

The Diffraction of Light by Metallic Screens.

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1. *Introduction.*

Gouy* discovered that when a metallic screen with a sharp and highly-polished edge is held in the path of a pencil of light, its boundary appears as a luminous line diffracting light through large angles, both into the region of shadow (interior diffraction) and into the region of light (exterior diffraction). He noticed further that this diffracted light is strongly polarised, but in perpendicular planes in the two regions mentioned; the colour of the diffracted light and its state of polarisation depend in a remarkable manner on the material of the screen and on the extent to which its edge is rounded off in the process of polishing. When the edge is viewed through a double-image-prism from within the shadow, only that image appears coloured which is more intense and is polarised with the magnetic vector parallel to the edge. The second image which is fainter and is polarised with the electric vector parallel to the edge, appears perfectly white. When the incident light is polarised in any arbitrary azimuth, the diffracted light is found to exhibit elliptic polarisation. These and other results have been confirmed by later observers.†

Gouy's experimental results were discussed by Poincaré on the basis of the electromagnetic theory of light in two memoirs published in the "Acta Mathematica."‡ The special case of an ideal screen (plane or wedge-shaped), supposed perfectly-reflecting and having a sharp edge, is amenable to complete theoretical treatment, and was dealt with by Poincaré himself, and later in a rigorous manner by Sommerfeld,§ and following him by numerous other mathematicians. The behaviour of actual metallic screens, however, differs considerably from that found theoretically for this ideal case. Though attempts have been made by Poincaré himself in the memoirs quoted, and later also by Epstein,|| to take the nature of the screen and the rounding of its edge into account, it cannot be said that Gouy's observations have so far received a complete or satisfactory explanation. We propose in this paper to discuss more particularly the

* 'Ann. Chim. et Physique,' vol. 8, p. 145 (1886).

† W. Wien, 'Wied. Ann.,' vol. 28, p. 117 (1886).

‡ 'Acta Mathematica,' vol. 16, p. 297 (1892), and vol. 20, p. 313 (1896).

§ 'Math. Annalen,' vol. 47, p. 317 (1896).

|| 'Diss. Munich'; also 'Encyklop. d. math. Wissensch.,' vol. V (3), p. 491.

influence of the material of the screen on the diffraction by a sharp edge, and to show how it may be explained in a very simple manner. The case of rounded edges is reserved for discussion in a separate paper.

In the fifth section of his first memoir, Poincaré discussed the electromagnetic boundary conditions at the surface of an imperfectly conducting screen, and made the important remark that the extreme smallness of the depth to which an optical disturbance penetrates into any actual metal, should considerably simplify the theory. In his actual attempt, however, to discuss the problem of diffraction by an imperfectly conducting screen, he made no use of the elegant mathematical methods and results contained in the earlier parts of his memoir, and contented himself with a qualitative discussion on the basis of the Kirchoff formulation of Huygens' Principle. The treatment given does not, as was indeed remarked by Poincaré himself, appear capable of leading to quantitative results. In the course of our paper, we shall show how it is possible to apply the Fresnel-Huygens' Principle with success to the problem of diffraction by imperfectly-conducting screens. It is more convenient, however, to base our treatment in the first instance on a modification of the known exact solutions for the case of perfectly-reflecting screens or wedges.

2. *Theory.*

Sommerfeld's solution of the wave-equation in cylindrical co-ordinates for the case of a semi-infinite screen which is a perfect reflector and lies in the xz -plane, with its edge along the z -axis, is

$$u = F(\rho, \phi, \phi_0) \mp \bar{F}(\rho, \phi, -\phi_0). \tag{1}$$

The upper (minus) sign refers to the case in which the plane of polarisation and the plane of incidence are parallel to each other; (we shall refer to this as the \parallel^2 case), and u then denotes the electric force parallel to the edge. The lower (plus) sign refers to the case in which the plane of polarisation of the incident light is perpendicular to the plane of incidence (we shall refer to this as the \perp^r case), u then denoting the magnetic force parallel to the edge.

