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The "Ectara"

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1. Lord Rayleigh has described an experiment in which a tuning-fork of frequency 256 say, is mounted upon a resonance-box corresponding to a frequency 512, and when strongly bowed, causes the latter to emit a tone of frequency 512, the primary tone of frequency 256 being partly or entirely suppressed. The effect is attributed to the curvature of the paths of points on the tuning-fork. Instances, in Acoustics of vibrating systems emitting tones of frequency higher than their own, on account of some such peculiarity inherent in them, not being common, I was induced to investigate for others.

2. In the theory of the small oscillations of a stretched string, it is assumed as a working hypothesis, that the tension of the string is constant thoroughout the oscillation. When metallic wires are employed, it is obvious that, since these have a high Young's Modulus, the amplitude of the oscillation should be very small indeed, if this assumption is to be valid. With larger amplitudes, such as can be obtained by plucking or bowing, we should expect a periodical fluctuation of tension, since, if its extremities are kept fixed, the length of the wire cannot be constant throughout an oscillation. With proper arrangements, the periodic fluctuation of tension can be rendered evident.

3. A thin board (figure 1) has its four corners screwed into a rigid frame, leaving the central part capable of vibration. A metallic wire is attached perpendicularly to the centre of the board by a screw, nut, and washer. The wire, which is horizontal, passes over a hook, also attached to the rigid frame, and is weighted at the end. On plucking or bowing the wire, so that it vibrates in one loop, a loud note is emitted by the sounding board. It is easily shown in the following way, that the pitch-number of this note is not the same as the frequency of vibration of the wire; being much higher, in fact exactly the octave of it. An exactly similar wire of the same length is stretched on a Sonometer and its tension adjusted so that its note is in unison with the note emitted by the sounding board in the first case. It is then seen that its tension is much greater than that of the first. On equalising the tensions, it is found that the Sonometer note is an octave below the other.

4. It is obvious that in the arrangement shown in figure 1 the wire being perpendicular to the sounding board, the oscillations of the former cannot excite the latter in the ordinary way. The vibration of the sounding board is therefore caused entirely by the periodic variation of tension above referred to.

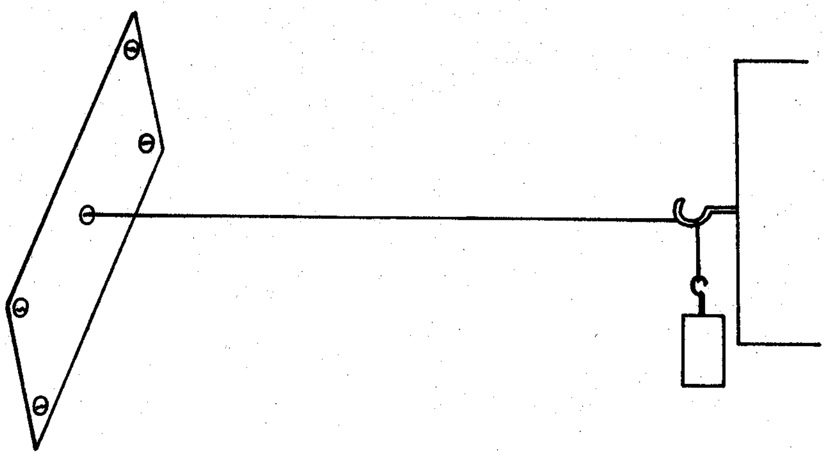

Figure 1

5. Strange as it may seem, there is in actual use in India, a musical instrument, a rather crude one, it is true, the working arrangements of which are, in essential, the same as those shown in figure 1. It is styled the 'Gopijantra' or oftener the 'Ectara,' and is chiefly used, I find, by vocalists—those of the poorer sort—for striking key notes and marking time. The users of the instrument are apparently totally unaware of its unusual characteristics. I have had one of these instruments constructed for me, a diagramatic representation of which is given in figure 2.

