BLACK HOLES ARE NOT FOREVER

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In this article we introduce the idea of a black hole and
then discuss the various types of black holes and their important
characteristics. A description is given of important processes
involving black holes. We elucidate the conceptual problems
that arise in the thermodynamics of systems containing black
holes as a result of the no hair result and the area theorem.
We next bring out the analogy between laws of black hole mechanics
and ordinary thermodynamics that culminated in the discovery of
the Hawking radiation. We conclude by indicating the unresolved
questions that these results raise.

The life of every star is a continuous struggle between the
attractive force of gravitation and a resistant internal pressure.
In young stars the pressure is thermal in origin while in white
dwarfs it is quantum mechanical. In the latter case the degeneracy
pressure of electrons can support stars upto the famous
Chandrasekhar limit of 1.4 $M_\odot$. More massive stars may find peace
as neutron stars where the support comes from degeneracy pressure
of neutrons. The theoretical mass limit for neutron stars is
uncertain because of uncertainties in the equation of state of
high density matter beyond 10^{14} gms/cm^3. However other general
considerations including stability aspects imply that the limits

cannot be higher than 3-5 $M_\odot$. What then is the fate of cores more massive than the above limits? Unless radically new physics intervenes in all such cases, gravity should win and matter undergo catastrophic collapse and end up as black holes. Gravity eventually wins since it is a long range force that can never be screened.

Mathematically a black hole is a particular solution of the field equations of gravitation that is asymptotically flat and contains an event horizon. The event horizon separates the spacetime into two disjoint regions one of which is causally inaccessible to the other. Technically, an event horizon is a null surface, i.e., a surface whose normal is a null vector. Every null surface is a one way membrane. A familiar example of a null surface is a wavefront of light. The black hole event horizon is different in that it is of finite extent, i.e., a compact null surface. Even with a quasi-Newtonian viewpoint one can appreciate the possible existence of such a surface. The crucial point is that according to general relativity gravity affects the propagation of light. Its energy and hence frequency will be decreased on propagation through a gravitational field. This is called the gravitational redshift of radiation. Bearing this in mind consider a massive star and ask for the minimum velocity an object should have to escape the gravitational field of a star. The condition is

$$KE \gtrsim PE \quad \text{i.e.} \quad v_e^2 \gtrsim \frac{2GM}{R} \ ,$$

where KE is the kinetic energy and PE the potential energy, with the other symbols having their usual meaning. Since special relativity introduces an upper limit to the speed of propagation of all physical signals c, it is clear that something peculiar will heppen if V_e the escape velocity from the star exceeds c. This will happen if

$$R_s < \frac{2GM}{c^2} \quad .$$

In Newtonian terms no radiation can escape the star, though it can fall into it. The surface of this object is thus a one-way membrane. The general relativistic analysis of this situation is obtained from the famous Schwarzschild solution. It turns out that the surface $r = \frac{2GM}{c^2}$ is a null surface and so forms an event horizon. This surface has another additional property. One finds that as the star approaches its gravitational or Schwarzschild radius, light emitted from it is redshifted or degraded without limit and hence not visible to the external observer. The region inside the horizon once the star has shrunk away is empty, cut off and inaccessible to the exterior universe. The object appears black and we have a black hole with event horizon at r_s separating events that can be viewed by a distant observer from those that he cannot however long he waits. For a black hole as massive as the sun the Schwarzschild radius is about 3 km. A black hole is not necessarily a high density object. A black hole of mass 10^{11} $M_\odot$ would be less dense than air!