$$F(\rho, \phi, \phi_0) = \left(\frac{i}{\pi}\right)^{\frac{1}{2}} e^{i\frac{2\pi t}{T}} e^{ik\rho \cos(\phi-\phi_0)} \int_{-\infty}^{\tau} e^{-i\lambda^2} d\lambda, \tag{2}$$

where

$$\tau = \sqrt{2k\rho} \cdot \cos \frac{1}{2}(\phi - \phi_0).$$

The expression (2) has the property that when τ is positive and sufficiently large, which is the case when $\pi + \phi_0 > \phi > 0$, the function tends to the limiting value $e^{i\frac{2\pi t}{T}} e^{ik\rho \cos(\phi-\phi_0)}$, which represents a train of plane waves incident on

the screen in a direction making an angle ϕ_0 with its plane; when τ is negative and sufficiently large, which is the case when $\pi + \phi_0 < \phi < 2\pi$, the function tends to the limit zero. $F(\rho, \phi, -\phi_0)$ is obtained by writing $-\phi_0$ for ϕ_0 . When $\pi - \phi_0 > \phi > 0$ it tends to the limit $e^{i\frac{2\pi t}{T}} e^{ik\rho \cos(\phi + \phi_0)}$, which represents a plane train of waves reflected from the screen. When $\pi - \phi_0 < \phi < 2\pi$, $F(\rho, \phi, -\phi_0)$ tends to the limit zero.

Now the solutions (1) satisfy the conditions $u = 0$ and $\frac{\partial u}{\partial \phi} = 0$ respectively, on both faces of the screen, supposed to be infinitely thin and perfectly reflecting, that is, when $\phi = 0$ and also when $\phi = 2\pi$. Now any actual screen to be opaque must be of finite thickness, hence a solution of the form (1) or any simple modification of it cannot be expected to represent the behaviour of such screens *completely*. Nevertheless, as already mentioned above, a metallic screen of a thickness which is only a small fraction of a wave-length, is practically opaque, and this makes it possible to represent its behaviour with a high degree of accuracy by a comparatively simple modification of (1). Consider expressions of the form

$$u = F(\rho, \phi, \phi_0) - (C_s + iD_s) \cdot F(\rho, \phi, -\phi_0) \quad (3)$$

and

$$u = F(\rho, \phi, \phi_0) + (C_p + iD_p) \cdot F(\rho, \phi, -\phi_0) \quad (4)$$

in which the (numerical) factors $C_s + iD_s$ and $C_p + iD_p$ are so chosen that they represent the amplitude of the wave reflected at the illuminated face of the particular screen used, for the particular angle of incidence under consideration. Equation (3) refers to the \parallel^i case, and (4) to the \perp^r case. Since $C_s + iD_s$ and $C_p + iD_p$ are functions only of the angle of incidence ϕ_0 , (3) and (4) continue to satisfy the wave-equation and represent distributions of light and shadow of the same general character as those indicated by (1), with this difference, however, that the disturbance on the illuminated face of the screen expressed by (3) and (4) will have the actual values corresponding to the screen used, while (1) corresponds to a screen with properties which cannot be physically realised. We are therefore justified in expecting that (3) and (4) would represent the disturbance throughout the whole field in the actual problem much more accurately than the Sommerfeld formulæ.

To understand the physical significance of the formulæ, it is best to use the asymptotic expressions for the functions for large values of ρ . We have, when $\cos \frac{1}{2}(\phi - \phi_0)$ is positive,

$$F(\rho, \phi, \phi_0) \sim e^{i\frac{2\pi t}{T}} e^{ik\rho \cos(\phi - \phi_0)} + \frac{i^{3/2} e^{i\frac{2\pi t}{T}} e^{-ik\rho}}{2\sqrt{2\pi k\rho} \cdot \cos \frac{1}{2}(\phi - \phi_0)}, \quad (5)$$

while when $\cos \frac{1}{2}(\phi - \phi_0)$ is negative, there is a similar expression in which the first term is missing. Similarly when $\cos \frac{1}{2}(\phi + \phi_0)$ is positive, we have

$$F(\rho, \phi, -\phi_0) \sim e^{i\frac{2\pi t}{T}} e^{ik\rho \cos(\phi + \phi_0)} + \frac{i^{3/2} e^{i\frac{2\pi t}{T}} e^{-ik\rho}}{2\sqrt{2\pi k\rho} \cdot \cos \frac{1}{2}(\phi + \phi_0)}, \quad (6)$$

