CHAPTER IV

FERRONEMATICS IN MAGNETIC AND ELECTRIC FIELD

4.1 Introduction

To observe the magnetic field effects in liquid crystals we usually require rather large fields, of the order of $10^4 G$. This is due to their weak diamagnetic anisotropy. In **1970** Brochard and de Gennes suggested the possibility of obtaining stable "ferronematic" and "ferrocholesteric" phases with a uniform suspension of ferromagnetic grains in the host liquid crystal [1]. They also worked out the effects of an external magnetic 'field on these systems under the assumption that diamagnetic anisotropy of the host liquid crystal can be neglected compared to the magnetization of the grains. Such ferronematic phases were made in the laboratory first by Rault at *al* [2] and later by others [3,4,5] using needle like ferromagnetic grains. Similar ferro systems with plate like grains have also been made [7]. In all these systems the magnetization of the grains appears to be quite small (of the order of 10^{-4}G). This is further supported by the experiments of Chen and Amer [5] who studied the Freederickzs transitions in a ferronematic, with magnetization M perpendicular to the nematic director, in homeotropic geometry and with the applied field parallel to the magnetization. Their results indicate that the effects of diamagnetic anisotropy of the host cannot be ignored. This prompted us to work out the implications of elastic anisotropy in such systems. We have considered the effects of elastic anisotropy also.

In this chapter we work out [6] the effects of magnetic and electric fields on ferronematics in the classical Freederickzs geometries. The free energy density for a ferronematic having magnetization M in an applied electric and magnetic field [depicted in Fig 4.1]:

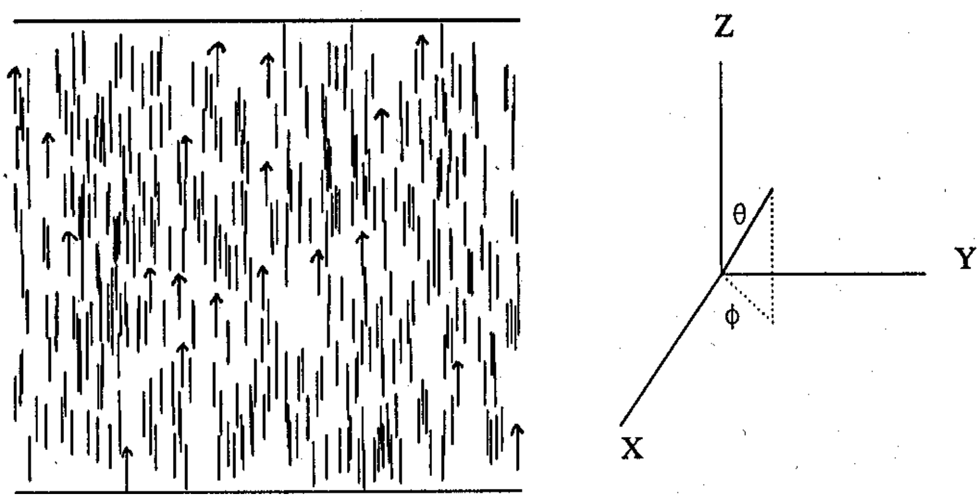

Fig 4.1: Ferronematic in homeotropic geometry with the magnetic field applied along the z-direction

$$F_d = \frac{k_{11}}{2}(\nabla.\mathbf{n})^2 + \frac{k_{22}}{2}(\mathbf{n}.\nabla \times \mathbf{n})^2 + \frac{k_{33}}{2}(\mathbf{n} \times (\nabla \times \mathbf{n}))^2 - \frac{\chi_a}{2}(\mathbf{n}.\mathbf{H})^2 - \mathbf{M}.\mathbf{H} - \frac{\epsilon_a}{8\pi}(\mathbf{n}.\mathbf{E})^2 \quad (4.1)$$

where

M	=	Magnetization
χ_a	=	Positive diamagnetic anisotropy
ϵ_a	=	Positive dielectric anisotropy
H	=	Magnetic field
E	=	Electric field
k_{11}, k_{22}, k_{33},		are the splay, twist, and bend elastic constants respectively

We consider a ferronematic with M along the director n. Further we assume the magnetic and electric fields to be parallel to the undistorted director, with the magnetic field antiparallel to the initial magnetization.

4.2 Homeotropic Geometry

In this geometry which is shown in Fig 4.1 the distortion is given by n = ($\sin\theta\cos\phi$, $\sin\theta\sin\phi$, $\cos\theta$). This distortion is cylindrically symmetric about the z-axis and results in what are called umbilics [8]. For small θ distortions we get

$$\theta = \theta_m(r)\sin(\pi z/d)$$

$$\phi = \pm\tan^{-1}\frac{y}{x}$$

where d is the sample thickness. θ_m being the distortion at the center.

Then for regions far away from the core of the umbilic *i.e.*,$r >> \xi$ (where ξ is the coherence length) the average free energy density from 4.1 is:

$$\bar{F}_d = \frac{1}{d}\int_0^d F_d \mathrm{d}z$$

$$= F_0 + \frac{\alpha}{2}\theta_m^2 + \frac{\beta}{4}\theta_m^4 + \frac{\gamma}{6}\theta_m^6 \quad (4.2)$$

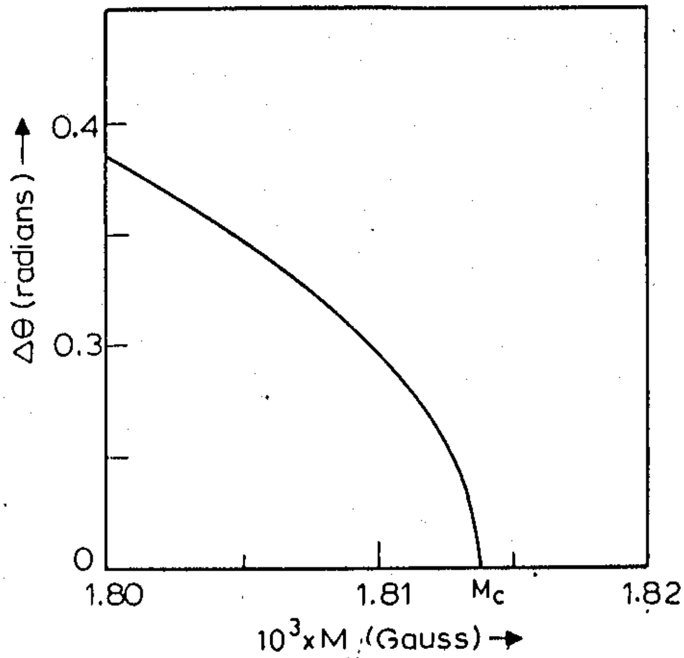

Fig 4.2: Jump **A0** at the transition as a function of M for the homeotropic geometry $d = 20\,\mu m$, $k_{11} = k_{22} = k_{33} = 0.5 \times 10^{-6}$ dynes, $M = 1.8 \times 10^{-3}$ Gauss, $\chi_a = 0.5 \times 10^{-6}$ cgs units

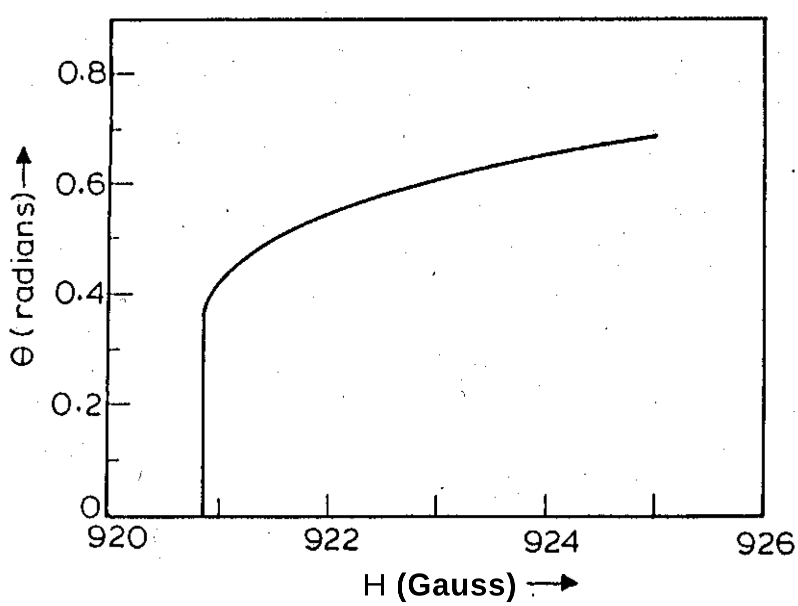

Fig **4.3:** Onset of first order Freedericksz transition in the homeotropic geometry and for parameters shown in **Fig.2** where

$$\alpha = \frac{1}{2}\left[\frac{k_{33}\pi^2}{d^2} - MH + \chi_a H^2 + \frac{\epsilon_a E^2}{4\pi}\right] \tag{4.3}$$

$$\beta = \frac{1}{4}\left[\frac{(k_{11}-k_{33})\pi^2}{d^2} + \frac{MH}{4} - \chi_a H^2 - \frac{\epsilon_a E^2}{4\pi}\right] \tag{4.4}$$

$$\gamma = \frac{1}{144}\left[\frac{(k_{33}-k_{11})\pi^2}{d^2} - \frac{MH}{16} + \chi_a H^2 + \frac{\epsilon_a E^2}{4\pi}\right] \tag{4.5}$$

Notice that **4.2** has the form of a Landau free expansion. We keep terms up to sixth power and allow β to take negative values also.

4.2.1 Magnetic field effects

Effect of χ_a

.First we work out the effects of magnetic fields only. Let us for simplicity assume k $= K_{11} = K_{22} = K_{33}$ then the equation system **4.3-4.5** will take the form

$$\alpha = \frac{1}{2}\left[\frac{k\pi^2}{d^2} - MH + \chi_a H^2\right] \tag{4.6}$$

$$\beta = \frac{1}{4}\left[\frac{mH}{4} - \chi_a H^2\right] \tag{4.7}$$

$$\gamma = \frac{1}{144}\left[-\frac{MH}{16} + \chi_a H^2\right] \tag{4.8}$$

We find that for values of M less than the critical value $M_c = (4\pi/d)(k\chi_a/3)^{1/2}$, β is negative and y is positive resulting in a first order transition to the umblic configuration. And for higher values of magnetization β is positive giving a second order transition. Therefore as M changes we find a tricritical behavior at M_c. Fig 4.2 shows the variation of $\Delta\theta_m$, the jump in the order parameter at transition as a fuction of M. In Fig 4.3 is shown the variation of θ_m with H in the first order transition. It should be remarked that this variation of θ_m is only approximate since the harmonic solution that we have assumed is not strictly valid when the sixth power term is included.

