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THE PROPAGATION OF LIGHT IN ABSORBING BIAXIAL CRYSTALS

II. Experimental

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§ 1. INTRODUCTION

THE mineral iolite available in South India shows a very pronounced pleochroism: certain specimens of this material appear practically colourless and transparent in certain orientations, but show an intense blue colour in certain others. Accordingly, a plate cut normal to one of the two optic axes of this orthorhombic mineral was found very suitable for qualitatively verifying some of the rather unexpected results regarding the properties of the singular axes in absorbing crystals that had been derived in the theoretical treatment of Part I. As the plate also exhibited in a striking manner the other well-known phenomena characteristic of absorbing biaxial crystals, it was thought worthwhile to publish illustrations of these phenomena (together with a qualitative verification of some of the more detailed aspects of the theory that had hitherto not been convincingly demonstrated). Broadly speaking, sections 2–4 do not cover original ground, and the phenomena they deal with are also explained, for example, in Pockels' *Lehrbuch.*[1]

In Part I of this paper[2] it was shown that the optical behaviour of absorbing crystals not possessing optical activity could be regarded as due to the effects of linear birefringence and linear dichroism superposed continuously along the depth of the medium.* The peculiar features in biaxial crystals arise because here the principal planes for the usual operation of birefringence do not coincide with those for the operation of dichroism. In spite of the simplification which such a method of consideration represents over the more rigorous electromagnetic theory, the features of propagation in an absorbing biaxial medium are still somewhat complex. Therefore, in explaining the experimental phenomena involved we shall start by assuming the elementary theory of Mallard; and we shall broadly indicate in §§ 2–4, how the experimental results themselves point out the need for a more refined

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* See Note at the end of the paper for references to some earlier work.[3]

theory, as well as to some of the results of such a theory. (The theoretical treatment in many standard texts[4–7] finally amount in practice to the assumptions of Mallard's theory—except for a slight change in the form of the absorption ellipsoid which may be neglected for directions near an optic axis.)

The photographs (Figs. 2–15) illustrated in this paper were all taken with sodium light illumination in convergent light; the simplest method of observing the phenomena which they reproduce is by holding the plate close to the eye and looking towards an extended source of light through the plate —there being arrangements for introducing auxiliary appliances (such as polaroids and quarter-wave plates) both in front of and behind the crystal. Each point in the convergent light figure obviously corresponds to a definite direction of propagation: to visualise this we may represent all directions as passing through the centre of a sphere and defined by their intersection with the spherical surface. The region surrounding an optic axis may then be approximated by a plane (the plane of the paper in Fig. 1), the central point O in the convergent light figure corresponding to the optic axis, and the horizontal line $X_k X_k'$ to the axial plane. (Since the axial angle is not small, the second optic axis does not appear in the figure.)

§ 2. PROPERTY OF THE OPTIC AXES IN ABSORBING CRYSTALS

When a plate of an absorbing biaxial crystal cut normal to an optic axis is examined in convergent light between crossed polaroids, the optic axial direction does not in general appear extinguished (as in transparent crystals) but shows two extinction positions as the crossed polaroids are rotated round together—its behaviour in this respect being similar to that of a *non*-axial direction in a transparent specimen. In the case of an orthorhombic crystal, these extinction positions occur when the vibration-direction of the polariser lies either along or perpendicular to the axial plane; the appearance of the figure which is the same in both cases, is given in Fig. 2, the dark isogyre lying along the axial plane as in transparent crystals. On the other hand Fig. 3 shows that for an intermediate position where the vibration-directions of the crossed polaroids make angles of 45° with the axial plane, the optic axial direction is not extinguished by a vertical isogyre.

Along the optic axis, therefore, two linearly polarised vibrations are propagated. If we take the elliptic section of the absorption ellipsoid normal to the optic axis, these two waves are linearly polarised along the principal radii of this section—which lie parallel to OX_k and OY_k in the case of the orthorhombic crystal iolite. The characteristic feature distinguishing these two waves is not any difference in their refractive indices, but the fact that they have different coefficients of extinction k_1 and k_2, where $1/\sqrt{k_1}$ and

$1/\sqrt{k_2}$ are the principal radii of the absorption ellipsoid lying along OX_k and OY_k respectively; the absorption for the vertical vibration-direction is much greater than that for the horizontal vibration-direction in the case of iolite

The existence of dichroism along the optic axial direction is sometimes[4, 5] expressed by the statement that the absorption of a wave travelling along an optic axis depends on its plane of polarisation. This statement is however somewhat misleading for if a wave incident in the direction of an optic axis has its vibration-direction inclined at an arbitrary angle to the axial plane, the disturbance is not propagated with a single specific coefficient of absorption; on the other hand, it is split up into two waves with different coefficients of absorption polarised along and perpendicular to the axial plane, and the vibration-direction of the wave emerging from the plate will not be the same as that of the incident wave but would have turned towards the less absorbed component—towards the axial plane in the case of iolite. This is the reason why the optic axial direction does not in general appear extinguished between crossed polaroids. According to this explanation the optic axial direction can be extinguished by rotating the analyser away from the crossed position, and this has been done in Fig. 4. The extinction would not have been complete if there had been any (sensible) difference in the refractive indices of the waves.

