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Shear dynamo problem: Quasilinear kinematic theory

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Large-scale dynamo action due to turbulence in the presence of a linear shear flow is studied. Our treatment is quasilinear and kinematic but is nonperturbative in the shear strength. We derive the integrodifferential equation for the evolution of the mean magnetic field by systematic use of the shearing coordinate transformation and the Galilean invariance of the linear shear flow. For nonhelical turbulence the time evolution of the cross-shear components of the mean field does not depend on any other components excepting themselves. This is valid for any Galilean-invariant velocity field, independent of its dynamics. Hence the shear-current assisted dynamo is essentially absent, although large-scale nonhelical dynamo action is not ruled out.

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Shear flows and turbulence are ubiquitous in astrophysical systems. Recent work suggests that the presence of shear may open new pathways to the operation of large-scale dynamos [1–5]. We present a theory of dynamo action in a shear flow of an incompressible fluid which has random velocity fluctuations due either to freely decaying turbulence or generated through external forcing. Of particular interest is the case of nonhelical large-scale dynamo action in shear flows. Several direct simulations show that large-scale fields can grow from small seed fields under the combined action of nonhelical turbulence and background shear flow [1,2]. However, the interpretation of how such a dynamo works is not yet clear. One possibility that has attracted much attention is the shear-current effect [4], in which extra components of the mean electromotive force (EMF) arise due to shear, which couple components of the mean magnetic field parallel and perpendicular to the shear flow. However there is no convergence yet on whether the sign of the relevant coupling term is such as to obtain a dynamo; some analytic calculations [6,7] and numerical experiments [1] find that the sign of the shear-current term is unfavorable for dynamo action. Moreover, analytic calculations treat shear as a small perturbation. We are interested here in studying the shear dynamo without such a restriction.

Our theory is "local" in character: In the laboratory frame we consider a background shear flow whose velocity is unidirectional (along the X_2 axis) and varies linearly in an orthogonal direction (the X_1 axis). The linear shear flow has a basic symmetry relating to measurements made by a special subset of all observers, who may be called co-moving observers. This symmetry is the invariance of the equations with respect to a group of transformations that is a subgroup of the full Galilean group. It may be referred to as "shear–restricted Galilean invariance," or Galilean invariance (GI). We introduce and explore the consequences of GI velocity fluctuations; not only are these compatible with the underlying symmetry of the problem, but they are expected to arise naturally. This has profound consequences for dynamo action because the transport coefficients that define the mean EMF become spatially homogeneous in spite of the shear flow. Systematic use of the shearing transformation allows us to develop a theory that is nonperturbative in the strength of the background shear. However, we ignore the complications associated with nonlinear interactions, hence MHD turbulence and the small-scale dynamo; so our theory is quasilinear in nature, equivalent to the "first-order smoothing approximation" (FOSA).

Let $(\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3)$ be the unit vectors of a Cartesian coordinate system in the laboratory frame, $\boldsymbol{X}=(X_1,X_2,X_3)$ be the position vector, and τ be the time. The fluid velocity is given by $(-2AX_1\boldsymbol{e}_2+\boldsymbol{v})$, where A is the shear parameter and $\boldsymbol{v}(\boldsymbol{X},\tau)$ is a randomly fluctuating velocity field, which is incompressible $(\boldsymbol{\nabla}\cdot\boldsymbol{v}=0)$ and has zero mean $(\langle\boldsymbol{v}\rangle=\boldsymbol{0})$. The magnetic field has a large-scale (mean-field) component $\boldsymbol{B}(\boldsymbol{X},\tau)$ and a fluctuating field, $\boldsymbol{b}$, with zero mean $(\langle\boldsymbol{b}\rangle=\boldsymbol{0})$. The evolution of the mean field is governed by

$$\left(\frac{\partial}{\partial\tau}-2AX_1\frac{\partial}{\partial X_2}\right)\boldsymbol{B}+2AB_1\boldsymbol{e}_2=\boldsymbol{\nabla}\times\mathcal{E}+\eta\nabla^2\boldsymbol{B}, \qquad (1)$$