CHAPTER II

NEMATIC DEFECTS IN MAGNETIC FIELD

2.1 Introduction

In this chapter we look at the effects of a magnetic field on the structure and properties of some defects in nematic liquid crystals. The **diamagnetic anisotropy** χ_a of a nematic can be positive or negative. Usually nematics with rod like molecules have positive χ_a and those with disc like molecules have negative χ_a. We consider both the cases. When a sample of nematic with $\chi_a > 0$ is placed in a uniform magnetic field the director n aligns parallel to the magnetic field. While in the case of $\chi_a < 0$ nematics the director aligns in a plane perpendicular to the magnetic field. In both the cases since n is apolar solutions with n and **-n** have the same energy. Two such solutions get connected by a domain wall inside which the director turns through an angle π. These walls were first discussed by Helfrich [1] and are called Helfrich walls or planar *solitons*. We can have bend rich, splay rich or pure twist walls. Volovik and Mineev showed that these walls can end in disclination lines of half integral strength [2,3]. Two Helfrich walls can get interconnected through a disclination. Volovik and Mineev also showed the possibility of having cylindrical domains ending in point singularities. These has been named as linear solitons. Ranganath [1] predicted that in uniform magnetic field point singularities can result in discs inside which the director turns through π. In addition the symmetry of nematic liquid crystals allows one to consider cylindrical shell structures -Bubble domains- connecting the inside and outside uniform regions by twist or bend cylinderical shell.

It may be remarked in passing that such structures can exist in biaxial nematics as well. The simplest of biaxial nematics have orthorhombic symmetry and have three directors a, b and c. The possible structures of various types of solitons in these,systems was discussed by Ranganath [5].

In this chapter we have considerd two types of magnetic fields. (1)The uniform magnetic field H_z acting in the z direction and (2)that of a circular magnetic field

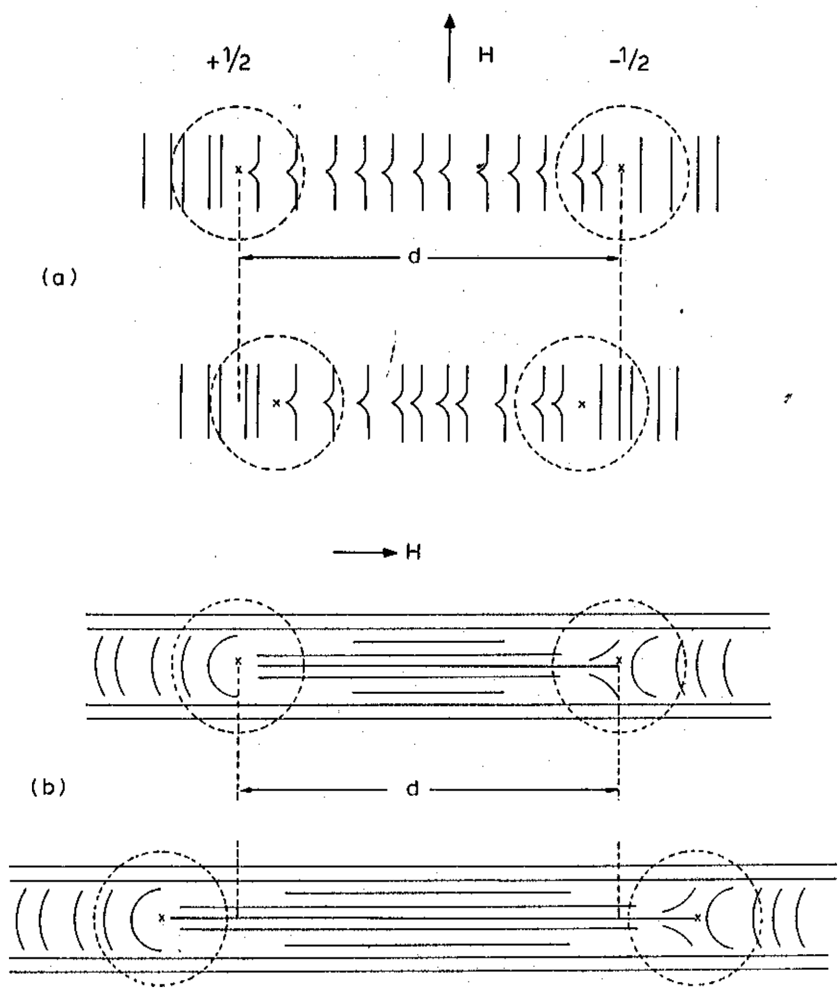

Fig 2.1: A pair of unlike disclinations of strength 1/2 in a magnetic field acting normal to the disclination lines (a) Field perpendicular to the line joining the disclinations, (b) Field along the line joining disclinations.

H_α generated by a linear current element [1]. The effects of elastic anisotropy, ie.,$k_{11} \neq k_{22} \neq k_{33}$ have also been worked out.

2.2 Interaction between disclinations

Let us consider the case of $\pm 1/2$ disclination line in a nematic with χ_a positive. It was said earlier that this disclination line gets transformed to a domain wall terminating in a singular line under the action of a magnetic field H. The wall thickness is of the order of the magnetic coherence length $\xi = (k/\chi_a H)^{1/2}$ where $k = k_{11} = k_{22} = k_{33}$ is the elastic constant. We consider the interaction between such disclinations.

Consider now the case of a magnetic field H acting perpendicular to the line connecting two unlike $1/2$ wedge disclinations. The field induced structure is shown in Fig 2.1a. The two singularities are connected by a splay wall. The energy per unit area of these walls is given by E = $2H(k\chi_a)^{1/2}$ [1]. It is clear from the figure that by moving the the two disclinations towards or away from each other the director pattern inside the dashed circles of radius t or at far off distances is not much affected. But the size of the connecting wall is altered by moving the two disclinations towards each other. We assume the distance of separation to be much larger than ξ. Then for a change of distance Ad the energy of the wall configuration is decreased (to a good approximation) by A E = energy per unit area of the wall × Ad *i.e.,* A E $= 2H(k\chi_a)^{1/2}\Delta d$. Hence the change in total energy is proportional to the change in the distance of separation Ad between two disclinations. Therefore two unlike disclinations attract with a force that is independent of distance separating them.

We now consider the case when the magnetic field is acting parallel to the line joining two unlike line disclinations. In this case we get a planar bend soliton extending on either side to infinity and away from the disclinations. This is shown in Fig 2.1b. The energy per unit area of this soliton is E = $\chi_a H^2/2$. Most of the region between the two defects is free from director distortion. To a good approximation we see that small displacements of the two defects do not change the director field either inside the dashed circles or at infinity. Hence by moving the two disclination away from each other by Ad the change in energy is given by:

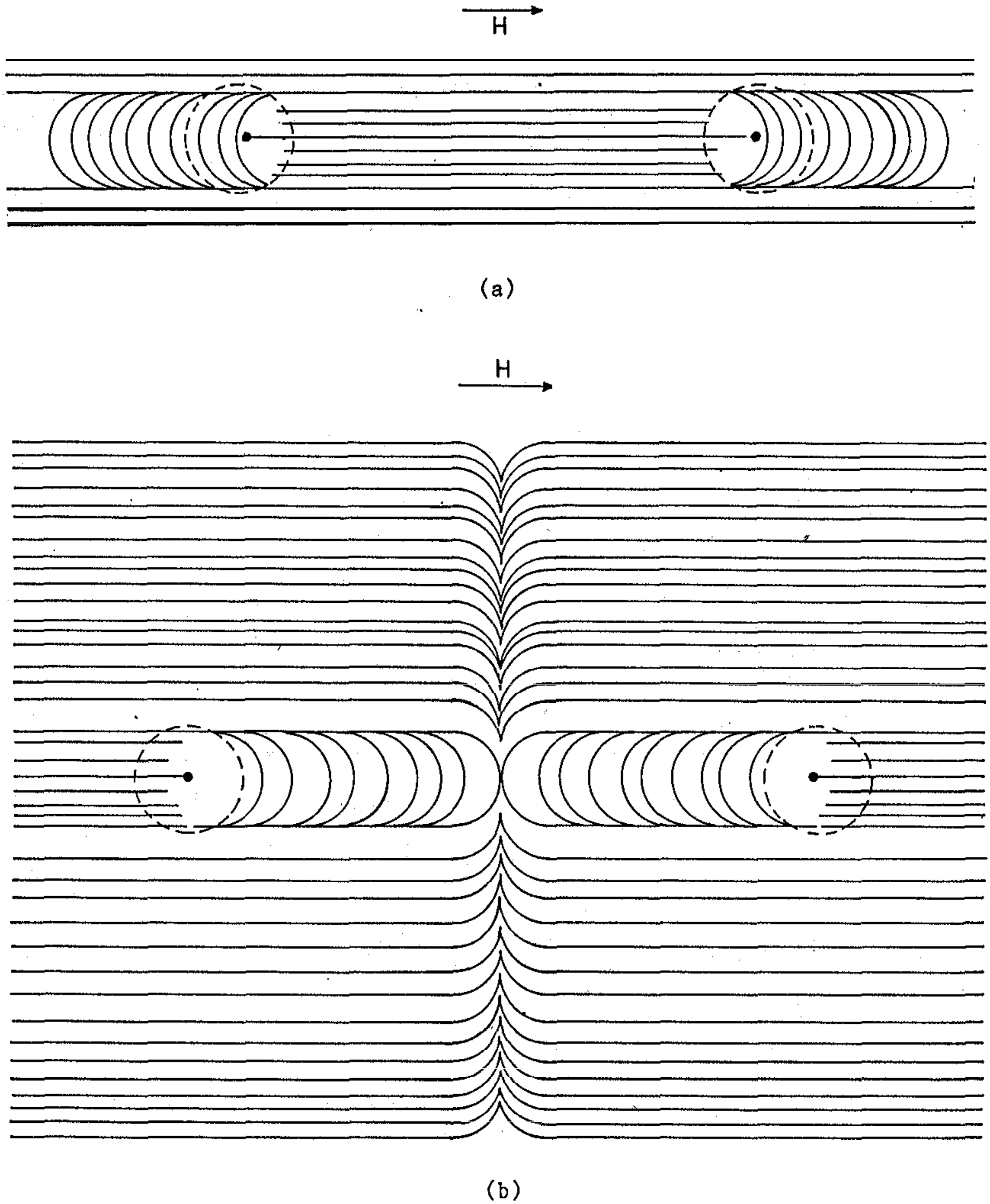

Fig 2.2: Pairs of like disclinations of strength $1/2$ in a magnetic field acting along the line connecting them . The force of interaction is independent of the distance of separation and can be repulsive (a) or attractive (b).