An estimate of the dichroism along the axial direction will be usefu later, and this can be made if, in Fig. 4, the inclinations ψ and ψ' that thel incident and emergent vibrations, make with the axial plane are known. These angles were estimated to be roughly (90° – 12½°) and 12½° respectively. From equation (1) of Part I we then get $kz = 3$ approximately, where k is the difference in the coefficients of absorption and z the thickness.

[The azimuths of the polariser and analyser had not been directly noted during the experiment. Hence ψ and ψ' were indirectly determined by noting that the two diameters along which the ring system is absent in Fig. 4, are inclined at roughly 25° to the axial plane. We then use the fact that these diameters represent directions where one of the principal planes of refraction coincide with the vibration-direction either of the polariser or of the analyser. (Since the diameters where this occurs are not highly inclined to the axial plane, the waves here may be regarded as linearly polarised.)]

§ 3. BREWSTER'S BRUSHES

With neither polariser or analyser introduced, the convergent light figure reveals the two dark Brewster's absorption brushes which appear in the plane perpendicular to the axial plane (Fig. 5).

The broad features of this phenomena are explicable on Mallard's theory. In Fig. 1 in the text, each double-headed arrow at the border gives the vibration-direction of the faster wave for all points on the corresponding radius of the field of view (assuming Mallard's theory). Now the absorption coefficients for the two vibrations propagated in any direction are determined by the intercepts that these vibrations make on an absorption ellipsoid; and though the orientations of these vibrations remain constant along any one diameter, they turn round rapidly with the diameter—giving rise to the Brewster's brushes.

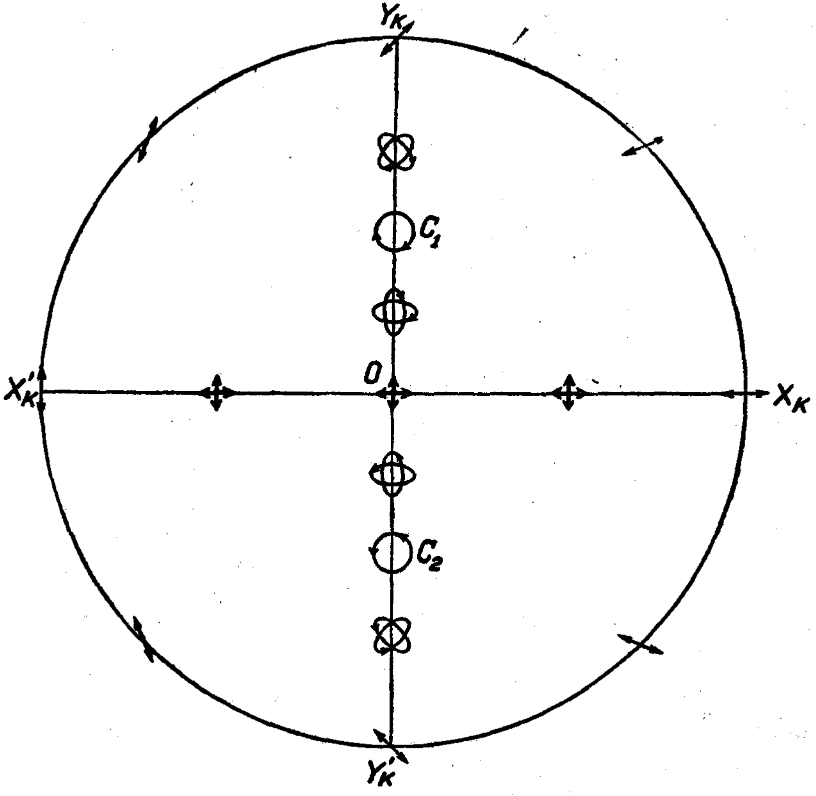

FIG. 1

X_k, X_k' = axial plane; O = optic axis; C_1, C_2 = singular axes.

The elliptical section of the absorption ellipsoid made by the plane of the paper in Fig. 1, may also be taken as the section normal to any direction of propagation in the small angular range covered by the convergent light figure. The mean of the two absorption coefficients may then be considered a constant for all directions in the angular range under consideration.

(On Mallard's theory this follows from a property of any two perpendicular radii of an ellipse, and in the more exact theory by the equations 15 of Part I.) Under this condition it may be easily shown that the sum of the intensities of the two waves emerging from the plate will be *minimum* along those directions where the two waves (into which the incident unpolarised light is split) are absorbed equally strongly; and that the sum of the intensities of these waves will be *maximum* along directions where the two waves differ in their absorption coefficients to the maximum extent.

Now the dichroism will be greatest where the waves are linearly polarised along the principal planes of absorption OX_k and OY_k—a situation which occurs for directions along the axial plane in the case of an orthorhombic crystal like iolite. For directions in the perpendicular plane Y_kY_k' the principal planes of refraction are inclined at 45° to the principal planes of absorption; hence the dichroism should be zero for all points on Y_kY_k' (the optic axial direction O being excepted in the case of Mallard's theory, the small strip C_1C_2 being excepted according to the exact theory). This explains the occurrence of the Brewster's brushes in the plane perpendicular to the axial plane.

§ 4. PHENOMENA NOT EXPLICABLE ON THE ELEMENTARY THEORY

(*a*) *The elliptical polarisation of the waves.*—Referring to Fig. 3, we note that when the plate is viewed between crossed polaroids set in the diagonal position it is not the optic axial direction *alone* that remains unextinguished; the region of non-extinction extends over a finite vertical strip passing through the optic axis, and, in fact, the extinction along the vertical ' isogyre ' becomes perfect only at the boundary of the figure.

