ON THE SPECTRUM OF NEUTRAL HELIUM. II

By C. V. RAMAN AND A. S. GANESAN

ABSTRACT

A rejoinder to Dr. Silberstein's reply.—The two points raised by Dr. Silberstein in reply to the criticisms of his combination formula are answered. The figures have been recalculated, taking the maximum limit of the quantum numbers to be that proposed by Dr. Silberstein himself, and the number of fortuitous coincidences to be expected between the observed lines of the helium spectrum and those given by the formula is calculated and found to agree fairly well with the actual number. The view that the coincidences noticed are due to mere chance is maintained.

In his rejoinder[1] to our criticism of his paper[2] on this subject Dr. Silberstein has attempted to dispose of our contention that the approximate coincidences between some of the lines of the helium spectrum and those given by his combination formula are purely fortuitous. He raises two points in his rejoinder. The first is that we took the limits of the quantum numbers to be

$$3 \leqq n \leqq 9 \text{ and } 4 \leqq m \leqq 32 \tag{1}$$

and thus obtained a high estimate of the probability of chance coincidences. To find whether this reply meets our objection, we have recalculated our figures, confining ourselves to the limits

$$3 \leqq n \leqq 8 \text{ and } 4 \leqq m \leqq 20 \tag{2}$$

now proposed by Dr. Silberstein himself as suitable for a test of the correctness of his views. Further to clinch the matter, a calculation has been made, as explained below, of the number of fortuitous coincidences to be expected.

The first step in the work is to make a table of all the lines given by the combination formula, satisfying the conditions (2) and lying in the frequency interval

$$19800 < \gamma < 37800. \tag{3}$$

We find there are 760 lines, not 631 as per Silberstein. It is found further that the lines are not distributed uniformly throughout

[1] *Astrophysical Journal*, **57**, 240, 1923.

[2] *Ibid.*, **56**, 119, 1922.

the whole interval of frequency. There are numerous small gaps and a small number of relatively large gaps. Thus one-third the number of gaps is less than 10, about two-thirds the number are less than 25, while only 10 per cent of the gaps exceed 50. Within the frequency limits given by (3), which are separated by a gap of 18,000, there are 96 observed lines of the helium spectrum. To determine how many chance coincidences may be expected between these and the 760 lines given by the formula, we may imagine the latter to be represented by points on a straight line, and that 96 shots are fired at random so as to hit the line. If there are N gaps of average interval x, the expectation of the number of shots that will hit some of these gaps is evidently

$$\frac{Nx}{18000}\times 96.$$

Further, if any shot falls within a frequency gap x, the maximum distance between it and the nearest point on the straight line is $x/2$. Actually all distances between 0 and $x/2$ are equally probable, and if y be the number of shots that may be expected to hit the gap x, we may divide them further into n groups of y/n shots each, each group hitting the line at distances of $\frac{x}{2(n+1)}, \frac{2x}{2(n+1)}, \ldots \frac{nx}{2(n+1)}$ from the nearest point. In practice it is sufficient to take n moderately large, say 5. In this way, by taking the different gaps in the line, it is possible to work out the number of shots that may be expected to hit the line within a specified distance from the nearest point. The argument may be illustrated by the following example. The number of frequency gaps between 20 and 25 is 83. The aggregate gap interval is 1925. The number of lines that may be expected to fall within this gap is

$$\frac{1925}{18000}\times 96=10.$$

Of these 10 lines we can expect 2 to fit the nearest one with an error of 1.9, 2 with an error of 3.8, 2 with 5.7, 2 with 7.6, and 2 with an error of 9.5. Calculating in this way, we have constructed the following table, giving the relations between the permissible error and the fits calculated and actual.

TABLE I

Permissible Error	Fortuitous Fits		Permissible Error	Fortuitous Fits		Permissible Error	Fortuitous Fits	
	Calculated	Actual		Calculated	Actual		Calculated	Actual
1.....	3	7	11	61	65	21	85	87
2.....	9	15	12	65	69	22	87	87
3.....	17	24	13	68	70	23	88	87
4.....	27	33	14	72	73	24	89	88
5.....	33	42	15	73	74	25	90	89
6.....	43	47	16	77	76	26	92	89
7.....	49	49	17	79	79	27	93	90
8.....	54	51	18	81	80	28	93	92
9.....	56	55	19	82	83	29	95	92
10.....	60	63	20	83	87	30	96	93

The table shows that except for small errors less than 3, the agreement between the calculated and the actual number of fits is nearly perfect. Hence, even confining ourselves to the conditions given in (2), we find that the coincidences are accidental, and our original contention still holds.

Silberstein says that "44 lines coincide with observed lines whose total is 96. The mean deviation is $|\delta\nu| = 2 \cdot 5.7$. Now a straightforward computation will show that the probability of such an event considered as a fortuitous set of coincidences is . . . . well below 10^{-13}." It is a matter for regret that Dr. Silberstein does not explain clearly in his reply to our criticism what his "straightforward method of computation" is. In view of the figures given above, it would seem that there must be some fundamental error which vitiates his method of calculation. Perhaps he has overlooked the fact that the theoretical lines according to his formula are not uniformly distributed, but fall into groups, and that this must profoundly affect the probability of random coincidences. But if his formula claims to explain the spectrum of helium, why should 44 frequencies alone coincide and not all the 96?

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