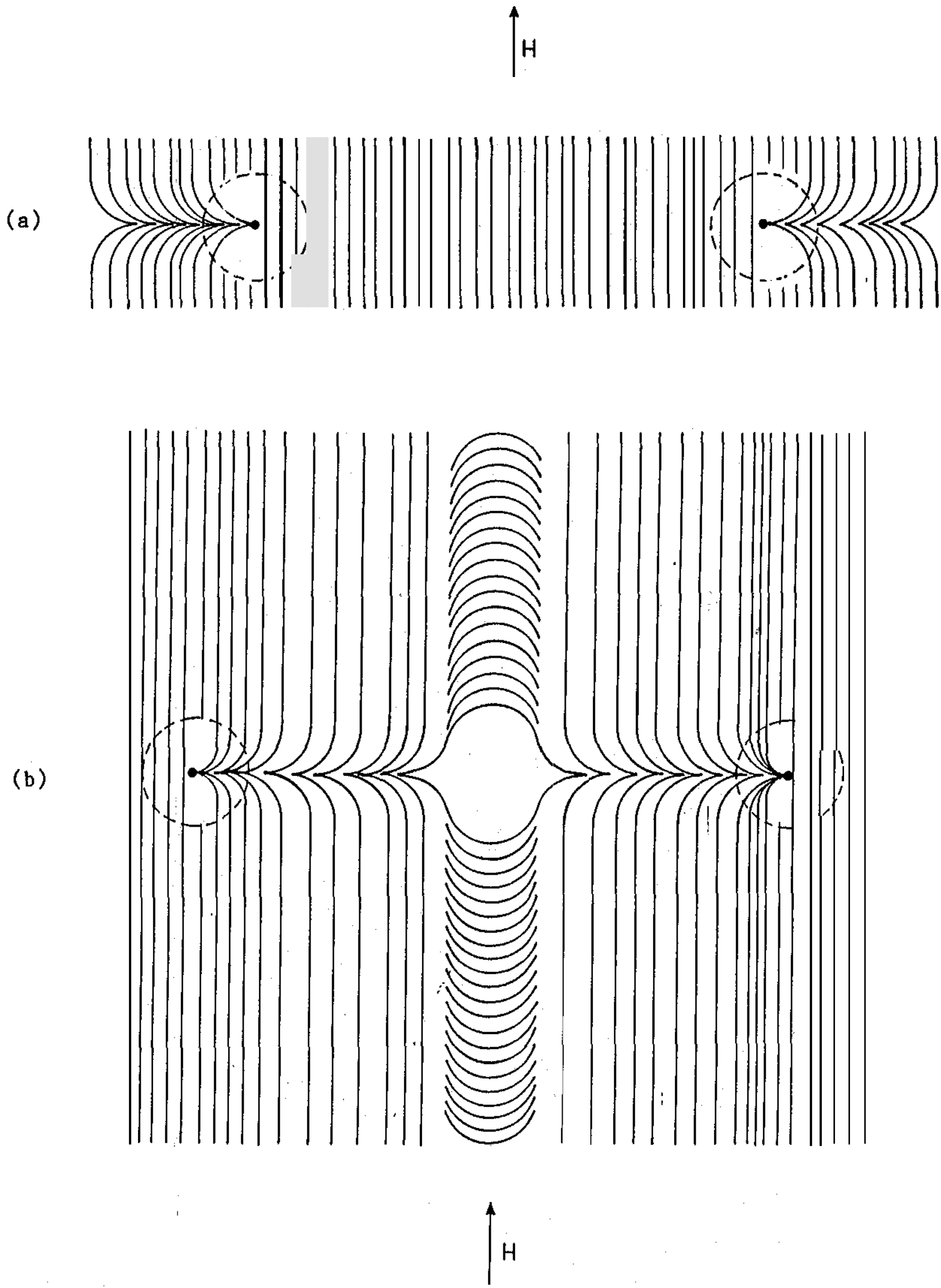

Fig 2.3: Pairs of like disclinations of strength 1/2 in a magnetic field perpendicular to the line connecting them. The interaction between them being repulsive (a) and attractive(b)

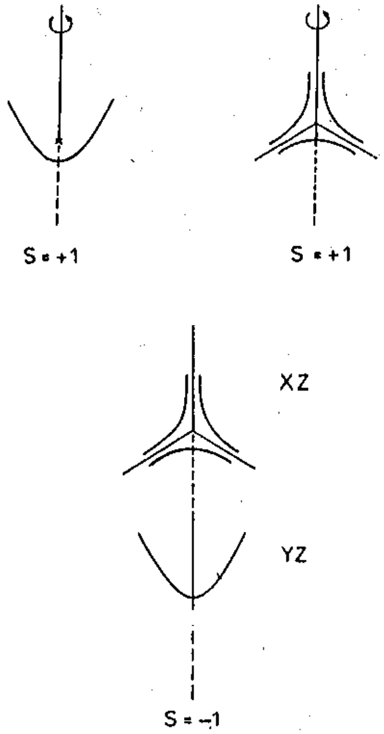

Fig 2.4: Poincarè defects of different strengths with line singularities (dashed line) ending at a point inside the material.

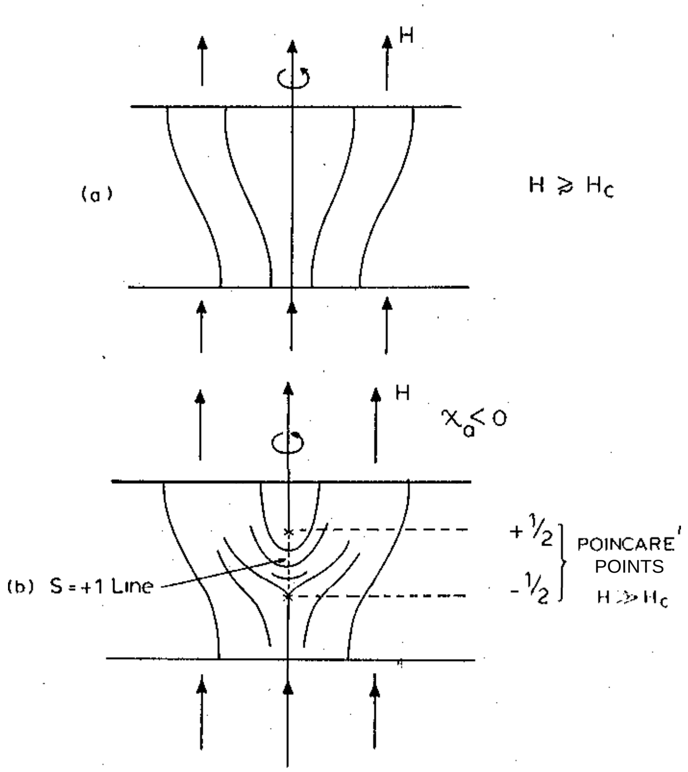

Fig 2.5: Generation of Poincarè defects in a nematic with negative diamagnetic anisotropy (a) before the Freedericksz threshold. (b) At much higher fields

$$\Delta E = \frac{-\chi_a H^2}{2} \Delta d$$

Thus in this geometry we find that the field favors repulsion between two unlike disclinations. The force of repulsion being independent of distance. This is an unusual interaction, since in general two unlike defects always attract each other.

In the same way we see that two disclination of opposite strength can experience an attractive or repulsive force in the presence of a magnetic field. The strengths of the attractive and repulsive forces are, however, different.

In the case of two like defects we have four walls being generated by a magnetic field. This is shown in Fig 2.2 and Fig 2.3 for magnetic field acting parallel and perpendicular to the line joining the defects. However, the interaction between the disclinations can be repulsive or attractive depending upon the geometry. Here again the force of interaction is independent of distance of separation.

In the above analysis we have ignored the formal elastic interaction. A calculation .of the net interaction taking both contributions into account is not easy. However we can make some approximate estimates. We know that the elastic free energy density varies as $k(\Delta\theta)^2$ while the magnetic energy varies as $\chi_a H^2 \sin^2\theta$. Thus over distances d less than ξ , to a good approximation the magnetic energy can be neglected compared to the elastic energy which dominates. Thus the distance independent interaction law will be valid only when d >> ξ.

Very similar arguments can be applied to nematics in electric field where the dielectric anisotropy ϵ_a aligns the director to the field

2.3 Poincarè structures

In all the experimental situations known to date a line singularity is found to end either on itself forming a loop or on the surface of the sample. However it is known that a line singularity can also be terminated in a pair of half point disclinations [7,6]. While working out the defects in nematics, by Poincarè's technique Nabarro found that structures of the type shown in Fig 2.4 are also allowed. The first two structures have cylindrical symmetry with S = $\pm 1/2$ director pattern in the meridional plane. We can also have = $+1/2$ structures in one plane and $-1/2$ in the orthogonal plane. Poincarii structures are very unique. Firstly they are singular for

$z < 0$ and are strictly non-singular for $z > 0$, i.e., a singular line ends in the body of the material. Secondly the line singularity has a strength of ± 1. Such topological defects have not been experimentally seen so far. In this section we suggest a method of generating such defects.

If a magnetic field is applied parallel to the director of a homeotropically aligned nematic with $\chi_a < 0$, it will undergo a Fredericks transition at a critical field given by $Hc = (\pi^2 k/\chi_a d^2)^{1/2}$. Just above this threshold we get cylindrical nonsingular structure shown in Fig 2.5a. It should be noticed that in the central region the director is still opposing the magnetic torque. Hence at fields much higher than the critical field this structure can break down to a singular structure shown in Fig 2.5b. Here a $S = +1$ line singularity is shown to end in a pair of unlike Poincark half point singularities.

This phenomena can be expected in nematic discotics since they usually have negative diamagnetic anisotropy. In rod-like nematics it is easier to get systems with negative dielectric anisotropy than negative diamagnetic anisotropy. Here the above arguments are valid *mutantis* mutandis in the presence of an electric field, provided the system is free of ions.

