Reprinted from "The Proceedings of the Indian Academy of Sciences", Vol. XLIX, No. 6, Sec. A, 1959

CAUSTICS FORMED BY DIFFRACTION AND THE GEOMETRIC THEORY OF DIFFRACTION PATTERNS

BY SIR C. V. RAMAN

(*Memoir No.* 116 *from the Raman Research Institute, Bangalore*-6)

Received May 27, 1959

1. INTRODUCTION

IT has long been known that the diffraction patterns of the *Fraunhofer class* exhibit, in certain simple cases, a recognizable relation to the form of the apertures giving rise to them. For example, the pattern of a triangular aperture resembles a six-rayed star; that of an elliptic aperture exhibits elliptic rings of the same eccentricity, but turned through a right angle, and so on. Investigations[1, 2] by the present author and his collaborators made and published many years ago revealed that geometric relationships which are essentially of the same nature are also exhibited by diffraction patterns of the *Fresnel class*. It was shown in the publications quoted that in the Fresnel patterns of an elliptic aperture there appears a concentration of the diffracted light along the evolute of the cross-section of the emergent pencil. The effect is in the nature of a caustic and indeed exhibits the characters usually associated with caustics. A similar effect is observable in the Fresnel patterns of elliptic discs, and indeed quite generally in the diffraction patterns of apertures or obstacles with undulating boundaries. The origin of the effect was further elucidated by investigations[3, 4, 5] made by the author and his co-workers in which the technique of the Foucault test was employed to determine the characters of the radiation from diffracting boundaries in various circumstances. It was observed that the elements of a diffracting boundary act as sources of secondary radiations which appear with maximum intensity in a plane normal to themselves; from each element, *two streams* of diffracted radiation emerge which are in opposite phases and appear respectively on either side of the direction of incidence of the light on the boundary.

From the facts stated, it follows that it should be possible to develop a theory of diffraction patterns on a purely geometric basis. Indeed, this was done successfully at the time[6] for the case of the Fraunhofer pattern of a semicircular aperture—the so-called heliometer diffraction figures. That it could also be done for *Fresnel patterns* was evident, but the matter was not followed up until many years later. The basic ideas of the theory were expounded in the second chapter of the author's "Lectures on Optics" written out and printed in the year 1943 but not then given out for publication. Basing himself on the ideas set out therein, Y. V. Kathavate[7] worked out in detail the cases of a great many Fresnel diffraction patterns and his results were published in a series of six papers which appeared together in the April 1945 issue of these *Proceedings*. Many excellent photographs were reproduced with his papers which exhibit a highly satisfactory concordance with the results of the geometric theory. Elliptic obstacles and apertures give Fresnel patterns of great beauty and afford perhaps the most striking examples of the power and elegance of the geometric method in treating diffraction problems. Kathavate had indeed dealt with these cases in his fifth paper, but it was felt that this work had not covered them adequately and that it would, therefore, be useful to consider them afresh. That indeed is the object of the present paper.

2. OBSERVATION OF THE PATTERNS

The simple device of holding a circular aperture or disc obliquely in the path of a beam of light enables us to obtain the equivalent of an elliptic aperture or screen of the desired eccentricity. It is necessary, for obtaining the best results, that the metallic sheet carrying the aperture or the metallic disc should be very thin and that the circular edge has been accurately machined and has a smooth polish. The patterns depend very much on the eccentricity; by progressively altering the tilt, the entire sequence ranging from a circular aperture or disc to an elliptic aperture or disc of high eccentricity can be readily studied. Kathavate's paper reproduces a series of photographs obtained in this manner of the Fresnel patterns observed in the region of shadow of discs and in the region of the light transmitted by apertures. His photographs, however, do not show the phenomena outside the region of the transmitted light in the case of elliptic apertures of moderate or large eccentricity. To fill this gap, the series of six photographs reproduced in Plate XIV were recorded in the present investigation. An aperture of 5 mm. diameter in a very thin steel sheet was used. The source of light was a pinhole illuminated by sunlight filtered through a red glass. The source was at a distance of 136 centimetres from the aperture and the divergent beam emerging therefrom was received on a photographic plate held 168 centimetres away from it. The tilt of the aperture was progressively altered, being quite large for Fig. (*a*) and quite small for Fig. (*f*). The exposures given were very short in the case of Figs. (*e*) and (*f*). They were much larger for Figs. (*b*), (*c*) and (*d*), while quite a long exposure was given for Fig. (*a*) recorded with the largest tilt.

