# Investigation of wave propagation in random and specially structured media

by

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Thesis submitted to the Jawaharlal Nehru University
for the award of the degree of
Doctor of Philosophy

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***December 2006***

*Dedicated to*

my parents

# DECLARATION

I hereby declare that the work reported in this thesis has been independently carried out by me at the Raman Research Institute, Bangalore, under the supervision of Dr. Hema Ramachandran. The subject matter presented in this thesis has not previously formed the basis of the award of any degree, diploma, associateship, fellowship or any other similar title.

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# CERTIFICATE

This is to certify that the thesis entitled Investigation of wave propagation in random and specially structured media, submitted by Divya Sharma for the award of the degree of Doctor of Philosphy of Jawaharlal Nehru University, is her original work. This has not been published or submitted to any other University for any other degree or diploma.

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# Acknowledgements

This is perhaps the most challenging but also the most important part of my Thesis, as I strongly feel that this Thesis would have been impossible without people who supported me and believed in me. While I feel glad in expressing my heartfelt gratitude to all such people, I am afraid, lest I might fail to do proper justice to them. I, nonetheless, try ...

First of all, I extend my overwhelming thanks to my research supervisor Dr. Hema Ramachandran, who was more like a friend than a guide to me for the entire duration of our scientific collaboration. Her amazing experimental skills and remarkable ability to maintain her composure, think and act shrewdly at critical stages of our experiments/work highly impressed me. Most importantly, she is a kind person. In fact, it would not be an exaggeration to say that Hema was there, wherever and whenever I needed her; be it an academic or a personal exigency. Thus, she ensured that I am not "scattered" off the "light scattering" experiments !! I sincerely hope and wish that I continue to benefit by maintaining this close scientific touch with her in future too.

I express my whole-hearted gratitude to Prof N. Kumar, whose abundant scientific passion and excitement was a perennial source of inspiration and encouragement. I thoroughly enjoyed long stretches of discussions regarding my work with him; each one of which was not only highly informative but also left me with stronger scientific spirit and curiosity. Above all his great sense of humour and liveliness made our discussions, usually accompanied by cups of tea at RRI canteen, very healthy and friendly apart from being an encouraging rejuvenation for me. I sincerely thank him for his contribution to my Thesis.

I express my deepest appreciation to all the members of the Light and Matter Physics (LAMP) Group as each one of them in their own distinct way, make this group an ideal one to pursue one's research. While Prof. R. Srinivasan and Dr. Andal were highly accessible for discussions regarding my work, Dr. Reji and Dr. Sadiq were prompt in their critical comments on varied issues/topics. I am specially indebted to

Meena, Sanjay, Raghavan, Jimmy, Ajay, Srihari, Amudha, Nandan, Karthik, Srikanth, Archana, Uday, Dhanalakshmi and Preethi for providing a highly scientific, cooperative, healthy, fun-filled and enjoyable laboratory environment. I thank Latha, Shiva and Manju for their help in maintaining the LAMP group.

Further, I enjoyed the company of my peer group at RRI especially Arun, Anija, Amit, Resmi, Atish, Chandreyee and Dipanjan. Arun's sincerity to work, Resmi's caring and loving attitude, Anija's golden silence and perennial smile, Amit and Atish's lively, humorous and playful nature, Chandreyee's readiness to help and Dipanjan's happy go lucky spirit, made our group special.

I was highly benefited from the computing knowledge and expertise of Ravi BT. His rare quality of going out of his ways to help others has always impressed me. Apart from the enormous academic help that I sought from him, I found a trustworthy and a reliable friend in him. He saw me through the ups and downs of my academic career and was not merely a patient listener to my cribs but was in fact, a sane and sound adviser too. Thanks a lot for everything, Ravi !!

Just before I commenced writing my Thesis, I found a true and sincere friend in Sowmya. As our collaboration made the LAMP pantry a hotter place to enjoy hot "adrak Wali" tea, we were soon joined by Sudipto, Jyoti, Shreoysi, Tridib, Gayathri and Ritayon in enjoying cups of tea, while having heated arguments and discussions on varied social, political and economic issues. I take this opportunity to thank all of them for their pleasant, enthusiastic company and the interesting discussions.

