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Huygens' principle and the phenomena of total reflection

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Abstract. In this paper the phenomena of total reflection are considered, *de novo*, from the standpoint of the principle of Huygens, no use whatever being made of the Fresnel formulae for reflection and refraction. Huygens' principle enables us to evaluate the disturbance appearing in the second medium when light is incident on the boundary between two media and is totally reflected into the first medium. The disturbance takes the form of a superficial wave moving parallel to the boundary. The existence of such a superficial wave is then shown to involve, as a necessary consequence, an *acceleration* of the reflected wave with reference to the incident wave, the acceleration being zero at critical incidence and increasing to half an oscillation at grazing incidence. The intensity of the superficial wave is at critical incidence greater for the component having the magnetic vector parallel to the surface, but diminishes more rapidly with increasing incidence than for the component having the electric vector parallel to the surface; the phase-advance reaches its maximum value correspondingly sooner. The phase-angle between the two components is evaluated and found to be an *acute* angle, in agreement with the classical treatment based on the Fresnel formulae, but in disagreement with the conclusions of Lord Kelvin and Schuster. The source of error in the Kelvin–Schuster treatment is pointed out. Experimental evidence regarding the magnitude of the phase-advance of each component separately is available and is in agreement with the classical theory.

Finally, a method is described by which the distribution of intensity, state of polarisation, and direction of flow of energy in the superficial wave may be studied experimentally.

1. Introduction

In chapter III of his celebrated treatise on light, Christian Huygens applied his principle of the superposition of elementary wavelets to the explanation of the phenomena of reflection and refraction, and showed that the absence of a refracted wave and the increased intensity of reflection for incidences exceeding the critical angle follow from his principle as simple consequences. The principle of Huygens is thus the natural avenue of approach to the phenomena of total reflection from the standpoint of wave-theory. Following Fresnel, however, the treatment of total reflection usually given is based on the formulae for reflection and refraction obtained by him, a suitable mathematical interpretation being given to the angles of refraction which become imaginary when the incidence exceeds the critical angle. While this method may be mathematically elegant, it leaves the physical aspects of the problem somewhat obscure. Moreover, there has been some controversy about the actual magnitude and sign of the changes of phase occurring in total reflection. Lord Kelvin*, in his *Baltimore Lectures*, discussed the subject at great length on the basis of the mechanical theory of light and claimed that the classical interpretation accepted for 80 years was in error. His views were supported by Schuster† in his book on optics, while Bevan's observations‡ on Lloyd's fringes in internal reflection and Drude's discussion§ of the Fresnel formulae on the electromagnetic theory, on the other hand, appeared to support the classical interpretation. Prof. Schuster has, however, reiterated his views in the new edition of his book‖, and in a recent paper¶ claims that Drude's electromagnetic treatment is in error. In view of these facts, it would seem that an independent treatment of total reflection, based directly on physical principles, and making no use whatever of Fresnel's formulae must be of value. It is proposed in this paper to supply such a treatment.

2. The superficial wave in the second medium

The method of approach which we shall adopt is that indicated by the author in a recent paper**. We regard the disturbance in the second medium as arising from the superposition of the wavelets radiated from different elements of the bounding surface and determine it by evaluating the integral which expresses the result of such superposition.

In figure 1 the plane of incidence is taken to be the plane of the paper. The origin of co-ordinates O is taken to be on the surface at which total reflection occurs, the latter coinciding with the xy-plane ($z = 0$), and the plane of incidence with the xz-plane. In accordance with the principle of Huygens, the effect at a point P in the second medium (co-ordinates x, o, z) is to be found by superposing the effects of the wavelets radiated from different elements of the bounding surface.

Drop a perpendicular PO_1 on the surface and divide the surface into circular zones with O_1 as centre. Denoting O_1P_1 by ρ, where P_1 is any arbitrary point on the surface, we have $PP_1 = r = (z^2 + \rho^2)^{1/2}$. An element of area on the surface is

*Kelvin, *Baltimore Lectures*, 1904 (386–406).
†A. Schuster, *Theory of Optics*, 2nd edn., p. 54.
‡P V Bevan, *Philos. Mag.* **14** 1907 (503).
§P Drude, *Theory of Optics*, English translation, pp. 278–284.
‖A Schuster and J W Nicholson, *Theory of Optics*, 3rd edn., p. 54.
¶A Schuster, *Proc. R. Soc. London* **A107** 1925 (15).
C V Raman, *Proc. Indian Assoc. Cultiv. Sci.*, **9 1926 (271).