where $\mathcal{E}=\langle\boldsymbol{v}\times\boldsymbol{b}\rangle$ is the mean EMF. Our goal is to calculate $\mathcal{E}$ in terms of the statistical properties of the fluctuating velocity field, which we will do using quasilinear theory. This means solving the equation for $\boldsymbol{b}$ by dropping terms that are quadratic in the fluctuations. We also drop the resistive term, assuming that the correlation times are small compared to the resistive time scale. So our theory is applicable when FOSA is valid [8]. Then $\boldsymbol{b}$ obeys

$$\left(\frac{\partial}{\partial\tau}-2AX_1\frac{\partial}{\partial X_2}\right)\boldsymbol{b}+2Ab_1\boldsymbol{e}_2=\boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}). \qquad (2)$$

It proves convenient to exchange spatial inhomogeneity for temporal inhomogeneity so we get rid of the $(X_1\partial/\partial X_2)$ term through a shearing transformation to new space-time variables,

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$$x_1 = X_1, \quad x_2 = X_2 + 2A\tau X_1, \quad x_3 = X_3, \quad t = \tau. \tag{3}$$

We also define new variables, $\boldsymbol{H}(\boldsymbol{x},t)=\boldsymbol{B}(\boldsymbol{X},\tau)$, $\boldsymbol{h}(\boldsymbol{x},t)=\boldsymbol{b}(\boldsymbol{X},\tau)$, and $\boldsymbol{u}(\boldsymbol{x},t)=\boldsymbol{v}(\boldsymbol{X},\tau)$, which are componentwise equal to the old variables.

Then Eq. (2) becomes

$$\frac{\partial \boldsymbol{h}}{\partial t} + 2Ah_1\boldsymbol{e}_2 = \left(\boldsymbol{H}\cdot\frac{\partial}{\partial \boldsymbol{x}} + 2AtH_1\frac{\partial}{\partial x_2}\right)\boldsymbol{u} - \left(\boldsymbol{u}\cdot\frac{\partial}{\partial \boldsymbol{x}} + 2Atu_1\frac{\partial}{\partial x_2}\right)\boldsymbol{H}. \tag{4}$$

Not only do sheared coordinates get rid of spatial inhomogeneity, but in quasilinear theory the evolution Eq. (4) does not contain spatial derivatives of $\boldsymbol{h}(\boldsymbol{x},t)$. The equations for h_1 and h_3 can be integrated directly. The h_1 so obtained can be substituted in the equation for h_2: there occur double-time integrals which can be manipulated to give expressions with only single-time integrals, by interchaging the order of the integrals. Then the particular solution for $\boldsymbol{h}(\boldsymbol{x},t)$ is given in component form by

$$h_m = \int_0^t dt'[u'_{ml} - 2A(t-t')\delta_{m2}u'_{1l}][H'_l + 2At'\delta_{l2}H'_1] - \int_0^t dt'[u'_l + 2At'\delta_{l2}u'_1][H'_{ml} - 2A(t-t')\delta_{m2}H'_{1l}], \tag{5}$$

where primes denote evaluation at space-time point $(\boldsymbol{x},t')$. We have also used notation $u_{ml}=(\partial u_m/\partial x_l)$ and $H_{ml}=(\partial H_m/\partial x_l)$.