An important inference may be drawn from this phenomenon. Along the optic axial direction (where the birefringence is zero) the two waves propagated are linearly polarised along the principal planes of absorption (*i.e.*, along the principal radii of the normal section of the absorption ellipsoid); for directions appreciably inclined to the optic axis (where the birefringence becomes notable) the waves may be regarded as linearly polarised along the principal planes of refraction. But the transition between these two extreme stages (*i.e.*, from waves polarised along the principal planes of absorption to waves polarised along the principal planes of refraction) occurs in some continuous fashion—not discontinuously, as soon as the optic axial direction is departed from, as is required by Mallard's theory. This is also to be expected on the simple ground that the phenomena that are physically observable along the optic axial direction are not really explained by postulating a discontinuous property for the exact axial direction, since we are always concerned with pencils of finite (even if small) convergence. For directions along the axial plane the principal planes of refraction and absorption coincide so that the horizontal isogyre in Fig. 2 is dark and continuous.

Further, those points in the immediate vicinity of the optic axis O, which lie along the vertical line Y_kY_k', cannot be extinguished even on rotating the crossed polaroids together, away from the diagonal setting—which means that the waves here must be *elliptically* polarised. This property resembles that of transparent optically active birefringent crystals where two extinction positions cannot be obtained between crossed polaroids.

(*b*) *Phenomena with polariser or analyser alone.*—For any direction along the axial plane the two waves are polarised along and perpendicular to the axial plane; the two waves have respectively the least and greatest absorption coefficients of all the waves travelling along any direction in the field of view. Hence if a polariser be set with its vibration-direction perpendicular to the axial plane, a dark brush appears along the axial plane, as in Fig. 9; while if it is set along the axial plane, a white brush appears in the same position as in Fig. 6. These two photographs incidentally demonstrate the existence of pleochroism along the axial direction.

The most interesting feature about Figs. 6 and 9, is, however, the occurrence of the idiophanic interference rings, which appear conspicuous in the plane perpendicular to the axial plane. As the polariser is turned from its position along the axial plane in Fig. 6 to that perpendicular to the axial plane in Fig. 9, the figure changes in a manner which appears to depend on the absorption of the specimen. In the present case, the diameter along which the rings are most conspicuous first turns with the polariser (Fig. 7); next a dark brush gets detached from this diameter and turns round towards the axial plane, while the ring system again turns back towards a position perpendicular to the axial plane (Fig. 8).

The occurrence of idiophanic interference rings cannot be expected if the states of polarisation propagated along any general direction are two orthogonal linear vibrations (as assumed in Mallard's theory), or alternatively two orthogonal elliptic vibrations—as occurs in transparent optically active crystals. For even when two such beams are completely coherent (a condition automatically ensured by polarising the incident light), they will be incapable of interference with one another—unless brought to the same state of vibration by the use of a suitable analyser. The two vibrations propagated along any general direction may therefore be assumed to be two *non-orthogonal* elliptic vibrations, propagated with different velocities and absorption coefficients. (Two elliptic vibrations having their major axes crossed and their ellipticities equal, and which are described in opposite senses are said to be orthogonal; any two vibrations not satisfying this condition are non-orthogonal.)

Figure 10 shows the photograph obtained by using an analyser alone, set in the same position as the polariser in Fig. 8, the exposures in the two cases being identical. The two photographs appear identical. It is possible to deduce from this that the non-orthogonal vibrations propagated along any general direction must be of the form of two similarly rotating elliptic vibrations having their major axes crossed and their ellipticities equal. If the non-orthogonal vibrations were of other forms (as for example, two linear vibrations inclined at an angle different from 90°) the observed identity of the effects obtained with a polariser alone and an analyser alone could not, in general, be expected.†

Along the axial plane X_kX_k', the principal planes of refraction and absorption are coincident, so that the vibrations for directions of propagation in this plane are linear and orthogonal, as indicated in Fig. 1. This explains the absence of the idiophanic rings along the axial plane in Figs. 6–9.

As we proceed outwards from the optic axis along the radius OY_k, the change in the states of polarisation of the two waves may be easily followed on the Poincaré sphere by applying the principle of superposition developed in Part I. For these directions the principal planes of refraction and absorption have their maximum mutual inclination of 45°, and effects connected with the ellipticity may be expected to be most pronounced. Referring to Fig. 3 in Part I of this paper, as the linear birefringence increases from zero, the two states of polarisation (initially coincident with the principal planes of absorption X_k and Y_k respectively) will move downwards along the arcs X_kC_r and Y_kC_r respectively till both states become coincident with the pole C_r when the linear birefringence δ becomes equal to the linear dichroism k; as the birefringence further increases, the two states again become distinct and move upwards along the meridians of longitude C_rX_r and C_rY_r respectively.

