detectors and assess their relative performance in two different ways: at a *fixed SNR*, with the aim of understanding how the bandwidth improves parameter estimation, and for a *fixed source*, to gauge the importance of sensitivity. Errors in parameter estimation at a fixed SNR are smaller for VIRGO than for both initial and advanced LIGO. This is because of the larger bandwidth over which it observes the signals. However, for sources at a fixed distance it is advanced LIGO that achieves the lowest errors owing to its greater sensitivity. Finally, we compute the amplitude corrections due to the 'frequency-sweep' in the Fourier domain representation of the waveform within the stationary phase approximation and discuss its implication on parameter estimation. We find that the amplitude corrections change the errors in $\mathcal{M}$ and η by less than 10% for initial LIGO at a signal-to-noise ratio of 10. Our analysis makes explicit the significance of higher PN order modelling of the inspiralling compact binary on parameter estimation. These results are finally compared against those obtained for the case of third generation European Gravitational Observatory (EGO).

Laser Interferometer Space Antenna (LISA) will routinely observe coalescences of supermassive black hole (BH) binaries up to very high redshifts. LISA can measure mass parameters of such coalescences to a relative accuracy of $10^{-4} - 10^{-6}$, for sources at a distance of 3 Gpc. The problem of parameter estimation of massive nonspinning binary black holes using post-Newtonian (PN) phasing formula is studied in the context of LISA. Specifically, the performance of the 3.5PN templates is contrasted against its 2PN counterpart using a waveform which is averaged over the LISA pattern functions. The improvement due to the higher order PN corrections to the phasing formula is examined by calculating the errors in the estimation of mass parameters at each order. The estimation of the mass parameters $\mathcal{M}$ and η are significantly enhanced by using the 3.5PN waveform instead of the 2PN one. For an equal mass binary of $2 \times 10^6 M_\odot$ at a luminosity distance of 3 Gpc, the improvement in chirp mass is $\sim$ 11% and that of η is $\sim$ 39%. Estimation of coalescence time t_c worsens by 43%. The improvement is larger for the unequal mass binary mergers. These results are compared to the ones obtained using a non-pattern averaged waveform. The errors depend very much on the location and orientation of the source and general conclusions cannot be drawn without performing Monte Carlo simulations. The effect of the choice of the lower frequency cut-off for LISA on the parameter estimation is also studied.

Though most of the sources will be in circular orbits by the time the GWs emitted by the system enter the sensitivity band of the laser interferometers, astrophysical scenarios such as Kozai mechanism could produce binaries which have nonzero eccentricity. Studies have shown that filtering the signal from an eccentric binary with circular orbit templates could significantly degrade the SNR. For constructing a phasing formula for eccentric binaries one has to compute the energy and angular momentum fluxes carried away by the GWs and thence compute how the orbital elements evolve with time.

In chapter 6 the instantaneous terms in the 3PN angular momentum flux from the inspiral phase of a binary system of compact objects moving in quasi-elliptical orbits is computed using the Multipolar post-Minkowskian wave generation formalism. Using the 3PN quasi-Keplerian representation of elliptical orbits obtained recently, the angular momentum flux is averaged over the binary's orbit. The evolution of orbital elements under 3PN gravitational radiation reaction is studied in the quasi-elliptic case. The angular momentum flux provided here has to be supplemented with the hereditary part to obtain the final input needed for the construction of templates for binaries moving in elliptical orbits, a class of sources for both the space based detectors and the ground based ones.

These investigations are carried out in collaboration with the thesis advisor Prof. Bala R Iyer, Prof. Luc Blanchet, Prof. B S Sathyaprakash, Moh'd S S Qusailah and Pranesh A Sundararajan. This thesis is based on the following papers.

1. **K G Arun**, L Blanchet, B R Iyer and M S S Qusailah:
   The 2.5PN gravitational wave polarizations from inspiralling compact binaries in circular orbits
   Class. Quant. Grav. **21**, 3771 (2004), gr-qc/0404085.

2. **K G Arun**, B R Iyer, B S Sathyaprakash and P A Sundararajan :
   Parameter estimation of inspiralling compact binaries using 3.5 post-Newtonian gravitational wave phasing: The non-spinning case
   Phys. Rev. **D 71**, 084008 (2005), gr-qc/0411146.

3. **K G Arun**:
   Parameter estimation of coalescing supermassive black hole binaries with LISA.
   Phys. Rev. **D 74**, 024025 (2006), gr-qc/0605021.

4. **K G Arun**, L Blanchet, B R Iyer and M S S Qusailah:
   Inspiralling compact binaries in quasi-elliptical orbits: 3PN angular momentum flux and the evolution of orbital elements
   (Manuscript in Preparation).