The expression in Eq. (5) for $\boldsymbol{h}$ should be substituted in $\mathcal{E}=\langle \boldsymbol{v}\times\boldsymbol{b}\rangle=\langle \boldsymbol{u}\times\boldsymbol{h}\rangle$. Following standard procedure, we allow $\langle\,\rangle$ to act only on the velocity variables but not the mean field; symbolically, it is assumed that $\langle \boldsymbol{uuH}\rangle=\langle \boldsymbol{uu}\rangle\boldsymbol{H}$. After interchanging the dummy indices (l,m) in the last term, we find that the mean EMF is

$$\mathcal{E}_i = \int_0^t dt'[\hat{\alpha}_{il} - 2A(t-t')\hat{\beta}_{il}][H'_l + 2At'\delta_{l2}H'_1] - \int_0^t dt'[\hat{\eta}_{iml} + 2At'\delta_{m2}\hat{\eta}_{i1l}][H'_{lm} - 2A(t-t')\delta_{l2}H'_{1m}], \tag{6}$$

where the transport coefficients $(\hat{\alpha},\hat{\beta},\hat{\eta})$ are defined in terms of the $\boldsymbol{uu}$ velocity correlators by

$$\hat{\alpha}_{il}(\boldsymbol{x},t,t') = \epsilon_{ijm}\langle u_j(\boldsymbol{x},t)u_{ml}(\boldsymbol{x},t')\rangle,$$
$$\hat{\beta}_{il}(\boldsymbol{x},t,t') = \epsilon_{ij2}\langle u_j(\boldsymbol{x},t)u_{1l}(\boldsymbol{x},t')\rangle,$$
$$\hat{\eta}_{iml}(\boldsymbol{x},t,t') = \epsilon_{ijl}\langle u_j(\boldsymbol{x},t)u_m(\boldsymbol{x},t')\rangle. \tag{7}$$

It is physically more transparent to consider velocity statistics in terms of the $\boldsymbol{vv}$ velocity correlators because this is referred to the laboratory frame, instead of the sheared coordinates. By definition,

$$u_m(\boldsymbol{x},t) = v_m[\boldsymbol{X}(\boldsymbol{x},t),t], \tag{8}$$

where $\boldsymbol{X}(\boldsymbol{x},t)=(x_1,x_2-2Atx_1,x_3)$ is the inverse of the shearing transformation given in Eq. (3). The velocity gradient u_{ml} is

$$u_{ml} = \left(\frac{\partial}{\partial X_l} - 2A\tau\delta_{l1}\frac{\partial}{\partial X_2}\right)v_m = v_{ml} - 2A\tau\delta_{l1}v_{m2}, \tag{9}$$

where $v_{ml}=(\partial v_m/\partial X_l)$. Using Eqs. (8) and (9) in Eq. (7),

$$\hat{\alpha}_{il}(\boldsymbol{x},t,t') = \epsilon_{ijm}[\langle v_j(\boldsymbol{X},t)v_{ml}(\boldsymbol{X}'t')\rangle - 2At'\delta_{l1}\langle v_j(\boldsymbol{X},t)v_{m2}(\boldsymbol{X}',t')\rangle\rangle],$$

$$\hat{\beta}_{il}(\boldsymbol{x},t,t') = \epsilon_{ij2}[\langle v_j(\boldsymbol{X},t)v_{1l}(\boldsymbol{X}',t')\rangle - 2At'\delta_{l1}\langle v_j(\boldsymbol{X},t)v_{12}(\boldsymbol{X}',t')\rangle],$$

$$\hat{\eta}_{iml}(\boldsymbol{x},t,t') = \epsilon_{ijl}\langle v_j(\boldsymbol{X},t)v_m(\boldsymbol{X}',t')\rangle, \tag{10}$$

where the quantities $\boldsymbol{X}=(x_1,x_2-2Atx_1,x_3)$ and $\boldsymbol{X}'=(x_1,x_2-2At'x_1,x_3)$.

