Chapter 6

Concluding summary and future directions

In this thesis, we have obtained the 2PN "instantaneous" corrections to the gravitational waveform and the far-zone fluxes for binaries in general orbits using the 2PN accurate BDI formalism. Using the expressions for the far-zone fluxes, we have obtained the 2PN "instantaneous" corrections to the evolution of the important orbital elements of the 2PN accurate generalized quasi-Keplerian parametrization for elliptical orbits. We also obtained all the "instantaneous" 2PN contributions to the gravitational wave polarizations for binaries in elliptical orbits. These expressions form one of the basic inputs to tackle the "wave generation problem" and the "radiation reaction problem" for the construction of the 2PN theoretical templates for binaries in quasi-eccentric orbits. We also deduced the gravitational radiation reaction to the second post-Newtonian order beyond the quadrupole approximation – 4.5PN terms in the relative equations of motion – using the refined balance method proposed by Iyer and Will, which involves the balance of the energy and the angular momentum. This completes the project on the computation of all the relevant analytical expressions required to construct the 'ready to use' 2PN accurate search templates for compact binaries in quasi-elliptical orbits.

There exists a number of theoretical issues which need to be addressed in the near future before one can explicitly provide the 'ready to use' 2PN accurate search templates for compact binaries in quasi-elliptical orbits. We briefly discuss them in what follows.

1. The 2PN accurate expressions for $h_\times$ and h_+ involve the 'tail' terms at the 1.5PN and the 2PN orders. In principle, these should be computable numerically in a straightforward manner following [128]. This should be added to our analytical results to obtain the complete 2PN accurate results.

2. To obtain the 2PN accurate expressions for $h_+(t)$ and $h_\times(t)$ for moderately eccentric binaries, we require the expansion of the eccentric anomaly in terms of a Fourier-Bessel series of the mean anomaly to the 2PN order.

3. It is well known from text books of celestial mechanics that the Fourier-Bessel expansion of the eccentric anomaly in terms of the mean anomaly is not in general applicable for the values of eccentricities greater than 0.66. For higher values of eccentricities, we may require a 2PN extension of the techniques employed in celestial mechanics for the orbital description of comets having very high eccentricities.

4. To obtain the simple relativistic 'timing formula' for the radio-wave observation of binary pulsars, Damour and Deruelle [147] introduced a 'proper-time eccentricity' associated with the object being timed. This expression for the 'proper-time eccentricity' involves along with the masses of the two compact objects in the binary, the three eccentricities and the orbital frequency associated with the quasi-Keplerian parametrization of the orbit. One may also want to look into the construction of a suitable 'eccentricity' for phasing of gravitational waves from compact binaries in quasi-elliptical orbits before venturing to explicitly construct 'ready-to-use' search templates for such binaries.

5. To obtain the 2PN accurate 'ready to use' search templates for compact binaries in quasi-elliptical orbits, i.e., elliptical orbits with radiation reaction, we

need to numerically integrate the 2PN accurate coupled equations for $< \frac{d\omega}{dt} >$ and $< \frac{de_r}{dt} >$ together with the 2PN accurate expansion of the eccentric anomaly in terms the mean anomaly obtained in item (2).

Finally, the refined balance procedure of Iyer and Will may be generalized to include the effects of the linear momentum flux motion of the center of mass and consequently on the individual accelerations of the two bodies of the compact binary of arbitrary mass ratio, moving in general orbits [158].

Currently, all these projects are under investigation [146, 158] and the explicit construction of the 2PN accurate theoretical templates for inspiraling compact binaries moving in *quasi-elliptical* orbits should be completed within the next year or two, well before the LIGO and VIRGO come on line.

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[154] For consistency of notation we have renamed the variables in IW in this paper. The following is a 'map' between the IW variable and the corresponding

notation used here, denoted as 'old' $\to$ 'new'.

$A_{\frac{5}{2}} \to A_{2.5}, B_{\frac{5}{2}} \to B_{2.5}, A_{\frac{7}{2}} \to A_{3.5}, B_{\frac{7}{2}} \to B_{3.5}, c_i \to f_i, d_i \to g_i,$ $\delta_i \to \xi_i, \varepsilon_i \to \rho_i,$ where $i = 1\ldots6$, $\gamma \to \alpha_1, \delta \to \alpha_2, \beta \to \alpha_3$ and $\kappa \to \beta_1, \alpha \to \beta_2, \epsilon \to \beta_3$.

[155] The combination $\eta(m/r^2)(m/r)$ chosen as prefactor in the IW ansatz for acceleration is proportional to $m_1 m_2$ or G^2 while the combination $\eta(m/r^2)$ chosen in the Appendix is proportional to $m_1 m_2 (m_1 + m_2)^{-1}$. Unlike the former choice, the latter is not a power series in m_1 and m_2 and hence while the former is the 'physically relevant solution' the latter is of 'mathematical interest'.

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