Special interest attaches to the study of the transition from the Fresnel to the Fraunhofer class of diffraction pattern in the case of elliptic apertures. Accordingly, the series of eight photographs reproduced in Plate XV was obtained to exhibit this transition. The patterns were recorded with the same aperture of 5 mm. diameter set at the same obliquity in all the cases. The source of light was a fine pinhole illuminated by sunlight filtered through a red glass. A long-focus telescope objective of about 5 feet focal length was used to form a focussed image of the pinhole some 30 feet away from the objective on the other side. The light passed through the objective and then through the aperture and was received on a photographic plate held at an appropriate distance behind the aperture. This distance in the series of eight photographs was 18 inches, 3, 6, 15, 19, 22, 26 and 30 feet respectively.

For the convenience of the reader, two of the plates of Kathavate's paper are reproduced with the present communication as Plates XVI and XVII respectively. These, together with Plate XIV and XV recorded in the present study, form the experimental material which the reader of the paper can compare with the results of the theory.

3. Geometric Theory of the Patterns

The geometric theory of diffraction patterns was expounded in the second chapter of the author's "Lectures on Optics" already referred to. The theory was further discussed in these *Proceedings* by G. N. Ramachandran[8] and also by Kathavate in the first of his series of six papers. Its application to numerous cases by graphical methods was also worked out by Kathavate. It is therefore unnecessary to go over the same ground here. For our present purpose, we commence by considering the simple case of a Fresnel pattern of a circular disc as it illustrates the ideas on which the theory is based very clearly. As is well known, the shadow thrown by a circular disc held in a divergent pencil of light exhibits a central spot surrounded by concentric dark and bright rings. The entire pattern can be deduced if we recognize that the edge of the disc is itself the source of the diffracted radiations which enter the region of the shadow. The radiations from the entire circumference arrive at the centre in the same phase and accordingly give the maximum possible intensity at that point. As we move away from the centre of the pattern in any direction, the elements of the boundary at the two ends of a diameter of the disc become effectively the sources of the radiation reaching the point of observation. The distances traversed by them being different, a phase difference arises and accordingly we would observe interferences. As has been shown by G. N. Ramachandran,[8] the positions of the circular rings around the central bright spot can be quantitatively deduced from these considerations.

With appropriate modifications, arguments of the same general nature enable us to deduce the characters of the diffraction pattern in the shadow of an elliptic disc. This pattern arises from the radiations diffracted by the boundary and entering into the region under consideration. It is useful to have a drawing in which the elliptic boundary of the shadow is shown and a series of normals are drawn to it. We proceed on the basis that each element of the diffracting boundary of the actual screen sends out radiations having the maximum intensity in the plane normal to itself and hence may indeed be regarded as effective only in that plane. Hence, to find the part of the boundary which is operative at any given point of observation on the screen, we draw a normal from that point to the boundary of the shadow. The length of the normal thus drawn is indicative both of the amplitude and the phase of the radiation reaching the point of observation from the corresponding point on the actual boundary. The amplitude decreases rapidly as the length of the normal increases, while the phase alters progressively. By superposing the radiations from the various points on the boundary which are operative at any particular point of observation, we derive the effects observable at that point. Thus finally, the features of the pattern over the entire area can be envisaged.

Two diagrams have been drawn in the manner indicated above. Figure 1 in the text below refers to the case in which the ratio of the major axis to the minor axis of the ellipse is 4: 3. The evolute of the ellipse in this case falls entirely within the ellipse itself. Figure 2 in the text below refers to the case in which the ratio of the major to the minor axis is as 2: 1. In the latter case, only part of the evolute and its cusps situated on the major axis appear within the ellipse, while the rest of the evolute along with the two other cusps lies outside the ellipse on a prolongation of the minor axis. The following features emerge from these diagrams. From any point outside the evolute, only two normals can be drawn to the elliptic boundary. On the other hand, from points within the figure of the evolute four normals can be drawn to the boundary. At any point on the evolute itself, these become coincident in pairs and hence here again, effectively only two normals can be drawn.

Thus from the diagrams we can draw the following general inferences. Within the figure of the evolute we have four sources operative situated at the points on the boundary, the superposition of which will give us a characteristic interference pattern. At points outside the evolute only two sources on the boundary would be operative and these again would give us an interference pattern of a relatively simpler type. The diagrams show, however, that special situations arise at and near the four cusps of the evolute and also

