# CHAPTER 9

## SECULAR EVOLUTION OF NEUTRON STAR MAGNETIC FIELDS

In this chapter we shall attempt to highlight some aspects of the evolution of the magnetic fields of neutron stars from a study we have made of recently discovered millisecond pulsars. Even though the subject of millisecond pulsars appears unrelated to the main theme of the thesis, they have both clarified and raised some fundamental issues concerning the origin, the nature and the evolution of magnetic fields of neutron stars - a subject to which the whole of Chapter 3 was devoted.

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## 9.1 OVERVIEW AND SUMMARY

The magnetic.field of a pulsar is inferred from its observed rotation period P and slowdown rate $\dot{P}$ assuming magnetic dipole radiation or an equivalent mechanism to be responsible for its energy loss. Assuming a dipole slowdown law, the magnetic field can be expressed as:

$$B_* = 1.01 \times 10^{12} \text{ Gauss} \left(\frac{P}{1 \sec}\right)^{1/2} \left(\frac{\dot{P}}{10^{-15} \, s s^{-1}}\right)^{1/2}.$$

Here we have taken a value of $10^{45}$ gm $cm^2$ for the moment of inertia of the pulsar. The magnetic field derived this way refers to the dipole component at the surface of the neutron star. In chapter 2 we remarked that almost all observed pulsars have derived magnetic fields in the range $10^{11}$ to $10^{13.5}$ gauss. It is natural to ask what is the origin of this field and whether and how do they evolve in time. In chapter 3 we have discussed the question of the origin of pulsar magnetic fields, and have concluded that it is most likely that pulsar fields are either amplified "fossil" fields, or are generated very quickly after the neutron star formation. In this chapter we shall discuss the long term evolution of the magnetic fields of neutron stars, once they are created.

It has been argued for a long time that pulsar magnetic fields decay over a timescale of a few million years. The main argument for this comes from the comparison of the kinetic ages of pulsars with their spindown ages (Gunn and Ostriker 1970). Pulsars are born from massive stars which are distributed within a distance $\sim 60$ pc from the Galactic plane. In the process of formation, however, pulsars acquire space velocities $\sim 100$ km/s, which enable them to travel up to much greater distances from the Galactic plane. Thus pulsars achieve their observed scale height of $\sim 400$ pc (Manchester and Taylor 1977) during their lifetime of a few million gears. The distance of a pulsar from the Galactic plane is therefore an indicator of its age. The age derived this way is called the "kinetic age" of the pulsar. The spindown age of a pulsar is defined as the ratio of its present period and period derivative:

$$\tau_s \equiv \frac{P}{2\dot{P}} .$$

This equals the true age of the pulsar if its initial rotation period was much smaller than the present period, and if its magnetic field has remained constant since its birth. It is found that the kinetic ages and the spindown ages agree reasonably well for pulsars less than a few million years old. However, for pulsars with larger kinetic ages, the spindown ages significantly exceed the kinetic ages. While the kinetic ages of pulsars are less than $\sim 10^7$ years, their spindown ages have values in excess of $10^8$ years. Since the spindown age of a pulsar is inversely proportional to the square of its magnetic field, the observed discrepancy between kinetic and spindown ages suggests that the magnetic fields of pulsars must decay in a timescale of a few million years.*

Evidence of such a field decay is also apparent in the field period distribution of observed pulsars shown in fig. 9.1. As a pulsar ages and loses energy, its rotation slows down. As long as the magnetic field is constant, it moves horizontally in this diagram. However, as the period lengthens, its luminosity drops, and when the electrical voltage generated near its surface ($V \propto B/P^2$) falls below a certain value, the pulsar stops functioning (Ruderman and Sutherland 1975). This critical voltage is represented as the "death line" in fig. 9.1. Pulsars are not seen below this

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*One must remember, however, that the discrepancy between the kinetic and spindown ages described above in fact indicates a decay of the braking torque on the pulsar, which we have interpreted as the decay of the magnetic field. It has been argued that the decay of the braking torque may be caused by reasons other than the decay of the field strength, for example, it may be due to the alignment of the magnetic dipole axis of the pulsar with its rotation axis (see e.g. Kundt 1981). However, while an orthogonal rotator may lose energy by magnetic dipole radiation, an aligned rotator also may lose a similar amount of energy due to particle currents (Goldreich and Julian 1969). Thus it is not clear whether alignment (or counter-alignment: see Beskin et.al. 1983) of the magnetic dipole axis and the rotation axis of the neutron star can achieve the observed "torque decay".

