Chapter 2

Defects in a nematic with a latent lattice

2.1 Introduction

The introductory chapter dealt with solitons in intrinsically achiral systems like nematics. **A** cholesteric liquid crystal on application of an external magnetic field perpendicular to the helical axis, develops a non-uniform twist in the form of a lattice of π twist solitons. Further, at a threshold field, the state gets completely unwound leading to a nematic like director configuration. **A** nematic state can also be obtained by deforming a cholesteric in a field applied along the twist axis.

In either case, the lattice order that was manifest previously at a lower field influences the structure and energetics of solitons that are created in the unwound state. This unwound state is referred to as a latent lattice.

We study in this chapter the solitons permitted in such a ***nematic*** state but with a latent lattice order. For our discussion we have considered two different geometries.

2.2 Planar solitons in H perpendicular to the twist axis

A cholesteric with positive diamagnetic anisotropy ($\chi_a > 0$), can be unwound by the application of a magnetic field perpendicular to the axis of the helix [1, 2]. This is a well known transition and the transformation is mediated, as said before, by the formation of π solitons. At a certain critical field H_c, given by

$$H_c = \frac{\pi q_0}{2}\sqrt{\frac{K_{22}}{\chi_a}} \tag{2.1}$$

the solitons become infinetly seperated from each other and we get a nematic state. This process of cholesteric to nematic phase transition is shown in Figure 2.1. The nematic thus obtained from the cholesteric however has a memory of the lattice. This can be seen by lowering the field in the nematic state and below the critical field the π solitons are spontaneously generated. We consider here, solitons in the unwound cholesteric i.e., in the nematic state.

2.2.1 Defects in an unwound cholesteric

The free energy density of distortion of a cholesteric liquid crystal in the presence of an external magnetic field can be written as

$$F = \frac{K_{11}}{2}(\nabla\cdot\mathbf{n})^2+\frac{K_{22}}{2}(\mathbf{n}\cdot\nabla\times\mathbf{n}-q_0)^2+\frac{K_{33}}{2}(\mathbf{n}\times(\nabla\times\mathbf{n}))^2-\frac{\chi_a}{2}(\mathbf{H}\cdot\mathbf{n})^2-(\frac{K_{22}}{2}q_0^2) \tag{2.2}$$

Here $q_0 = 2\pi/\mathrm{P}$ with P as the pitch of the cholesteric and $\chi_a > 0$. For H perpendicular to the twist axis and with $n_x = \cos\phi$ and $n_y = \sin\phi$ the free

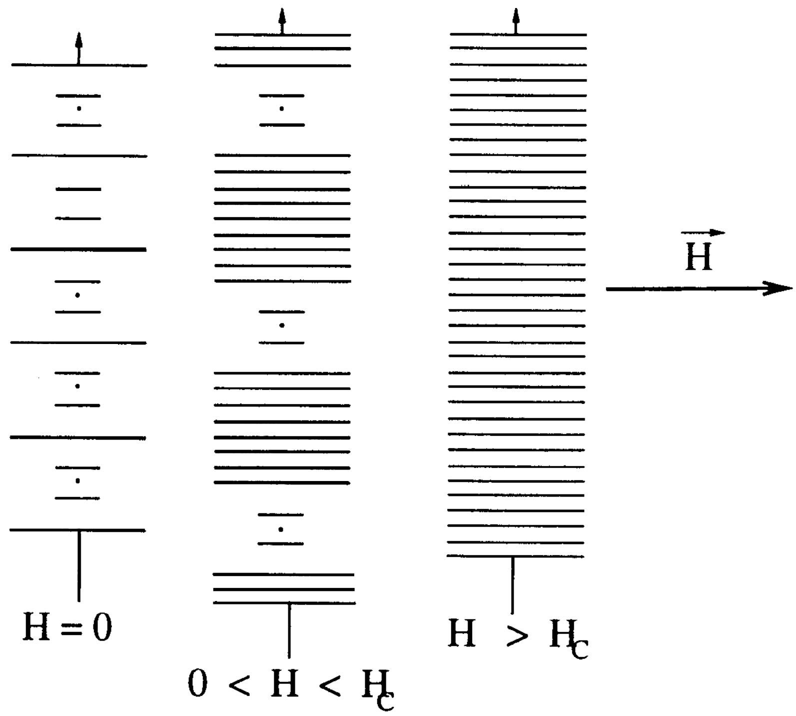


Figure 2.1: *Unwinding of* $\chi_a > 0$ ***cholesteric*** *helix in a magnetic field H perpendicular to the twist axis*

energy density is given by

$$F = \frac{K_{22}}{2}(\phi_z - q_0)^2 + \frac{1}{2}\chi_a H^2 (\sin\phi)^2 - \frac{K_{22}}{2} q_0^2 \tag{2.3}$$