My academic acquaintance to RRI was brought out by the Summer School. Recalling those memorable days at RRI, I thank Sushil, Anshu, Viswa, Pani, Ashish, Rekhesh, Amitesh, Niruj, Murthy and Reddy for their'wonderful company.

I enjoyed the company of Ram and G.B. Suresh during our regularly, irregular music sessions at RRI and while organizing the RRI Badminton tournaments. Further, I benefited from the regular yoga classes held by Shantha at RRI clinic. I thank Ram, Suresh and Shantha for their nice and hearty company.

I would like to acknowledge my debt to RRI administration, library staff, computer section, workshop, canteen staff, security staff and hostel cooks who in their own ways and capacities made my stay at RRI, very comfortable and memorable.

Last but not the least, I am much beholden to my family for it being a constant source of inspiration and encouragement to me throughout my life. I am extremely grateful to my elder brother Rahul and elder sisters Rashmi and Swati, who have been my best friends till date, for their patience and affectionate care. Further, I am very much obliged to my sister-in-law Jyoti and brothers-in law Ashutosh and Ankur, who have become an integral part of our family within no time and have given me a sense of belongingness, affection and emotional support. Finally, I express my whole-hearted gratitude to my parents who have all throughout supported my dreams and aspirations, even if it amounted to sacrificing their own comforts and pleasures.

To **them,** I **dedicate this thesis.**

# Preface

This Thesis describes the experimental, theoretical and the numerical simulation work carried out on Random Amplifying Medium (RAM). By RAM, we mean here an optically pumped lasing (active or amplifying) medium with spatial (essentially quenched) disorder. In general, disorder is understood as introduced through a dielectric medium ($\epsilon(\omega)$) with complex refractive index ($n(\omega) = \sqrt{\epsilon(\omega)} \equiv n'(\omega) - in''(\omega)$). Here, the real part $n'(\omega)$ provides the randomness while the imaginary part $n''(\omega)$ gives the amplification. In general, both could be random. In this Thesis, we have specifically considered two distinct classes of RAMs, the direct or D-RAM and its reverse or R-RAM. A D-RAM consists of continuous active medium in which discrete, passive scatterers are randomly embedded. In contrast, in an R-RAM, it is the scatterers which are active while the continuum medium is passive.

The emphasis of the Thesis is, primarily, on studying the statistical fluctuations in the emitted intensity from RAMs. Here the statistical fluctuations are over the ensemble of different microscopic realizations of the randomness or disorder for the given macroscopic disordered sample (hence, "sample-to- sample" fluctuations)[1]. Normally, these statistical fluctuations exhibit Gaussian statistics, that is the observed intensity fluctuates about a well-defined mean with a finite variance. However, we have observed a crossover from the Gaussian to the non-Gaussian, or Lévy statistics with diverging variances for intensity fluctuations. The crossover is as a function of scattering and gain parameters characterizing the RAM; these parameters, thus, provide a means for tuning the Lévy exponent.

The Thesis comprises six chapters; their contents are stated below. References made in

[1]These fluctuations are not to be confused with an intrinsic quantum/classical statistical intensity fluctuations in the emission of light.

each chapter are listed out at the end of that chapter.

Chapter 1 reviews the present state of research in RAMs and provides an elementary introduction to both the Gaussian and the Lévy statistics. The distinguishing features are described through some physical examples.

Chapter 2 discusses our theoretical and experimental studies on dye-scatterer RAMs. We analytically show that in the limit of high gain and strong scattering, Lévy statistical fluctuations in the emitted intensity should be observed. We attribute these to the "sample-to-sample" fluctuations which in turn give rise to certain rare, long paths which dominate the intensity due to high gain in the system. We experimentally demonstrate the "Gaussian-to-Lévy" statistical crossover in dye-scatterer RAMs. By altering the scattering and gain parameters, the tunability of Lévy exponent was achieved.

In order to make a parametric study of the crossover over a wide range of RAM parameters that are not easily accessible in experiments, a Monte Carlo simulation of the photon diffusion in a RAM is performed. The simulation and its results which corroborate our model calculation and physical experiments of the previous chapter, are discussed in Chapter 3. In the limit of very high gain, an interesting "Lévy microscope effect" is observed, where the mean intensity is dominated by a single largest event.