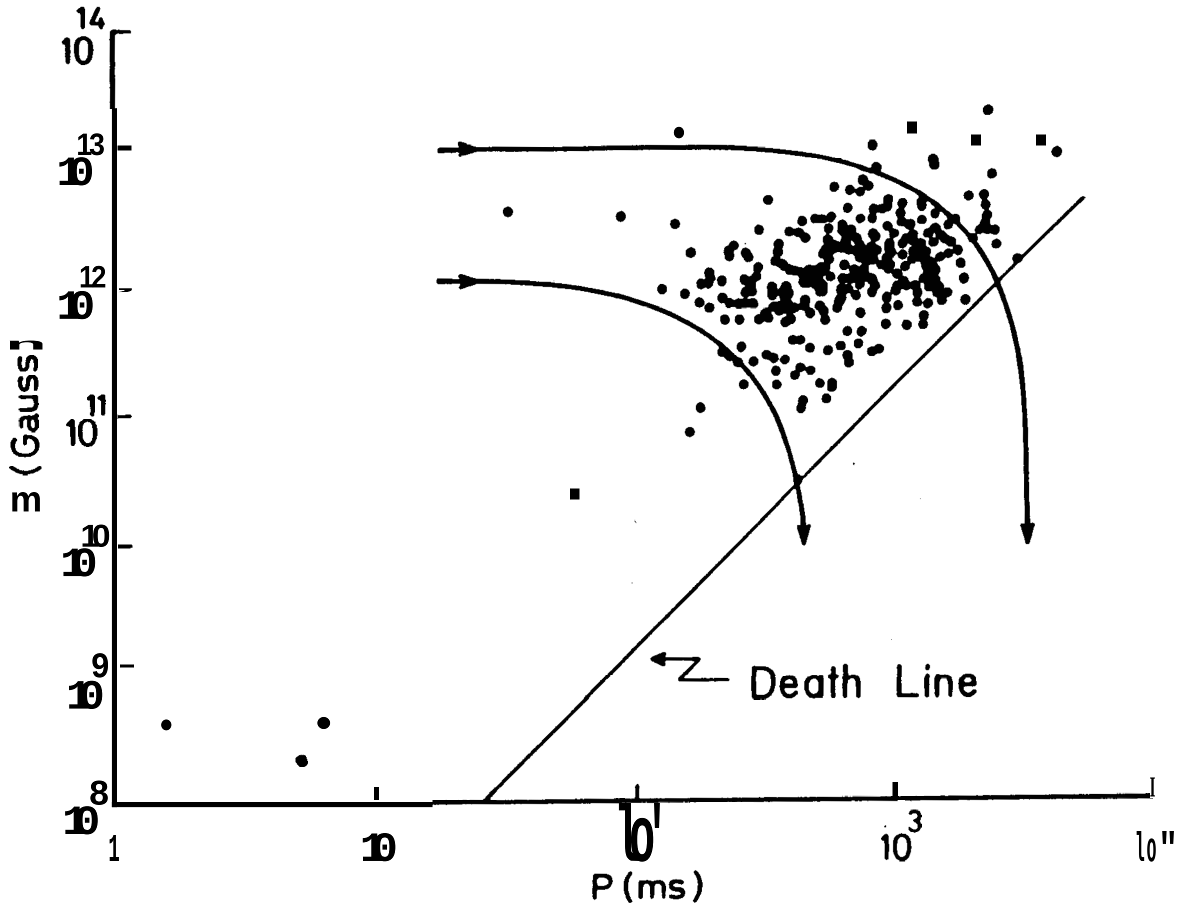


Fig. 9.1: The distribution of spin periods and derived magnetic fields of observed pulsars. The trajectories of pulsars with field decay and the Death line are also shown.

line. It is also noticeable from fig. 9.1 that as pulsars approach the death line, the fraction of low field pulsars among them increases. This again suggests that pulsar magnetic fields must diminish with age (Radhakrishnan 1982). A careful analysis of all these observations yields a timescale for the field decay process, and the current estimates lie in the range $\sim$ 4-8 million years (Lyne, Manchester and Taylor 1985; Chevalier and Emmering 1986).

Although a satisfactory theoretical explanation for this decay is lacking, it is usually assumed that the decay proceeds exponentially with time - as would be expected from, say, ohmic diffusion (Flowers and Ruderman 1977). We have shown in fig. 9.1 two typical trajectories of pulsars in the field-period diagram assuming an exponential decay of the magnetic field with an e-folding time of 4 million years. It may be seen that this fits closely with the observed distribution of pulsars. Lengthening of the rotation period dominates the evolution of a pulsar till a few million years, after which its field begins to decay and it moves down vertically in this diagram. Within a few decay timescales it drops below the death line and stops functioning as a pulsar. Thus a lifetime $\sim 10^7$ years for a pulsar can be understood in terms of field decay with a characteristic timescale $\sim$ a few million years.

An immediate conclusion of this scenario is that in about $10^8$ years the magnetic field of the neutron star will fall to $\lesssim 10^7$ qauss. Thus it came as a surprise when it turned out that the three millisecond pulsars are $3\ 10^9$ years old! This has raised some very interesting questions concerning the magnetic fields of neutron stars which we feel are worth stating. But first we shall outline the arguments that led to the conclusion that the millisecond pulsars are $\gtrsim 10^9$ years old. The details can be found in the original paper which is bound along with this thesis.