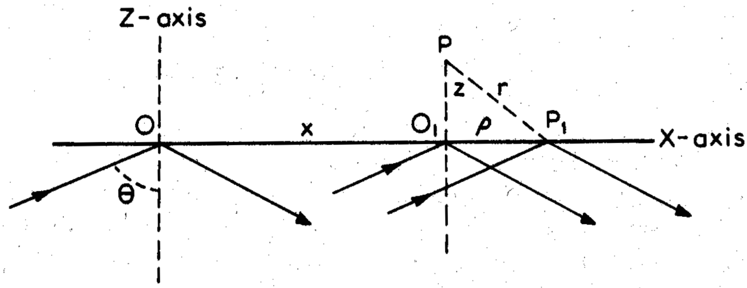

Figure 1

$\rho d\rho d\phi$ and the projection of ρ on the x-axis is $\rho\cos\phi$. If a train of light-waves of period T is incident on the boundary between two media in which the velocities of light are respectively V_1 and V_2, the refractive index of the second medium relatively to the first, V_1/V_2, being n, we may express the disturbance in the first medium due to the incident waves in the form

$$A\cos\left\{\frac{2\pi t}{T}-\frac{2\pi(x\sin\theta+z\cos\theta)}{V_1T}-\tfrac{1}{2}\delta\right\}, \tag{1}$$

where θ is the angle of incidence.

The disturbance due to the wavelet radiating from the elementary area $\rho d\rho d\phi$ at P_1 and reaching P may be assumed to be of the form

$$\frac{k_1A}{V_2T}\cos\left\{\frac{2\pi t}{T}-\frac{2\pi(x+\rho\cos\phi)\sin\theta}{V_1T}-\frac{2\pi r}{V_2T}\right\}\frac{\rho d\rho d\phi}{r}. \tag{2}$$

It will be seen from (1) and (2) that we have assumed a difference of phase $\tfrac{1}{2}\delta$ (for the present undetermined) between the disturbance incident on any element of the surface and the secondary wavelet starting out from it. Such a difference of phase can conceivably exist, though, of course, its value may be zero in special cases. The numerical factor k_1 appearing in the amplitude of the wavelet is also for the present undetermined. The whole effect at the point P is found by integrating (2) over the entire area of the surface. Integrating first with respect to ϕ between the limits 0 and 2π, the result appears in the form

$$A_1\cos\frac{2\pi}{T}\left(t-\frac{x\sin\theta}{V_1}\right)+A_2\sin\frac{2\pi}{T}\left(t-\frac{x\sin\theta}{V_1}\right), \tag{3}$$

where

$$A_1=\frac{k_2}{V_2T}\int_0^\infty A\cdot J_0\left(\frac{2\pi\rho\sin\theta}{V_1T}\right)\cos\frac{2\pi r}{V_2T}\cdot\frac{\rho d\rho}{r}, \tag{4}$$

and

$$A_2 = \frac{k_2}{V_2 T}\int_0^\infty A\cdot J_0\left(\frac{2\pi\rho\sin\theta}{V_1 T}\right)\sin\frac{2\pi r}{V_2 T}\cdot\frac{\rho d\rho}{r}, \tag{5}$$

where k_2 is proportional to k_1 and is therefore undetermined for the present.

Integrals (4) and (5) are of standard types which have been evaluated by H Lamb*. When $\sin\theta$ is greater than V_1/V_2 or n, which is the condition for total reflection, the integral (5) vanishes, and the integral (4) reduces to the form

$$A_1 = \sigma A\cdot\exp\left(-\frac{2\pi}{TV_2}\cdot\sqrt{\frac{\sin^2\theta}{n^2}-1}\cdot z\right),$$

where σ is a numerical constant which remains to be determined. The disturbance in the second medium is thus of the form

$$\sigma A\cdot\exp\left(-\frac{2\pi}{TV_2}\cdot\sqrt{\frac{\sin^2\theta}{n^2}-1}\cdot z\right)\cdot\cos\frac{2\pi}{T}\left(t-\frac{x\sin\theta}{nV_2}\right), \tag{6}$$

which expresses a wave of amplitude σA at the surface but decreasing exponentially with z and propagated parallel to the surface along the x-axis. We shall consider the magnitude of the energy-flow in the superficial wave a little later, but it is obvious from (6) that it is entirely parallel to the surface, and thus the energy-flux *across* any element of area of the surface must be zero.