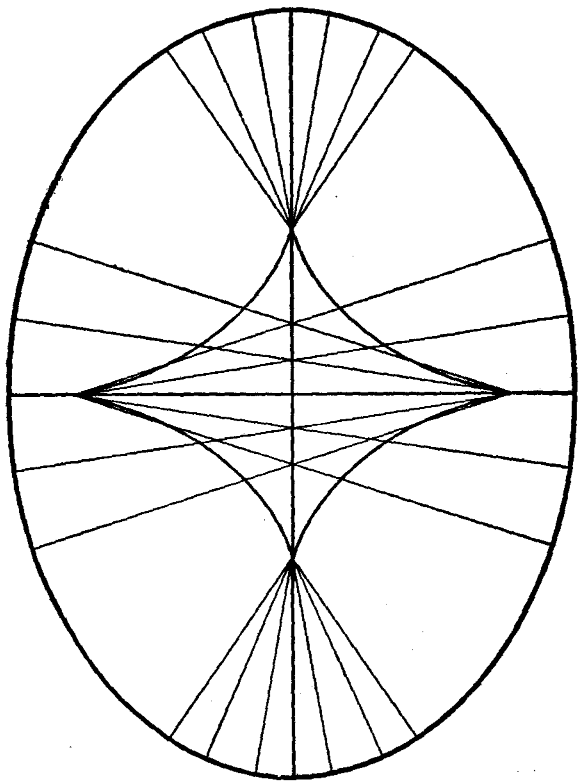


FIG. 1

along the evolute itself. The two cusps on the *major axis,* as can be seen from the diagram, are effectively the foci of the diffracted radiations from a considerable length of the boundary on either side of the terminations of the major axis. It follows that the resultant intensities at these two cusps must be very large. A similar situation would also appear at the two cusps on the *minor axis.* But since the convergence of the diffracted rays is less and the angles of diffraction are larger in the latter case, the intensity of illumination at the cusps on the minor axis would be less pronounced and would also diminish rapidly with increasing ellipticity of the boundary. The evolute being itself the locus of the foci of the diffracted radiations, it would be a region in which the resultant intensity is large. Further, since the radiations reaching their foci at the cusps would diverge again, there would be a noticeable concentration of light on the prolongation of the minor axis beyond the cusps located on it.