We can arrive at some general conclusions for delta-correlated-in-time velocity fields. Let the two-point correlator be taken between space-time points $(\boldsymbol{R},\tau)$ and $(\boldsymbol{R}',\tau')$ be $\langle v_i(\boldsymbol{R},\tau)v_j(\boldsymbol{R}',\tau')\rangle=\delta(\tau-\tau')T_{ij}(\boldsymbol{R},\boldsymbol{R}',\tau)$. We define $T_{ijl}(\boldsymbol{R},\tau)=(\partial T_{ij}/\partial R'_l)_{\boldsymbol{R}'=\boldsymbol{R}}$. The delta function ensures that $\boldsymbol{X}$ and $\boldsymbol{X}'$ occurring in the velocity correlators of Eq. (10) are equal to each other. So $\langle v_i(\boldsymbol{X},t)v_j(\boldsymbol{X}',t')\rangle=\delta(t-t')T_{ij}(\boldsymbol{X},\boldsymbol{X},t)$ and $\langle v_i(\boldsymbol{X},t)v_{jl}(\boldsymbol{X}',t')\rangle=\delta(t-t')T_{ijl}(\boldsymbol{X},t)$. The integrals over time in Eq. (6) can all be performed, so the mean EMF is

$$\mathcal{E}_i = \epsilon_{ijm}[T_{jml} - 2At\delta_{l1}T_{jm2}][H_l + 2At\delta_{l2}H_1] - \epsilon_{ijl}[T_{jm} + 2At\delta_{m2}T_{j1}]H_{lm}. \tag{11}$$

It is useful to write the EMF in terms of the original variables and laboratory-frame coordinates. To this end we transform

$$H_{lm} = \left(\frac{\partial}{\partial X_m} - 2A\tau\delta_{m1}\frac{\partial}{\partial X_2}\right)B_l = B_{lm} - 2A\tau\delta_{m1}B_{l2}, \tag{12}$$

where $B_{lm}=(\partial B_l/\partial X_m)$. Then the explicit dependence of $\mathcal{E}_i$ on the shear parameter A cancels out, and mean EMF assumes the simple form

$$\mathcal{E}_i = \epsilon_{ijm}T_{jml}B_l - \epsilon_{ijl}T_{jm}B_{lm}, \tag{13}$$

which is the familiar expression obtained in the absence of shear. Thus, shear needs time to manifest and, to see the effects of shear explicitly, it is necessary to consider nonzero correlation times. Henceforth we consider velocity statistics with finite correlation times.

The linear shear flow has a basic symmetry relating to measurements made by a special subset of all observers. We define a co-moving observer as one whose velocity with respect to the laboratory frame is equal to the velocity of the background shear flow, and whose Cartesian coordinate axes are aligned with those of the laboratory frame. A co-moving observer can be labeled by the coordinates, $\boldsymbol{\xi}=(\xi_1,\xi_2,\xi_3)$, of her origin at time $\tau=0$. Different labels identify different co-moving observers and vice versa. As the labels run over all possible values, they exhaust the set of all co-moving observers. The origin of the coordinate axes of a co-moving observer translates with uniform velocity; its position with respect to the origin of the laboratory frame is given by

$$\boldsymbol{X}_c(\tau) = (\xi_1, \xi_2 - 2A\tau\xi_1, \xi_3). \tag{14}$$

An event with space-time coordinates $(\boldsymbol{X},\tau)$ in the laboratory frame has space-time coordinates $(\widetilde{\boldsymbol{X}},\widetilde{\tau})$ with respect to the co-moving observer, given by

$$\widetilde{\boldsymbol{X}} = \boldsymbol{X} - \boldsymbol{X}_c(\tau), \quad \widetilde{\tau} = \tau - \tau_0, \tag{15}$$

where the arbitrary constant τ_0 allows for translation in time as well.