3. Change of phase in total reflection

Since there is no energy-flux *across* the boundary, it follows that the amplitudes of the incident and reflected waves must be equal. It can be easily seen, moreover, that the existence of a superficial wave of the form shown in (6) involves as a necessary consequence a difference in the phases of the incident and reflected waves at the boundary. To prove this, we have only to consider the continuity of the disturbance on the two sides of the boundary. It is convenient to consider separately the cases in which the electric vector and the magnetic vector in the incident waves are respectively parallel to the surface of separation between the two media.

Case 1. Light polarised in the plane of incidence. In this case we are concerned with the electric force E_y parallel to y, and identify the amplitude A appearing in our formulae with the amplitude of this vector. In order that E_y may have the same value on both sides of the boundary, we must have, when $z = 0$, the sum of the electric forces in the incident and reflected waves equal to that in the

*H Lamb, *Proc. London Math. Soc.* 7 1909 (140).

superficial wave. In order to obtain this result, using the expressions (1) and (6) already derived for the incident and superficial waves respectively, we are compelled to assume that the electric force in the reflected wave is given by

$$A\cos\left\{\frac{2\pi t}{T}+\frac{2\pi(x\sin\theta-z\cos\theta)}{V_1 T}+\tfrac{1}{2}\delta\right\}, \tag{7}$$

and find that the factor σ appearing in the amplitude of the superficial wave is connected with the phase-advance δ of the reflected wave relatively to the incident wave by the simple relation

$$\sigma = 2\cos\tfrac{1}{2}\delta. \tag{8}$$

We shall use σ_s and δ_s to signify the values of σ and δ in the present case. To find a second relation connecting them, we note that the component of the magnetic force H_x parallel to the surface must also be continuous at the boundary. We must therefore have $(\partial E_y/\partial z)$ the same on both sides of the boundary. Differentiating (1), (6), and (7) with respect to z, and applying this condition, we obtain very readily

$$\sigma_s^2 = \frac{4\cos^2\theta}{1-n^2}, \text{ and } \tan\tfrac{1}{2}\delta_s = \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}. \tag{9}$$

At the critical incidence $\sin\theta = n$, and therefore $\frac{1}{2}\delta_s$ is either zero or any multiple of π. The angle δ_s is either zero or any multiple of 2π. We therefore take δ_s to be zero at the critical incidence. For incidences greater than the critical angle, $\tan\frac{1}{2}\delta_s$ is positive, and becomes infinite at grazing incidence. Accordingly, δ_s is positive and increases from 0 to π, as we pass from critical to grazing incidence.

Case 2. *Light polarised at right angles to the plane of incidence.* In this case the magnetic force is parallel to the y-axis, and we identify it with the vector A appearing in our formula. Since H_y must be the same on both sides of the boundary, we obtain the same expressions (7) for the reflected wave as in the preceding case, A being now understood to refer to the magnetic force in the incident wave. Since the component of the electric force E_x parallel to x must be the same on both sides of the boundary, $(\partial H_y/\partial z)$ in the second medium must be n^2 times as large as its value in the first medium. Differentiating (1), (6), and (7) with respect to z, and applying this condition, we obtain a second relation between σ_p and δ_p in addition to the relation $\sigma_p = 2\cos\frac{1}{2}\delta_p$ deduced as in the preceding case. In this way, we find

$$\sigma_p^2 = \frac{4n^4\cos^2\theta}{(n^4-n^2)+\sin^2\theta(1-n^4)} \text{ and } \tan\tfrac{1}{2}\delta_p = \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}. \tag{10}$$

Arguing exactly as the preceding case, we see that the phase of the reflected wave is *advanced* relatively to that of the incident wave by a quantity δ_p which is zero at critical incidence and increases to π at grazing incidence.