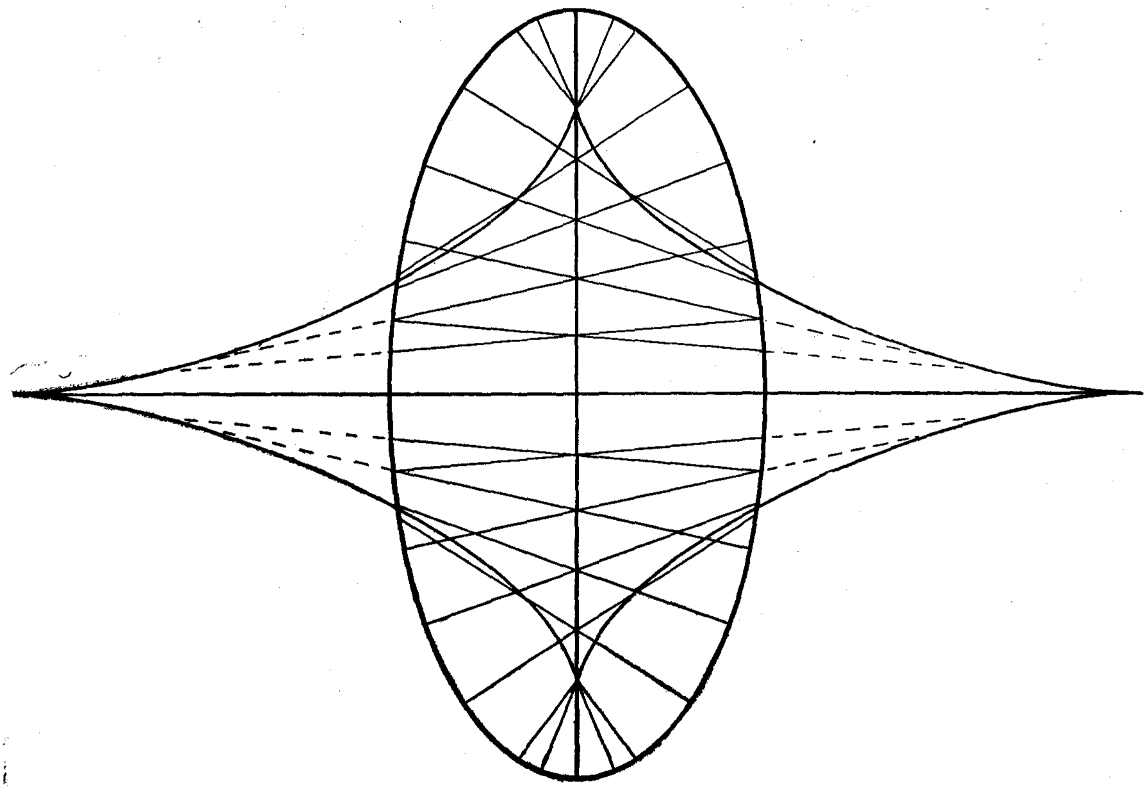


FIG. 2

4. THE FRESNEL PATTERNS OF ELLIPTIC DISCS

We now proceed to consider the nature of the interference patterns observable within the shadow of an elliptic disc. As has already been remarked, these would be of an altogether different nature in the region within the evolute and outside it. Provided the ellipticity of the disc is not too great, the situation in the region outside the evolute would be analogous to that in the shadow of a circular disc; in other words, we would have interference rings which are closed curves appearing in that region. But they would be neither circular nor of uniform intensity. It is easily seen that an interference ring of any given order would cut the minor axis at points further away from the centre of the pattern than its intersections with the major axis. The interference rings would therefore be of approximately elliptic form, their major axis coinciding with the minor axis of the shadow and *vice versa.* They would be most conspicuous at and near the points where they cross the minor axis and scarcely visible where they cut across the major axis, the reason for such difference being that the radiations which interfere would be approximately of equal intensity in the former case and very unequal in the latter case.

We now proceed to consider the interference pattern inside the region of the evolute. At and near the centre of the pattern, four sources of diffracted radiation would be effective which lie on the boundary at or near the ends of the major and minor axes of the ellipse. The radiations from the ends of each axis would arrive at the centre in the same phase, but such phase would be different for the two axes and would also vary as we move away from the centre in either direction. Hence the interference pattern due to the superposition of the effects of the four sources would be an array of spots arranged in regular order along equidistant lines parallel to the major axis of the ellipse, and similarly also on lines parallel to the minor axis. Since the ends of the major axis are further away from each other and from the point of observation than the ends of the minor axis, the array of spots on lines parallel to the major axis would be spaced wider apart and would also be more conspicuous than the arrays on lines parallel to the minor axis. It should also be remarked that since two normals can be drawn to the boundary from any point on the evolute itself and two sources of diffracted radiation are therefore effective thereon, the evolute itself would not appear as a continuous curve of high luminosity but would be cut across by the interference bands parallel to the major axis referred to above.

A few remarks will suffice regarding the case in which the elliptic disc is of large eccentricity. A substantial part of the evolute would then lie outside the shadow and we are not concerned with it here. It is obvious that in these circumstances, the diffraction pattern within the shadow would exhibit prominently a set of interference bands which are for the most part equidistant and parallel to the major axis but which would be splayed out near their ends, as the sources of diffracted radiation giving the interferences then approach each other. The pattern would be crossed by another set of bands parallel to the minor axis, but these would be much closer together and far less conspicuous than the bands parallel to the major axis, since the sources of diffracted radiation giving rise to them are further apart and effectively much weaker in the region under consideration.

The reader may compare the foregoing results with the patterns reproduced in Plate XVI which exhibit the patterns within the shadow of elliptic discs of six different ellipticities.

5. THE FRESNEL PATTERNS OF ELLIPTIC APERTURES

Two distinct regions have to be considered in the discussion of Fresnel patterns of elliptic apertures. These may be designated as the internal and external regions respectively. In the first or internal region the radiations diffracted by the boundary are superposed on the geometric pencil of light emerging through the aperture and the pattern observed is determined by their interferences. The second or external region is the region of shadow which contains only the diffracted radiations from the boundary. The discussions given earlier for the case of an elliptic disc are helpful in dealing with the present problem. Indeed, the results of that discussion may be taken over *mutatis-mutandis* in considering the internal pattern; we have to remember, however, that the phases of the diffracted radiations would all be reversed and that in the final result the interferences with the transmitted pencil of light have also to be taken into account.

The diffraction pattern in the exterior region may be deduced in a simple fashion. If the ellipticity is small or moderate, the pattern would consist of closed interference curves. These would approximate to ellipses in their configuration, their major axis being parallel to the minor axis of the aperture and *vice versa*. Kathavate verified this result experimentally (see Plate XI of his paper). It is evident for reasons already explained that the visibility of the rings would be much greater in the vicinity of the minor axis than of the major axis.

If the ellipticity be considerable, the figure of the evolute and its prolongation beyond the two cusps on the minor axis would extend into the region of the shadow. This extension would be then the most noticeable feature of the external diffraction pattern, since the intensity would be greatest along the figure of the evolute. This is evident in Figs. (*a*), (*b*), (*c*) and (*d*) in Plate XIV. These also exhibit other features which we shall proceed to consider. We may remark in the first place that a few interference fringes are to be noticed alongside the evolute running parallel to it. This is a general feature of all caustics and does not therefore need special explanation. In the region enclosed by the evolute and also cutting across it at regular intervals, a regular sequence of bands appears, these being transverse to the minor axis of the ellipse and exhibiting a marked curvature. They arise from the superposition of the diffracted radiations from the two sides of the ellipse and are conspicuous because the interfering radiations are of comparable intensity. The narrowing of the ellipse as we move from its centre towards the ends causes the curvature of the bands. They are most intense at small angles of diffraction and fade away as we proceed towards the cusps of the evolute. Finally, it may be remarked that outside the figure of the evolute and cutting across its prolongation on the minor axis beyond the cusps, the elliptic rings referred to above should also manifest themselves, though only very feebly.