Let $[\widetilde{\boldsymbol{B}}(\widetilde{\boldsymbol{X}},\widetilde{\tau}),\widetilde{\boldsymbol{b}}(\widetilde{\boldsymbol{X}},\widetilde{\tau}),\widetilde{\boldsymbol{v}}(\widetilde{\boldsymbol{X}},\widetilde{\tau})]$ denote the mean, the fluctuating magnetic fields, and the fluctuating velocity field, respectively, as measured by the co-moving observer. They are all equal to the respective quantities measured in the laboratory frame: $[\widetilde{\boldsymbol{B}}(\widetilde{\boldsymbol{X}},\widetilde{\tau}),\widetilde{\boldsymbol{b}}(\widetilde{\boldsymbol{X}},\widetilde{\tau}),\widetilde{\boldsymbol{v}}(\widetilde{\boldsymbol{X}},\widetilde{\tau})] = [\boldsymbol{B}(\boldsymbol{X},\tau),\boldsymbol{b}(\boldsymbol{X},\tau),\boldsymbol{v}(\boldsymbol{X},\tau)]$. That this must be true may be understood as follows. Magnetic fields are invariant under nonrelativistic boosts so the mean and fluctuating magnetic fields must be the same in both frames. To see that the fluctuating velocity fields must also be the same in both frames, we note that the total fluid velocity measured by the co-moving observer is, by definition, equal to $[-2A\widetilde{X}\boldsymbol{e}_2+\widetilde{\boldsymbol{v}}(\widetilde{\boldsymbol{X}},\widetilde{\tau})]$. This must be equal to the difference between the velocity in the laboratory frame, $[-2AX\boldsymbol{e}_2+\boldsymbol{v}(\boldsymbol{X},\tau)]$ and $(-2A\xi_1\boldsymbol{e}_2)$, which is the velocity of the co-moving observer with respect to the laboratory frame. Using $\widetilde{X}=X-\xi_1$, we see that $\widetilde{\boldsymbol{v}}(\widetilde{\boldsymbol{X}},\widetilde{\tau})=\boldsymbol{v}(\boldsymbol{X},\tau)$. Equations (1) and (2) are invariant under the simultaneous transformations of space-time coordinates and fields discussed above. We note that this symmetry property is actually invariance under a subset of the full ten-parameter Galilean group, parametrized by the five quantities $(\xi_1,\xi_2,\xi_3,\tau_0,A)$; for brevity we refer to this restricted symmetry as Galilean invariance, or simply GI. There is a fundamental difference between the coordinate transformations associated with GI [Eq. (15)] and the shearing transformation [Eq. (3)]. The former relates different co-moving observers, whereas the latter describes a time-dependent distortion of the coordinate axes of one observer. Moreover, the relationship between old and new variables is homogeneous for the Galilean transformation, whereas it is inhomogeneous for the shearing transformation.

Naturally occurring processes lead to G-invariant velocity statistics. Let the observer in the laboratory frame correlate v_i at space-time location $(\boldsymbol{R},\tau)$ with v_j at location $(\boldsymbol{R}',\tau')$. Now consider a co-moving observer, the position vector of whose origin is given by $\boldsymbol{X}_c(\tau)$ of Eq. (14). An identical experiment performed by this observer must yield the same results, the measurements now made at the space-time points denoted by $[\boldsymbol{R}+\boldsymbol{X}_c(\tau),\tau]$ and $[\boldsymbol{R}'+\boldsymbol{X}_c(\tau'),\tau']$ in the laboratory-frame variables. Therefore, a GI two-point velocity correlator must satisfy the condition

$$\langle v_i(\boldsymbol{R},\tau)v_j(\boldsymbol{R}',\tau')\rangle = \langle v_i(\boldsymbol{R}+\boldsymbol{X}_c(\tau),\tau)v_j(\boldsymbol{R}'+\boldsymbol{X}_c(\tau'),\tau')\rangle \tag{16}$$

for all $(\boldsymbol{R},\boldsymbol{R}',\tau,\tau',\boldsymbol{\xi})$. We also have

$$\langle v_i(\boldsymbol{R},\tau)v_{jl}(\boldsymbol{R}',\tau')\rangle = \langle v_i[\boldsymbol{R}+\boldsymbol{X}_c(\tau),\tau]v_{jl}[\boldsymbol{R}'+\boldsymbol{X}_c(\tau'),\tau']\rangle. \tag{17}$$