2.4 Bubble domains

As mentioned earlier Bubble domains are cylindrical shell structures connecting the inside and outside regions through twist, splay or bend deformation. We have investigated such structures in the presence of an all circular magnetic field of $H_\alpha = A/r$ (generated by a linear current element A) in a nematic with $\chi_a < 0$.

In cylindrical polar coordinates (r, θ, ϕ) the director n is given by

n = [$\sin\theta$ cos(ϕ − a), sin0 sin($ − a), $\cos\theta$]

The distortion free energy density is:

$$F = \frac{k}{2}[(\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2] + \frac{\chi_a A^2}{2r^2} \sin^2 0 \sin^2(\phi - a)$$

Minimization of the total energy $\int F\,dv$ energy with respect to θ results in

$$k[\nabla^2\theta - \sin\theta\cos\theta(\nabla\phi)^2] - \frac{\chi_a A^2}{r^2}\sin\theta\cos\theta\sin^2(\$ - a) = 0 \qquad (2.1)$$

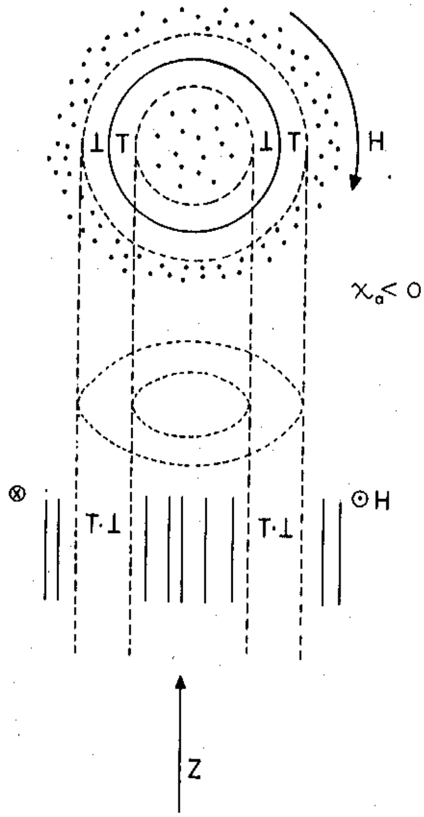

Fig 2.6: **A** twist bubble domain in a nematic with $\chi_a < 0$, in an ail circular field. The dashed lines are the boundary of the domain

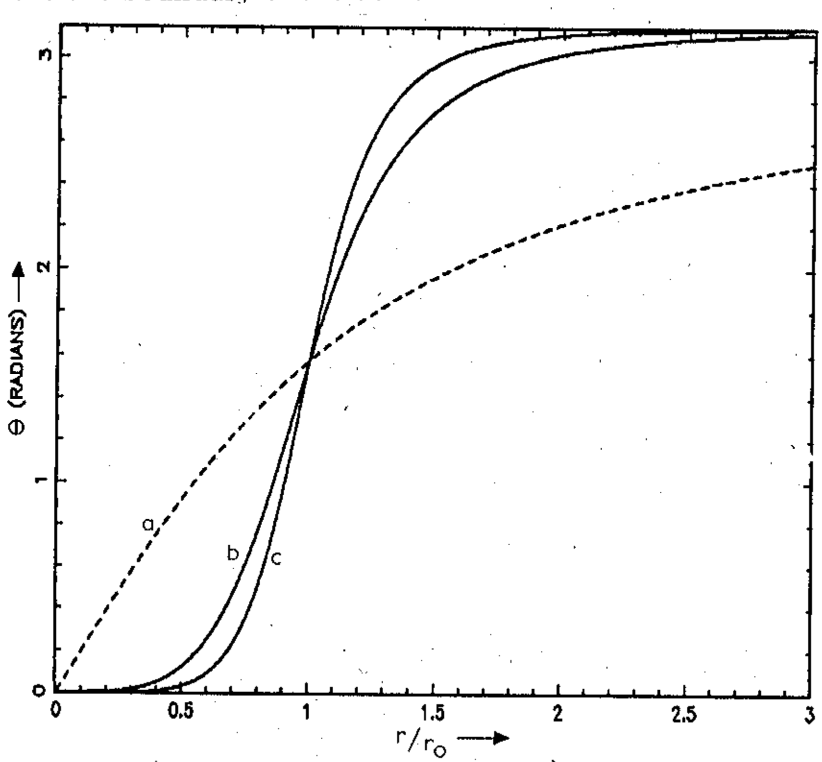

Fig 2.7: The director tilt θ in a twist bubble domain as a function of the distance from the center. (a) with $\eta = 1$. (b) with $\eta = 4$ and (c)$\eta = 6$

similarly minimizatibn with respect to ϕ gives

$$k(\nabla^2\phi) \quad \frac{\chi_a A^2}{r^2} \sin(\phi-\alpha)\ \cos(\phi-\alpha)=0 \qquad (2.2)$$

The solution satisfying both the equations (2.1) and (2.2) with the boundary conditions

$\theta=0$ at r = O and $\theta=\pi$ at $r=\infty$ is

$\theta = 2\, tan^{-1}(r/r_o)^\eta$ and ϕ = a + $\pi/2$

where

η = $[1 + \chi_a A^2/k]^{1/2}$

I-Iere r_o is the point at which 0 becomes $\pi/2$. This solution represents a Bloch bubble domain which has been depicted in Fig 2.6. The variation of θ with respect to r/r_o is shown in Fig 2.7 for different values of η. We see that the width of the cylindrical domain wall decreases as the field increases. Thus in such a field the 'bubble domain is a natural soliton solution. The total energy of this structure per unit height is found to be $4\pi k\eta$. Interestingly the energy is independent of its radius r. At high fields $\eta \rightarrow [\chi_a A^2/k]^{1/2}$ and the energy become $4\pi A(k\chi_a)^{1/2}$, which is $2\pi r$ times the energy/area of the planar soliton obtained in uniform fields.

Bubble domains can also exist in diamagnetically positive materials. Here for all circular fields $A < (k/\chi_a)^{1/2}$ we find a collapsed +1 all circular disclination [9]. This becomes a planar all circular singular structure at a critical value of **A** given by $(k/\chi_a)^{1/2}$. On this structure we can now impose a twist-bubble or an inplanar bend-bubble domains. We consider here the case of twist bubble domain where the boundary conditions are:

$$\theta = -\pi/2 \text{ at } r=0 \ \&\ \theta=\pi/2 \text{ at } r=\infty$$

tlie equations of equilibrium

$$k[V^2 0 + \sin\theta\ \cos\theta(\nabla\phi)^2] - \chi_a A^2/r^2\ \sin\theta\ \cos\theta\ \sin^2(+ - a) = 0$$

and

$$k(\nabla^2\phi) + \chi_a A^2/r^2\ \sin(\phi-\alpha)\ \cos(\phi-\alpha)=0$$

yield the following solutions.

$$\phi = \alpha + \pi/2 \ \& \ \theta = 2\, tan^{-1}[r/r_o]^{\eta} - \pi/2$$

where $\eta = [\chi_a A^2/k - 1]^{1/2}$

The energy of the bubble domains in this case is $4\pi k\eta + E_c$, where E_c is the energy of the singular core.

It should be mentioned in passing that although it is'also possible to have such bubble domains in a uniform H_z field, an energy analysis is not possible in a linear theory of elasticity.

To conclude, we notice that effect of an external field on defects are non trivial. And in some cases the field can induce some defects. In either case we find not only interesting but in some cases even unexpected results.

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CHAPTER III

SMECTIC AND DISCOTIC DEFECTS IN A MAGNETIC FIELD

3.1 Introduction

In the previous chapter we saw that a magnetic field not only chgnges the structure of nematic defects considerably but can also lead to new defects in nematics. It must be remarked that these structures were possible because nematics have no lattice order. But in smectics and discotics lattice ordering is an additional constraint, since it is difficult to bend the director in a smectic or splay it in a discotic without introducing dislocations. In this chapter we work out the effects of a magnetic field on defects in smectic and discotics. We also investigate the structure of field induced defects in these systems. As in the case of nematics the effects of uniform linear as well as circular magnetic fields on systems with $\chi_a < 0$ and $\chi_a > 0$ will be analyzed here too. At the end of the chapter we briefly discuss the effects of a magnetic field on the core structure of disclinations in smectics.

3.2 Smectic A - Screw and helical dislocations

Smectic A has rod like molecules stacked in layers with molecular long axis preferentially aligned along the layer normal. By virtue of its lattice structure edge and screw dislocations are possible in smectics. But in many ways these are different from the edge and screw dislocations in crystals. This is due to the fact that smectics have not only lattice elasticity due to layers but also nematic like curvature elasticity. For example smectic edge dislocations have finite self energy even in an infinite sample and smectic screw dislocations have no self energy at all. Since not many studies exist on the effect of fields on such defects this will be addressed to here.

Within the linear theory of elasticity the free energy density of smectic A upto second order in deformations can be written as [1],:

$$F_l = \frac{B}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{k_{11}}{2}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]^2 \tag{3.1}$$

where
u=the layer displacement
B=the elastic constant for lattice dilation ($\sim 10^6 - 10^7$ dynes/cm^2)
k_{11}= the elastic constant for layer curvature ($\sim 10^{-6}$ dynes)

The minimization of the total free energy with respect to u gives:

$$\frac{\partial^2 u}{\partial z^2} = \lambda^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 u$$

where $\lambda^2 = k_{11}/B$. This permits solutions of the form

$$u = \frac{b}{2\pi}\tan^{-1}(y/x) \tag{3.2}$$

Here b is called strength of the dislocation. It is an integral multiple of the layer spacing. It is clear that equation **3.2** represents a screw disloction with u changing by b as one goes once around the dislocation line. Since it does not involve lattice dialatation $\partial u/\partial z$ or layer curvature $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2$ it is easily seen that this screw dislocation solution in smectic-A has no self energy. Also since superposition of such solutions is also a solution the interaction energy between any two dislocations is also zero.