Here $\phi_z = \frac{\partial\phi}{\partial z}$. This leads to an equation of equilibrium given by

$$\phi_{zz} = \frac{\chi_a H^2}{K} \sin\phi\cos\phi \tag{2.4}$$

where $\phi_{zz} = \partial^2\phi/\partial z^2$. This permits a π- twist and π anti-twist soliton solution described respectively by

$$\phi = 2\tan^{-1}(\exp[\frac{\pm z}{\xi}]) \tag{2.5}$$

with

$$\xi = \sqrt{\frac{K_{22}}{\chi_a H^2}} \tag{2.6}$$

A twist soliton (TS) has the same sense of twist as the parent cholesteric while the anti-twist soliton (ATS) has an opposite sense of twist. The total distortion energy per unit area of these solitons are easy to work out. We get

$$E_{ATS} = K_{22}[(2/\xi) + q_0\pi]$$

$$E_{TS} = K_{22}[(2/\xi) - q_0\pi]$$

For $(\pi/2)q_0\xi > 1$ i.e., for H $< (\pi/2)q_0\sqrt{K_{22}/\chi_a}$, the energy E_{TS} of a twist soliton becomes negative indicating a spontaneous generation of such twist solitons leading to a soliton lattice. However for $(\pi/2)q_0\xi < 1$, we have the undistorted nematic to be of the lowest energy with the ATS having a higher energy compared to that of TS. An interesting possibility exists in such nematics. Here a bend soliton can also connect the same base states as that associated with a TS or an ATS. This is shown in Figure *2.2.* This bend soliton can become energetically favourable compared to an ATS. For this to happen the energy E per unit area of the bend soliton should be lower than that of the ATS which leads to the condition

$$\sqrt{\frac{\overline{K}}{K_{22}}} - 1 < \frac{\pi}{2} q_0 \xi \tag{2.7}$$

where $\overline{K}$ is the bend or splay elastic constant. This together with the condition $(\pi/2)q_0\xi < 1$ for the nematic state leads to

$$\sqrt{\frac{\overline{K}}{K_{22}}} - 1 < \frac{\pi}{2} q_0 \xi < 1 \tag{2.8}$$

Hence for $\overline{K} < 4K_{22}$ a bend soliton is favourable compared to an $\boldsymbol{ATS}$. However in a given situation this occurs only upto a field H' above H_C. At H'

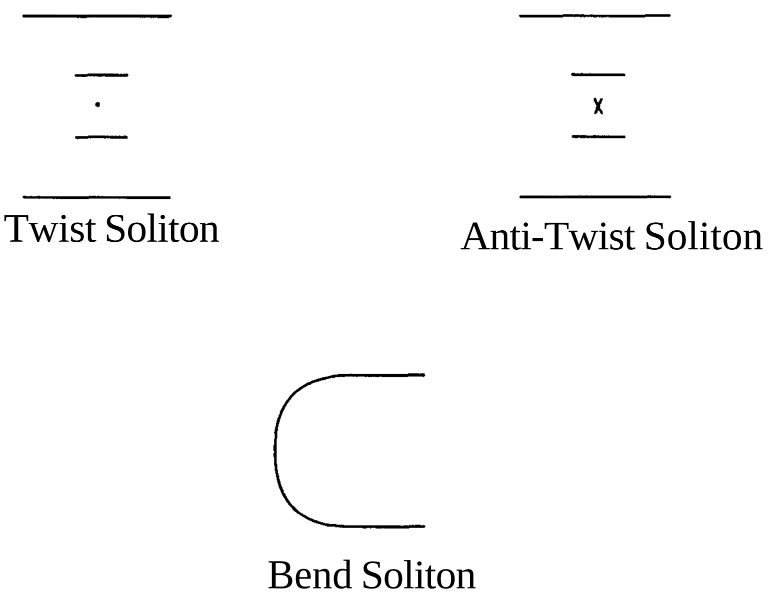
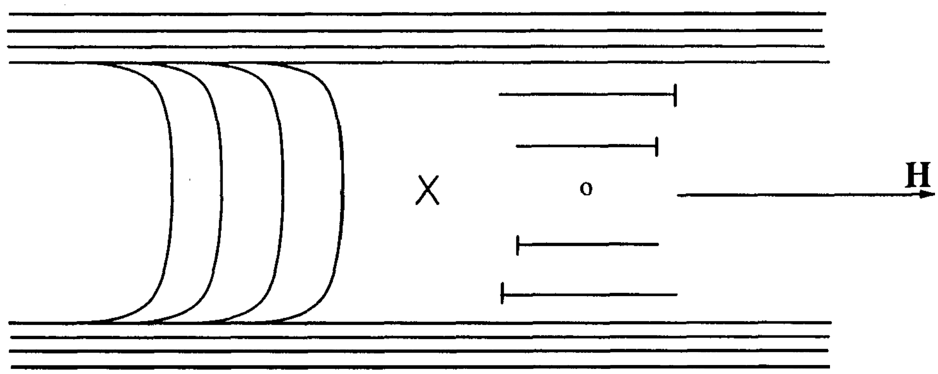