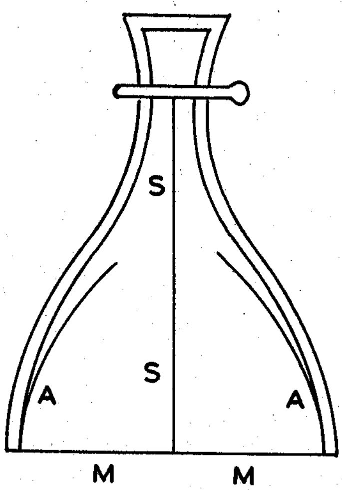

Figure 2

A is the thick shell of a gourd that has been cut across in two places. Over the lower section is stretched a membrane M. A metallic wire is attached perpendicularly to the membrane at its centre, passes through the upper section of the gourd and is kept stretched by a key. The pieces of wood that attach the key and its block to the gourd, are flexible. On pressing these flexible pieces of wood inward, the tension of the string is relaxed: if they are pulled outwards, the tension is increased.

6. The periodic variation of tension can easily be calculated in the case when the vibrating wire has both its ends fixed. The length of the arc of the sine curve $y = a\sin(\pi x/l)$ from node to node, is

$$\int_0^l \left[1 + \frac{\pi^2 a^2}{l^2}\cos^2\frac{\pi x}{l}\right]^{1/2} dx = \int_0^l \left[1 + \frac{\pi^2 a^2}{2l^2}\cos^2\frac{\pi x}{l}\right] dx \ldots\text{approx.}$$

$$= l + \tfrac{1}{4}\frac{\pi^2 a^2}{l}.$$

If $a = b\sin pt$ the length of the arc is

$$l + \frac{\pi^2 b^2}{8l} - \frac{\pi^2 b^2}{8l}\cos 2pt.$$

If T be the tension of the wire in the zero position and H be the modulus of extension of the wire, then the tension of the wire at any instant is

$$T + H\left(\frac{\pi^2 b^2}{8l^2} - \frac{\pi^2 b^2}{8l^2}\cos 2pt\right).$$

From this, it is obvious that the average tension depends to some extent on the amplitude of the oscillation and that the periodic element is of double the frequency of the vibrations of the wire. Of course, what is effective in forcing the oscillation of the sounder, is not the tension of the wire but the component normal to the plane of sounder, which if θ be the angle between the normal and the wire at its extremity, may be written as

$$T\left(1 - \frac{\theta^2}{2}\right) + H\left(\frac{\pi^2 b^2}{8l^2} - \frac{\pi^2 b^2}{8l^2}\cos 2pt\right).$$

Since $\cos\theta$ is nearly equal to unity and the second term is small: that is as

$$T + H\left(\frac{\pi^2 b^2}{8l^2} - \frac{\pi^2 b^2}{8l^2}\cos 2pt\right) + T\frac{\pi^2 b^2}{4l^2}\cos 2pt - T\frac{\pi^2 b^2}{4l^2},$$

the fourth term is obviously the well known 'pressure of the incident and reflected waves'. The third is its periodic complement which, it may be noticed, is of double frequency. Since T is small compared with H, the last two terms may be neglected in comparison to the second. We revert therefore to the original expression.

7. Experiments with the 'Ectara,' of the kind described in para 3 conclusively proved that the pitch of the note emitted by it, was *twice the frequency of the oscillations of the wire.* With the use of a resonator it was however possible to hear a faint note of the same frequency as the oscillation of the wire along with the bulk of the note which was of twice that frequency.*

It now remains to consider the dynamical theory of the instrument in detail. The ideal 'Ectara' would be constructed on the following lines: The vibrating wire would be perfectly inextensible and be fixed at one end and would be attached at the other end to the centre of a perfectly rigid massless plane sheet constrained to move in a direction parallel to the wire. This sheet would be backed by an inertia-less system (such as a long, excessively fine spring) keeping the tension of the vibrating wire absolutely constant. Under the circumstances, the oscillations of the wire, if in sine loops and even when of considerable amplitude, would be of normal character and the plane sheet would be constrained to execute in a direction perpendicular to itself, normal oscillations of double the frequency. If the oscillations of the wire be represented by $y = b \sin(\pi x/l) \sin pt$, the oscillations of the sheet would be given by $X = -(\pi^2 b^2/8l) \cos 2pt$. If the oscillations of the wire be compound and be represented by $b_1 \sin(\pi x/l) \sin(pt + \varepsilon_1) + b_2 \sin(2\pi x/l) \sin(2pt + \varepsilon_2) + \text{etc.}$ it may be shown by integration that the oscillation of the sheet would be given by

$$X = -\left[\frac{\pi^2 b_1^2}{8l} \cos(2pt + 2\varepsilon_1) + \frac{4\pi^2 b_2^2}{8l} \cos(4pt + 2\varepsilon_2) + \text{etc.}\right].$$