and a similar expression in which the first term is missing, if $\cos \frac{1}{2}(\phi + \phi_0)$ is negative. The general expression for the light diffracted by the edge is either

$$E_z = \frac{i^{3/2} e^{i\frac{2\pi t}{T}} e^{-ik\rho}}{2\sqrt{2\pi k\rho}} \left[\frac{1}{\cos \frac{1}{2}(\phi - \phi_0)} - \frac{C_s + iD_s}{\cos \frac{1}{2}(\phi + \phi_0)} \right] \quad (7)$$

or

$$H_z = \frac{i^{3/2} e^{i\frac{2\pi t}{T}} e^{-ik\rho}}{2\sqrt{2\pi k\rho}} \left[\frac{1}{\cos \frac{1}{2}(\phi - \phi_0)} + \frac{C_p + iD_p}{\cos \frac{1}{2}(\phi + \phi_0)} \right]. \quad (8)$$

3. An Alternative Treatment.

The formulæ (7) and (8) may also be derived in the following manner based on the Fresnel-Huygens' Principle. It is readily shown that the wave-equation in cylindrical co-ordinates,

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial u}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0, \quad (9)$$

is exactly satisfied by putting

$$u = e^{i\frac{2\pi t}{T}} \frac{e^{-ik\rho}}{2\sqrt{2\pi k\rho}} \cos \frac{1}{2}(\phi + \epsilon), \quad (10)$$

which represents a cylindrical wave of Poisson's type diverging from the z -axis. A plane wave incident on the xz -plane in the direction ϕ_0 is transmitted through the part of the plane to the left of the edge and is reflected from the part on the right. To find the disturbance diverging from the edge of the screen, at any point very distant from it, we divide the area of the xz -plane adjacent to the edge into half-period strips parallel to it on either side, and show in the usual way that the effect of the transmitted wave reduces to one-half of the first half-period strip on one side of the edge, and that the effect of the reflected wave reduces to that of a similar strip on the other side. Assuming that each of these strips is the origin of a cylindrical wave of the type appearing in (10), we may write the total disturbance at (ρ, ϕ) diverging from the origin, in the form

$$u = e^{i\frac{2\pi t}{T}} \frac{e^{-ik\rho}}{2\sqrt{2\pi k\rho}} [A_0 \cos \frac{1}{2}(\phi + \alpha) + B_0 \cos \frac{1}{2}(\phi + \beta)], \quad (11)$$

where A_0 , B_0 , α and β have to be so chosen as to give the amplitudes and phases of the divergent waves correctly. Now the width of either half-period strip is easily shown to be

$$\frac{\lambda}{4 \cos \frac{1}{2}(\phi - \phi_0) \cos \frac{1}{2}(\phi + \phi_0)},$$

and we may assume, as is usual in the elementary diffraction theory, that the amplitudes A_0 and B_0 are proportional to this width. It is necessary that the term in (11) proportional to A_0 , contributed by the transmitted wave, remains finite when $\phi = \pi - \phi_0$, and that proportional to B_0 , contributed by the reflected wave, remains finite when $\phi = \pi + \phi_0$, as these directions do not coincide with the respective directions of travel of these waves. We are thus obliged to assume that $\alpha = \phi_0$ and that $\beta = -\phi_0$. The equation (11) then reduces to

$$u = e^{i\frac{2\pi t}{T}} \frac{e^{-ik\rho}}{2\sqrt{2\pi k\rho}} \left[\frac{A}{\cos \frac{1}{2}(\phi - \phi_0)} + \frac{B}{\cos \frac{1}{2}(\phi + \phi_0)} \right], \quad (12)$$

where A and B are suitably chosen constants. As in the elementary diffraction theory we write

$$A = e^{-i\frac{\pi}{4}} = i^{3/2},$$

which expresses the difference of path of $\lambda/8$ between a parent plane wave and the divergent cylindrical wave from a lamina strip cut out of it. The value of B relatively to A evidently depends on the change of amplitude and phase occurring in reflection at the surface of the screen. We write therefore

$$B/A = -(C_s + iD_s) \quad \text{or} \quad +(C_p + iD_p),$$

according to the state of polarisation of the incident beam. The formulæ (7) and (8) are then reproduced.