Figure **2.2:** *Twist, Anti-twist and bend solitons in a nematic obtained by unwinding a cholesteric with* $\chi_a > 0$

the inequality (2.7) is just violated. For H > H', the ATS is favoured *en*-ergetically. However, in the one constant approximation $\overline{K} = K_{22}$, in such a nematic a bend soliton is always favoured to an ATS. It may be mentioned that in a normal nematic a bend soliton is not favoured since it always has a higher energy than the TS or ATS since $K_{22} < \overline{K}$.

Thus in $\chi_a > 0$ nematics, obtained from an unwinding of a cholesteric, we have two solitons namely the bend soliton and the twist or the anti-twist soliton connecting the same base states. From the above discussions, we find that the ATS and a bend soliton can sometimes have comparable energies. Therefore we can think of a λ disclination line **[3]** connecting these two pla-



A Bend wall and a Twist wall connected by a λ line

Figure **2.3:** *A λ line between a bend soliton and a splay soliton denoted by x.*

nar solitons. Interestingly, such lines cannot exist in normal nematics since $K_{22} < K_{33}$.

It may be mentioned that in χ_a negative nematics it is not possible to have both bend and twist soliton as solutions with the same base states, in a magnetic field alone. However, if dielectric anisotropy $\epsilon_a < 0$, then in crossed electric and magnetic fields, it is possible to stabilize a bend soliton or a twist soliton between the same base states. Since their energies can be independently varied by altering the strengths of the fields, it is possible to have the energy of the bend soliton to be less than the energy of a anti-twist soliton.

In conclusion unlike the usual nematics, in these nematics which have a latent lattice symmetry we find some new results.

Intertwined solitons in an unwound cholesterics

In these structures, one can also think of creating intertwined solitons of the type discussed in the introductory chapter. The differential equations in this case are

$$\nabla^2\theta = \sin\theta\cos\theta[(\nabla\phi)^2 - 2q_0\nabla\phi - E_2(\sin\phi)^2] \tag{2.9}$$

$$\nabla^2\phi = -E_2\sin\phi\cos\phi - \cot\theta[2(\nabla\theta.(\nabla\phi - q_0))] \tag{2.10}$$

Here $E_2 = \chi_a H^2/K$. The linear term in the gradient of ϕ is absent in the case of normal nematics (Chapter I). The presence of this linear term leads to a completely different answer in this case. When solved the equations give the ϕ and θ profile as shown in Figure 2.4. We see that ϕ continues to be a π-twist soliton. However, 8 is not any more soliton like. This is the case even for very small values of q_0. We can infer from this, that though this is a nematic state got from unwinding a cholesterics, the intertwined solitons which are possible in normal nematics are not permitted here.

2.3 Note on electric field effects

A similar argument, can be carried through for the energetics of soliton states in nematics in an electric field. An induced electric polarisation results under distortions in a nematic. This is termed the flexo-electric effect [4, 5]. In general, a splay distortion leads to an induced electric polarisation along the director and a bend distortion leads to induced polarisation perpendicular to the director. One consequence of this is that, in an external static electric field one can produce alternating regions of periodic splay and bend distortions [4].