The most general black hole solution to Einstein's field equations is the Kerr-Newmann family that describes axisymmetric,

matter free spacetimes and represents black holes that in general
rotate and carry an electric charge. This three parameter
family is labelled by mass M, angular momentum J and charge Q.
In general relativity all that you want to know about a solution
is contained in the line element. For the Kerr Newmann black hole
we have

$$ds^2 = \frac{\Delta}{\Sigma} (dt - a \sin^2\theta \, d\phi)^2$$

$$- \frac{\sin^2\theta}{\Sigma} ((r^2+a^2)d\phi - adt)^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 \; ;$$

$$J = Ma; \quad \Delta = r^2+a^2-2Mr+Q^2 \; ; \quad \Sigma = r^2+a^2 \cos^2\theta .$$

Some special cases are: if Q = 0, but J ≠ 0, the hole is
not spherically symmetric and is called the Kerr black hole.
If J=0, Q ≠ 0, we obtain the Reissner Nordstrom black hole. If
J = Q = 0 the solution reduces to the famous Schwarzschild solu-
tion. The latter two solutions are spherically symmetric. The
infinite redshift surface is located at

$$r = r_\infty = m + \sqrt{m^2-a^2 \cos^2\theta - Q^2} \; .$$

In general if a ≠ o, r_∞ is not a null surface. In this
case the horizon is located at

$$r = r_h = m + \sqrt{m^2 - a^2 - Q^2} \; .$$

The region between the above two surfaces was called the
ergosphere by Penrose since as we shall show later energy can be
extracted using this region of a rotating black hole.

Unlike in Newtonian gravitation, in general relativity a rotating source tends to drag the spacetime round it. Consequently, when one looks at the wavefront of light flashed at different points not only are they pulled towards the source, but also swept in the direction of its rotation. The infinite redshift surface also locates the static limit beyond which one cannot resist the effect of the rotational dragging even with the help of external forces.

Black holes in general are detectable only indirectly by their gravitational interaction with nearby matter. However in 1969 Penrose invented a procedure to extract the rotational energy of a Kerr black hole exploiting the existence of 'negative energy' trajectories in the ergosphere. What one does is to drop a particle in the ergosphere so that it splits into two parts there. One fragment is then captured in one of the negative energy orbits while the other then escapes to infinity with more energy than the incoming particle. This is called the Penrose process and is of great theoretical rather than practical interest. Still it has not prevented people from constructing scenarios of civilizations without energy or ecological problems.

The next development was the realization that there exists a wave analog of the Penrose process. Waves incident on rotating or charged black holes are scattered with increased amplitude in certain modes. On a particle description this corresponds to an increase in particle number or stimulated emission. This phenomenon of amplification of reflection from black holes

is called superradiance. In both the Penrose process and super-radiance, the energy is extracted at the cost of the rotational energy of the black hole. It should thus be clear that after all the rotational energy has been extracted one is left with a non-rotating black hole and no further energy can be mined. It has been shown that all bosonic waves superradiate while Fermionic waves do not. For Fermions the Pauli exclusion principle forbids superradiance since more than one particle cannot be accommodated in any mode.

Associated with the above stimulated emission there should obtain a corresponding spontaneous emission of particles in the classical superradiant modes. Like all spontaneous processes this is a typical quantum mechanical effect and is referred to as the Zeldovich-Starobinsky-Unruh emission. This leads to a spontaneous loss of angular momentum or charge from the black hole (Though Fermion fields do not superradiate, they exhibit the ZSU emission). Thus a rotating or charged black hole may be viewed as an excited state for black hole solutions but once the rotational or electromagnetic energy is radiated off, the ergo-sphere disappears and no further energy can be obtained. The Schwarzschild black hole thus appears like the ground state for black hole solutions but this as we shall see soon leads to conceptual problems.

The conceptual problems trace back to certain questions concerning thermodynamics in the presence of black holes which we now discuss. When a body undergoes gravitational collapse to

form a black hole very few items of information survive to tell the outside observer what the black hole was made up of. The effect of collapse is to impose a type of coarse graining as far as measurements from far are concerned. All information of the internal state of matter is washed away and we rapidly have a quasi-stationary state characterised by the three parameters mass, angular momentum and charge. Wheeler summarized this by a famous aphorism "Black holes have no hair". Black hole baldness leads to a serious problem for the following reason. If one takes a piece of matter and transfers it into a black hole then from the associated change in M, J and Q of the black hole one cannot estimate the entropy of the packet dropped inside the black hole. Thus an external examiner with no inside information can never be sure if the second law of thermodynamics is obeyed. We say that the second law is transcended, i.e., it loses all its predictive power. This problem can be solved only if in some way one can define an entropy associated with the black hole. Insight along this route was provided by the area theorem due to Hawking that "the surface area of the boundary of the horizon of any black hole cannot decrease and will always increase in any dynamical process". This result points out a formal analogy between the black hole area and thermodynamic entropy both of which increase. To proceed further it can be shown starting from the expression for the area of the horizon,