Physically this means that as we proceed outwards along the radius OY_k, the two vibrations (initially polarised along the principal planes of absorption) open out into two right elliptic vibrations which become two identical circular vibrations at C_1; further on these split up into two elliptic vibrations with major axes along the principal planes of refraction, these elliptic vibrations tending to the form of two orthogonal linear vibrations at the

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† This will be proved in a later paper dealing with absorbing crystals which are also optically active, where the effects obtained with polariser and analyser are different,

border of the field of view. For directions on the other side of the axial plane the vibrations are left-elliptic. The velocities of the waves are equal for all directions lying within the small vertical strip C_1C_2 (as may be seen by setting $2\phi = \pi/2$ in Equation 14, Part I). But for all other directions in the vertical plane Y_kY_k', the absorption coefficients of the waves are equal, the velocities being, in general, different (as may be seen by setting $2\psi = \pi/2$ in Equation 15, Part I).

For any given angular distance from the axial direction, the ellipticity of the waves (and hence their departure from orthogonality) is maximum for directions lying in the vertical plane Y_kY_k'. Consequently the idiophanic rings may be expected to be most prominently seen, if the polariser is adjusted such that the ring system lies perpendicular to the axial plane as in Figs. 6 and 9. (This effect does not appear to be very pronounced.) The considerations of the previous paragraph show that the rings will appear in the vertical plane when the polariser is set along or perpendicular to the axial plane; for the two waves propagated along any direction in the vertical plane Y_kY_k' (excluding the small strip C_1C_2) will then have equal amplitudes and will consequently be capable of showing the maximum interference effects. (The result of this paragraph has been proved by means of a more exact mathematical treatment in Pockels.)

(c) *Interference effects in Brewster's brushes.*—Figure 5 shows distinct traces of periodic maxima and minima in the Brewster's brushes, in the region near the optic axis. The occurrence of such interference effects may be broadly expected on the following grounds. When unpolarised light is split into two orthogonally polarised vibrations, these vibrations will be completely incoherent; but when the incident unpolarised light is being split into two *non-orthogonally* polarised vibrations, as in the present case, the two vibrations must necessarily be considered as partially coherent—for if they were incoherent it can be shown that the resultant beam would be partially polarised and not unpolarised (see *e.g.*, reference 8). Since the beams are partially coherent (to an extent which depends on their departure from orthogonality), they will be capable of restricted interference with one another (to an extent which depends again on their departure from orthogonality). The interference effects are therefore very feeble, being noticeable only in the immediate vicinity of the singular axes—unlike the idiophanic rings which are fairly prominent because complete coherence of the beams is ensured by polarising the incident light. It can be shown[‡] that the maxima

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[‡] This will be shown incidentally in a later paper dealing with absorbing crystals possessing optical activity.

and minima in the Brewster's brushes must occur in the same position as those in the idiophanic rings obtained by setting the polariser along the axial plane, and this is verified by comparison with Fig. 6.

The explanation of the Brewster's brushes given in §2 is however still applicable. For though the intensity emerging along any direction is not merely equal to the sum of the intensities of the two waves emerging in that direction (because of an additional term expressing their interference), it is this sum that broadly locates the position of the brushes.

From the standpoint of the method of superposition we may also regard the interference effects as arising because the crystal itself acts both as polariser and analyser—and a similar explanation could also be given for the idiophanic rings.

§ 5. THE SINGULAR AXES

(*a*) *Examination between circular polariser and crossed circular analyser.*—The point C_1 represents a direction where only a right-circular vibration can be propagated unchanged; while the point C_2 represents a direction where only a left-circular vibration can be propagated without change of form. These two directions are termed the Windungsachen or singular axes. We shall now give a direct confirmation of the existence of these axes; the only previous experimental work dealing directly with the property of these axes was, as we shall see, not interpreted in a correct manner.

In front of the crystal plate is kept a 'left-circular polariser' (*i.e.*, an arrangement which transmits left-circularly polarised light when unpolarised light is incident on it). Behind the crystal is kept a 'right-circular analyser' (*i.e.*, an arrangement which completely cuts off left-circularly polarised light). Under such conditions a transparent crystal will show a system of circular rings with a black spot at the optic axis, where the incident circularly polarised light can be propagated unchanged. In the present case the dark spot occurs slightly to one side of the optic axis (Fig. 11), corresponding to the direction of the lower singular axis C_2. (This is seen better in Fig. 16, which was taken with another plate of iolite having less total absorption.) When the crystal plate is viewed between a right-circular polariser and a crossed circular analyser, the other singular axis C_1 appears extinguished (Fig. 12). The angle between the singular axes is seen to be very small, and has been exaggerated in Fig. 1.

(A quarter-wave plate preceded by a polaroid at 45° to the principal planes, together constitute a convenient circular polariser; a quarter-wave plate followed by a polaroid at 45° to the principal planes, similarly constitute a circular analyser.)

In Part I, Section 4 *c*, it was shown that the vibrations propagated in any direction are right- or left-elliptic according as the principal plane of refraction OX_r (corresponding to the slower wave, in the absence of absorption) makes a negative or positive acute angle with the principal plane of absorption OX_k (corresponding to the less absorbed component in the absence of birefringence). Accordingly in Fig. 1, the sense of description of the elliptic vibrations are correct only if we assume that the arrows at the border give the *faster* vibration-direction (for all points on the corresponding radius) in the absence of absorption. That this assumption was indeed correct, was verified by using the fact that at the border of the field of view the refractive indices and states of polarisation are substantially the same as in the absence of absorption. The polariser and analyser were set along and perpendicular to the axial plane respectively, as in Fig. 2, and a quarter-wave plate was introduced immediately behind the crystal plate with its slow vibration-direction making an angle of + 45° with respect to OX_k. The rings along OY_k moved outwards indicating a diminution of the phase retardation for such directions, while those appearing on the line OY_k' moved towards the centre. This confirmed that the right- and left-circular singular axes occur respectively on the sides of the axial plane that are to be expected according to theory.