3.2.1 Field induced effects on screw dislocations

In this section we will study the energetics of screw dislocations in the presence of an uniform magnetic field applied along normal to the layers(z-axis). The free energy density for $\chi_a > 0$ material is:

$$F_d = F_l + \frac{\chi_a H^2}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] \tag{3.3}$$

and the equation of equilibrium is:

$$B\,\frac{\partial^2 u}{\partial z^2} - k_{11}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]^2 u + \chi_a H^2\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] = 0 \tag{3.4}$$

Even in this case the screw dislocation described by:

$$u = \frac{b}{2\pi}\tan^{-1}(y/x)$$

is a solution. But this time it has a self energy

$$E_0 = \frac{\chi_a H^2 b^2}{4\pi}\ln\frac{R}{r_c}$$

due to the second them in **3.3.** When two such dislocations of strengths $b_1$andb_2 are centered at positions $(x_1, y_1)$and(x_2, y_2) respectively, the net solution is a linear combination of the individual solutions and is:

$$u = \frac{b_1}{2\pi}\tan^{-1}\frac{(y-y_1)}{(x-x_1)} + \frac{b_2}{2\pi}\tan^{-1}\frac{(y-y_2)}{(x-x_2)} \tag{3.5}$$

substituting equation **3.5** in equation **3.3** and integrating over the sample we find the total energy E_0 to be

$$E = \frac{b_1^2 + b_2^2}{4\pi}\chi_a H^2 \ln\frac{R}{r_c} + \left(\frac{b_1 b_2}{2\pi}\right)\chi_a H^2 \ln(2R/d) \tag{3.6}$$

The first term represents the total self energy and the second term gives the interaction energy E_i. Here R is the sample size , r_c is the radius of the singular core of the dislocations and d is the distance of separation between the dislocations.

Hence we find, in a magnetic field, like screw dislocations to repel and unlike dislocations to attract with a force prapotional to $1/d$. This interaction law is exactly the same as that between screw dislocations in crystals.

A similar interaction law is also to be expected in a nonlinear smectic. Here the free energy will have a nonlinear term given by

$$F_{nl} = \frac{B}{2}\left(\frac{\partial u}{\partial z}\right)\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] \tag{3.7}$$

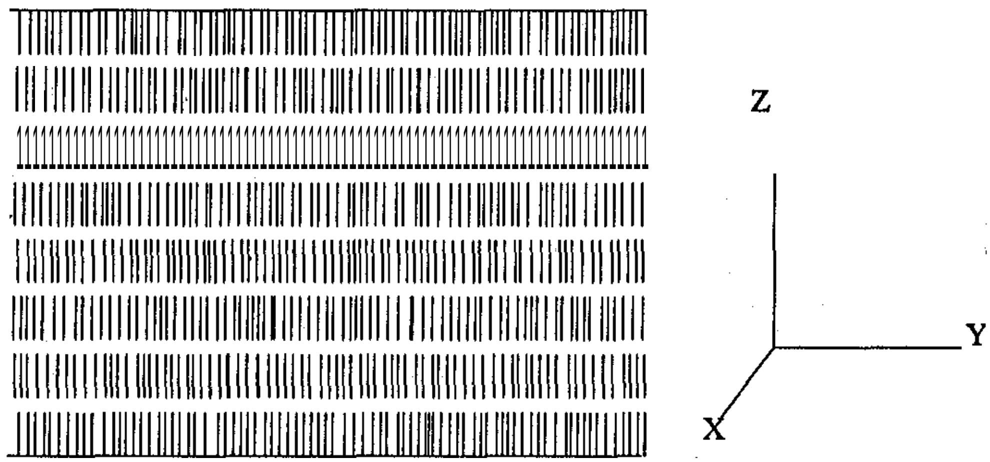

Fig 3.1: Smectic **A** with layers aligned parallel to the sample boundary

F_{nl} is similar to the magnetic term in equation (3.3). Comparing equations (3.3) and (3.5) we can predict the force of interaction in this case to be proportional to imposed lattice compression $\partial u/\partial z$.

3.2.2 Field induced defects

Hz field

In an homeotropically aligned sample (shown in Fig 3.1) with $\chi_a < 0$ in a magnetic field applied along the z direction, the free energy density takes the form:

$$F_d = \frac{B}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{k_{11}}{2}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]^2 - \frac{\chi_a H^2}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] \tag{3.8}$$

We can easily see that equation (3.2) which represents a screw dislocation solution is a solution in this case also. The self energy is then given by:

$$E = -\frac{\chi_a H^2 b^2}{4\pi}\ln(R/r_c) \tag{3.9}$$

This energy is negative *i.e.* it is possible to lower the energy by creating many screw dislocations. Hence the structure develops an instability through a proliferation of screw dislocations. It should be mentioned that (3.2) is not the only solution to equation of equilibrium. But solutions with contribution from the elastic term should be of higher energy and a threshold will be needed to excite them.

. It should he mentioned that in a non-linear smectic we can expect a similar instability under a lattice dilatation $\partial u/\partial z$.

H_α field

Here we will investigate in the same geometry a material with $\chi_a > 0$ in an all circular magnetic field $H_\alpha = A/r$ (generated by the linear current element A) acting parallel to the layers. Let us introduce a perturbation in the director field of the form

$$n_r = 0, \quad n_\alpha = \frac{-1}{r}\left(\frac{\partial u}{\partial \alpha}\right), \quad n_z = 1 - \frac{1}{2r^2}\left(\frac{\partial u}{\partial \alpha}\right)^2$$

Then the free energy density is given by:

$$F_d = \frac{k_{11}}{2r^4}\left(\frac{\partial^2 u}{\partial \alpha^2}\right)^2$$

Minimizing the total free energy leads to

$$\frac{k_{11}}{r^4}\frac{\partial^4 u}{\partial \alpha^4} + \frac{\chi_a A^2}{r^4}\frac{\partial^2 u}{\partial \alpha^2} = 0 \tag{3.10}$$

This equation again permits the screw dislocation solution given by

$$u = \frac{b}{2\pi}\alpha \tag{3.11}$$

The energy per unit length is now

$$E = -\frac{\chi_a A^2 b^2}{8\pi}\left[\frac{1}{r_c^2} - \frac{1}{R^2}\right]$$

since $R > r_c$ this energy is negative. Thus the system is unstable against such perturbation. Thus energy can he lowered by creating screw dislocations. We can therefore expect a proliferation of screw dislocations in an H_α field. Notice that this screw dislocation is different from the classical one in the sense that its energy is finite even for an infinite sample (R $\to \infty$).

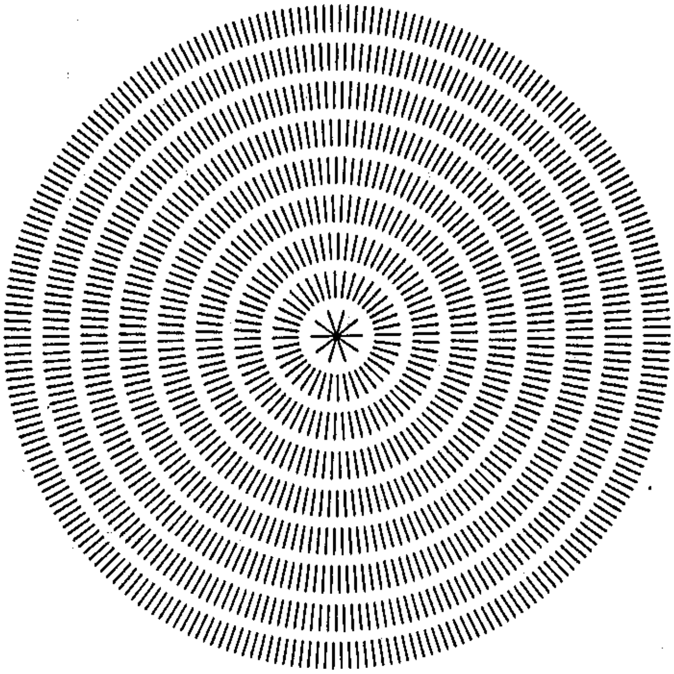

Fig **3.2:** **A** wedge disclination of strength $1 is smectic **A**

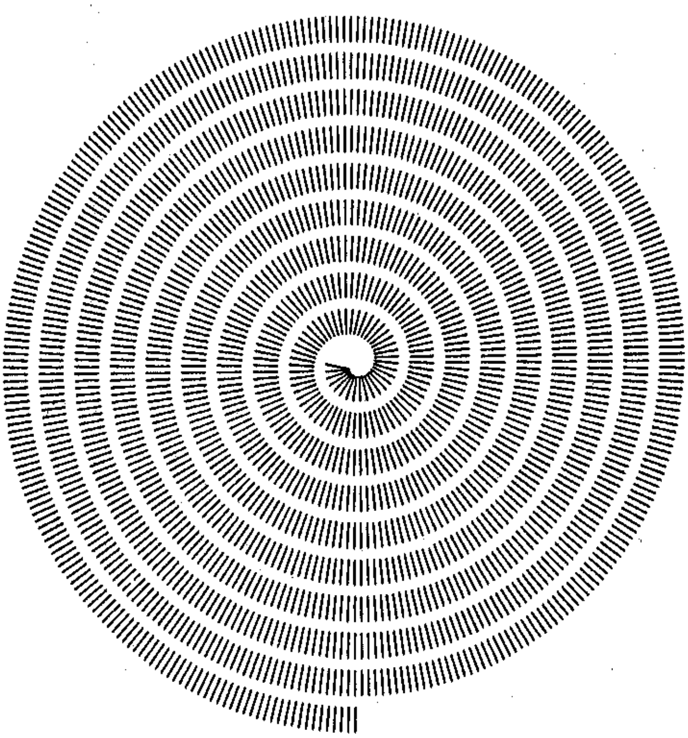

Fig **3.3:** Spiral distortion in Smectic **A** with the helical wrapping of layers

3.2.3 Spiral Instability of a disclination in an H_α field

Now we study the effect of the circular field on a $+1$ wedge disclination in smectic **A** shown in Fig **3.2.** This has smectic layers in concentric cylinders about the disclination line. Consider the disclination to be along the z direction with the field acting parallel to the smectic layers. When the molecules have positive diamagnetic anisotropy we can expect a perturbation of the form

$$n_r = 1 - \frac{1}{2r^2}\left(\frac{\partial u}{\partial \alpha}\right)^2, \quad n_\alpha = -\frac{1}{r}\left(\frac{\partial u}{\partial \alpha}\right), \quad n_z = 0$$

Then the free energy density up to second order in u is:

$$F_d = \frac{k_{11}}{2}\left[\frac{1}{r^2} + \frac{1}{r^4}\left(\frac{\partial u}{\partial \alpha}\right)^2\right] - \frac{\chi_a A^2}{2r^2}\left[\frac{-1}{r}\frac{\partial u}{\partial \alpha}\right]^2 \tag{3.12}$$

It can be easily verified that a solution of the form

$$u = \frac{a_0 N}{2\pi}\alpha \tag{3.13}$$

where a_0 is the layer spacing and N an integer minimizes this energy. This solution represents a spiral distortion, resulting in a helical wrapping of smectic layers as shown in Fig **3.3.** Substituting for **u** in equation **(3.12)** we find the extra energy due to the spiral distortion to be

$$\delta F = \left[k_{11} - \chi_a A^2\right]\frac{1}{2r^4}\left(\frac{\partial u}{\partial \alpha}\right)^2$$

This becomes negative for $A > (k_{11}/\chi_a)^{1/2}$making the system unstable against such distortions. This is a new type of dislocation. Such a structure was first described by Kleman and Parodi [2]. They looked at the stability of a structure with layers stacked in concentric cylinders against the helical structure. But energy calculations ruled out such a possibility. Also this spiral disclination is very different from that described by Bouligand **[3]** where the layers end on a cylindrical boundary.