4. *Explanation of Ellipticity of the Diffracted Light.*

From expressions (7) and (8) the ellipticity of the light diffracted through large angles, when the incident light is polarised in any arbitrary azimuth, follows as an immediate consequence. We shall consider first the case of normal incidence on a plane screen. We have then

$$C_s + iD_s = C_p + iD_p = \frac{n(1 - i\kappa) - 1}{n(1 - i\kappa) + 1}.$$

Taking for the case of a steel edge and for $\lambda = 5.80 \times 10^{-5}$ cm., $n\kappa = 3.24$ and $n = 2.46$, we have $C_s + iD_s = C_p + iD_p = 0.69 - i \times 0.29$. With these numerical values and writing the expressions in the square brackets on the right-

hand side of (7) and (8) in the form $F_s + iG_s$ and $F_p + iG_p$ respectively, the values of $F_s^2 + G_s^2$ and of $F_p^2 + G_p^2$ for various angles of observation, and the phase differences between these two components are shown in Table I.

Gouy noticed that with a sharp steel edge, the light diffracted into the region of shadow shows no sensible ellipticity when the deviation is less than 45° . For larger deviations, it becomes sensible, the \parallel^r component being in advance of the \perp^r component, the difference of path being, however, always numerically less than $\lambda/4$. It will be seen that this is in general agreement with the figures for the difference of path shown in the fifth column of Table I. The table also indicates the interesting result that in the region of exterior diffraction, the path difference changes sign and increases in a continuous manner up to the boundary of reflection, when it becomes half a wave-length, while according to the Sommerfeld formulæ there is a sudden reversal of phase when the diffracted ray lies in the continuation of the plane of the screen.

Table I.—Diffraction by Steel Edge : Normal Incidence.

\parallel^r and \perp^r indicate plane of polarisation parallel and perpendicular respectively to the plane of incidence.

$$\lambda = 5.80 \times 10^{-5} \text{ cm.}$$

Region of observation.	Direction of diffracted ray ϕ .	Intensity of \parallel^r component $F_s^2 + G_s^2$.	Intensity of \perp^r component $F_p^2 + G_p^2$.	Difference of path between the components $\frac{\delta_p - \delta_s}{\lambda}$.	Difference of path according to Sommerfeld formulæ.
Boundary of reflection....	90	—	—	0.50	0.50
Exterior diffraction (region of illumination)	120	15.0	3.9	0.45	0.50
	150	6.7	0.38	0.35	0.50
	180	5.9	0.36	0.15	—
	210	8.0	1.55	0.06	0
	240	21.1	10.0	0.03	0
Boundary of shadow.....	270	—	—	0	0
Interior diffraction (region of shadow)	285	49	70	-0.01	0
	300	10.0	21.1	-0.03	0
	315	3.6	11.4	-0.04	0
	330	1.55	8.0	-0.06	0
	345	0.72	6.4	-0.09	0
	360	0.36	5.9	-0.15	—

5. *Effect of Oblique Incidence.*

When the light is incident on the screen obliquely at an angle θ (measured as usual from the normal) we have

$$C_s + iD_s = -\frac{\cos \theta - \sqrt{n^2(1 - i\kappa)^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2(1 - i\kappa)^2 - \sin^2 \theta}}, \quad (13)$$

and

$$C_p + iD_p = \frac{n^2(1 - i\kappa)^2 \cos \theta - \sqrt{n^2(1 - i\kappa)^2 - \sin^2 \theta}}{n^2(1 - i\kappa)^2 \cos \theta + \sqrt{n^2(1 - i\kappa)^2 - \sin^2 \theta}}, \quad (14)$$

At normal incidence (13) and (14) are equal, and at grazing incidence they are again equal but opposite in sign. At the principal incidence (which lies between 70° and 80° for most metals) the difference of path between the \parallel and \perp components of the reflected wave amounts to $\lambda/4$ and then rapidly diminishes to 0 as grazing incidence is approached. With the help of the formulæ (7), (8), (13) and (14) the intensity of the two components of the diffracted light and their phase-difference can be calculated for any angle of incidence and of observation. Let us first consider moderately oblique incidence. Two cases have naturally to be distinguished, viz., when the incidence is from the screen-side of the normal, and when it is from the farther side. Tables II and III give the values for a steel screen when $\phi_0 = 135^\circ$ and 45° , respectively, for yellow light. It can be seen that the effects observed in interior and exterior diffraction are no longer similar to each other.