8. The complications that may occur in practice, can be classified as (*a*) the wire may be extensible, (*b*) the sounding surface and its system have inertia, (*c*) the forces acting on the sounder apart from the tension of the vibrating wire, which in practice are due to its own flexure or deformation, may be variable and (*d*) the vibrations of the sounder are damped on account of the emission of energy. These may all be considered together. The sounding board or membrane may, as an approximation, be considered equivalent to a mass M with a spring q. These two quantities may conceivably vary with the frequency of the oscillations forcing the sounding board or membrane. Let T and l be the tension and length respectively of the wire, when the apparatus is at rest. The force exerted on the representative mass M and therefore also its fraction, is equal to T. Let X_1 be the displacement at any instant during the oscillation, of M from its equilibrium position. The force exerted during M by its spring is $T + qX_1$.

The wire at any instant is $l + Cl - Cl \cos 2pt - X_1$ where $C = (\pi^2 b^2/8l^2)$. The

*The tone having the same frequency as that of the oscillation of the wire was heard much more strongly when the wire executed its vibrations in the plane on the flexible rods, (figure 2), than when these took place in a perpendicular plane. In fact, in the first case there was very often an uncertainty of a whole octave in the pitch of the note heard by the unassisted ear.

tension of the wire at any instant, is therefore

$$T + CH - CH\cos 2pt - \frac{HX_1}{l}.$$

The equation of motion of mass M is therefore

$$\left[MD^2 + kD + \left(q + \frac{H}{l}\right)\right]X_1 = CH - CH\cos 2pt$$

k being the damping factor. Putting $X = X_1 - (CH)/[q + (H/l)]$ the equation becomes

$$\left[MD^2 + kD + q + \frac{H}{l}\right]X = -CH\cos 2pt.$$

$$\therefore \quad X = \frac{-CH\cos(2pt + \varepsilon)}{\left\{\left[4Mp^2 - q - \frac{H}{l}\right]^2 + 4k^2p^2\right\}^{1/2}}$$

where

$$\varepsilon = \tan^{-1}\frac{2kp}{4Mp^2 - \left(q + \frac{H}{l}\right)}.$$

Then

$$\varepsilon = \frac{\pi}{2},$$

when

$$2p = \frac{\left(q + \frac{H}{l}\right)^{1/2}}{M}$$

that is, when the free period (without damping) of the system, sounding board—extensible wire, for oscillations parallel to the length of the wire, is half that of the transverse oscillation of the wire.

9. The expression for the phase of the oscillation of the sounding board was verified experimentally for the case in which H/l was very large compared with $4Mp^2$ or with q. In this case $\varepsilon = 0$, and $X \equiv -C\cos 2pt$, which is also the result in the case of the ideal Ectara considered above. A brass wire was stretched between one side of a frame and the centre of a stout piece of twine stretched across the frame and kept deflected by the tension of the wire. The oscillation of the wire executed in the plane of the frame. A piece of paper pinned to the centre of the twine emitted a note the frequency of which was obviously twice that of the

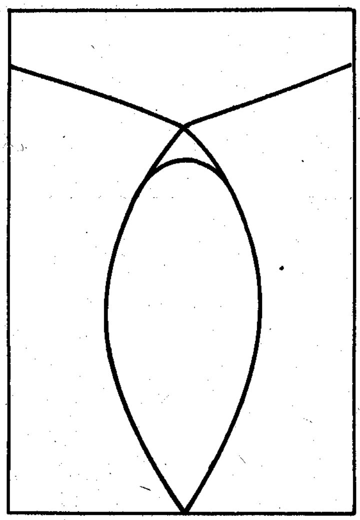

Figure 3

oscillations of the wire. Any point on the wire had two movements, one transverse to the wire, and the other twice the frequency parallel to it, the latter being zero at the fixed end and maximum at the end attached to the twine.

It was observed that the paths described, in the plane of the frame by points near this end were parabolas with their concavity directed, way from the movable end of the wire. It may easily be shown that the path of the compound motion $X = -C\cos 2pt$, $y = b\sin pt$ is a parabola.

P.S. The experiments described above bear upon the existence of variations of tensions in the free oscillations of a wire, if these are not of small amplitude. The results obtained have no direct connection with the case in which the oscillation of a wire is maintained by periodically varying its tension.