3.3 Smectic C

Structurally these are similar to smectic A, but with the molecules tilted by an angle θ in a particular direction with respect to layer normal. This results in an additional degree of freedom. We find a weak curvature elasticity associated with azimuthal variations *i.e.,* distortion in the projection vector c. Also topological defects similar to the ones in smectic A can be generated here. Again many of the field effects are the same. In addition we find that in an H_α field, a disclination in the c-director(projection of the molecular axis to the plane of the layer) can be expected.

For small layer displacements u, with $\chi_a > 0$ the free energy density F (by ignoring coupling terms of Saupe theory) is given by:

$$F = \frac{k}{2}\theta^2(\nabla\phi)^2 - \frac{\chi_a A^2}{2r^2}\left[\frac{1}{r}\left(\frac{\partial u}{\partial \alpha}\right) + \theta\sin(\phi-\alpha)\right]^2 + \frac{k_{11}}{2r^4}\left[\frac{\partial^2 u}{\partial \alpha^2}\right]^2 \qquad (3.14)$$

Where

θ = the tilt angle

ϕ = azimuth of the c-director

k = the elastic constant for the in-plane bend or splay in the c-director

Minimization of the total energy with respect to ϕ and u leads to the following equations:

$$r k\theta^2\, \nabla^2\phi = \frac{\chi_a A^2}{r}\left[\frac{1}{r}\left(\frac{\partial u}{\partial \alpha}\right) + \theta\sin(\phi-\alpha)\right]\cos(\phi-\alpha)$$

$$\frac{k_{11}}{r^3}\frac{\partial^4 u}{\partial \alpha^4} = -\frac{\chi_a A^2}{r^2}\left[\frac{1}{r}\left(\frac{\partial u}{\partial \alpha}\right) + \theta\sin(\phi-\alpha)\right]\left[\frac{1}{r}\left(\frac{\partial^2 u}{\partial \alpha^2}\right) - \theta\cos(\phi-\alpha)\right]$$

These can be solved to get

$$\phi = \alpha + \frac{\pi}{2} \qquad (3.15)$$

and

$$u = \frac{a_0 N}{2\pi\alpha} \qquad (3.16)$$

Here again a_0 is the layer spacing and N is an integer.

Equation (3.15) represents a disclination in the c-director while (3.16) is the familiar screw dislocation. Substituting (3.15) and (3.16) in (3.14) we get the free energy density to be

$$F = \frac{\theta^2}{2r^2}\left[k - \chi_a A^2\right] - \frac{\chi_a A^2}{2r^2}\left[\left(\frac{b}{2\pi}\right)^2 + \frac{b\theta}{2\pi r}\right] \tag{3.17}$$

The second term in (3.17) is due to the screw dislocation. Since this term is negative at any value of **A** the uniform structure is unstable against screw dilocations. Moreover for fields $A >$ A, $(= (k/\chi_a)^{1/2})$ this energy if further lowered by the creation of a disclination in the c-director. The net distortion will have the features of both a screw dislocation and a disclination. Such composite structures are called dispirations [5,6,7].

'3.4 Columnar discotics

These mesophases are made up of disc like molecules stacked in columns with the columns arranged in a two dimensional lattice. In each columns the molecules are irregularly positioned *i.e.* fluid like order prevails [4]. Discotics have $\chi_a < 0$ with the director n perpendicular to the disc plane. In the simplest of the columnar discotics the lattice has hexagonal symmetry. One of the many possible defect states in this system has columns bent around into concentric circles [3] with the director forming an all circular pattern i.e.,$n_\alpha = 1$. In this section we will investigate the stability of this defect configuration in the presence of an all circular field H_α acting parallel to the column axis *i.e.* to the director n

Let us assume a perturbation of the form:

$$n_r = -\frac{1}{r}\left(\frac{\partial u}{\partial \alpha}\right), \; n_\alpha = 1 - \frac{1}{2r^2}\left(\frac{\partial u}{\partial \alpha}\right)^2, \; n_z = 0$$

where u is the displacement of the layers perpendicular to n and the singular line. The resulting structure represents various columnar circles in any given plane getting interconnected to form a spiral., Neglecting lattice distortions the change in free energy density due to this perturbations is

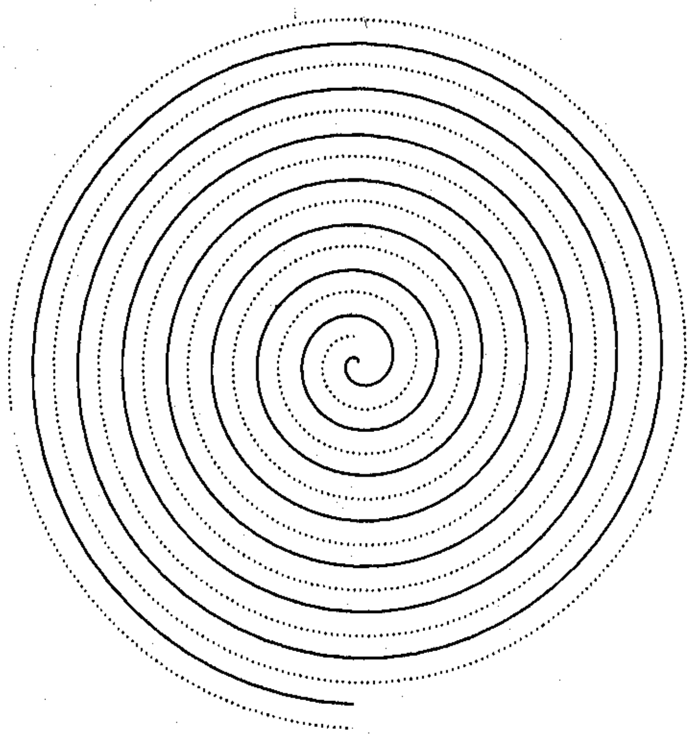

Fig **3.4:** Two neighboring columns in the spiral distortion in columnar discotics

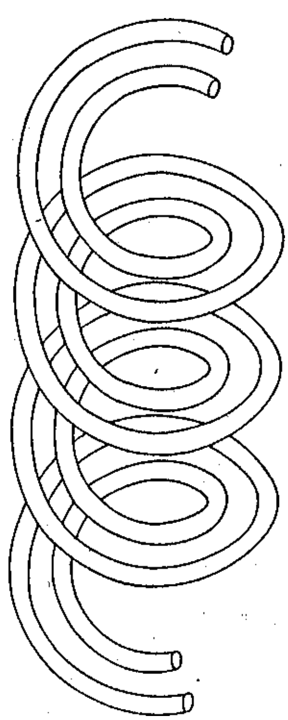

Fig **3.5:** Coaxial helices of discotic columns

$$\delta F_d = \frac{1}{2r^4}\left(\frac{\partial u}{\partial \alpha}\right)^2 (k_{33} - \chi_a A^2) \tag{3.18}$$

where k_{33} is the bend elastic constant

We see that a solution of the form u $= (b_\perp/2\pi)\alpha$ minimises the total free energy. Here $b_\perp$ is an integral multiple of the lattice spacing of the columns perpendicular to the singular line. We see from *(3.18)* that such *a* distortion will lower the free energy above a threshold field given by $A_, = (k_{33}/\chi_a)^{1/2}$. When this sets in the various circular1 columns in any given plane will get connected into a continuous spiral as shown in Fig *3.4.* Thus above this field a smectic like spiral dislocation will exist in addition to the disclination in the director n. The resulting distortion is again a dispiration in wich a classical disclination is associated with a spiral dislocation.

We will now investigate the effect of an another form of perturbation of the same system leading to a coaxial helical columns.

$$n_r = 0,\ n_\alpha = 1 - \frac{1}{2r^2}\left(\frac{\partial u}{\partial \alpha}\right)^2,\ n_z = -\frac{1}{r}\left(\frac{\partial u}{\partial \alpha}\right)$$

Again neglecting the lattice distortions, the change in the free energy density is:

$$\delta F = \frac{1}{2r^4}\left(\frac{\partial u}{\partial \alpha}\right)^2 (2k_{33} - \chi_a A^2) \tag{3.19}$$

From equation *(3.19)* we see that above the critical field given by $A_, = [2k_{33}/\chi_a]^{1/2}$ we get u $= \left[(b_{||}/2\pi)\alpha\right]$ as the lowest energy solution. Here $b_{||}$ is an integral multiple of the layer spacing parallel to the singular line. The resulting structure illustrated in Fig *3.5.* This has a helical connection between the circular columns of different planes,i.e., we get coaxial helices.

It should be remarked that these two structures are very different from the spiral columns around a cylinder proposed by Kleman [8] and a helical distortion around a helix discussed by Bouligand *[3]*for columnar discotics, on the basis of the theory of developable domains.

3.5 The core of a disclination near A-C transition.

Many smectic C materials undergo on heating a second order phase transition to smectic A at a definite temperature. Near such A-C transitions the tilt angle θ smoothly and continuously goes to zero. It is important to recognize that θ also becomes a function of the applied field. Moreover in a disclination in the c field there will be at the center a singularity in the c vector. This singularity gets lifted through a smooth decrease in the tilt angle which vanishes at $r = 0$. Hence θ will also be a function of r in the disclinations. It's variation is just the same as the variation of order parameter in a superfluid quantum vertex [9]. In this section we discuss the field effects on the variation of θ near the core of a disclination on both the smectic A side and the smectic C side of the transition point. We use a Landau theory with 0 as the order parameter.