We have now to consider the nature of the internal patterns in which the radiations diffracted by the boundary and the incident pencil of light as it emerges from the aperture are superposed. It is evident that the radiations diffracted through small angles by the elements of the boundary and interfering with the primary beam would give rise to fringes forming closed curves adjacent to the boundary and running parallel to it. Further inside the patterns, however, the disposition of the interferences would be profoundly altered, firstly by the convergence of the diffracted radiation to foci at the four cusps of the evolute, and elsewhere by the superposition of the diffracted radiations from two or even four elements of the boundary at particular points within the patterns. Since at the two cusps on the major axis the intensity of the diffracted radiation is large and indeed comparable with that of the primary beam, the interferences with the latter at and near these foci should be very conspicuous; a disposition of the fringes in their vicinity along arcs of circles centred around them is also to be expected. Then again, the superposition of the radiations from opposite sides of the elliptic boundary would result in cancelling out the influence of the curvature of that boundary on the disposition of the interferences within the pattern. Such an effect should be particularly conspicuous in respect of the parts of the boundary which run nearly parallel to the major axis of the ellipse. Thus, we should expect the fringes of the usual type running parallel to the elliptic boundary to be replaced by a series of straight bands parallel to the major axis, their number and disposition being determined by the ellipticity and other relevant circumstances. Since the effect of four sources is superposed on the primary beam at and near the centre of the pattern, the bands parallel to the major axis would in that region be crossed by another set of bands parallel to the minor axis which would be less conspicuous and also more closely spaced.

The foregoing results are very clearly illustrated by the patterns reproduced as Figs. (*e*) and (*f*) in Plate XIV and by the array of fifteen pictures in Plate XVII taken from Kathavate's paper. The latter were recorded by him with elliptic apertures of different sizes and eccentricities. They were obtained by making three sharp circular holes in a metal sheet, with radii respectively equal to 6·9 mm., 5·3 mm. and 4·8 mm. By tilting the sheet with respect to the incident beam, three diffraction patterns could be photographed simultaneously. Thus with five different tilts, fifteen photographs in all were obtained. In the reproduction, all the fifteen figures were enlarged to nearly the same size.