The millisecond pulsars are believed to have originated from mass transfer binary systems (Radhakrishnan and Srinivasan 1982; Alpar et.al. 1982; Henrichs and van den Heuvel 1983). Such a neutron star is born functioning as a normal pulsar with a high magnetic field ($\sim 10^{12}$ gauss), and after a few million years, ceases to function for the reasons detailed above. At this stage the spin period of the neutron star would be several seconds, and its magnetic field $\lesssim 10^{11}$ gauss. After some more time, its companion star .in the binary system evolves and grows in size. This initiates a transfer of mass on to the neutron star from its companion. The orbital angular momentum of the accreted material deposited on the neutron star spins it up to a short rotation period. We have shown in fig. 9.2 a typical such evolutionary track. The minimum rotation period the neutron star can achieve this way turns out to be a function only of the magnetic field of the neutron star, apart from its mass and

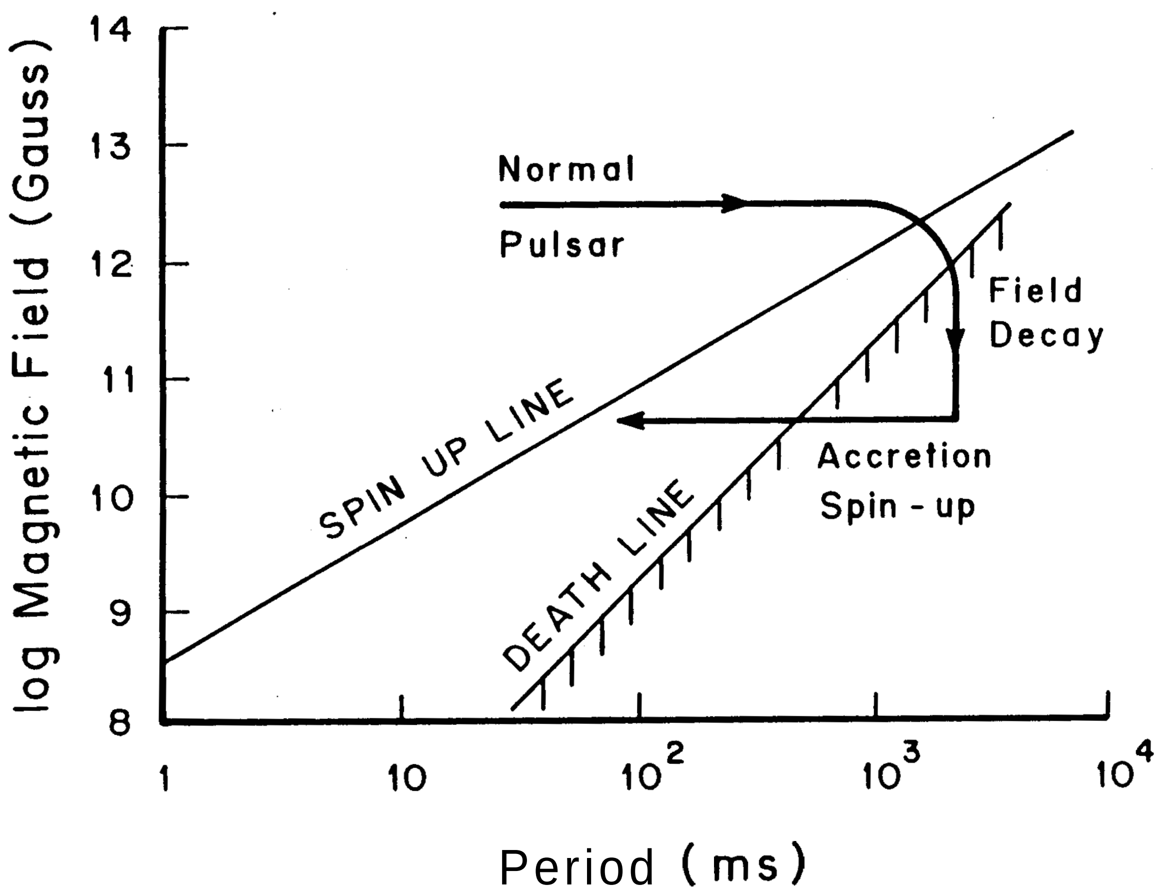


Fig. 9.2: The evolution of a neutron star in a close binary system. After the birth of the neutron star it functions as a normal pulsar till it crosses the death line due to spin down and field decay. Later the companion star starts transferring matter on to the neutron star, spinning it up. After the accretion phase is over, the neutron star will again function as a short period pulsar.

radius (Srinivasan and van den Heuvel 1982). This minimum period after spin up is shown in fig. 9.2 as the "spin-up" line. We can see from this figure that in order to be spun up to a few milliseconds, the magnetic field of the neutron star must be rather low: $\sim$ a few times $10^8$ gauss, the same as the observed field of the millisecond pulsars. When the mass transfer ceases, this neutron star will start functioning as a pulsar again.

For the spin up of the neutron star to proceed to such short periods one more condition must be satisfied, namely, that the total rotational energy deposited on the neutron star by the accreted matter must be $\sim 10^{52}$ ergs. This requires a prolonged duration for mass transfer: $10^7$ -$10^8$ years. The companion star should be able to maintain the mass transfer for such a long time. The only systems that can meet this requirement are those in which the companion of the neutron star is a low mass ($\sim 1 M_{\odot}$) giant star (Webbink, Rappaport and Savonije 1983; see van den Heuvel, 1984 for a review). Mass transfer in these systems is driven by the nuclear evolution of the low mass giant. There are known examples of such systems, and they are called "Low Mass X-ray Binaries" (LMXBs).