If we now set $\boldsymbol{R}=\boldsymbol{R}'=\boldsymbol{0}$, $\tau=t$, $\tau'=t'$ and $\boldsymbol{\xi}=\boldsymbol{x}$, we will have $\boldsymbol{X}_c(\tau)=(x_1,x_2-2Atx_1,x_3)$ and $\boldsymbol{X}_c(\tau')=(x_1,x_2-2At'x_1,x_3)$. Therefore $\boldsymbol{X}_c(\tau)$ and $\boldsymbol{X}_c(\tau')$ are equal to $\mathbf{X}$ and $\boldsymbol{X}'$, which are the quantities that enter as arguments in the velocity correlators of Eqs. (10) defining the transport coefficients. Hence, [reading Eqs. (16) and (17) from right to left], we see that

$$\langle v_i(\boldsymbol{X},t)v_j(\boldsymbol{X}',t')\rangle = \langle v_i(\boldsymbol{0},t)v_j(\boldsymbol{0},t')\rangle = R_{ij}(t,t'),$$

$$\langle v_i(\boldsymbol{X},t)v_{jl}(\boldsymbol{X}',t')\rangle = \langle v_i(\boldsymbol{0},t)v_{jl}(\boldsymbol{0},t')\rangle = S_{ijl}(t,t') \tag{18}$$

are independent of space, and are given by the functions $R_{ij}(t,t')$ and $S_{ijl}(t,t')$. Symmetry and incompressiblity imply that $R_{ij}(t,t')=R_{ji}(t',t)$ and $S_{ijj}(t,t')=0$. Using Eqs. (18) in Eqs. (10), we find that the GI transport coefficients

$$\hat{\alpha}_{il}(t,t') = \epsilon_{ijm}[S_{jml}(t,t') - 2At'\delta_{l1}S_{jm2}(t,t')],$$

$$\hat{\beta}_{il}(t,t') = \epsilon_{ij2}[S_{j1l}(t,t') - 2At'\delta_{l1}S_{j12}(t,t')],$$

$$\hat{\eta}_{iml}(t,t') = \epsilon_{ijl}R_{jm}(t,t') \tag{19}$$

are also independent of space.

Galilean invariance is the fundamental reason that the velocity correlators, hence, the transport coefficients, are independent of space. The derivation given above is purely mathematical, relying on the basic freedom of choice of parameters $(\boldsymbol{R},\boldsymbol{R}',\tau,\tau',\boldsymbol{\xi})$, but we can also understand the results more physically. $\boldsymbol{X}$ and $\boldsymbol{X}'$ can be thought of as the location of the origin of a co-moving observer at times t and t', respectively. GI implies that the velocity correlators measured by the co-moving observer at her origin at times t and t' must be equal to the velocity correlators measured by any co-moving observer at her origin at times t and t'. In particular, this must be true for the observer in the laboratory frame, which explains Eqs. (18), consequently Eqs. (19). We can derive an expression for the GI mean EMF by using Eqs. (19) for the transport coefficients in Eq. (6), and simplifying the integrands. We define

$$C_{jml}(t,t') = S_{jml}(t,t') - 2A(t-t')\delta_{m2}S_{j1l}(t,t'),$$

$$D_{jm}(t,t') = R_{jm}(t,t') + 2At'\delta_{m2}R_{j1}(t,t'). \tag{20}$$

Then the mean EMF, $\mathcal{E}(\boldsymbol{x},t)$, can be written compactly as

$$\mathcal{E}_i = \epsilon_{ijm}\int_0^t dt'\,C_{jml}(t,t')H'_l - \int_0^t dt'[\epsilon_{ijl} - 2A(t-t')\delta_{l1}\epsilon_{ij2}]D_{jm}(t,t')H'_{lm}. \tag{21}$$