6. Transition from the Fresnel to the Fraunhofer Patterns

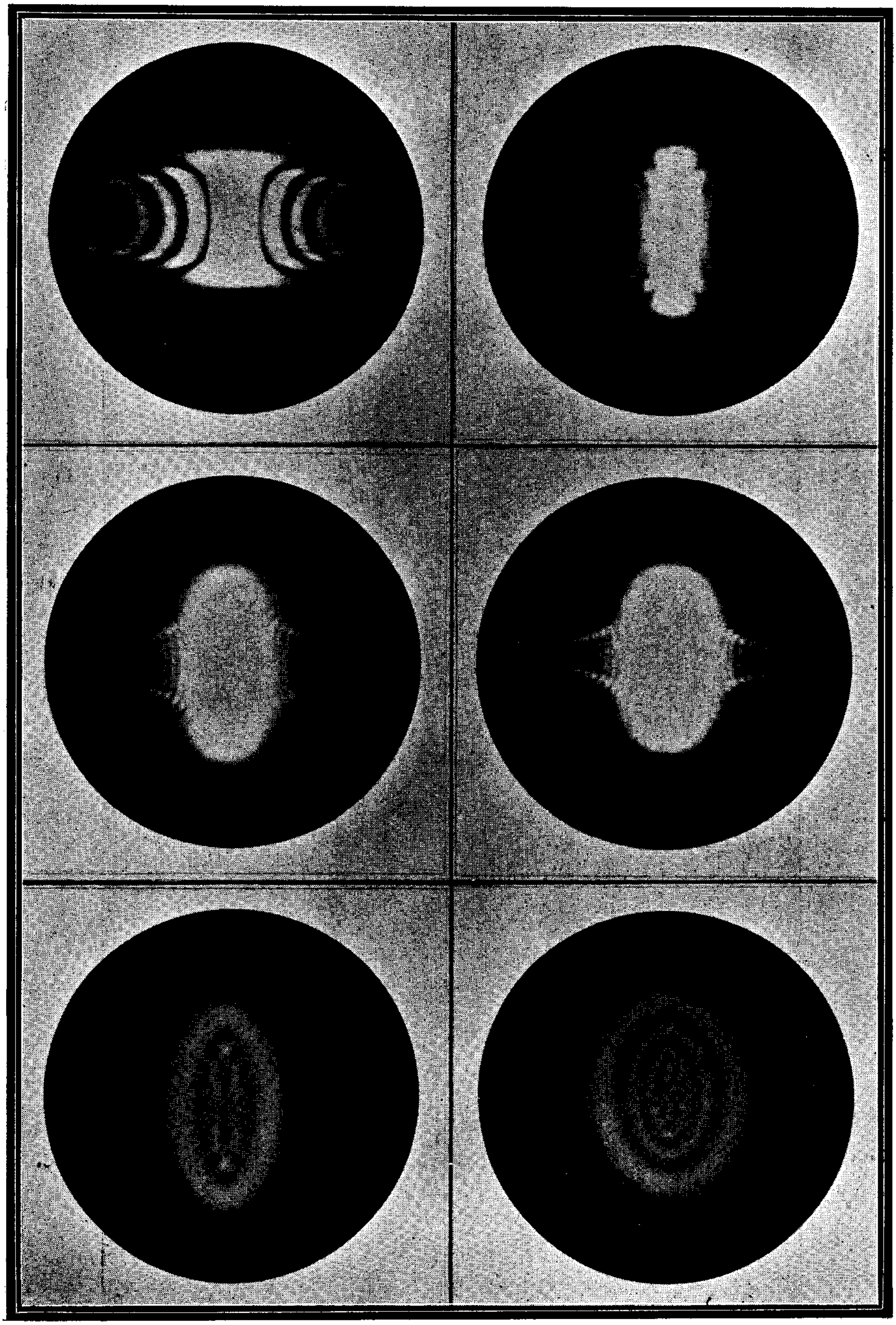
The Fresnel pattern of an elliptic aperture observed in its immediate vicinity is an elliptic patch of light orientated in the same manner as the aperture itself. The Fraunhofer pattern observed at the focus of the lens which concentrates the light passing through the aperture is also an elliptic patch of light, but turned round through a right angle and surrounded by elliptic rings of the same shape and orientation. The successive stages of this transformation are illustrated by the series of eight photographs reproduced in Plate XV. The features seen in the photographs are readily understood in terms of the preceding discussion of the nature of the Fresnel patterns themselves. As we proceed from the aperture towards the focus of the lens, the cross-section of the geometric pencil diminishes progressively, and what has been referred to as the interior pattern contracts to a point and ultimately vanishes. What survives is the exterior pattern and it is the progressive alteration in the nature of that pattern that is represented in the series of photographs. In Fig. 2 of Plate XV we see the evolute and its accompanying fringes outside the elliptic patch of light. An important role is played by the very faint extension of this evolute beyond its cusps. A brush of light extends outwards along the minor axis; as can be seen from the series of photographs, this is interrupted by interferences crossing it, due to their intersection with the elliptic rings which are a characteristic feature of the exterior patterns; these are however much too feeble to give any other evidence of their presence in the first four photographs of the sequence. They are barely visible in the fifth photograph, but are much better seen in the sixth. The seventh and the eighth photographs show the final transformation of the brush into a complete set of elliptic rings.

7. Summary

The Fresnel patterns of elliptic discs and apertures afford a striking example of the power of the geometric method in treating the theory of diffraction patterns. They are discussed in detail and a series of photographs exhibiting the features indicated by the theory are reproduced.

8. References

1. Raman, C. V. .. *Phys. Rev.*, 1919, **13**, 259.
2. Mitra, S. K. .. *Phil. Mag.*, 1919, **38**, 289.
3. Raman, C. V. and Ghosh, P. N. *Nature*, 1918, **102**, 205.