§ 6. EFFECTS WITH INCIDENT CIRCULARLY POLARISED LIGHT

When the incident light is circularly polarised it is propagated without change of form along one of the singular axes. In Part I it was shown that for the direction corresponding to the other singular axis, the emergent intensity will not be either zero or negligible (as had been thought by Voigt[1]), but should in fact be greater than the intensity emerging in the direction of the first singular axis. This was confirmed in the following manner. When examined between a left-circular polariser and a crossed circular analyser, we have seen that the singular axis C_2 where the incident vibration is propagated without change of form appears as a dark spot (Fig. 11). If now the circular analyser is removed, the same singular axis appears darker than the other singular axis (which appears within a bright spot), as may be seen in Fig. 13.

Similarly Fig. 14 shows the appearance presented when the incident light is right-circularly polarised. It will be seen by comparison with Fig. 12, that the upper singular axis C_1 where the incident vibration is propagated without change of form appears the darker of the two. Along the other singular axis C_2 the state of vibration alters progressively along the depth of the medium, according to the theoretical analysis of Part I. The emergent vibration in this direction will be elliptically polarised, the major axis of the vibration lying along OX_k (which is the principal plane corresponding to the less absorbed component). Hence if a quarter-wave plate is introduced behind the crystal plate with its principal planes along OX_k and OY_k, the elliptic vibration emerging in the direction of the lower singular axis C_2 will be reduced to a linear vibration at an angle θ to the axial plane—where $\tan \theta$ is the ratio of the minor to the major axis of the elliptic vibration. Thus by introducing a polaroid behind the quarter-wave plate it should be possible to extinguish the singular axis C_2. To verify this, the point C_2 was marked with an ink spot on the ground glass of the camera, after having determined its exact location by the position of the black spot in the arrangement of Fig. 11. By rotating the polaroid it was found that a point of extinction lying on the vertical line Y_kY_k' could be taken continuously *through* the singular axis C_2 (see next paragraph). Figure 15 shows the photograph taken when C_2 has been extinguished in this manner. The angle θ was roughly estimated at 25°; this is roughly the magnitude to be expected since from Equation 18 of Part I we should have: $\tan(\pi/4 + \theta) = kz$, where kz has already been estimated to have a value near 3 (refer § 2).

More generally (with the incident vibration right circularly polarized, as before) the elliptic vibration emerging along any direction on the vertical plane Y_kY_k' will have its principal axes along OX_k and OY_k. This is a consequence of the fact that for any such direction the principal planes of refraction and absorption are inclined at 45° to one another—the corresponding diameters X_rY_r and X_kY_k on the Poincaré sphere being at right angles to one another as in Fig. 3 of Part I; in that figure, if C_r represents the incident vibration, the movements ds_r and ds_k will always lie along the same line-element, so that the emergent vibration will necessarily be represented by some point on the arc $C_rX_kC_lC_r$. Hence if a quarter-wave plate is introduced with its principal planes along OX_k and OY_k (as described in the last paragraph) the vibrations emerging along all the directions in the vertical plane Y_kY_k' will be reduced to linear vibrations (with various azimuths of polarisation). The polaroid behind this quarter-wave plate was set at 45° to the principal planes in such a position that the upper singular axis C_1 where the incident vibration is propagated unchanged appeared extinguished. By rotating the polaroid away from this position in the proper direction, the point of extinction could be moved continuously downwards *through* the lower singular axis C_2.

§ 7. EXAMINATION BETWEEN ELLIPTIC POLARISER AND CROSSED ELLIPTIC ANALYSER

It would be desirable to confirm that the principal axes of the two elliptic vibrations propagated without change of form along any direction, do not (in general) coincide with the principal planes of refraction—for even the behaviour of the idiophanic rings can be sufficiently well explained without taking this factor into account (see Pockels). We shall describe an experiment confirming this for directions represented by the points on C_1C_2, where the principal axes of the elliptic vibrations should lie parallel to OX_k and OY_k whereas the principal planes of refraction are inclined at 45° to these directions.

The arrangement used was that described by Ramachandran and Chandrasekharan[9] in another connection. The crystal was kept between crossed polaroids which could be rotated around together. A quarter-wave plate was set immediately behind the first polaroid, with its principal planes along OX_k and OY_k; another quarter-wave plate was set immediately in front of the second polaroid with its principal planes turned by 90° with respect to the first quarter-wave plate. If the crystal plate were absent, then obviously the elliptically polarised light produced by the arrangement in front would be automatically cut out completely by the arrangement behind, independent of the setting of the ganged polaroids. In the presence of the crystal plate, any direction where the incident elliptic vibration is propagated without change of form will appear extinguished. (The principal axes of the incident elliptic vibration will always lie along OX_k and OY_k.)

Starting with the polariser set along the axial plane, the optic axial direction O appears extinguished as in Fig. 2. As the ganged polaroids are rotated away from this setting in the proper direction, the point of extinction on the line Y_kY_k' moves upwards from O till it reaches the singular axis C_1 (polaroid at 45° to principal planes of $\lambda/4$ plate). As the ganged polaroids are rotated still further the point of extinction retraces its path, moving from C_1 back to O (polariser set perpendicular to axial plane). Further rotation through 90° causes the extinction to move upto C_2 and back to O.

