On the Whispering-Gallery Phenomenon.

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1. *Introduction.*

The theory of the curious and interesting effects observed in the Whispering Gallery at the base of the dome in St. Paul's Cathedral was discussed long ago by the late Lord Rayleigh in his treatise on Sound,* and more fully, from the point of view of wave-propagation, in two recent papers.† Expressed in general terms, the explanation put forward is: the sound-waves cling to the concave surface of the wall and travel circumferentially along it, suffering a diminution of intensity relatively small in comparison with that of waves spherically diverging from the source. The theory was supported by the observation that the effects under consideration are most noticeable when a *directed* source of sound is used and placed close to the curved surface, with its maximum emission in a direction tangential to it. An experimental illustration of the theory was also put forward,‡ using a semicircular reflector of sheet metal, with a bird-call as a source of sound of short wave-length, and a high-pressure sensitive flame as detector. With this apparatus, a small obstacle placed close to the concave surface is competent to intercept most of the effect. Some recent experiments of Barton and Kilby,§ illustrating the matter further, by the aid of dust-figures produced by electric sparks, may also be mentioned.

On account of the great interest of the subject, not only from a purely acoustical standpoint, but also in view of analogous effects in other branches of physics, *e.g.*, the propagation of electric or seismic waves round the earth, it occurred to the authors that it would be worth while to make a closer investigation of the phenomena and of the explanations that have been advanced, and this paper describes the results obtained.

* Vol. 2, 2nd edition, pp. 126–128.

† 'Phil. Mag.,' vol. 20, p. 1001 (1910); 'Scientific Papers,' vol. 5, p. 617; 'Phil. Mag.,' vol. 27, p. 100 (1914); 'Scientific Papers,' vol. 6.

‡ 'Roy. Inst. Proc.,' January, 1904; 'Scientific Papers,' vol. 5, p. 171; see also Sir William Bragg's 'The World of Sound,' pp. 84–86.

§ 'Phil. Mag.,' vol. 24 (1912).

2. *The Optical Analogue of the Whispering-Gallery Effect.*

In view of the remark by Rayleigh in a footnote to his paper, as reprinted in his collected works,* that his theory should be applicable equally for electro-magnetic waves, as in the case of sound, it will be appropriate here to refer to the results of some observations on the curvilinear propagation of light along a reflecting surface, initiated by one of us (C. V. R.), and carried out by Mr. Bidhubhusan Ray at Calcutta. The experiment was intended to find how the effect due to a light-source placed on a reflector alters with the curvature of the surface. It was carried out in the following way. A strip of plane mirror, about 100 cm. in length and 5 cm. broad, rested on two wooden supports near its ends. A razor edge, placed on the mirror near one end, formed an exceedingly fine slit between it and the surface on which the light of a mercury lamp with green ray filter was concentrated. Two light wooden bridges, placed on the mirror near its two ends, and equally loaded, enabled a curvature of variable magnitude to be imposed on the mirror. It was very interesting to watch the luminous effect at the distant end of the mirror through an eyepiece as the curvature was gradually increased. When the surface was quite plane, there was only a very faint general illumination of the field, the edge of the mirror being, however, a perfectly black line. When a slight curvature is put on, there is a very rapid increase in the luminosity of the field, a bright band of light flashing out next to the surface of the mirror, which continues to be seen as a line of zero illumination. With further increase of curvature this band contracts in width, and is followed by a second bright band, separated from it by a dark band. Then a third bright band appears, preceded by another dark band, and so on, the number of bands and their sharpness increasing, and their width decreasing with the increase in the curvature of the mirror. More and more light is heaped up, as it were, near the surface of the mirror. Some striking differences also appear in the intensity of the different bands, and ultimately also in their spacing, and it then becomes possible to differentiate the head of the caustic formed by a single reflection at the surface of the mirror from the train of feebler and narrower bands following it, in which also appear variations of intensity, which might be interpreted as due to the superposition of caustics of second and higher orders

According to Rayleigh's theory, the light vector in the experiments described above should be zero on the surface of the mirror, and on moving away from it should increase to a maximum and then decrease again rapidly, remaining very small for all further displacements. As we have seen above,

* 'Scientific Papers,' vol. 5, p. 617.

this agrees with what is observed when the radius of curvature of the mirror is exceedingly large (practically infinite) in comparison with the wave-length of light, and with a smaller radius we get not merely a single maximum adjacent to the surface, but several such maxima, separated from each other by intervening minima. Accepting this as a guide, we see that in the acoustical case, where the radius of the reflecting surface is a relatively modest multiple of the wave-length, we should get, not a single narrow belt of maximum intensity skirting the wall of the gallery, but several belts of sound alternating with silence.