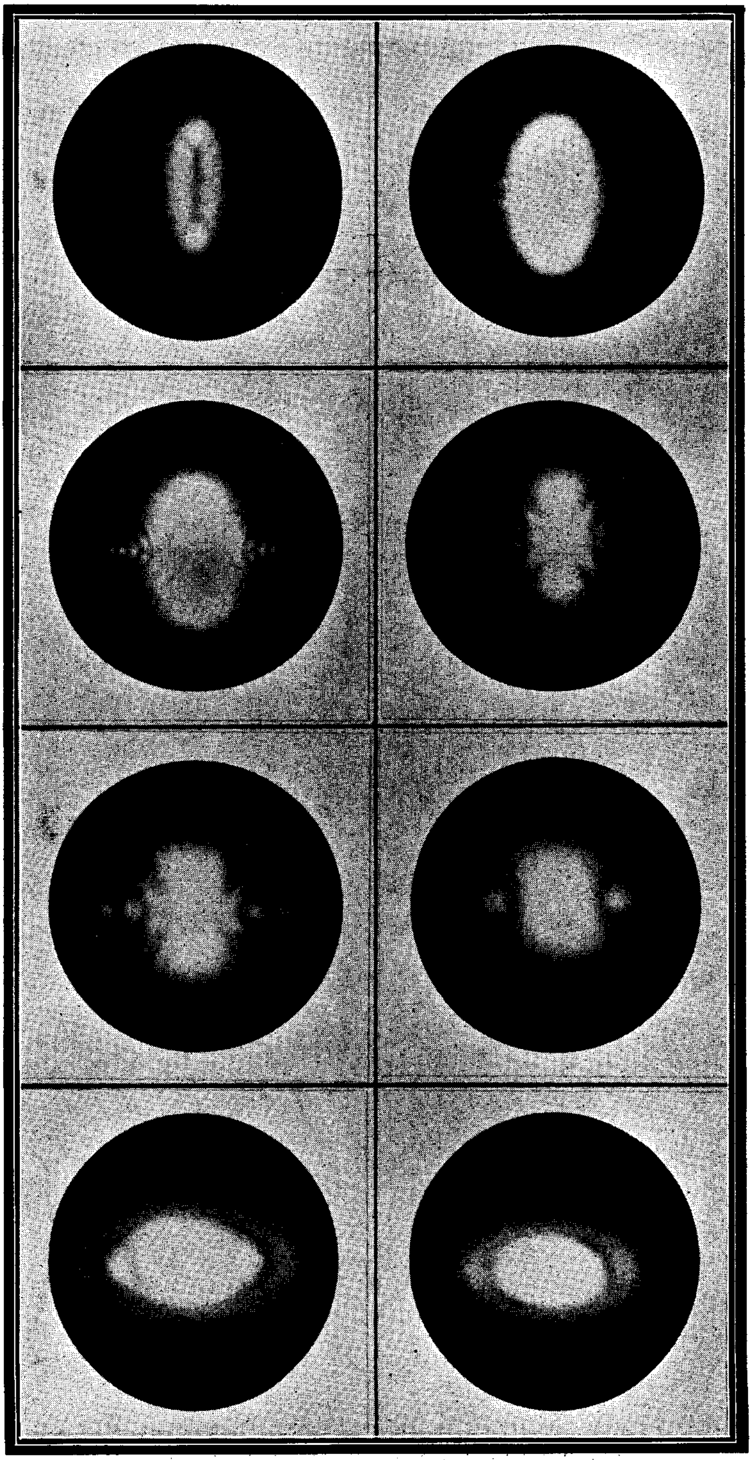
Fresnel Diffraction Patterns of Elliptic Apertures.