Table II.—Diffraction by a Steel Edge. $\phi_0 = 135^\circ$.

Region of observation.	Direction of diffracted ray ϕ .	Intensity of \parallel component $F_s^2 + G_s^2$.	Intensity of \perp component $F_p^2 + G_p^2$.	Difference of path between the components $\frac{\delta_p - \delta_s}{\lambda}$.
Exterior diffraction	135	4.6	0.30	0.23
	180	3.8	0.35	0.14
	225	4.9	0.8	0.09
	270	12	4.0	0.04
	285	24	11	0.03
	300	75	48	0.02
Boundary of shadow.....	315	—	—	0
Region of shadow	330	41	75	-0.02
	345	6.0	27	-0.05
	360	0.7	18	-0.17

In the case considered in Table II, the region of shadow is of much smaller angular width, and the degree of polarisation and also the ellipticity increase rapidly as we approach the plane of the screen ($\phi = 360^\circ$). On the other hand, in Table III the region of shadow is much wider, and the polarisation and ellipticity of the diffracted light increase less rapidly with increasing deviation of the diffracted ray. In exterior diffraction these effects are reversed.

Table III.—Diffraction by a Steel Edge. $\phi_0 = 45^\circ$.

Region of observation.	Direction of diffracted ray ϕ .	Intensity of \parallel^i component $F_s^2 + G_s^2$.	Intensity of \perp^r component $F_p^2 + G_p^2$.	Difference of path between the components $\frac{\delta_p - \delta_s}{\lambda}$.
Exterior diffraction	135	—	—	0.45
	165	28	2.3	0.32
	195	31	8.4	0.06
Boundary of shadow	225	—	—	0
Region of shadow	255	9.6	22	-0.03
	285	1.5	7.0	-0.06
	315	0.5	4.2	-0.09
	345	0.2	3.3	-0.14
	360	0.1	3.1	-0.17

6. *Diminished Intensity of Diffracted Light.*

If we compare the figures shown in Tables II and III with those calculated from Sommerfeld's formulæ for a perfectly reflecting screen, we find that the effect of imperfect conductivity of the screen is not only to introduce elliptic polarisation, but also to diminish the total intensity of the diffracted light and the ratio of the components of vibration for specified angles of incidence and diffraction. In fact, these effects are all closely related to one another, and become the more striking when the incidence is very oblique.

In Table IV the case of oblique incidence on a steel edge has been worked out and shown. It is assumed that $\phi_0 = 170^\circ$, that is, only 10° short of grazing incidence. The figures for the steel edge and for a perfectly reflecting screen are shown side by side for comparison, and it will be seen that the intensity of the \perp^r component is diminished to one-fifth of its value, on the surface of the screen, by reason of the imperfect conductivity, while that of the \parallel^i component is very slightly increased. Nevertheless, the ratio of the \perp^r and \parallel^i components remains large, showing that even at such incidences the polarisation remains large. We have to approach grazing incidence still more closely before the

Table IV.—Diffraction by a Steel Edge. $\phi_0 = 170^\circ$.

Region of Shadow.

Direction of diffracted ray ϕ .	Intensity of \parallel' component for a perfect conductor.	Intensity of \parallel' component for steel $F_s^2 + G_s^2$.	Intensity of \perp' component for steel $F_p^2 + G_p^2$.	Intensity of \perp' component for a perfect conductor.	Difference of path between the components for steel $\frac{\delta_p - \delta_s}{\lambda}$.
0	—	—	—	—	0
350	—	—	—	—	—
352	2590	2600	3100	4060	-0.01
354	460	480	750	1280	-0.02
356	119	128	313	746	-0.04
358	23	28	170	571	-0.07
360	0	0.9	109	527	-0.23

diminution of the \perp' component is such as to produce a striking diminution of the completeness of polarisation. From formulæ (7), (8), (13) and (14) it follows that, at grazing incidence, the diffracted light should be unpolarised at all angles.