In order to esatablish the crucial role of rare long amplifying paths in causing Lévy statistical fluctuations, we created a specially structured RAM consisting of random aggregate of active (dye-doped) fiber segments embedded in a passive bulk. We term this an F-RAM (Fiber- Random Amplifying Medium) and showed it as an optical realization of the Arrhenius cascade model in Chapter 4. The F-RAM is shown to exhibit the crossover from the Gaussian to the Lévy statistics with tunable Lévy exponent, by means of experiments. Further, the theoretical treatment described in Chapter 2 for a dye-scatterer RAM an extended using the "effective-medium" theory to describe an F-RAM.

Chapter 5 discusses the interesting case of a reverse RAM (R-RAM) (active scatterers (discrete) embedded in passive bulk (continuum)). The Mie scattering theory was applied using negative absorption or gain to model the amplifying scatterers. Unlike conventional dye-scatterer RAMs (discussed in Chapter 2) where enhanced scattering always favours light amplification, we find by Monte Carlo simulation of photon diffusion in an R-RAM, that this is not always true in R-RAMs. In particular, we observe a non-monotonic dependence of the overall gain on the refractive index mismatch for a suitable range of parameter values characterizing the R-RAM. We attribute this observation as arising due to the competition between the effectiveness of the individual amplifying scattering events and the frequency of these multiple scattering events in an R-RAM. This in turn is due to the reversal of roles of the scatterers and the propagation medium in an R-RAM as distinct from the D-RAM.

Finally, we conclude the Thesis by discussing in Chapter 6, some finer points of the work presented herein and some open questions that may provide directions for future studies.

As appendices, we present certain identities (appendix A), computer codes (appendices B and C) and scattering coefficients for an amplifying slab (appendix D), at the end of the Thesis.

The work presented in the Thesis has appeared in the following publications :

# Publications

1. *Lévy statistical fluctuations from a random amplifying medium.*
   Divya Sharma, Hema Ramachandran and N. Kumar
   Fluctuation and Noise Letters, **6**, L95-L101 (2006).

2. *Lévy statistics of emission from a novel random amplifying medium : an optical realization of the Arrhenius cascade.*
   Divya Sharma, Hema Ramachandran and N. Kumar
   Optics Letters, **31**, 1806-1808 (2006).

3. *Gaussian-to-Lévy statistical crossover in random amplifying medium : Monte Carlo simulation.*
   Divya Sharma, Hema Ramachandran and N. Kumar
   Accepted for publication in Optics Communications.

4. *Interplay of disorder and amplification in passive medium with active scatterers : Monte Carlo simulation.*
   Divya Sharma and N. Kumar
   To be communicated.

## Papers presented in Conference Proceedings

1. *Enhancement of the cooling by frequency up-conversion through multiple scattering.*
   Divya Sharma and Hema Ramachandran
   National Laser Symposium, Dec. 22-24, 2003, Indian Institute of Technology (IIT) Kharagpur, Kolkata, India.

2. *Random Amplifying Medium : Lévy laser.*
   Divya Sharma, Hema Ramachandran and N. Kumar
   International Conference on optoelectronics, fiber optics and photonics, Dec. 9-11, 2004, Cochin University of Science and Technology (CUSAT), Cochin, India.

3. *Lévy statistical fluctuations from a random amplifying medium.*
   Divya Sharma, Hema Ramachandran and N. Kumar
   National Laser Symposium, Dec. 7-10, 2005, Vellore Institute of Technology (VIT), Chennai, India

4. *Tunable Lévy exponent of emission statistics of a fiber-random amplifying medium.*
   Divya Sharma, Hema Ramachandran and N. Kumar
   International Conference on Optics and Optoelectronics, Dec. 12-15, 2005, Instruments Research and Development Establishment (IRDE), Uttaranchal, India

5. *Lévy statistics in optics : Violation of the Central Limit Theorem.*
   Divya Sharma, Hema Ramachandran and N. Kumar
   International Conference on Current Developments in Atomic, Molecular and Optical Physics with applications, March 21-23, 2006, Delhi University, Delhi.

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