4. Difference of phase of components

Our investigation has shown that the incident and reflected waves are in the same phase at critical incidence, and are opposed in phase at grazing incidence. This is true both in Case 1 and Case 2 and hence the angle $(\delta_p - \delta_s)$, which represents the difference in the phase-advance in the two cases, is zero both at critical incidence and at grazing incidence. Since $n^2 < 1$, it can be seen from (9) and (10) that at intermediate incidences $\tan\frac{1}{2}\delta_p$ is greater than $\tan\frac{1}{2}\delta_s$. In other words, the phase-advance in Case 2 is greater than in Case 1, and $(\delta_p - \delta_s)$ is positive. From (9) and (10) we derive the following formulae:

$$\tan\delta_s = \frac{2\cos\theta\sqrt{\sin^2\theta - n^2}}{(n^2-1)+2\cos^2\theta}, \qquad \cos\delta_s = \frac{n^2-1+2\cos^2\theta}{1-n^2}, \tag{11}$$

$$\tan\delta_p = \frac{2n^2\cos\theta\sqrt{\sin^2\theta - n^2}}{(n^4+1)\cos^2\theta-(1-n^2)}, \quad \cos\delta_p = \frac{(n^4+1)\cos^2\theta-(1-n^2)}{(n^4-1)\cos^2\theta+(1-n^2)}, \tag{12}$$

$$\tan\tfrac{1}{2}(\delta_p-\delta_s) = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta},$$

$$\cos(\delta_p-\delta_s) = \frac{2\sin^4\theta-(n^2+1)\sin^2\theta+n^2}{(1+n^2)\sin^2\theta-n^2}. \tag{13}$$

Formula (13) is identical with that derived by Drude. From our formulae it follows that $(\delta_p - \delta_s)$ for glass is always an acute angle, in agreement with what had generally been accepted as true. According to Kelvin and Schuster, however, the phase of the reflected wave is advanced in Case 1, and retarded in Case 2, and the difference in phase of the two components is represented by an obtuse angle. Since the method adopted in the foregoing treatment is based directly on first principles and gives the quantities under consideration in an entirely unambiguous manner, it would seem that the grounds on which Kelvin and Schuster base their criticism of the classical treatment must be invalid. We shall presently see that this is actually the case.

5. The Kelvin–Schuster treatment

For our purpose it is sufficient to examine the argument in the form in which it is presented by Schuster in his recent paper in which he criticises Drude's treatment, which is, in fact, substantially the same as that put forward by Kelvin. It rests on the supposed necessity for assuming that the well-known Fresnel coefficients of reflection for the parallel and perpendicular components of vibration must be both numerically and algebraically equal to each other in the limiting case of normal incidence. The argument rests on a fallacy which will become clear when we recollect that in order to fix the direction of a ray in a unique manner, we require to know the positive direction of two *vectors*, namely the electric and magnetic vectors. The direction of the ray is perpendicular to both, and the phase of the oscillation, in it may be determined indifferently from the phase of either. In the case of normal incidence, the direction of the ray is reversed on reflection, so that if the electric vectors in the incident and reflected pencils are in the same direction, the magnetic vectors are opposed to each other, and *vice versa.* If, as proposed by Schuster, the positive direction of a vector is always to be so chosen that the positive directions of the vector for the incident and reflected pencils become coincident for normal incidence, we should be led to the absurd conclusion that the phase of a ray on normal reflection is electrically reversed but is magnetically unaltered. It is necessary, in fact, in order to obtain correct results, to assume a rule of signs for vectors lying in the plane of incidence which is exactly opposite to that suggested by Schuster. The scheme adopted by Drude is the correct one, and accordingly gives correct results. All ambiguity or difficulty may be avoided, however, by taking as the phase of a pencil of light, the phase of the vector (electric or magnetic as the case may be) which has a direction perpendicular to the plane of incidence. The direction of such vector remains invariable when the angle of incidence is altered, and no special convention as to sign is necessary. As a matter of fact, the Fresnel formulae for reflection and refraction can easily be derived by considering only the vector (electric or magnetic as the case may be) perpendicular to the plane of incidence and writing down the condition for its continuity at the boundary, as also for the equality of the normal flux of energy through unit area of the boundary on either side of it. The two Fresnel coefficients then appear with *opposite* signs, and since the phase of the reflected ray must be identical for the electric as for the magnetic vibration, the same result must also be valid when we consider the vectors lying in the plane of incidence.