The foregoing considerations help to explain, at least in part, the interesting observation of Gouy that for given directions of the incident and diffracted rays, the intensity of the diffracted light is a *maximum* when the plane of the screen bisects the angle between these two directions. According to Sommerfeld's formulæ the intensity should be a *minimum* for this position of the screen. Owing, however, to the imperfect conductivity of actual screens, as we have seen, the intensity falls off in approaching the extreme cases in which the light is incident grazingly on the screen from either direction. As we shall see later, other circumstances, as, for instance, the finite angle formed by the faces of the screen at the edge, or the actual rounding off of the latter, would also operate in the direction of diminishing the intensity of the diffracted light in the two extreme positions of the screen. Hence the intermediate position for the screen actually gives the maximum instead of the minimum intensity for the diffracted rays in the particular direction.

7. Explanation of Diffraction Colours.

The wave-length enters in the expression for the intensity of the diffracted light in two distinct ways. Referring to (7) and (8) it will be seen that the intensity is inversely proportional to k , that is, proportional to the wave-length. The longer wave-lengths would thus tend to be more prominent in

the light diffracted by the edge than in the incident light. This effect would operate both on the parallel and perpendicular components of vibration in the diffracted light. The colour of the diffracted light is also influenced, and in an entirely different way, by the factors $C_s + iD_s$ and $C_p + iD_p$ appearing in (7) and (8), which are, in general, functions of the wave-length of the incident light. If we confine ourselves to the case of normal incidence, $C_s + iD_s$ and $C_p + iD_p$ are identical in magnitude. But the former appears with a negative sign in (7) and the latter with a positive sign in (8). Hence, if a particular wave-length appears with a strengthened amplitude in (8) it will appear with a weakened amplitude in (7), and *vice versa*. This, taken together with the proportionality to λ already mentioned, furnishes an explanation of the difference in the colour of the parallel and perpendicular components of the light diffracted into the region of shadow, which was discovered by Gouy for metals such as copper and gold. It can easily be seen that in the region of shadow, the longer wave-lengths which are strongly reflected by the metal would be much enhanced in the perpendicular component, while the corresponding weakening in the parallel component would be almost insensible. In the region of exterior diffraction, these features are interchanged.

Table V.—Gold Screen. $\phi_0 = 90^\circ$ (Normal Incidence).

Direction of diffracted ray ϕ .	$(F_s^2 + G_s^2) \lambda \times 10^5$, for $\lambda \times 10^5 =$			$(F_p^2 + G_p^2) \lambda \times 10^5$, for $\lambda \times 10^5 =$		
	4.20.	5.80.	7.00.	4.20.	5.80.	7.00.
270	—	—	—	—	—	—
285	217	282	326	279	408	508
300	48.3	59.0	64.4	80.4	125	159
315	19.1	22.1	22.1	41.8	68.3	88.7
330	9.56	10.7	9.50	28.1	48.5	63.9
345	5.44	6.36	4.87	22.1	40.2	53.6
360	3.41	4.86	3.48	19.5	37.6	50.7

In Table V the intensity of the diffracted light has been calculated for the case of a gold screen using the following data :—

$$\begin{aligned} \lambda &= 7.00 \times 10^{-5} \text{ cm.} ; & n &= 0.280 , & n\kappa &= 3.800 ; \\ \lambda &= 5.80 \times 10^{-5} \text{ cm.} ; & n &= 0.415 , & n\kappa &= 2.750 ; \\ \lambda &= 4.20 \times 10^{-5} \text{ cm.} ; & n &= 1.570 , & n\kappa &= 1.800. \end{aligned}$$