$$A = 4\pi (r_h^2 + a^2)$$

$$= 4\pi (2M)^2 \quad \text{for} \quad a = Q = 0,$$

that for two equilibrium states of a black hole differing by δM, δJ and δQ one has

$$\delta M = \frac{\kappa}{8\pi} \delta A + (\Omega_h \, \delta J + \phi_h \, \delta Q)$$

where κ is called the surface gravity, Ω_h the angular velocity and ϕ_h the electric potential of the horizon. This resembles the first law of thermodynamics that reads

$$du = Tds + dw$$

where du is the change in internal energy, Tds the heat transferred to the system and dw the work done on the system. Thus if some multiple of A is analogous to entropy a multiple of κ is analogous to temperature; the other two terms represent work done on the eystem in changing its angular momentum by δJ and charge by δQ. Even going beyond formal similarities it is clear that in the process of collapse and formation of a black hole a lot of information is lost down the horizon whose area thus gives a measure of the lost information. To quantify the lost information following Bekenskin we proceed as follows. Estimate the number of 'elementary' particles that go to make up the black hole and assume that each particle corresponds to one bit of information. Since classically the constituent mass may be made as small as possible one gets an infinite answer, i.e., the black hole entropy is infinite. However if the quantum nature of matter is taken into account one obtains a finite result. This is because, now, one can only use those particles whose Compton wave-length is smaller than the black hole radius which implies

that the mass $m > \dfrac{hc}{2GM}$.

This leads to $N_m = \dfrac{M}{n} = \dfrac{2G}{hc} M^2$ so that for a Schwarzschild black hole entropy scales as the square of M or the area of the horizon. This also implies that black hole entropy is not just the entropy of matter making up the black hole, for that would scale as M. Thus in addition to the formal analogy there appear physical reasons to think of A as representing the black hole entropy. Taking stock of the results we then have, analogous to the thermodynamic laws, the following laws of black hole mechanics.

<u>Zeroth law:</u> For stationary black holes the surface gravity κ is constant over the event horizon.

<u>First law:</u> $\delta M = \dfrac{\kappa}{8\pi} \delta A + (\Omega_h \, \delta J + \phi_h \, \delta Q)$

<u>Second law:</u> The surface area of the boundary of the horizon of any black hole cannot decrease.

<u>Third law:</u> It is impossible to reduce the surface gravity of the horizon to zero by a finite sequence of operations however idealized. Unlike the second law there is no rigorous proof of the third law though there exist plausible justifications. The proof of the second law and the justification of the third law are based on the unproven cosmic censorship conjecture due to Penrose that all singularities are covered by an event horizon, i.e., nature abhors a naked singularity. The proof or disproof of this conjecture is one of the outstanding problems of classical general relativity.

Serious questions arise if following Bekenstein the above analogy is taken as representing the thermodynamic parameters of the black hole. For what does the black hole temperature physically mean? To assign a temperature to something implies that the object can be in equilibrium with a heat bath at the same temperature. To be able to do so a black hole would have to emit heat energy at the same rate as it absorbs it. However the event horizon is a one way membrane. It lets radiation in but does not let anything out! This was the state of affairs in 1973 and the above laws were referred to as the laws of black hole mechanics by Bardeen, Carter and Hawking. The analogy was regarded as suggestive but it was emphasized that it could not represent the thermodynamic parameters of the black hole since otherwise it implied that radiation should come out of a black hole.