3. *Observations in St. Paul's Cathedral and in the Laboratory.*

By courtesy of the authorities of St. Paul's Cathedral we have been enabled to carry out some observations in the Whispering Gallery. Owing to the limitations of time, there was not sufficient opportunity to make any reliable *quantitative* observations in the gallery, but as to the general nature of the phenomena observable in it, there was no doubt whatsoever. With a steady source of sound placed on the wall of the gallery at one point and blown from a bellows at steady pressure, observations were made at different points in the gallery. Exceedingly well-marked fluctuations of intensity were perceptible as the observer's ear was moved radially away from the wall, sound alternating with comparative silence at distances of the same order as the wave-length of the sound. (At the minima, some of the overtones of the source could be heard while the fundamental was practically silent). These radial fluctuations of intensity could be observed practically at all points in the gallery. Very pronounced fluctuations of intensity were also noticeable when the observer's head was moved *circumferentially*, that is, parallel to the wall, and this latter effect, though detectable everywhere, was most marked near the far end of the diameter of the gallery in which the source was situated. The distances apart of the successive minima were about half the wave-length of the sound used. It was found that by using a fairly high-pitched source of sound and a sensitive flame run from a cylinder of gas under pressure, the radial and circumferential fluctuations of intensity could be demonstrated in the gallery.

It was found also that similar effects could be observed in the laboratory, using a bird-call or a Galton whistle, blown from a steady-pressure bellows as source, and a sensitive flame as detector, with a curved metal sheet as the reflecting wall. In one series of observations, a semicircular archway at the University College Laboratory was also successfully used to exhibit the effect. We should here mention an important difference which was noticed between the case in which a semicircular reflector is used and that of a complete circular reflector. It is obvious that in the latter case the sound can go right round the circumference in either direction, whereas in the case of the semicircular cylinder we are confined to the effects due to the propagation of sound in one direction only. In both cases radial and circumferential variations of intensity may be observed; but their character, *especially of the latter*, is different in the two cases. It was found in the case of the semicircular reflector that the circumferential variations of intensity did not occur at equidistant points, but were much more crowded together near the end of the diameter remote from the source. With the complete circular reflector, on the other hand, they were equidistant, suggesting thereby that the circumferential nodes and loops in the latter case were due mainly to the interference of waves travelling in opposite directions along the surface of the gallery.

4. *Discussion of the Theory.*

Rayleigh's theory is attractive in its simplicity, but as is evident from the foregoing experimental results, it does not fully explain the observed phenomena. The treatment is based on the assumption that the progressive wave can be represented by a term of which the amplitude factor is a Bessel function* of large integral order which vanishes *only* on the boundary, but *not* also at any internal points; in other words, that it is analogous to a stationary vibration with a large number of nodal diameters but no internal nodal circles. This assumption would be justifiable if the wave-motion were *strictly* circumferential, but in the actual circumstances of the problem this is not rigorously true, as a certain amount of divergence in the radiation from the source is inevitable. In attempting to develop a more complete theory, two distinct modes of approach suggest themselves, neither of which, however, is free from difficulties. The first method is to regard the illumination near the surface of the mirror as due to the superposition of the fringe-systems accompanying the caustics formed by one, two or more reflections, utilizing Airy's well-known investigation for the purpose. The second method is to treat the progressive disturbance as the resultant of two systems of stationary waves differing in phase existing in the space enclosed by the cylindrical wall, and to investigate the latter by the method of stationary vibrations on the basis of the strict dynamical theory and of the boundary conditions to be satisfied at the surface of the wall. Perhaps the difference between the two methods of approach here indicated is really less fundamental than might seem at first sight, for as has been shown by J. W. Nicholson, the Bessel functions of large but nearly equal order and argument involved in the solution of the

* Or its differential in the case of the velocity potential in the acoustical problem, and of the magnetic vector in the electromagnetic problem.

boundary-value problem are most easily evaluated by transforming them into integrals of Airy's type. The important point to be noticed is that the solution on the basis of strict dynamical theory would include vibrations having nodal *circles* as well as nodal *diameters.* Either mode of dealing with the problem thus indicates *radial* alternations of intensity of the kind observed in experiment. The *circumferential* alternations of intensity would arise in two ways: (*a*) by the superposition of progressive waves travelling in opposite directions round the gallery; and (*b*) by the superposition of progressive waves of identical frequency travelling in the same direction but of slightly differing *circumferential* wave-length, that is, having a larger or smaller number of *nodal circles.* The effect contemplated in (*a*) necessarily arises when we are dealing with sustained sounds in a circular gallery and may be regarded as simply representing a modified type of stationary vibration. The effect in (*b*) is most easily observed in experiments with a semi-circular gallery, as it then occurs by itself.

5. *Summary and Conclusion.*

The paper deals with some observations made in the Whispering-Gallery in St. Paul's Cathedral and also in laboratory experiments, which show that while Rayleigh's theory of the phenomenon is undoubtedly on the right lines, it does not offer a complete explanation of all that is observed. The optical analogue of the Whispering-Gallery is described, and it is shown that the effect contemplated by Rayleigh, that is, a single belt of maximum intensity close to the wall, is obtained only in the limiting case when the radius of the reflecting circle is practically infinite in comparison with the wave-length. For more moderate values of the radius of curvature, we get a succession of belts of alternately great and small intensity. Similar effects are also observed in the acoustical case. The explanation of the results is discussed, and it is shown that the slight inevitable deviation from the condition of strictly circumferential wave-propagation postulated by Rayleigh must give rise to such effects.

A considerable part of the experimental work described in this paper was carried out at University College, London, during the short visit of one of us to England. We wish to express our cordial thanks to Prof. A. W. Porter, F.R.S., for his kindly interest and encouragement and for the facilities he put at our disposal for the investigation.