The total number of Low Mass X-ray Binaries in our Galaxy is estimated to be $\lesssim$ 100 (McClintock and Rappaport 1985). The lifetimes of these low mass X-ray binaries are expected to be typically $\sim 10^8$ years. If the magnetic field of a spun up millisecond pulsar continues to decay in a timescale of a few

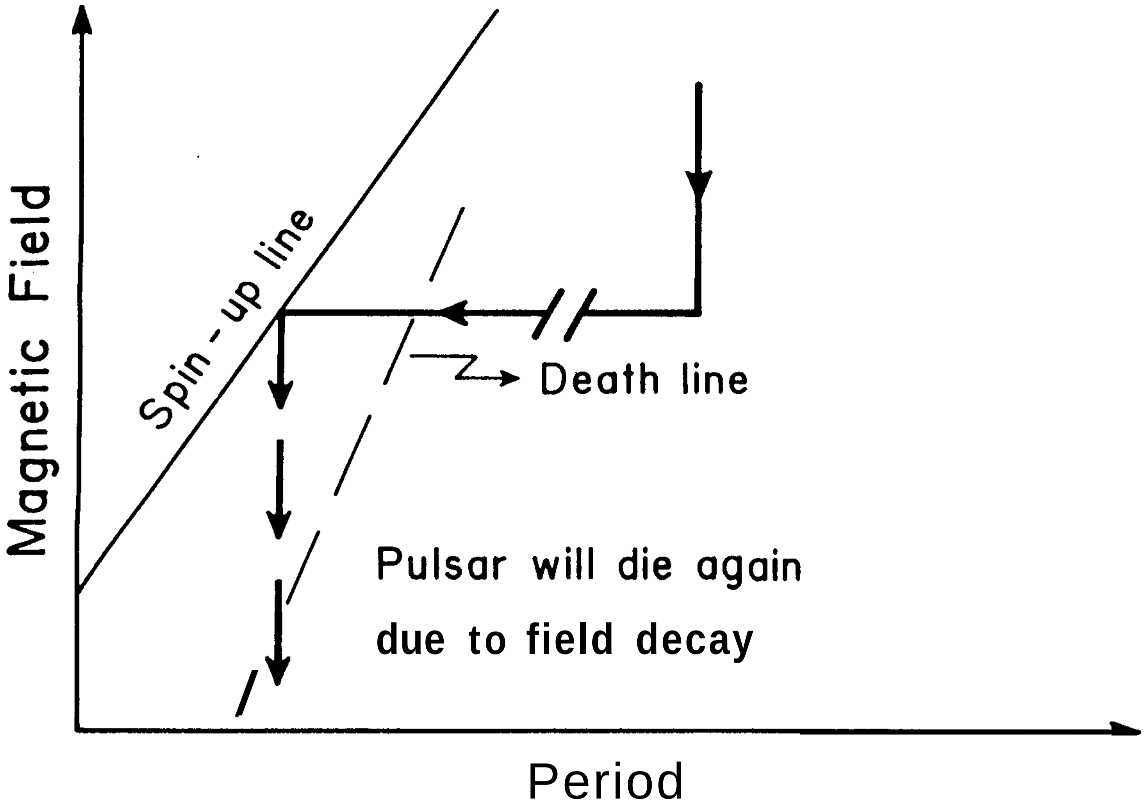
Magnetic Field
Spin - up line
Death line
Pulsar will die again
due to field decay
Period

Fig. 9.3

million years, then in less than $\sim 10^8$ years it will drop below the "death line" and stop functioning, or, in other words, the pulsar will "die again" (fig. 9.3). The lifetime of a millisecond pulsar would then be expected to be $\lesssim 10^8$ years, similar to the lifetimes of their progenitor systems, namely, the LMXBs. From this argument one would expect that the number of millisecond pulsars in the Galaxy to be similar to the number of LMXBs, i.e., $\sim 100$ or so.

The scale height of the population of LMXBs is $\sim 300$ pc. However, the three millisecond pulsars found so far lie within $\sim 20$ pc of the Galactic plane. One would, of course, expect the millisecond pulsars to have the same scale height as their progenitors. The location of the three pulsars detected so far must be a result of the limits of the regions surveyed by the instrument used to detect these pulsars, i.e. a selection effect. This immediately suggests that there must be many more unobserved millisecond pulsars in the Galaxy. In fact, scaling from the volume of the Galaxy in which they have been found to the volume they are expected to occupy yields a total number of $\sim 2000$-3000 millisecond pulsars in our Galaxy. This number is about 20-30 times the number of their progenitor systems, and therefore implies that the lifetime of millisecond pulsars should be $\sim 20$-30 times the lifetime of LMXBs. Thus,

**The millisecond pulsars must function for $\gtrsim 10^9$ years.**

The same conclusion has also been reached by van den Heuvel, van Paradijs and Taam (1986) based on similar arguments. Recently the White Dwarf companion of the 5.4 ms pulsar has been detected, and its low surface temperature ($\sim 6000$ K) suggests that its age is indeed more than $\sim 10^9$ years (Wright and Loh 1986). This is in agreement with the above conclusion, viz., that the millisecond pulsars are $\gtrsim 10^9$ years old. Clearly, this is not possible if the magnetic field continued to decay in a timescale of a few million years, as suggested by the observations of the normal pulsars.