3.5.1 Smectic C disclinations

Consider a ± 1 disclination in smectic C near A-C transition. Very near the transition the order parameter θ (tilt angle) is small. Then the free energy density for the core in the presence of a H_z field for $\chi_a > 0$ can be written as:

$$F_d = \frac{k}{2}\left[\left(\frac{\partial\theta}{\partial r}\right)^2 + \frac{\theta^2}{r^2}\right] + \frac{\acute{\alpha}}{2}\theta^2 + \frac{\acute{\beta}}{4}\theta^4 \qquad (3.20)$$

where

$$\acute{\alpha} = a + \chi_a H^2,\ \acute{\beta} = \beta - \frac{2}{3}\chi_a H^2;\ \alpha = \alpha_0(T - T_c) \text{ and } \beta > 0$$

α_0 and β are thermodynamic parameters of the classical Landau expansion. Minimisation of the total free energy leads to the Ginsburg-Pitavskii equation:

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial f}{\partial\xi}\right) - \frac{f}{\xi^2} + f(1 - f^2) = 0 \qquad (3.21)$$

where

$$= \frac{r}{\xi_0},\ \xi_0 = (-k/\acute{\alpha})^{1/2}\ f = \frac{\theta}{\theta_0},\ \theta_0 = (\acute{\alpha}/\acute{\beta})^{1/2} = \text{Tilt angle at r} \to \infty$$

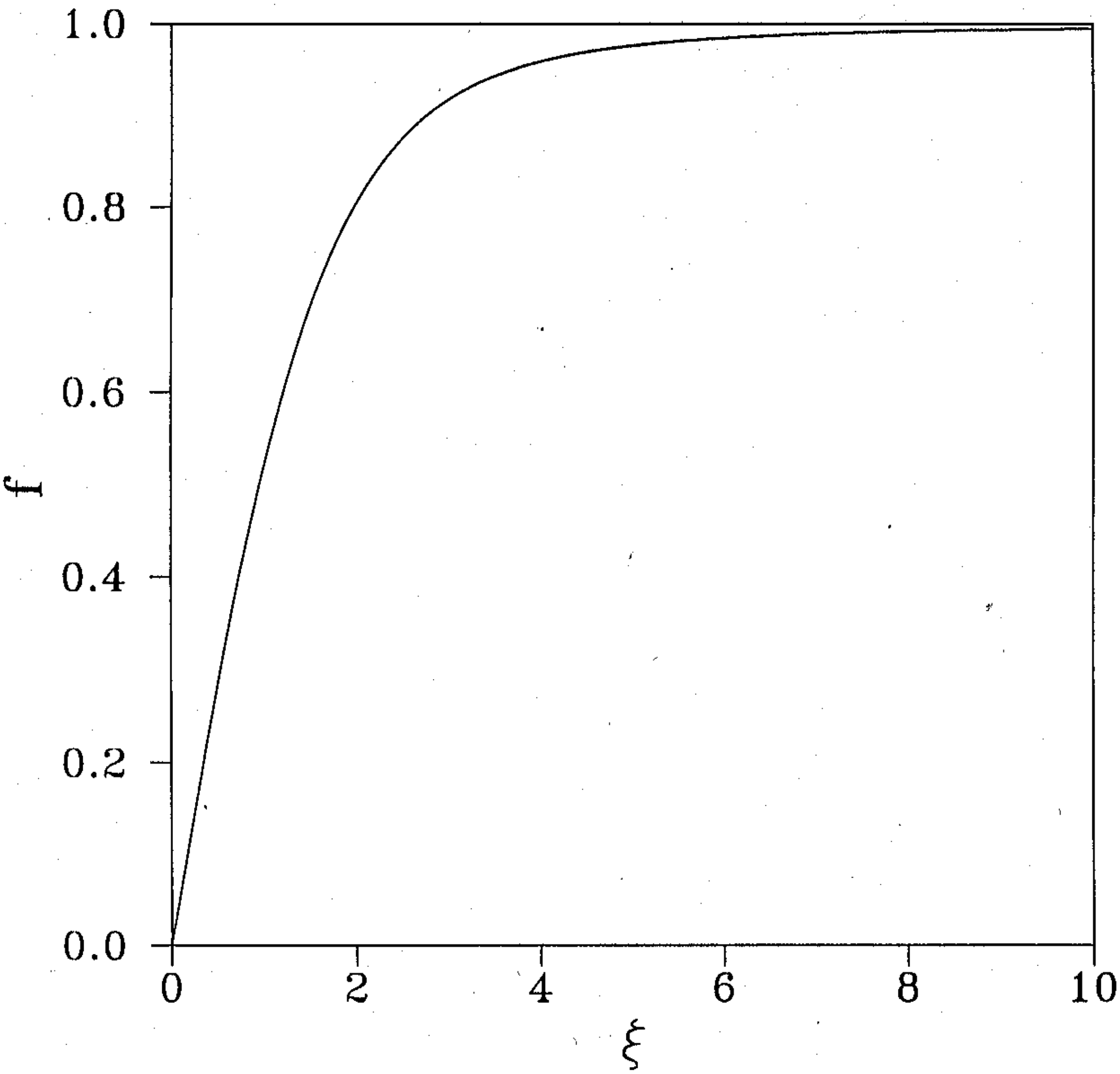

Fig **3.6:** The variation tilt angle in smectic C as a function of distance from the core of a +1 disclination (for $\chi_a = 10^{-7}$, $k = .5 \times 10^{-6}$)

In Fig **3.6** is shown the variation off as a function of ξ for $\acute{\beta} > 0$, the tilt angle drops to zero at the centre of the singularity. Most of the variation in θ takes place over a distance ξ_0. As H decreases ξ_0 increases slowly sweeping whole area. At the critical field H $= [|\alpha|/\chi_a]^{1/2}$ tilt angle becomes zero everywhere,i.e.,we get a smectic **A** state.

It is possible for $\acute{\beta}$ to be negative depending on the thermodynamic parameter β and the field strength H. When this happens we can speculate on the possibility of a first order transition to the smectic **A** state at a critical value of ξ.

3.5.2 Smectic A - field induced disclinations

Here if an all-circular field *i.e.* $H_\alpha = A/r$ acts parallel to the smectic layers in a $\chi_a > 0$ material, the free energy density will be of the form

$$F_d = \frac{k}{2}\left[(\nabla\theta)^2 + \frac{\theta^2}{r^2}\right] + \frac{\alpha\theta^2}{2} + \frac{\beta\theta^4}{2} + \frac{\chi_a A^2}{2r^4}\left(1 - \theta^2 + \frac{\theta^4}{3}\right) \qquad (3.22)$$

From equation **(3.22)** we see that due to the magnetic term the coefficient of θ^2 can be negative over a range of r. In this range of r, the field will induce a θ and we will have a smectic C-like disclination. **A** detailed analysis of this again leads to a Ginsburg-Pitaevskii type equation and the result is similar to the one shown if Fig 3.6.

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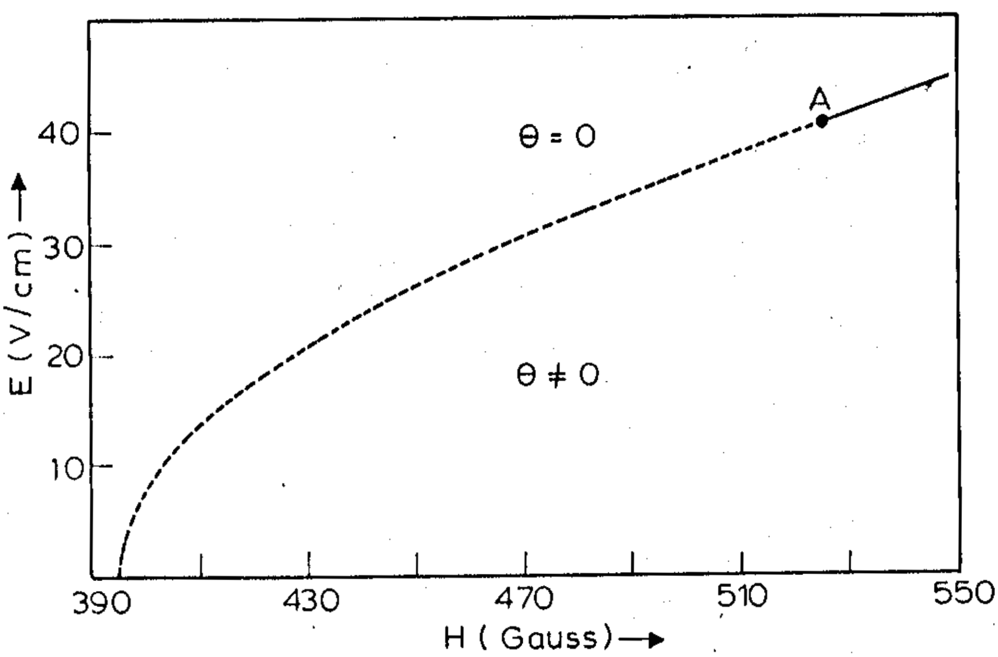

Fig 4.4: Tricritical behavior in parallel electric and magnetic fields in the homeotropic geometry. $d = 5\,\mu m$, $k_{11} = k_{22} = k_{33} = 10^{-6}$ dynes, $M = .1$ *Gauss*, $\epsilon_a = 0.1$ cgs units. The dashed line represents the second order transition and the full line the first order transition. The point **A** represents the tricritical point

If we give up the assumption that $k_{11} = k_{22} = k_{33}$ we find another interesting result. From Equation **4.4** we see that even when diamagnetic effects are ignored β changes sign at the Freedericksz transition threshold when $k_{33} = 4k_{11}/3$. Thus the transition is first order or second order depending on whether k_{33} is greater or less than this value resulting in a tricritical behavior.