The above experiment provides a direct confirmation of the theoretical result that the two elliptic vibrations propagated along these directions have their major axes crossed, have the same ellipticity and are described in the *same* sense. It also shows that the two elliptic vibrations gradually degenerate into two (identical) circular vibrations as the singular axis is approached.

The author is very grateful to Prof. Sir C. V. Raman for his keen interest in this investigation.

§ 8. SUMMARY

The phenomena shown in convergent light by a section plate of iolite cut normal to an optic axis are discussed and illustrated. Of particular interest among the photographs reproduced are those taken (*a*) between circular polariser and crossed circular analyser, demonstrating the existence of the two singular axes (a right-circular vibration alone being propagated unchanged along one singular axis, and a left-circular vibration alone along the other); (*b*) with a circular polariser alone, confirming the theoretical prediction in Part I that the singular axis where the incident circular vibration can be propagated unchanged should appear darker than the other; the state of the elliptic vibration emerging from the brighter singular axis was also qualitatively in accord with theory. Examination between elliptic polariser and crossed elliptic analyser demonstrated that along any general direction in an (optically inactive) absorbing biaxial crystal, two similarly rotating elliptic vibrations can be propagated unchanged—the major axes being crossed but not coincident with the principal planes of refraction.

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§ 10. DESCRIPTIVE NOTES ON THE PLATES

All photographs were recorded with a plate of iolite cut normal to an optic axis, the plate being held normally with the axial plane horizontal.

Phenomena with polariser and analyser

FIG. 2. Crossed polaroids: vibration directions along and perpendicular to axial plane. (Optic axis extinguished.)

FIG. 3. Crossed polaroids: vibration directions at 45° to axial plane. (Optic axis not extinguished.)

FIG. 4. Non-crossed polaroids: vibration directions equally inclined to axial plane. (Optic axis extinguished.)

Brewster's brushes and idiophanic rings

FIG. 5. Brewster's brushes, with neither polariser nor analyser. (Traces of interference rings seen.)

FIGS. 6, 7, 8 & 9. Idiophanic rings with polariser *alone* at various settings: along the axial plane in Fig. 6, perpendicular to the axial plane in Fig. 9, and at two intermediate orientations in Figs. 7 and 8.

FIG. 10. Idiophanic rings with analyser alone, set in the same position as the polariser in Fig. 8.

Properties of the singular axes

FIG. 11. Left-circular polariser and right-circular analyser. (Lower singular axis extinguished.)

FIG. 13. Left-circular polariser alone. (Lower singular axis appears darker than the other.)

FIG. 12. Right-circular polariser and left-circular analyser. (Upper singular axis extinguished.)

FIG. 14. Right-circular polariser alone. (Upper singular axis appears darker than the other.)

FIG. 15. Right-circular polariser and an elliptic analyser adjusted to cross out the elliptic vibration emerging from lower singular axis.

FIG. 16. Circular polariser and crossed circular analyser. (Photograph taken with a more lightly coloured plate: lower singular axis extinguished.)

NOTE

In Part I of this paper it was shown that the features of wave-propagation in absorbing biaxial crystals may be regarded as due to the effects of linear birefringence and linear dichroism superposed continuously along the depth of the material; this idea was worked out by a simple geometric method using the Poincaré sphere (as also by a more lengthy algebraic method). A series of papers published earlier by Clark Jones[3] dealing with 'A new calculus for the treatment of optical systems' has since then come to the notice of the author; the comprehensive calculus elaborated in the first half of that series uses the representation of any optical device by 2×2 complex matrix (operating on the electric vector of the incident light). In Part IV of that series one of the applications that has been made of the calculus is the derivation of the matrix for a plate of an absorbing crystal by a method which is physically equivalent to the method of superposition; the matrix corresponding to an infinitesimal path has been considered in Part VII.