This has raised several basic and interesting questions:

1. Does the magnetic field of a pulsar really decay, or should one try to understand the decay of the braking torque on a pulsar from a different point of view?

2. Why does the magnetic field decay? What is the characteristic timescale of the decay and does it depend upon the value of the field itself?

3. The derived magnetic fields of the $\sim 400$ or so pulsars discovered so far show a spread of about two orders of magnitude. And yet all the three millisecond pulsars discovered so far have almost identical field strength* (see fig. 9.1). How is this to be understood?

Clearly the answer to these questions will be related to the

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* The period derivative of the third millisecond pulsar has recently been measured (Taylor 1987), and it implies a magnetic field $\sim 2\times10^8$ gauss, very close to that of the other two millisecond pulsars.

nature of the magnetic fields of neutron stars. Is it due to macroscopic currents in the interior of the neutron stars, or is the field trapped in the vortex cores of the superconducting protons in the interior?

Flowers and Ruderman (1977) have suggested that the exterior poloidal field is due to toroidal current loops in the highly conducting fluid interior. But such a purely poloidal field is unstable because the current loops may realign themselves to minimize the far field. Flowers and Ruderman argued that such a field can be stabilized against realignment by a toroidal field which in turn is produced by poloidal currents. These stabilizing current loops are anchored in the solid crust of the neutron star. Thus if the outer layers crystallize before the dipole field diminishes due to the magnetohydrodynamic instability discussed above, the dipole field of the neutron star will be quite stable. However, Flowers and Ruderman point out that because of the finite conductivity of the crustal matter, the poloidal currents that close in the crust will decay due to ohmic dissipation, and they estimate the decay timescale to be $\sim$ a fewmillion years. This will result in a decay of the poloidal field with a similar characteristic time. This simple yet elegant mechanism for magnetic field decay appears to be in reasonable agreement with pulsar data. However, if such a decay continues indefinitely with the same timescale, one encounters the difficulty in explaining the magnetic fields of the millisecond pulsars which, as we have argued, must be $\gtrsim 10^9$ yr old.

If this is indeed the mechanism of the field decay, then what is it that arrests the decay?

According to the literature dealing with the interior of the neutron stars (e.g. Baym 1972,1977), the protons in the interior of neutron stars will be superconducting and it has been argued that they will form a type II superconductor. A type II superconductor has the property that it will allow a magnetic field to reside in it, provided the field strength is between two critical values. Fields with strengths less than the lower critical field will be expelled while those with strength larger than the upper critical field will destroy the superconductivity. In this intermediate range the magnetic field is confined in quantized flux tubes or vortices. Interestingly, estimates show that the value of the lower critical field is several orders of magnitude higher than the observed fields in pulsars, so that the field should be expelled from the interior. It has nevertheless been suggested that the magnetic field will be trapped in the vortices but that it will be in a "metastable state". If one tentatively accepts this picture, then it is even harder to understand why such a field would decay. Although a very elegant mechanism for the decay of such a field due to migration of these vortices to the surface has been suggested by Muslimov and Tsygan (1985), there is disagreement about the timescale of such a decay. Whereas Muslimov and Tsygan obtain timescale $\sim 10^6$ yr, Harvey, Ruderman and Shaham (1986) have pointed out that it must be much larger $\gtrsim 10^9$ yr. At any rate, if one invokes such a mechanism for the decay of the field then it is hard to understand why there is any residual field in the very old neutron stars.

We wish to suggest the following possible resolution of the difficulties mentioned in the preceding discussion. First about the nature of the field.

As we have remarked, since the observed fields of pulsars are very much less than the estimated lower critical field, the straightforward conclusion would have been that the field should be expelled. But since the field was due to electron currents in the interior, it is not energetically favourable for the proton superconductor to "expel" this field. It is because of this that the suggestion was made that the field would be trapped in vortices but that it would be a metastable state. In view of the observational clues one now has regarding the secular behaviour of the magnetic field, it appears to us more plausible that the onset of superconductivity in the interior will be delayed till the currents in the interior have dissipated. The picture we wish to suggest is the following. The field decays initially for exactly the same reason as suggested by Flowers and Ruderman, namely that the poloidal currents which close in the crust decay due to ohmic dissipation in a timescale which is appropriate to the conductivity of the crustal matter ($\tau_{\mathbf{decay}} \sim$ a few million years). After this, the poloidal current loops which are needed to stabilize the field will close entirely within the highly conducting interior. Baym (1972) estimates that the timescale of ohmic dissipation in the interior may be as long as $\sim 10^9$ yr. In this picture, the field will continue to decay, but now with a much longer timescale. Thus, there will be no difficulty in explaining the presence of very large fields in very old neutron stars. But a very basic problem still remains. The fact that all the three millisecond pulsars have a field strength of about $5 \times 10^8$ gauss suggests that if the scenario we are advancing is correct, then the change over between these two timescales occurs when the field decays to about its present value. We do not yet have any explanation for this. But it is interesting to note the following.