From the table it will be seen that in the region of shadow $(F_s^2 + G_s^2)\lambda$ has practically the same value for different wave-lengths, while $(F_p^2 + G_p^2)\lambda$

increases in value as we proceed towards the red end of the spectrum. The increase with wave-length is much more marked for large angles of diffraction than for small angles. Further, $F_p^2 + G_p^2$ is always greater than $F_s^2 + G_s^2$, the ratio between the two increasing with the angle of diffraction. From these facts, it follows that when the region of shadow is examined, the \parallel^i component of the diffracted light will be perfectly white, while the \perp^r component, which is in fact much stronger than the other, will exhibit an orange-yellow tint, the colouration being the more marked, the further we go into the region of shadow. The same colour effects will be noticeable also in the region of exterior diffraction, the \parallel^i and \perp^r components now, however, exchanging places.

8. Intensified Colours at Oblique Incidences.

While the variation with wave-length of the intensity of the \perp^r component shown in Table V is marked enough, it is not exceptionally large, being in fact of the same order of magnitude as the variation of the reflecting power of the metal with wave-length. This is in agreement with observation, for Gouy found that the sharpest metallic edges do not show particularly vivid colours by diffraction. When, however, the incidence on the screen is made oblique, the colours of the \perp^r component should become more lively. To understand why this should be the case, we have only to refer to section 6 above, in which it was shown that the imperfect reflectivity of the metal results in a diminution of the intensity of the diffracted light in comparison with the theoretical value for a perfectly reflecting screen, and that this diminution becomes the more marked as the incidence of the light on the screen becomes more oblique. Those wave-lengths, however, for which the reflecting power of the metal approaches unity, persist in nearly full strength in the \perp^r component of the diffracted light, and hence determine its colour in increasing measure as the obliquity of the screen is increased. It is to be noted also that the colour should appear at smaller deviations of the ray in interior diffraction, and at larger deviations in exterior diffraction, or *vice versa*, according to the position of the screen.

In Table VI the case of a gold screen, for a position of the screen 10° short of grazing incidence, has been worked out and the intensities of the \parallel^i and \perp^r components are shown for six different wave-lengths, the direction of observation considered being along the surface of the screen in the region of shadow. The normal reflecting power of the metal for the same wave-lengths is also shown for comparison.

Table VI.—Gold Edge. $\phi_0 = 170^\circ$, $\phi = 360^\circ$.

$\lambda \times 10^5$ (cm.).	Intensity of \parallel^l component $(F_s^2 + G_s^2) \lambda \times 10^5$.	Intensity of \perp^r component $(F_p^2 + G_p^2) \lambda \times 10^5$.	The reflection coefficient.
4.00	9.9	200	0.360
4.60	12.5	210	0.358
5.20	17.5	240	0.608
5.80	10.5	470	0.827
6.20	8.8	650	0.889
7.00	7.1	1000	0.930

It will be noticed that the intensity of the \parallel^l component varies but little with wave-length, while the \perp^r component shows large intensities in the orange and red regions in the spectrum. The effect in the latter case is of a highly selective character, becoming pronounced only for the wave-lengths for which the reflecting power of the metal approaches unity.

When the figures shown in Table VI are computed after the manner employed by the late Lord Rayleigh* for discussion of the colours of thin plates, and plotted in Maxwell's colour-triangle, it is found that the \parallel^l component is perfectly white, while the \perp^r component is of a rich orange-yellow colour.

The case of other metals may be worked out in a similar manner. In Table VII are given the reflection-coefficients and the colour of the diffracted light as observed by Gouy and Wien, for a number of metals. The general relationship between them is fairly clear from the figures. In the case of the whiter metals, of course, the colour is largely determined by the factor λ appearing in the expression for the intensity of the diffracted light.

Table VII.—Reflection Coefficients and Diffraction Colours.

For $\lambda \times 10^5 =$	4.20	4.50	5.00	5.50	6.00	6.50	7.00	Colour of \perp^r component of diffracted light.
Silver	0.866	0.905	0.913	0.927	0.926	0.935	0.946	Pale yellow.
Copper	0.33	0.37	0.44	0.48	0.72	0.80	0.83	Red.
Steel	0.52	0.54	0.55	0.55	0.55	0.56	0.58	Reddish white.
Platinum	0.518	0.547	0.584	0.611	0.642	0.663	0.69	Yellow.
Zinc	0.803	0.806	0.805	0.789	0.774	0.771	0.770	Colour insensible.
Tin	—	0.605	0.670	0.686	0.706	0.713	0.716	Greenish yellow.