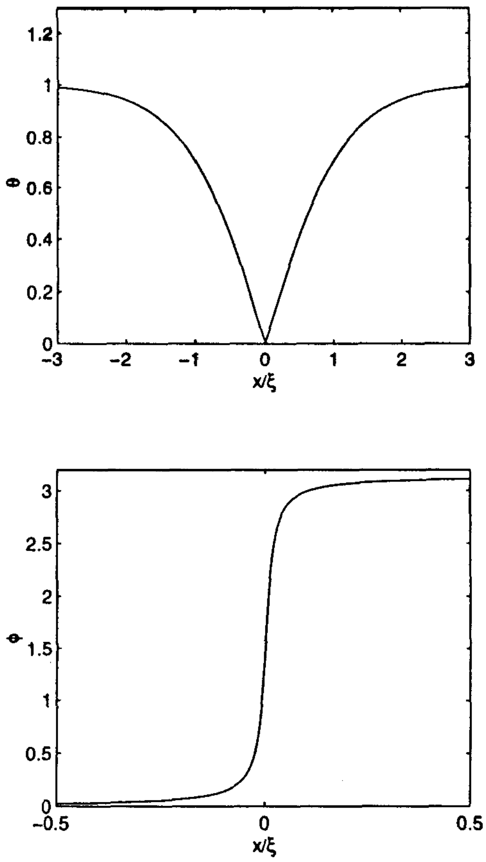


Figure 2.4: *θ and ϕ profiles of a coupled soliton state in an unwound nematics*

This is called the flexo-electric lattice. In such a nematic, if $|\epsilon_a| > \pi^3 e^{*2}/4\bar{K}$ where $e^* = e_1 - e_3$, with e_1 and e_3 as flexoelectric coefficients, then it can be shown that a spontaneous splay bend flexo electric lattice is not possible [6], [7]. This is another example for a nematic with a latent lattice order. In such nematics also, we can construct bend solitons of opposite bends. These have their flexo-electric polarization either along or opposite to the external electric field. We consider the bend soliton of lower energy. Its flexo polarization is along the external field. In view of the above analysis we can again consider a twist soliton or an anti-twist soliton in the very same geometry. These twist solitons are equally energetic. In this case, for the twist state or anti-twist state to be more favourable than the bend soliton, we have to satisfy the inequality

$$\frac{\pi}{2}e^*\sqrt{\frac{\pi}{\overline{K}\epsilon_a}} < [1-\sqrt{\frac{K_{22}}{\overline{K}}}] \tag{2.11}$$

Since K_{22} is invariably less than $\overline{K}$ this inequality is sufficient leading to an interesting conclusion that a twist soliton is preferred to the permitted bend soliton.

In a nematic obtained by unwinding a cholesteric in an electric field acting perpendicular to the twist axis, we find a totally different answer. In this case, above the threshold, there is no spontaneous twist, splay or bend . Hence, here a flexo-electrically permitted bend soliton can be made energetically favourable compared to an ATS by satisfying two inequalities.

$$\sqrt{\frac{\overline{K}}{K_{22}}}-1 < \frac{\pi q_0}{2}\sqrt{\frac{4\pi K_{22}}{\epsilon_a E^2}} < 1 \tag{2.12}$$

$$\frac{\pi e^*}{2}\sqrt{\frac{\pi}{\overline{K}\epsilon_a}} < 1 \tag{2.13}$$

For values of ϵ_a satisfying (2.13), the inequality (2.12) will be satisfied only in a range of fields $E_c < E < E'$ as in the case of magnetic field effects. Only then a bend soliton is favoured in such a nematic with a very large ϵ_a.

2.4 Planar solitons in H parallel to the twist axis

In a magnetic field parallel to the twist axis for a $\chi_a > 0$ cholesteric, the director n which was confined originally to the $x-y$ plane perpendicular to the twist axis will experience an out of plane deformation given by $n_x = \sin\theta\cos\phi$, $n_y = \sin\theta\sin\phi$ and $n_z = \cos\theta$. At sufficiently high magnetic fields the director everywhere goes over to the field direction i.e., along the twist axis resulting in a nematic state. We can construct soliton states in this nematic. Even, this is an example for a nematic with a latent lattice.

2.4.1 A giant soliton

The free energy density for this deformation in the one constant approximation of $K_{11} = K_{22} = K_{33} =$ K is given by

$$F = \frac{K}{2}[(\nabla\theta)^2 + (\sin\theta)^2(\phi_z^2 - 2q_0\phi_z + f)] \tag{2.14}$$

Here $f = \chi_a H^2/K$. This leads to the following equations of equilibrium

$$\nabla^2\theta = \sin\theta\cos\theta[(\phi_z)^2 - 2q_0\phi_z + f] \tag{2.15}$$

$$\nabla^2\phi = \cot\theta[2(\nabla\theta.(\nabla\phi - q_0)\hat{k})] \tag{2.16}$$