The atoms in the crust of a normal pulsar are expected to have a highly anisotropic charge distribution due to the influence of the strong magnetic fields of order $10^{12}$ gauss (Cohen, Lodenquai and Ruderman, 1970). However, when the field falls to a value $\sim 10^8$ gauss (Larmor radius Bohr radius for this field), it ceases to have a significant effect on the atomic structure, and as a result, the electrical properties of the crust may undergo a substantial modification. Since this transition value of the magnetic field is very close to the observed asymptotic fields of very old (millisecond) pulsars, it is worth asking if there is a possible connection between these two phenomena (J. Samuel, C.S. Shukre and V. Radhakrishnan, private communication).

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# ON THE IMPLICATION OF THE RECENTLY DISCOVERED 5 MILLISECOND BINARY PULSAR PSR 1855+09

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**ABSTRACT**

We argue that the extraordinary fact that all three known millisecond pulsars are very close to the galactic plane implies that there must be ~ **100** potentially observable millisecond pulsars within ~ 4 **kpc** from the Sun. Our other main conclusion is that the dipole magnetic fields of old neutron stars probably saturate around 5 **x** 10' gauss. .

## I. INTRODUCTION

**ONE of the** unsolved **puzzles** about the two fastest **pulsars**, the 1.5 **ms** pulsar (PSR **1937+21) and** the 6 ms **binary** pulsar (PSR 1953+29) concerns their location: ***why are these two pulsars, which must surely be very old, so close to the galactic plane?*** The recent discovery[1] of a 5 ms binary pulsar PSR **1855**+09, again very close to the plane, underscores this **puzzle**. We address this question and its implications in this article. Table I summarizes some of the relevant parameters of these three pulsars.

## II. PROGENITORS OF MILLISECOND PULSARS

It is now widely accepted that millisecond pulsars must have been spun up in mass transfer binary systems[2-4]. If one grants that **the** neutron star accretes from its companion at the Eddington rate for sufficiently long, then it can be spun up to ultra-short periods. The observed P and $\dot{P}$ for the **1.5 ms** pulsar and the 6 ms pulsar are consistent with this picture. Even though there is as yet no consensus on the details of such a recyclingscenario for the 1.5 ms pulsar·', the picture is much clearer for the 6 ms pulsar which is still in a binary. It has been very convincingly argued by Joss and **Rappaport**[5] that the progenitor for this latter system must have been **a** low-mass X-ray **binary** (LMXB); many such systems are known in our galaxy. These are believed to consist of neutron stars **accreting** from low mass giants. As they have argued, in such systems there is no difficulty in maintaining mass transfer at close to the Eddington rate for as long as ~ $10^8$ **years** or more.

PSR **1855+**09: Drawing from the **detailed** work of Webbink ***et al***[6], **we** wish to suggest that this pulsar, too, must have evolved from an LMXB. An initial orbital period ~ 1 day would lead to the presently observed orbital period of 12.33 days. In this case the mass transfer probably lasted for ~ $10^9$ years, implying that this pulsar must be even older than PSR 1953+29.

Why are all these three very old pulsars so close to the galactic plane? "Normal" pulsars have a scale height' **~350 pc,** much larger than the scale height of their progenitors, which is only **~60 pc**. This is easily understood if neutron stars acquire substantial velocities at birth. Even if millisecond pulsars are not crealed'with such velocities, one would expect them to have a scale height at least comparable to-that of their progenitors.

As already mentioned, the progenitors of millisecond pulsars, at least those which are in binary systems, must be LMXBs. Presently about 30 LMXBs are known in our galaxy and the total number[8-10] is probably —100. The 20 or so sources for which reasonably accurate distances are known clearly reveal that they have a scale height **~300 pc,** consistent with their belonging to the old disk population. It is, therefore, extraordinary that all **three** millisecond pulsars discovered are within **~25 pc** of the galactic plane.

## III. POSSIBLE WAYS OUT

This remarkable "coincidence" could in principle be rationalised under two exotic circumstances:

(a) The scale height of the very old neutron stars previously associated with LMXBs is indeed large, but for some reason they do not function as pulsars unless they are in an environment that obtains only very close to the galactic plane.