FIG. 1 FIG. 2

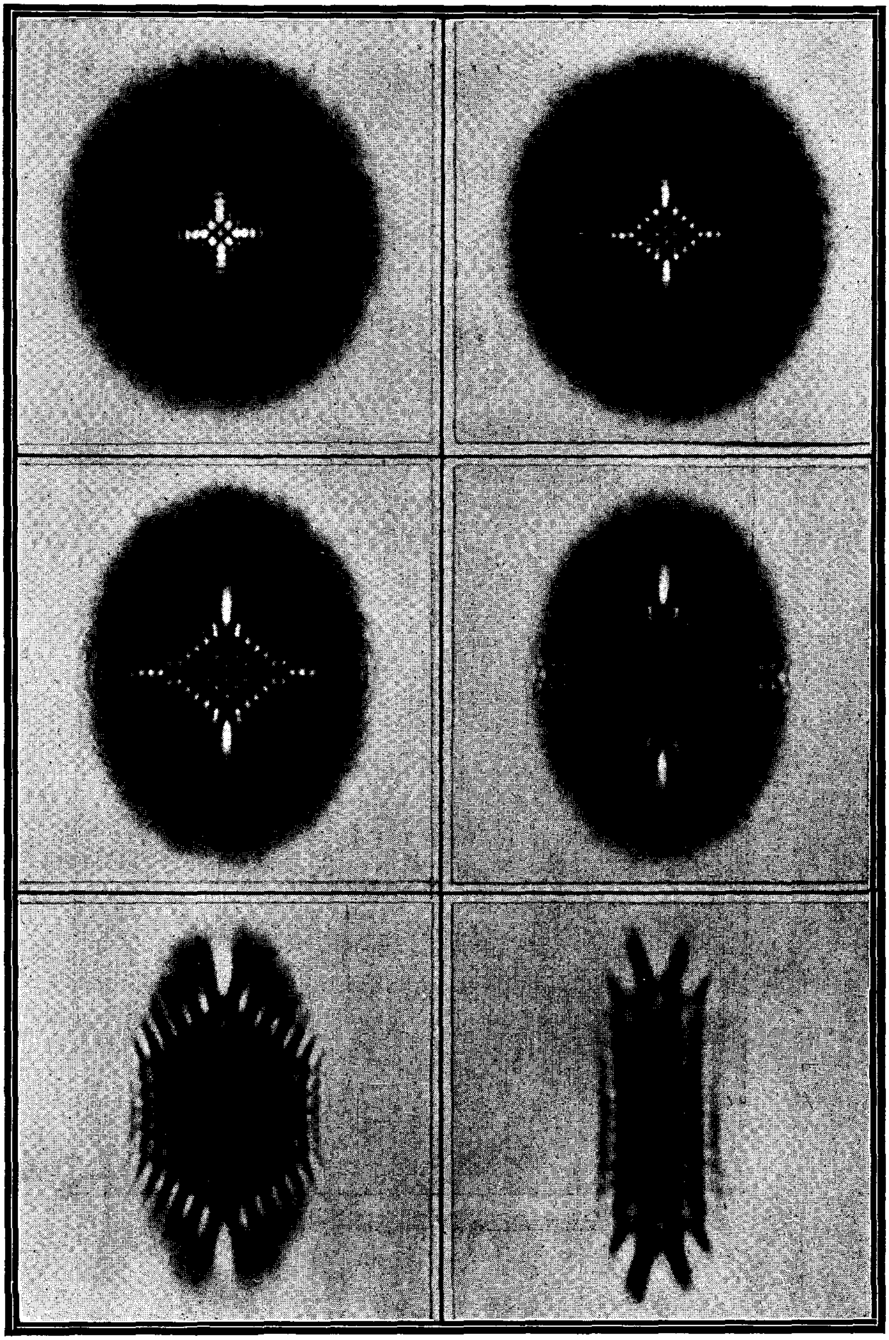
FIG. 3 FIG. 4

FIG. 5 FIG. 6

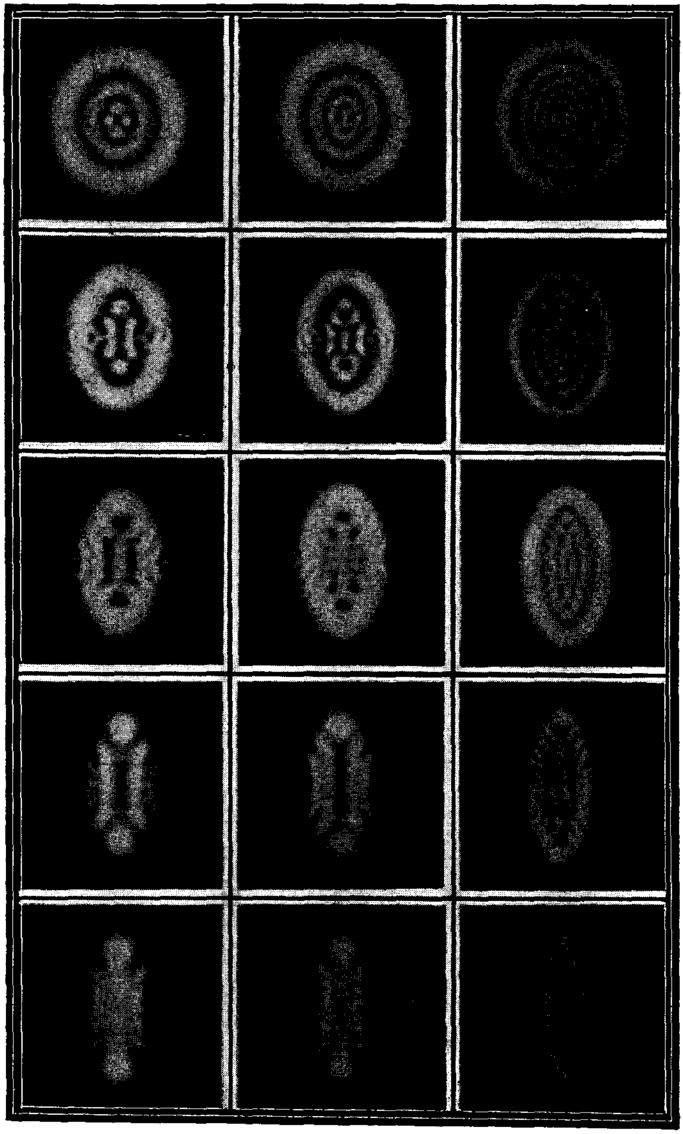
FIG. 7 FIG. 8



Transition from Fresnel to Fraunhofer Patterns.



Pattern in the Shadow of Elliptic Discs.



Transmission by Elliptic Apertures.

4. Ghosh, P. N. .. *Proc. Roy. Soc.*, 1919, **96 A**, 257.
5. Banerji, S. K. .. *Phil. Mag.*, 1919, **37**, 112.
6. Mitra, S. K. .. *Proc. Ind. Assocn. Cult. Sci.*, 1920, **6**, 1.
7. Kathavate, Y. V. .. *Proc. Ind. Acad. Sci.*, 1945, **21 A**, 177.
8. Ramachandran, G. N. .. *Ibid.*, 1945, **21**, 165.