4.2.2 Combined effect of electric and magnetic field

There are many important effects associated with Freedericksz transition in combined electric and magnetic field. For example a transition between the four states of distortions that are characterized by the uniform state, pure twist state, splay bend state and mixed state, in a homogeneously aligned nematic placed in crossed electric and magnetic fields that are orthogonal to the undistorted director was reported by Barbero *et.al.,* [9]. In view of this we have considered the same geometry in ferronematics also. Here we look at the effects of an electric field applied parallel 'to the magnetic field. For simplicity we assume $k_{11} = k_{22} = k_{33}$. we find that up to a critical value of the electric field $E_c = (2\pi/d)(k\pi/3\epsilon_a)^{1/2}$ the transition will be first order thus exhibiting tricritical behavior as in the case of M. This feature is illustrated in Fig **4.4.**

4.3 Homogeneous geometry

In this case also the magnetic field is antiparallel to M. Here we have both the out of plane *0* and in plane ϕ distortions. Just above the threshold we can assume θ and 4 to be small and given by the harmonic solutions:

$$\theta = \theta_m \sin \pi z/d$$
$$\phi = \phi_m \sin \pi z/d$$

Then the free energy density after averaging over the sample thickness is:

$$\bar{F}_d = F_0 + \frac{\alpha_1}{2}\theta_m^2 + \frac{\alpha_2}{2}\phi_m^2 + \frac{\beta_1}{4}\theta_m^4 + \frac{\beta_2}{4}\phi_m^4 + \frac{\delta}{2}\theta_m^2\phi_m^2 \tag{4.9}$$

here

$$\alpha_1 = \frac{1}{2}\left[\frac{k_{11}\pi^2}{d^2} - MH + \chi_a H^2 + \frac{\epsilon_a E^2}{4\pi}\right] \tag{4.10}$$

$$\alpha_2 = \frac{1}{2}\left[\frac{k_{22}\pi^2}{d^2} - MH + \chi_a H^2 + \frac{\epsilon_a E^2}{4\pi}\right] \quad (4.11)$$

$$\beta_1 = \frac{1}{4}\left[\frac{(k_{33}-k_{11})\pi^2}{d^2} + \frac{MH}{4} - \chi_a H^2 - \frac{\epsilon_a E^2}{4\pi}\right] \quad (4.12)$$

$$\beta_2 = \frac{1}{4}\left[\frac{MH}{4} - \chi_a H^2 - \frac{\epsilon_a E^2}{4\pi}\right] \quad (4.13)$$

$$\delta = \frac{1}{8}\left[\frac{(k_{33}-2k_{22})\pi^2}{d^2} + 3\left(\frac{MH}{2} - \chi_a H^2 - \frac{\epsilon_a E^2}{4\pi}\right)\right] \quad (4.14)$$

This expansion for $\bar{F}_d$ is similar to the generalized Lifshitz's expression for a two order parameter system [10].

When β_1 (or β_2) is negative , we also take the sixth power terms $(\gamma_1/6)\theta_m^6$ [or $(\gamma_2/6)\phi_m^6$] to 4.9 for stability reasons.

where:

$$\gamma_1 = \frac{1}{16}\left[\frac{(k_{11}-k_{33})\pi^2}{d^2} - \frac{MH}{24} + \frac{2}{3}\left(\chi_a H^2 + \frac{\epsilon_a E^2}{4\pi}\right)\right] \quad (4.15)$$

$$\gamma_2 = \frac{1}{16}\left[-\frac{MH}{24} + \frac{2}{3}\left(\chi_a H^2 + \frac{\epsilon_a E^2}{4\pi}\right)\right] \quad (4.16)$$

Lifshitz's theory predicts four possible states:

(a) $\theta = 0, \ \phi = 0$	when $a > 0$ (undistorted state)
(b) $\theta = 0, \ \phi \neq 0$ (c) $\theta \neq 0, \ \phi = 0$	when $a < 0, \ \beta < \lvert\delta\rvert$
(d) $\theta \neq 0, \ \phi \neq 0$	when $a < 0, \ \beta > \lvert\delta\rvert$

The possibility of obtaining these four states and transitions between them is investigated in the next section.

4.3.1 Magnetic field effects

We first look at the effect of χ_a in the one constant approximation. Numerical calculations show that condition (d) is never satisfied in this case and the transition

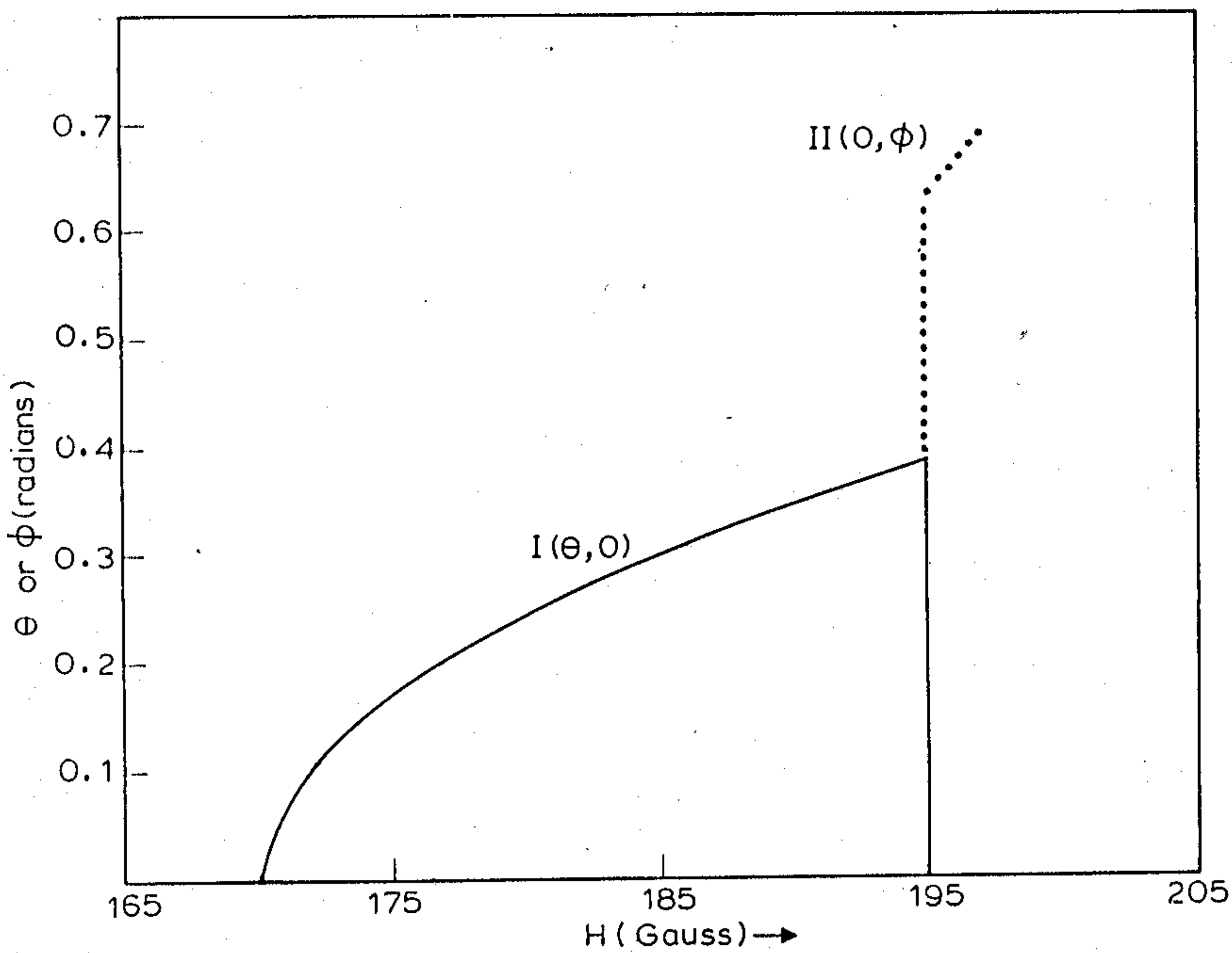

Fig 4.5: The classical second order Freedericksz transition with the second instability in the homogeneous geometry. The second transition is of first order and it is from splay-bend (represented as $(\theta, 0)$) to a twist configuration $(0, \phi)$. d $= 20\,\mu m$, $k_{11} =$ 0.55 x 10^{-6}, $k_{22} = 0.6$ x 10^{-6}, $k_{33} = 1.5$ x 10^{-6} dynes, M $= 8$ ***Gauss***, $\chi_a = 10^{-7}$ cgs units.

always takes place to states described by (b) or (c). Also in the one constant approximation (b) and (c) solutions are equally energetic. This transition to θ or 4 alone can be first or second order, depending on the value of M. Thus in this case it is possible to think of a new type of wall connecting the distortions θ and 4. This wall is quite different from the familiar Brochard walls which connect only degenerate θ (or ϕ). In the case of second order transition the wall will be continuous. In the case of Iirst order transitions the wall will have discontinuities in θ and ϕ.

Now we will assume diamagnetic terms to be absent and look at the effects of elastic anisotropy. A numerical calculations shows some interesting results. For example when $k_{11} < k_{22}$ we always get solution (c) ($\theta \neq 0, \phi = 0$). This transition is first order for $k_{33} < (4/3)k_{11}$ and second order for $k_{33} > (4/3)k_{11}$. For $k_{22} < k_{11}$ we always gel solution (b) ($\phi \neq 0,0 = 0$) whenever $k_{33} \geq k_{11}$ and this transition is always second order. However one can even get solution (c) *i.e.*, ($\phi = 0,0 \neq 0$), when $k_{22} < k_{11}$ provides k_{33} is very much smaller than k_{11} and the transition to this state is always first order.

When diamagnetic effects are also included we get solution (c) (4 = 0,0 # 0). for $k_{11} < k_{22}$ and $k_{33} \leq k_{11}$. Depending on the value of magnetization this transition will be first or second order. However, one can also get the solution (b) ($\phi \neq 0,0 = 0$) for $k_{11} < k_{22}$ provided k_{33} is very much smaller than k_{11} and the transition to this state is always first order. But in the case of $k_{22} < k_{11}$ the diamagnetic term plays no significant role and the results given in the previous section are valid.

Another interesting pbssibility in this geometry is the occurrence of a second thereshold at which the system goes from solution (b) to solution (c) or vice versa under certian conditions. It goes from ($\phi = 0,0 \neq 0$) to ($\phi \neq 0,0 = 0$) through a first order transition when $k_{11} < k_{22}$ and $k_{11} < k_{33}$. Similarly a first order transition from ($4 \neq 0, \theta = 0$) to ($4 = 0,0 \neq 0$) is also possible when $k_{11} > k_{22}$ and $k_{11} > k_{33}$. This is depicted in Fig 4.5

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