(b) For some unknown reason, the velocity dispersion

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Table 1 *The millisecond pulsars*

| Pulsar | Spin period (ms) | Magnetic field ($10^8$ gauss) | Orbital period (days) | Most likely companion mass ($M_\odot$) | Distance (kpc) | z* (pc) | Ref. |
|---|---|---|---|---|---|---|---|
| 1937+21 | 1.55 | 4.5 | — | — | −3–5 | ~−20 | (a) |
| 1953+29 | 6.13 | 4.5 | 117.3 | 0.2–0.5 | ~3 | ~24 | (b) |
| 1855+09 | 5.36 | — | 12.33 | 0.25–0.6 | ~0.4 | ~24 | (c) |

(a) Backer, D. C. 1984, *J. Astrophys. Astron.*, *5*, 187
(b) Rawley, L. A. et. al. 1986, *Nature, (London)*, *319*, 383
(c) Segelstein, D. J. el. al. 1986, *IAU Circ.* no. 4162
* Distance from the galactic plane

of these old pulsars *decreases* with time and they "settle down" near the plane.

At present we have no suggestions to offer concerning either of these possibilities. This leaves us with only one alternative: The proximity of the three known millisecond pulsars to the galactic plane is accidental, or possibly due to selection effects which we do not understand. If so, an immediate implication of this is that the sample of millisecond pulsars must be grossly incomplete even within the distances upto which they have been found. If the scale height Tor these pulsars is ~300 pc, like the LMXBs Trom which they must have come, then one would expect only ~10% of them to be within ~30 pc from the galactic plane.

This suggests that there should be at least ~30 millisecond pulsars in the region in which the three known pulsars lie, namely between the galactic longitudes 30° and 90° and within −4 kpc from the sun. This implies that within the above distance from the sun, there should be at least ~$10^2$ millisecond pulsars. This number must surely be a lower limit since it does not allow for the beaming factor of pulsars (taken to be ~5 for circular beams) and various other selection effects due to dispersion and luminosity .which are likely to be severe Tor ultrafast pulsars. Taking a conservative attitude, it is reasonable to speculate that there should be $\gtrsim 3 \times 10^2$ millisecond pulsars within about −4 kpc, implying a total number in the galaxy $\gtrsim 2 \times 10^3$.

This number is clearly discrepant with the number of LMXBs in the galaxy. As already mentioned, the number of LMXBs is not expected to be much larger than −100. Our basic premise, namely that these millisecond pulsars evolve from LMXBs, then implies that their lifetimes must be 20–30 times longer than the X-ray phase of these systems, which are believed to last for ~$10^8$ years. We thus conclude that millisecond pulsars must live for $\gtrsim 10^9$ years.

## IV. FIELD SATURATION

We shall now argue that such long lifetimes are possible for pulsars only if their magnetic dipole moments, after an initial decay, essentially saturate. It is possible to obtain a lower limit for the decay timescale of the dipole field in the following manner. It is now generally accepted that millisecond pulsars make their appearance in the B–P plane along the so-called "spin-up" or "equilibrium period" line ($P$ a $B^{6/7}$)[4]. They will die when they cross the "death tine" ($P \propto B^{1/2}$)[11,12]. From figure 1 it is clear that a pulsar spun up to the shortest period will have the longest lifetime, and will therefore give a lower limit to the timescale of possible field decay. The standard dipole braking law together with an exponential decay of the surface dipole field (with timescale $\tau_d$) yields for the lifetime of a 1 ms pulsar

$$t \approx 4.2 \times 10^9\,\mathrm{yr}\ \tau'_d \ln\left[\left\{1 + \sqrt{1 + 10^{-6}\,\tau'_d(1+\tau'_d)}\right\} \times (1+\tau'_d)^{-1}\right]$$

where $\tau'_d = \tau_d/(6.2 \times 10^8\,\mathrm{yr})$

From the above equation, it follows that a lifetime $t > 10^9$ years is possible only if $\tau_d \gtrsim 10^9$ years. It will be recalled that the statistics of normal pulsars strongly suggest that their magnetic fields decay with a characteristic time ~a few million years[13]. The apparent 1000-fold lengthening of the decay timescale is perhaps indicative of field saturation around $B \lesssim 10^9$ gauss. van den Heuvel, van Paradijs and Taam[14] have also recently arrived at the conclusion that fields must saturate from a similar argument, but they estimate the total number of millisecond pulsars in the galaxy from a different viewpoint. Motivated by the very large apparent age of the white dwarf companion of the binary radio pulsar PSR 0655+64, Kulkarni'' has advocated that magnetic fields of neutron stars **must** eventually saturate.

***An independent argument:*** As was mentioned in the beginning, it is very probable that the newly discovered millisecond pulsar evolved from a low mass X-ray binary with an initial orbital period $\lesssim 2$ days. The mass transfer in a system with such initial conditions lasts for almost $(6–10) \times 10^8$ years at a rate $\lesssim 10^{-9}$ $M_{\odot} yr^{-1}$, which is smaller than the Eddington rate for a standard neutron star. Consequently, when the binary finally **detaches**, the neutron star could be almost $\sim 10^9$ years old. This poses a problem: how come such an old neutron star still has a substantial field to enable it to function as a pulsar. The simplest explanation is that its field must have saturated.