The analytical necessity for so fixing the positive directions of vectors lying in the plane of incidence that the two Fresnel coefficients appear with opposite signs becomes very clear when we consider the case of a circularly-polarised ray incident normally on a surface and reflected from it. In this case, no doubt, both the components of the ray are reflected under the same conditions and it might seem, at first sight, that their relative phase should remain unaltered. In reality, however, owing to the reversal of the direction of the ray, if it has right-handed circular polarisation before reflection, it has left-handed circular polarisation after reflection, and this is analytically equivalent to a reversal of the phase of one of the components relatively to the other. It is precisely in order that the physical and analytical requirements of the case might both be complied with, that the signs of the vectors have to be taken in the manner chosen by Drude. The convention adopted by him is, in fact, merely the well-known Ampère rule of signs in another form. It has the effect of making vectors in the incident and reflected rays, which lie in the plane of incidence and which are both in the same phase and parallel to each other at grazing incidence, continue to be in the same phase, though (geometrically) oppositely directed at normal incidence unless, of course, in the interval a real physical change in their relative phase has occurred.

6. Polarisation and intensity of superficial wave

We have already had occasion to consider the components of the electric and magnetic vectors parallel to the boundary in the superficial wave and their relation to the expressions (1), (6) and (7). The components of the electric and magnetic forces perpendicular to the surface may be similarly found by applying the boundary conditions. It appears that the components E_y and H_x in Case 1 differ in phase by a quarter of an oscillation, and the components H_y and E_x in Case 2 similarly differ in phase. Hence, in either case, the energy-flow along the axis of z is zero. The components H_z and E_y in Case 1 and the components E_z and H_y in Case 2 have, however, identical phases, and therefore give rise to energy-flow parallel to the axis of x proportional to H_zE_y and E_zH_y respectively. The ratio of the energy-flow in the two cases is found to be

$$\frac{n^2\sigma_s^2}{\sigma_p^2}, \quad \text{and } not\ \frac{\sigma_s^2}{\sigma_p^2}.$$

Why the factor n^2 appears is most easily seen on considering the case in which the incidence is just at the critical angle; in this case $\sigma_s^2 = \sigma_p^2$, but in the emergent ray, which is parallel to the surface, the intensity of its s component is n^2 times that of the p component. In other words, since $n^2 < 1$, the superficial wave is polarised with the stronger component of the electric vector perpendicular to the surface.

When the expression for the flow of energy in the superficial wave is plotted against the angle of incidence θ, it is found to fall off rapidly from its maximum value for critical incidence to zero value at grazing incidence. In other words, the intensity of the superficial wave falls off to zero as the angle of incidence is increased. σ_p^2 falls off more quickly than σ_s^2. Since $\sigma_p = 2\cos\frac{1}{2}\delta_p$ and $\sigma_s = 2\cos\frac{1}{2}\delta_s$, the intimate relation between this fall in intensity and the change of phase in reflection will be readily understood.

7. Experimental study of superficial wave

The existence of a superficial wave has been demonstrated in different ways. Stokes used the method of observing Newton's rings between a prism and a lens, when the angle of incidence exceeded the critical angle, and Quincke also investigated it by a method very similar in principle. By placing small particles in contact with the surface and thereby causing them to scatter light, the superficial disturbance has also been made evident. This method is, however, only qualitative. The present writer has shown in a recent paper* that when a totally-reflecting surface is of finite extent, as indeed is inevitable, the superficial wave gives rise to diffraction effects which can be observed and photographed at great distances from the surface. The edges of the surface, in fact, act as sources of secondary radiation in the same way as the boundary of any fully illuminated diffracting aperture. In order, however, to observe the disturbance in the second medium at points very close to the surface itself and investigate it in detail, an entirely different method has been devised by the writer. This is indicated in figure 2.

AB is the hypotenuse of a right-angled glass prism ABC at which total reflection occurs. DE is a fresh "Gillette" razor-blade of which the sharp edge E is placed as nearly as possible parallel to the surface AB. A fine slow motion, such as that provided on interferometers, enables the razor-blade to be moved forward or backward so that the edge approaches or recedes from the face of the prism by fractions of a wavelength. The microscope M is focussed on the razor-edge; the latter is usually invisible unless some diffuse illumination is provided in the field of view. If the axis of the microscope be in the plane of incidence and the razor-blade be placed perpendicular to the plane of incidence and slowly advanced within a very small distance of the surface, it is seen as a luminous line in the microscope. The distance of approach or removal from the surface within which the luminosity of the edge is visible is a measure of the thickness of the layer within which the superficial disturbance in the second medium is sensible.