* Lord Rayleigh, 'Scientific Papers,' vol. 2, p. 498.

9. Diffraction by Metallic Wedges.

Poincaré considered the case of a perfectly reflecting wedge in his first memoir, and showed that if the surfaces of the wedge are given by the angles $\phi = 0$ and $\phi = \chi$, the rays diffracted from its edge have an amplitude proportional to

$$\left[\frac{1}{\cos \frac{\pi^2}{\chi} - \cos \frac{\pi}{\chi} (\phi - \phi_0)} \mp \frac{1}{\cos \frac{\pi^2}{\chi} - \cos \frac{\pi}{\chi} (\phi + \phi_0)} \right]. \quad (15)$$

As the result of a more elaborate analysis, Wiegrefe* found for the cylindrical wave diverging from a wedge-shaped edge the identical expression given by (15) with the multiplying factor

$$-i^{3/2} e^{i\frac{2\pi t}{T}} e^{-ik\rho} \sqrt{\frac{\lambda}{\rho}} \cdot \frac{\sin \frac{\pi^2}{\chi}}{2\chi}. \quad (16)$$

On putting $\chi = 2\pi$, the formulæ (15) and (16) reduce to those for the case of a perfectly reflecting plane screen. In the case of an imperfectly conducting wedge, we modify expression (15) and write it in the form.

$$\left[\frac{1}{\cos \frac{\pi^2}{\chi} - \cos \frac{\pi}{\chi} (\phi - \phi_0)} - \frac{C_s + iD_s}{\cos \frac{\pi^2}{\chi} - \cos \frac{\pi}{\chi} (\phi + \phi_0)} \right], \quad (17)$$

or

$$\left[\frac{1}{\cos \frac{\pi^2}{\chi} - \cos \frac{\pi}{\chi} (\phi - \phi_0)} + \frac{C_p + iD_p}{\cos \frac{\pi^2}{\chi} - \cos \frac{\pi}{\chi} (\phi + \phi_0)} \right], \quad (18)$$

where C_s, D_s, C_p, D_p have the same significance as previously, and are functions of the angle of incidence of the light on the illuminated side of the wedge.

From formula (15) it appears that along the two surfaces of the wedge $\phi = 0$ and $\phi = \chi$, the diffracted light should be completely polarised with the intensity of the \parallel^l component zero, and that of the \perp^r component finite. As the rear surface of the wedge limits the region of shadow, it follows as a consequence that the polarisation-effects should usually appear at smaller deviations of the diffracted ray in the case of a wedge than for a plane screen. When the imperfect reflecting power of the metal is taken into account, as in formulæ (17) and

* A. Wiegrefe, 'Ann. d. Physik,' vol. 39, p. 449 (1912).

(18), it would also follow, for the reasons stated, that the colours of the diffracted light should also be observable at smaller deviations and be generally more striking than for a plane screen.

10. *Summary.*

1. The paper contains a discussion of the observations of Gouy on the intensity, colour and polarisation of the light diffracted through large angles by metallic screens and wedges with polished edges. The well-known expressions due to Poincaré and Sommerfeld referring to diffraction by perfectly-reflecting screens and wedges, are modified so as to take into account the changes of phase and amplitude which occur when light is reflected at the surface of a metal. The modified formulæ are then discussed.

2. The formulæ show that when the incident light is plane-polarised in any arbitrary azimuth, the light diffracted through large angles is elliptically polarised, the sign of the ellipticity being different in interior and exterior diffraction.

3. In interior diffraction, the component polarised in the plane of incidence is white, while the perpendicular component is coloured, the colour depending on the nature of the metal. In exterior diffraction, these effects are reversed.

4. The effect of imperfect conductivity is to make the intensity of the diffracted light less and less as the incidence becomes more and more oblique. This diminution is least for the wave-length for which the reflection-coefficient is largest. The colour-effects arise in this way and therefore become more prominent at oblique incidences.