While discussing the evolutionary scenario for the **6 ms** pulsar, and faced with a similar difficulty, van den **Heuvel** and **Taam**[16] suggested that the neutron star was formed by the accretion induced collapse of a whitedwarf. This way the age of the neutron star needs to be only a fraction of the duration of the mass transfer phase, and the problem mentioned above may not arise. However, such an alternative may not work in the case of the present system. It is believed that at the relatively low accretion rate mentioned above, nova eruptions will prevent the white dwarf from approaching the Chandrasekhar limit''. If so, the neutron star must indeed be very old and one is therefore forced to conclude that the field must have saturated.

Flowers and Ruderman[1'] have argued that the current loops in the superconducting interior of the star are **stabilised** against rearrangements by currents flowing in **the crust**. Since thecrustal current is likely to decay due to ohmic dissipation in a **timescale** of a few million years, the decay **of** the dipole field takes place over the same **timescale**. It is not at all clear what the final value of the dipole field will be after a sufficiently long time. We feel that the dipole field will eventually saturate at some value, and this will be the same for all neutron stars. It is extraordinary that the recent measurement[1'] of $\dot{P}$ for the **6 ms** pulsar yields a value for its surface magnetic field which is almost identical to that of the 1.5 ms pulsar. Since the 6 ms pulsar is so far from the spin up line (figure I), we feel that this cannot be a coincidence, and $\sim 5 \times 10^8$ gauss may be the sort of field strength at which the dipole fields of neutron stars saturate. If the magnetic field of the **newly** discovered pulsar PSR **1855**+09 also turns out to be the same it will provide strong support for our conjecture. It is interesting that in an attempt to model superhigh energy radiation from old neutron stars in accreting binary systems, **Ruderman**[19] has suggested that their magnetic fields may be $\sim 10^8$ gauss. **The recently** discovered Quasi-Periodic Oscillations in some LMXBs may also be providing independent evidence that the neutron stars in them havea magnetic field $\gtrsim 5 \times 10^8$ **gauss**[20,21].

If **the** above **conjecture** is true, then it has a very interesting consequence for the evolution of pulsars processed in LMXBs. **Let** us assume that the duration **of** the mass **transfer** is longer than the time it would take for the dipole field to decay to its saturation value. This will probably be the case for all LMXBs with initial orbital periods $\leqslant$ **10 days,**[6] and if the decay time for the magnetic field continues to be $\sim$ a few million years **till** the field saturates. The neutron stars In such systems will be spun up to some point along the equilibrium period line and then they will dribble down along this line till a limiting period $\sim$ 1.5 ms is reached, corresponding to field saturation around $\sim 5 \times 10'$ gauss. **After** the mass transfer stops. and the neutron star starts functioningas a *pulsar*, its evolution will be ***sideways, i.e.*** the period will lengthen, as shown in figure 1. But given the age of the galaxy, the

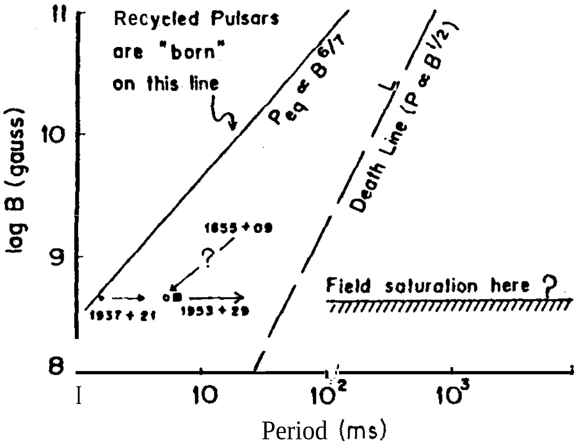


Figure **1**. The derived magnetic fields of the 1.5 ms pulsar (1937+21) and the 6 ms pulsar (1953+29) are shown. Also plotted are the "spin-up." line corresponding to the critical accretion rate ($\dot{M}_{\text{Eddington}}$) and the "death line" for pulsars. It is suggested that the magnetic fields of all old neutron stars, including that of PSR 1855+09, will be $\sim 5 \times 10^8$ **gauss**. The horizontal arrows represent the future evolutionary tracks of these pulsars.

maximum period reached will only be ~ 10 ms. Since $\frac{dP}{dt} \propto \frac{1}{P}$, there will be a "piling up" of these pulsars near $P \leqslant 10$ ms. The fact that two of the three millisecond pulsars discovered so far have period ~ 6 ms is not inconsistent with the above picture.

## V. CONCLUSIONS

1. The fact that all three known millisecond pulsars are so close to the galactic plane can only be a "selection" effect. Millisecond pulsars must have a scale height comparable to that of LMXBs from which they probably evolve and we predict that there should be ~ 100 potentially observable millisecond pulsars within — 4 kpc.
2. Such a large number of millisecond pulsars is consistent with the birthrate of LMXBs only if the lifetime of these pulsars is more than ~ $10^9$ years. This, in turn. would require that their magnetic fields do not decay indefinitely, but saturate.
3. We predict that such a saturation occurs around ~ 5 x $10^8$ gauss. This will be confirmed if the newly discovered pulsar turns out to have its field near this value.
4. We also predict that the majority of ultrafast pulsars will have periods in the interval 6–10 ms.

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