The principle of the method depends on the well-known property of a sharp and highly polished metallic edge to diffract a stream of light falling upon it

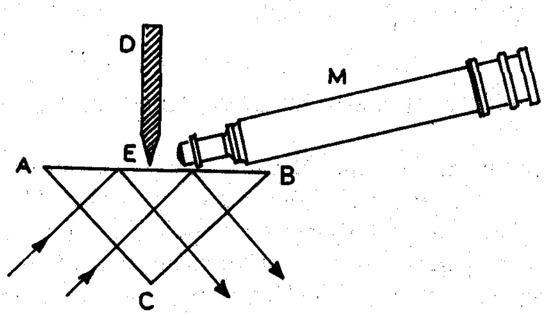

Figure 2

*C V Raman, *Philos. Mag.* **50** 1925 (812).

through considerable angles and to appear as a luminous line when viewed in suitable directions. Simple observations with a sharp razor-edge held in an ordinary parallel pencil of light show that the directions in which the luminosity of the edge is visible depend on the position of the edge relatively to the stream of rays. In the general case when the edge is inclined to the light-stream, the luminosity is visible only when the direction of observation is any generator of a cone having the edge for its axis and the direction of the undiffracted rays for another generator. When the stream of light falls normally on the diffracting edge, its luminosity is visible in any direction lying in a plane perpendicular to the edge.

From what has been stated above it is clear that the observations of the luminosity of the edge in the arrangement shown in figure 2, when it approaches sufficiently close to the surface, prove that there is actually a stream of energy in the second medium travelling parallel to the surface in the plane of incidence. The method enables the rate at which the intensity in the superficial wave decreases with increasing normal distance from the surface to be determined. It is found that, when the incidence is just at the critical angle, the intensity of the superficial wave is a maximum and is quite comparable with that of the incident and reflected beam. But as the incidence is increased the intensity falls off more or less quickly and becomes zero at very oblique incidences. The decrease of the intensity of the superficial wave with increasing normal distance from the surface is very rapid; when the incidence is not much greater than the critical angle, say about 50°, the luminosity is perceptible when the edge is within a wavelength or so from the surface. For larger angles of incidence, 60° or more, the decrease is far more rapid, and the luminosity is perceptible practically only when the razor-edge is in actual contact with the surface of the prism. When the incident light is unpolarised, the luminosity of the edge, observed as nearly as possible parallel to the surface, is found to be strongly polarised with the strong component of the electric vector perpendicular to the edge.

8. Direct measurement of phase-changes

It has already been mentioned that Bevan, by observations on Lloyd's fringes formed by internal reflection, verified the fact that the phase-differences δ_p and δ_s between the incident and totally-reflected waves amount to π at grazing incidence. In an investigation on the colours of mixed plates*, it was shown by the present writer that part of the light falling upon the curved edges of the air-bubbles contained in these plates is totally reflected at various angles between grazing and critical incidence and interferes with another part which also emerges after two refractions through the curved edges. The positions of the maxima and

*C V Raman and K Seshagiri Rao, *Philos. Mag.* **42** 1921 (679).

minima in these interferences depend on the phase of the totally-reflected light, and the good agreement found between the observed and calculated values is a confirmation of the correctness of the formula used for calculating the phase-difference. The method also enabled the difference $\delta_p - \delta_s$ to be directly observed and measured, and this was found to be an *acute* angle. It would appear quite practicable to arrange a simple interference experiment in which the phase-advances δ_p and δ_s may be measured separately and compared with each other.

9. Conclusion

The method of the knife-edge described above was shown to be practicable in observations made at the author's suggestion by Mr D P Acharya and was later more fully tested by Mr S C Sirkar. Further observations by this method (using stronger illumination), and an experimental study of the state of polarisation of light scattered by small particles in the vicinity of a totally reflecting surface, would appear to be called for.

In conclusion, the author has much pleasure in referring to the valuable assistance received by him from Mr K S Krishnan in the preparation of the paper.