FIG. 2

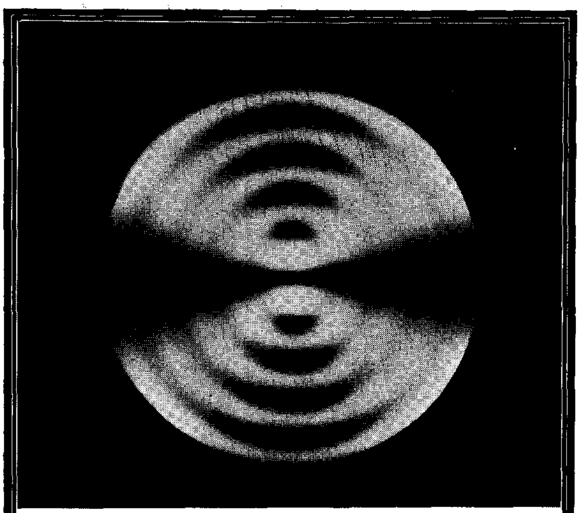


FIG. 3

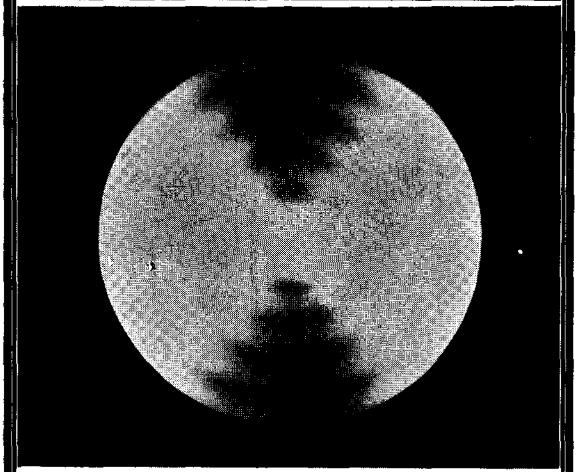
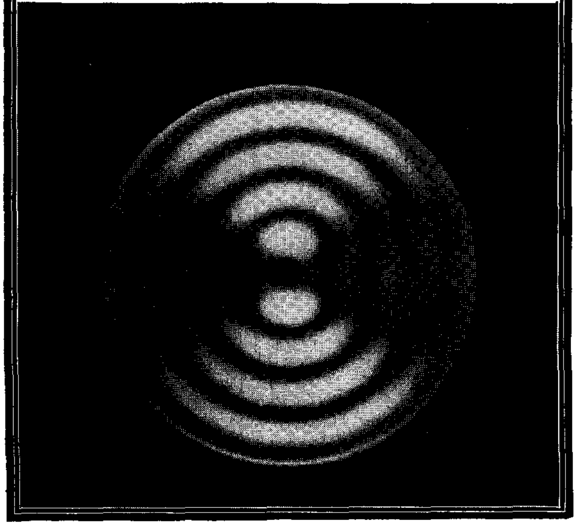