The equations of equilibrium then permit the following solutions

$$\phi = q_0 z \tag{2.17}$$

$$\theta = 2\tan^{-1}(\exp[\frac{z}{\eta_b}]) \tag{2.18}$$

where $\eta_b = \sqrt{1/(f - q_0^2)}$. Equation (2.18) describes a soliton which has a chirality as given by (2.17) with $\theta(-\infty) = 0$ and $\theta(+\infty) = \pi$. Its structure is schematically shown in Figure 2.5. Over a length of $2\eta_b$ the uniform state can be distorted to form a cholesteric like section. We call this a 'Packet Soliton' since a lattice is packed inside this soliton. This lattice has the pitch of the parent cholesteric i.e., $2\pi/q_0$. On decreasing the field this region of

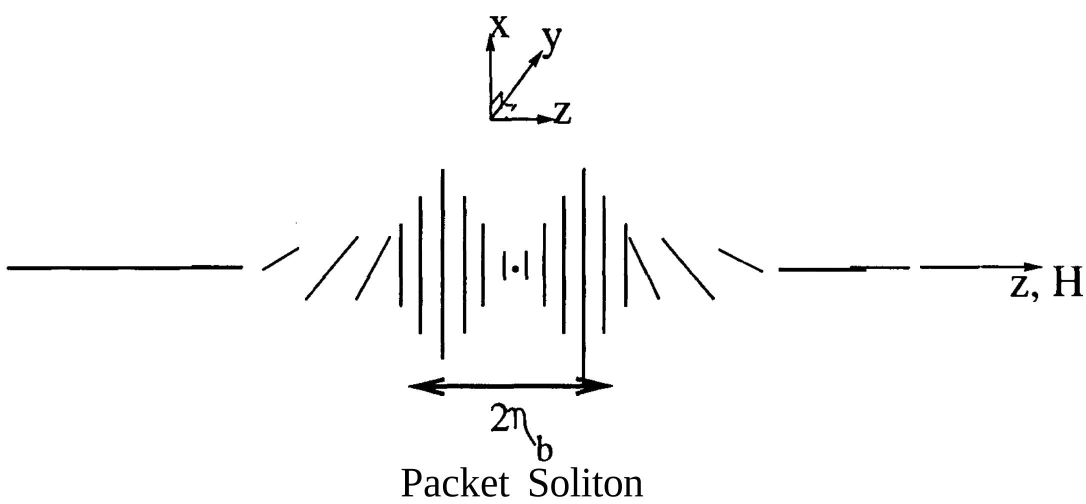
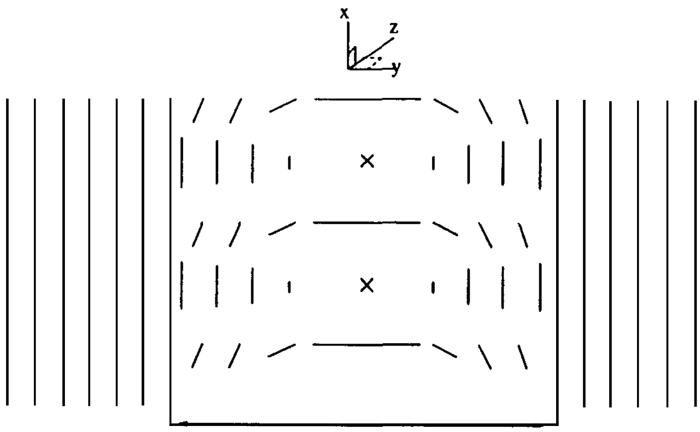


Figure 2.5: Structure *of* a longitudinal Packet soliton in a nematic.

width $2\eta_b$ grows and at the critical field given by $H_c = (2\pi/P)\sqrt{K/\chi_a}$ the entire structure transforms into a cholesteric. That is, there can be a single soliton mediated nematic to cholesteric transition. The energy of creation of this soliton continuously going to zero as H decreases to H_C.

From equation (*2.15)* and *(2.16)* we find that, a packet soliton with θ variations in a direction perpendicular to the twist axis i.e., $\theta = \theta(y)$ is also a possible solution. This is schematically shown in Figure *2.6.* This can be considered as an alternative stack of twist and bend-rich solitons. In the one constant approximation the widths and energies of both the transverse and longitudinal packet solitons are the same. However elastic anisotropy can decide as to which packet soliton triggers chirality in the nematic phase.



Packet Soliton

Figure 2.6: *Structure of a transverse Packet Soliton in a nematic.*

Such a giant soliton is also possible in a ferronematic got by distortion of a ferrocholesteric by the application of an external magnetic field parallel or perpendicular to the twist axis. If the grain migration is ignored and if $M >> \chi_a H$ we get essentially the same results.

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