Discussion

Mr T Smith: It is gratifying to find that the total reflection problem for a harmonic wave-train can be treated so thoroughly on the simple Huygenian principle. It would be still more satisfying to have a corresponding discussion in which events according to some of the modern quantum theories of light are considered; since these theories are still in their infancy we shall probably have to wait some time for an investigation of such special problems on these bases. Unlike the important question of phase, which relates essentially to the waves and is thus common to all theories, the question whether the real light energy, that is the light quantum, penetrates the second medium does not appear to be yet answered. It is obvious that there must be something in the nature of a superficial wave in the second medium—one can hardly imagine that conditions just on one side of a surface, while a train of waves is being reflected from the other side, are no different from those holding when no disturbance reaches the surface—but the energy of these waves is not necessarily identical with the energy of the light.

Mr J W Perry: The boundary condition here assumes the natural confines of the first and second media to be a common boundary surface. But it is found necessary to postulate a transition layer in order to explain the residual elliptic polarisation on reflection at the boundary. It would be of interest if from the experimental study of the results obtained some indication were found serving to corroborate the existence of the transition layer.

Mr E T Hanson: Prof. Raman is to be congratulated upon his ingenious and interesting method of attacking the problem of total reflection, which, strange to say, has been a stumbling-block to so many celebrated physicists and mathematicians. I am not, however, in entire agreement with him as to the advisability of using Huygens' principle in the way he has done. Kirchhoff's formula, which is derived by purely mathematical reasoning, might have been applied. But Huygens' principle is a concrete physical conception, which somewhat loses its meaning when reduced to the limiting equation (6) of Prof. Raman's paper, for, in this limiting case, no energy can be said to be radiated into space from the elementary secondary wavelets.

Drude's very careful treatment of the problem has always appeared to me to be excellent in every essential respect, and it is satisfactory to have Prof. Raman's alternative confirmation of its correctness, even though his method may be open to criticism.

Drude's attempt to explain how the energy passes from the first medium into the second medium and back into the first in the case of total reflection appears to me to be incorrect, but the matter is perhaps of small importance. One of the most interesting phenomena at, or near, total reflection, is that of the diffraction of a beam of light by a slit when the latter is placed inside the first medium and not too close to the surface at which total reflection takes place. The explanation of the observed phenomena by the use of Huygens' principle and Fresnel's formulae is very instructive.

With regard to the superficial wave itself, apart from diffraction, the following remarks may be of interest. For angles of incidence equal to and greater than the critical angle it is possible to combine the incident and reflected wave systems into a single plane wave of variable amplitude. In fact, in the incident space, there is a plane wave travelling parallel to the interface, the amplitude varying harmonically along the wave-front. In the refracted space there is a plane wave also travelling parallel to the interface, the amplitude decreasing exponentially from the interface. These two wave systems, derived by theory, satisfy all the conditions of the mathematical problem. Now it can be shown that at critical incidence no energy is transmitted to and from across the interface. The physical explanation appears to be, therefore, that the two systems of waves in the first medium adjust themselves to the velocity of a possible plane wave in the second medium, so that there is no reaction to the propagation of the electromagnetic displacements along the boundary.

Prof. Raman (communicated): Replying to Mr Perry's remark, it may be said that transition layers have but a small influence on the total reflection of light (Drude, *Wied. Ann.* **43** 1891 (126) and Maclaurin, "Theory of Light," pp. 62 and 82). They may, however, be important in the total reflection of X-rays, where the wavelength is much smaller. The effect of the transition layers in total reflection is experimentally evident in observations on the scattering of light by liquid surfaces (*Proc. R. Soc. London* **A109** 1925 (150)).

In reply to Mr Hanson, I may say that the simplest expression of Huygens' principle is employed, as it answers the purpose sufficiently and avoids unnecessary mathematical difficulties. It must be remembered in this connection that Kirchhoff's principle is only one way of formulating the propagation of waves in an uninterrupted medium and, even as such, is not unique. In the present case we are concerned with the effects occurring at the boundary between two media, and not with a single uninterrupted medium.

With regard to the question of energy it is clear that the elementary wavelets entering into the second medium attenuate each other's effects by interference so completely that the actual energy conveyed by them is an infinitesimal quantity. I would interpret formula (6) as being just a mathematical expression of this physically intelligible result.