The mean field Eq. (1) for $\boldsymbol{H}(\boldsymbol{x},t)$ is

$$\frac{\partial H_i}{\partial t} + 2A\,\delta_{i2}H_1 = (\boldsymbol{\nabla}\times\mathcal{E})_i + \eta\boldsymbol{\nabla}^2 H_i, \tag{22}$$

where $(\boldsymbol{\nabla})_p \equiv \partial/\partial X_p = (\partial/\partial x_p + 2At\delta_{p1}\,\partial/\partial x_2)$. We use Eq. (21) to evaluate $(\boldsymbol{\nabla}\times\mathcal{E})_i$ as follows:

$$(\boldsymbol{\nabla}\times\mathcal{E})_i = \int_0^t dt'[C_{iml}-C_{mil}][H'_{lm}+2At\delta_{m1}H'_{l2}] + \int_0^t dt' D_{jm}\{H'_{ijm}+2At\delta_{j1}H'_{i2m} - 2A(t-t')\delta_{i2}[H'_{1jm}+2At\delta_{j1}H'_{12m}]\}. \tag{23}$$

Equations (22) and (23) form a closed set of integrodifferential equations governing the dynamics of the mean field, $\boldsymbol{H}(\boldsymbol{x},t)$, valid for arbitrary values of A. The most visible properties of Eq. (23) for $(\boldsymbol{\nabla}\times\mathcal{E})$ are: (i) Only the part of $C_{iml}(t,t')$ that is antisymmetric in the indices (i,m) contributes. Indeed both S_{iml} and C_{iml} can vanish for nonhelical velocity fluctuations, in which case dynamo action is determined only by the D_{jm} terms. (ii) The $D_{jm}(t,t')$ terms are such that $(\boldsymbol{\nabla}\times\mathcal{E})_i$ involves only H_i for $i=1$ and $i=3$, whereas $(\boldsymbol{\nabla}\times\mathcal{E})_2$ depends on both H_2 and H_1. Together with the mean-field induction Eq. (22) this means that the equations determining the time evolution of H_1 and H_3 are closed. Thus $H_1(\boldsymbol{x},t)$ [or $H_3(\boldsymbol{x},t)$] can be computed by using only the initial data $H_1(\boldsymbol{x},0)$ [or $H_3(\boldsymbol{x},0)$]. The equation for H_2 involves both H_2 and H_1, and can then be solved.

The implications for the original field, $\boldsymbol{B}(\boldsymbol{X},\tau)$, can be read off because it is equal to $\boldsymbol{H}(\boldsymbol{x},t)$ componentwise [i.e., $B_i(\boldsymbol{X},\tau)=H_i(\boldsymbol{x},t)$]. Thus, the $D_{jm}(t,t')$ terms do not couple either B_1 or B_3 with any other components, excepting themselves. In demonstrating this, we have not assumed that either the shear is small, or that $\boldsymbol{H}(\boldsymbol{x},t)$ is such a slow function of time that it can be pulled out the time integrals in Eqs. (21) and (23). Comparing with earlier work (where, essentially, both assumptions have been made) we conclude that *there is no shear-current-assisted dynamo of the form discussed in* Refs. [4,6,7], *where there is explicit coupling of* B_2 and B_1 in the evolution equation for B_1. Our calculations are based on a nonperturbative treatment of shear, and this makes for a basic departure from earlier work which have treated shear perturbatively. Even when the shear is weak, two fluid elements which were close together initially would be separated by arbitrarily large distances at late times. Thus the two-time correlations, which appear naturally in the dynamo problem, have to be handled carefully in the presence of shear. Moreover, the perturbative treatment of shear is not guaranteed to preserve GI, which is a natural and fundamental ingredient of our non perturbative approach.

In conclusion we find that systematic use of the shearing coordinate transformation and the Galilean invariance of a linear shear flow allows us to develop a quasilinear theory of the shear dynamo which, we emphasize, is nonperturbative in the shear parameter. Specifically, we have proved that there is essentially no shear-current-assisted dynamo in the quasilinear limit when FOSA is applicable. Moreover, our results are valid for any GI velocity statistics, independent of the forces (Coriolis, buoyancy etc) governing the dynamics of the velocity field. However, large-scale nonhelical dynamos (i.e., with no initially imposed kinetic helicity) are not ruled out, and further progress requires developing a dynamical theory of velocity correlators in shear flows.

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