The Perturbative Evolution of
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DECLARATION

I hereby declare that the work presented in this thesis is entirely original, and has been carried out by me at the Raman Research Institute, Bangalore, under the auspices of the Department of Physics, Indian Institute of Science, Bangalore. I further declare that this has not formed the basis for the award of any degree, diploma, membership, associateship or similar title of any university or institution.

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Synopsis

Most models of the formation of the structure we see in the universe today are based on gravitational instability. While very large scales evolve rather simply in the linear theory, there is a significant amount of nonlinearity even at the largest scales for which reliable observations are available. Numerical simulations suffer from serious boundary effects on such large scales. Further, some conceptual issues in this regime are best approached by analytic work. The understanding of weak gravitational clustering is the main motivation for the work described in this thesis where the dynamical evolution of some statistical properties of disturbances in an expanding universe is studied theoretically. The main points discussed in the thesis are listed below.

Chapter II.

1. The universe is modeled as a system of particles interacting only through the Newtonian gravitational force. This is appropriate on scales much smaller than the horizon scale but large enough that gravity is the dominant force. We consider an ensemble of such systems and we set up the equations of the BBGKY hierarchy in the fluid limit to study the evolution of some of the statistical properties of such an ensemble. A convenient parameter is used for the evolution instead of the cosmic time.

2. The initial conditions are chosen such that the deviation of any of the systems from the uniform state can be characterized by a small parameter ϵ. The initial conditions are also such that all the members of the ensemble have a single streamed flow. These initial conditions allow us to associate powers of ϵ with various statistical quantities for the ensemble. We consider the second and the third equations of the BBGKY hierarchy. By taking velocity moments of these equations we obtain equations for perturbatively evolving the two point and the three point correlation functions. At the lowest order of perturbation these equations give us the linear evolution of the initial two point and three point correlation functions.

Chapter III.

3. We consider a situation where the initial disturbances are such that the density fluctuation is a random Gaussian field in a universe with the critical density. For these initial conditions the initial three point correlation function is zero. We calculate the nonlinearity-induced three point correlation function at the lowest order of perturbation for which it is non-zero. We obtain a general expression for this in

terms of the linear two point correlation function and its average over a sphere. This investigation brings out the limitations of the commonly used hierarchical form.

Chapter IV.

4. In general the evolution of the two point correlation is influenced by the three point correlation function. The **BBGKY** hierarchy equations are used to calculate perturbatively, the lowest order nonlinear correction to the two point correlation and the pair velocity for Gaussian initial conditions. Our formalism is valid even if the flow becomes multi-streamed as the evolution proceeds. We compare our results with the results obtained using the hydrodynamic equations which neglect pressure and other effects of multi-streaming. We find that the two match, indicating that there are no effects of multi-streaming at the lowest order of nonlinearity.

Chapter V.

5. We study the two point correlation induced at large scales for the case when it is initially zero there. Based on an analytic study confirmed by numerical results we conclude that this has a universal x^{-6} behaviour.

6. We numerically study a class of initial conditions where the power spectrum at small k has the form k^n with $0 < n \leq 3$ and we calculate the nonlinear correction to the two point correlation, its average over a sphere and the pair velocity over a large dynamical range. We find that at small separations the effect of the nonlinear term is to enhance the clustering whereas at intermediate scales it can act to either increase or decrease the clustering. We also find that the small scales significantly influence the evolution at large scales and this may lead to a possible early breakdown of linear theory at large scales due to spatial nonlocality. We obtain a simple fitting formula for the nonlinear corrections at large scales and we interpret this in terms of a diffusion process. We also investigate the case with $n = 0$ and we find that it differs from the other cases.

7. We use the perturbative calculations described above to numerically investigate a widely discussed universal relation between the pair velocity and the average of the two point correlation. We find that in the weakly nonlinear regime there is no universal relation between these two quantities.

Chapter VI.

8. The Zel'dovich approximation (ZA) is used to study some of the issues that have been studied perturbatively for the full gravitational dynamics (GD) in the previous chapters. We investigate whether it is possible to study perturbatively the transition between a single streamed flow and a multi-streamed flow. We do this by calculating the evolution of the two point correlation function using two meth-

ods: a. Distribution functions b. Hydrodynamic equations without pressure and vorticity. The latter method breaks down once multi-streaming occurs whereas the former does not. We find that the two methods give the same results to all orders in a perturbative expansion. We thus conclude that we cannot study the transition from a single stream flow to a multi-stream flow in a perturbative expansion. We expect this conclusion to hold even if we use the full GD instead of ZA, as already checked at the lowest order of nonlinearity.

9. We calculate nonperturbative expressions for the evolution of the two point correlation function, the pair velocity and its dispersion in the Zel'dovich approximation. We numerically investigate these formulae at various scales.

10. We use ZA to look analytically at the evolution of the two point correlation function at large spatial separations and we find that until the onset of multi-streaming the evolution can be described by a diffusion process where the linear evolution at large scales gets modified by the rearrangement of matter on small scales. We compare these results with the lowest order nonlinear results from GD. We find that the difference is only in the numerical value of the diffusion coefficient and we interpret this physically.

11. We also use ZA to study the induced three point correlation function. At the lowest order of nonlinearity we find that, as in the case of GD, the three point correlation does not necessarily have the hierarchical form. We also find that at large separations the effect of the higher order terms for the three point correlation function is very similar to that for the two point correlation and it can be described in terms of a diffusion process.

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