In early 1974 came the resolution with the work of Hawking. Using techniques of quantum field theory in classical spacetime, he showed that at late times a body collapsing to a black hole emits radiation in all modes with a characteristic thermal spectrum at a temperature proportional to κ as suggested by the thermo-dynamic analogy, $N_{wlm} = \Gamma_{wl} (\exp 8 \pi Mw \neq 1)^{-1}$. The result was so surprising that Hawking said 'I put a lot of effort in trying to get rid of this embarrasing effect. It refused to go away so that in the end I had to accept it'. Ironically what finally convinced him was the thermodynamic consistency that emerged. The quantum mechanical calculation determined the propor-

tionality constant between temperature and surface gravity and
hence entropy and area. It is

$$S \; = \; \frac{1}{4} \, kA \; = \; \frac{4\pi \, kG}{hc} \, M^2$$

$$T \; = \; \frac{h\kappa}{2\pi \, kc} \; = \; \frac{hc^3}{8\pi \, GkM} \; = \; 6\times10^{-8} \, (\frac{M_\odot}{M}) \; °K$$

for a = Q = 0. The Hawking radiation depends on the existence
of the collapse but not on its details. As h → o, T → o so that
the effect is quantum mechanical in origin. Thus classically
black holes are indeed black. Quantum mechanics causes all black
holes including the Schwarzschild black hole to radiate away.
The Schwarzschild black hole is not the ground state as we thought
it to be. Physically the temperature of the black hole corres-
ponds to the temperature of the quantum radiation from the black
hole. For a stellar mass black hole however T = 10^{-7} °K and in
the cosmic microwave background radiation of 3°K such holes will
absorb instead of radiate and grow. For such objects the Hawking
radiation is not significant. It is important for mini or
primordial black holes of mass 10$^{15-16}$ gms, which could have been
formed by density fluctuations in the very early universe. Such
a black hole is at a temperature of 10^{11} °K so that it will emit
radiation, lose mass and grow hotter in consequence. This is a
manifestation of the negative specific heat of self gravitating
systems. Though in the early stages only massless particles
like neutrinos, photons and gravitons are emitted, as the tempera-
ture increases other species of massive particles will be emitted

leading to a runaway explosion. In the final tenth of a second
the energy released is equivalent of the million megatonne
thermonuclear bombs.

Hawking radiation is an example of particle creation by
strong external fields. Vacuum fluctuations of the matter field
create virtual pairs outside the horizon of a black hole. If
the tidal force near the horizon can separate the virtual pairs
within a Compton wavelength, then the virtual pair can become
real by taking energy for the gravitational field. In this case
if one member of the pair falls in the hole the other can escape
to infinity and appear as energy emitted by the black hole.
Quantum evaporation entails a loss in mass and hence the area or
entropy of black hole. The decrease may be thought of as due to
a negative energy flux into the event horizon. This allows the
violation of the area theorem whose proof assumes positivity of
energy. However, the decrease in entropy of the black hole is
more than compensated by an increase in the entropy of the exterior
where one now has the Hawking radiation. The second law should
then be applied to the sum of ordinary entropy and black hole
entropy.

Hawking radiation opens up doors to new physics not as yet
clarified. For e.g., in conventional quantum mechanics a pure
state cannot evolve into a thermal state. Thus in a black hole
case there seems to be an additional degree of unpredictability
than contained in the uncertainty principle. This could have
profound implications for constructing a quantum theory of gravi-

tation. Hawking wrote "God not only plays with the dice he sometimes throws them where they cannot be seen". Further the thermal nature of the radiation seems related to the existence of horizons. Horizons arise even in flat spacetime for accelerated observers as also in cosmological models like de Sitter spacetime for a particular class of geodesic observers. In both these cases one again obtains a similar thermal radiation. However, in the de Sitter case one is faced with a new experience: an observer dependent quantum field theory not all aspects of which we understand. Further, like black hole entropy, can one go to define the entropy of an arbitrary, say cosmological, gravitational field? This question may shed light on the observation that unlike in non-gravitating systems where high entropy (disorder) is simple and low entropy (order) is complex in gravitating systems the reverse seems to be true.

In conclusion we have seen how the concept of a cold classical blak hole is inconsistent with the rest of physics and leads naturally to the idea of a black hole that is hot and glowing though sometimes feebly. Even a black hole is not for ever. Black hole evaporation raises a number of fundamental questions that still await resolution. In regard to these pristine objects it would not be an exaggeration to exclaim "Black but not so black and yet blacker than anything the mind contemplates".