FIG. 4



Phenomena with polariser and analyser

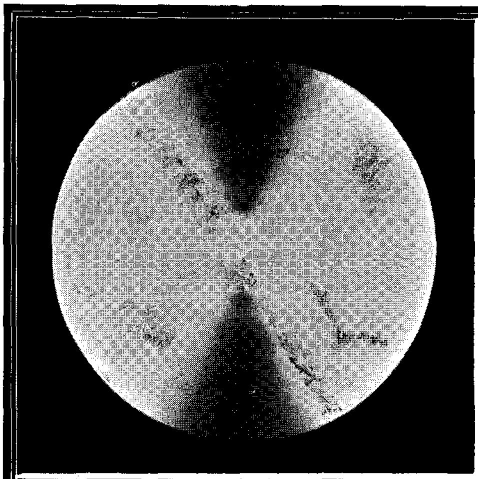


FIG. 5

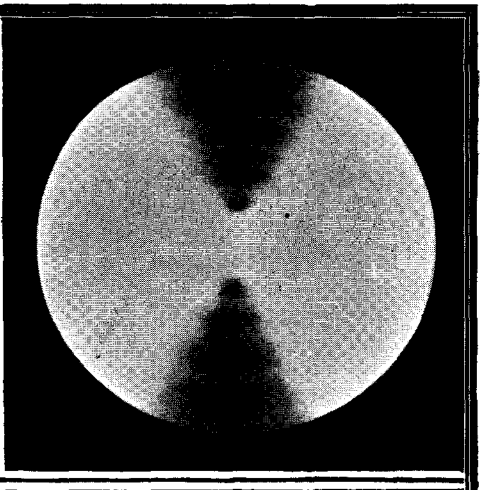


FIG. 6

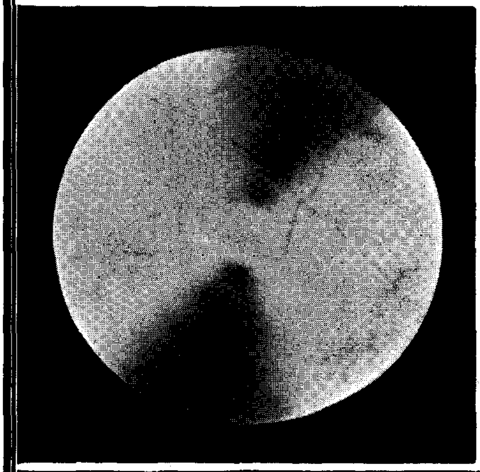


FIG. 7

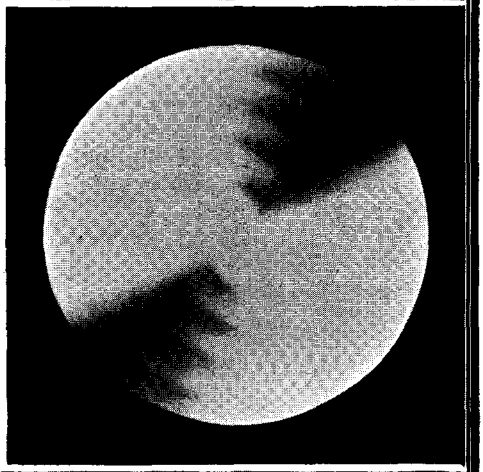


FIG. 8

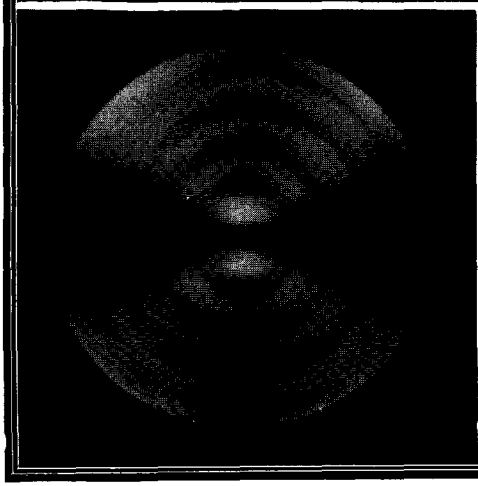


FIG. 9

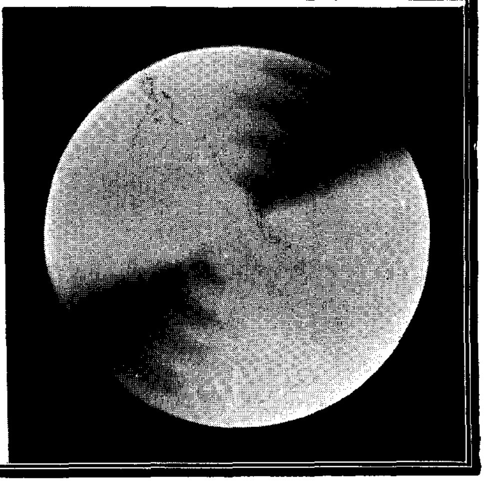


FIG. 10

Brewster's brushes and idiophanic rings

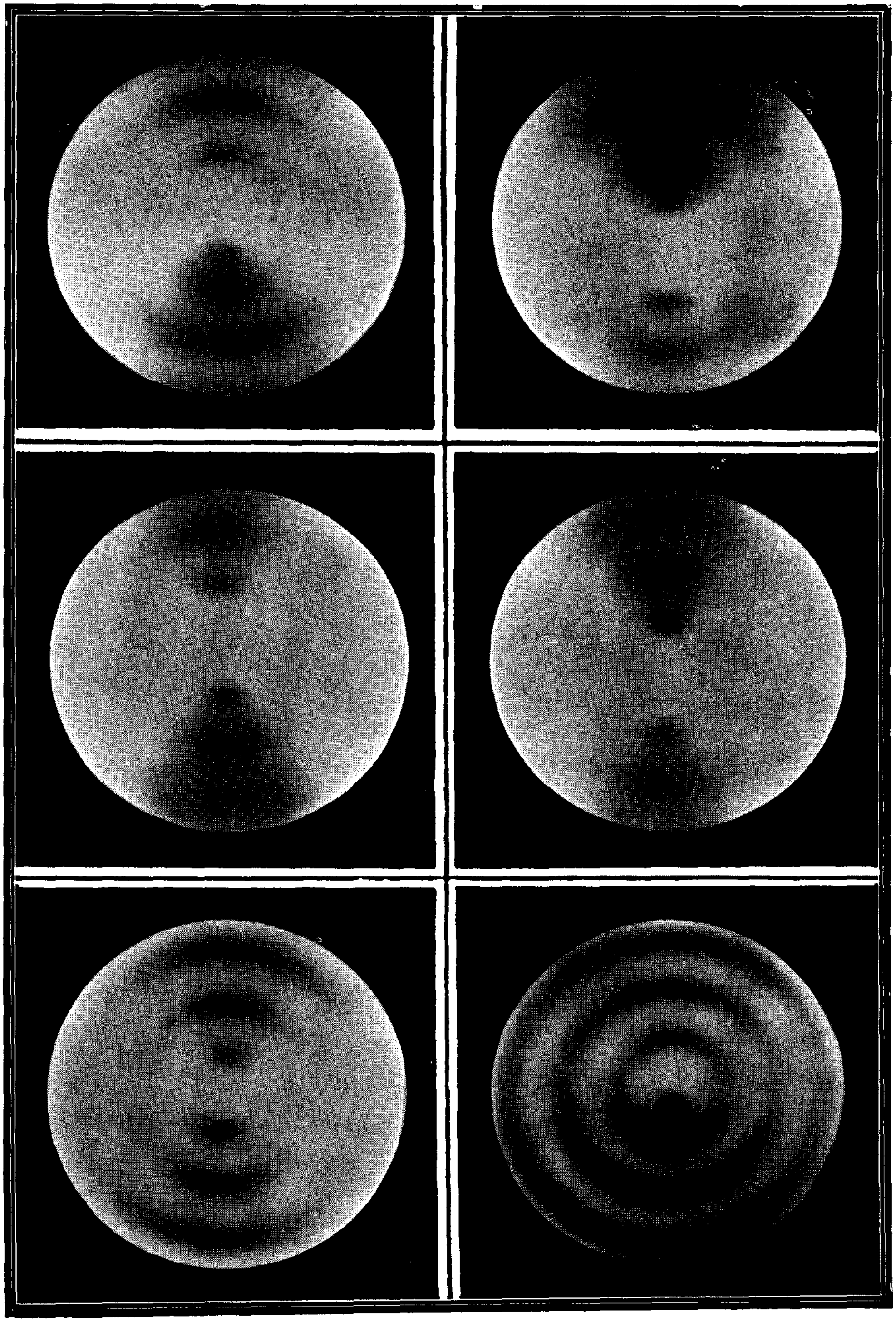


FIG. 11

FIG. 12

FIG. 13

FIG. 14

FIG. 15

FIG. 16

Properties of the singular axes