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Research Note

Statistics of refractive pulsar scintillation: effect of limited data length

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Abstract. Recent theoretical studies of refractive effects due to large scale ($\gtrsim 10^{12}$ cm) electron density fluctuations in the interstellar medium have provided a framework that has been used to estimate slow variations in the mean intensity, image size, etc. of pulsars and compact extragalactic sources. Mostly, the estimates of fluctuations in the various observables resulting from these studies correspond to observing times ($t_{\rm obs}$) which are much longer than the associated refractive time scales, ($t_{\rm ref}$). However, in practice the time span of a set of observations is often not much longer than the refractive time scales. In this paper we investigate the dependence of the apparent modulation in observed intensities on the ratio ($t_{\rm obs}/t_{\rm ref}$) by direct simulation. The apparent modulation reduces with ($t_{\rm obs}/t_{\rm ref}$) as expected. The variance on this estimate and the effect of using the sample mean rather than the (not directly available) true mean are both accounted for in our calculation (this is difficult to do by purely analytical methods).

We suggest that the apparent modulation index should be corrected suitably to obtain a more meaningful estimate of the strength of modulation. This correction is particularly relevant to recent observational studies of pulsar intensity modulation.

Key words: pulsars – refractive scintillations – modulation index

1. Introduction

It has been shown (Sieber, 1982; Rickett et al., 1984) that the observed slow fluctuations in the intensities of pulsars and compact extragalactic sources can be attributed to the refractive effects due to the long-wavelength electron-density fluctuations in the interstellar medium (ISM). Recent models (e.g. Romani et al., 1986, hereafter RNB; Coles et al., 1987) adopting power-law spectra for the density perturbations have been used to compute the fluctuations in mean intensity, image size, pulse width and pulse arrival times, along with their cross-correlation and fluctuation time scales. In practice, the observation period may not be much longer than the typical time scale of the fluctuations. In such cases, the amount of fluctuation derived from the observations may grossly underestimate the true strength of fluctuation. Existing theoretical studies (Rickett et al., 1984; Blandford and Narayan, 1985) do compute the structure function of intensity fluctuations, which is the mean value of $(I(t)-I(0))^2$. However, this cannot be interpreted directly in terms of the modulation index without the knowledge of the true mean intensity. This can differ considerably from the sample mean computed using data from only a limited time stretch. Further, it is useful to estimate the typical errors on the modulation index estimated in this manner from a short data stretch. The simulations presented in this paper are designed to deal with these two issues. If F is the flux density observed from a pulsar or compact extragalactic source then the apparent modulation index $m_{\rm a}$ is given by:

$$m_{\rm a}=\left[\frac{1}{t_{\rm obs}}\int_0^{t_{\rm obs}}(F(t)-\bar{F}_{\rm a})^2\,{\rm d}t\right]^{1/2}\Big/\bar{F}_{\rm a}, \tag{1}$$

where the apparent mean intensity is

$$\bar{F}_{\rm a}=\frac{1}{t_{\rm obs}}\int_0^{t_{\rm obs}}F(t)\,{\rm d}t,$$

and $t_{\rm obs}$ is the length of time over which observations are available. When $t_{\rm obs}\gg t_{\rm ref}$, where $t_{\rm ref}$ is the refractive time scale, then $\bar{F}_{\rm a}\to\bar{F}$, the true mean intensity of the object and $m_{\rm a}\to m$, the true modulation index.

Our present interest is to find the dependence of m and the error bars for "m" on ($t_{\rm obs}/t_{\rm ref}$). Any analytical approach for realistic models of $F(t)$, appears quite complex because one is dealing with the ratio of two fluctuating quantities. Therefore, we chose to simulate the required input function $F(t)$ using a suitable model for the process which is responsible for the refractive fluctuations. For this purpose, we adopt the approximate model for refractive fluctuations in intensity used by RNB.

In the RNB model the effects of refractive fluctuations in the ISM are treated as perturbations of an underlying bundle of rays scatter-broadened by the diffractive scale inhomogeneities. When averaged over a much longer time scale than that for diffractive scintillation, the image of a point source is shown to be essentially Gaussian with a characteristic angular radius θ. This Gaussian bundle will be focused, defocused, and tilted by density fluctuations on the scale of the "spot" or image size, $\sigma=\theta L$ (where L is the distance of the screen from the observer).

Thus, in this model the intensity received at a general point x (see Fig. 1) from unit area around $x+r$ on the screen is

$$F(r,x)=\frac{\bar{F}}{\pi\sigma^2}\exp\left\{-\left[\frac{L\eta+r}{\sigma}\right]^2\right\}, \tag{2}$$

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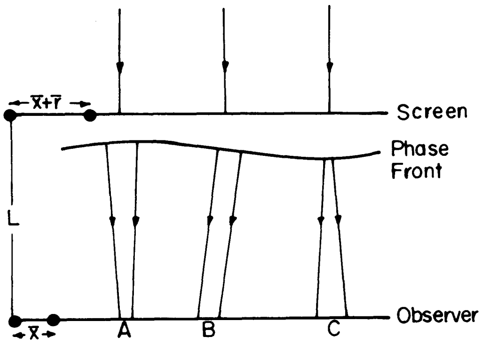


Fig. 1. A schematic showing the effect of a thin screen with a single long-wavelength sine-wave perturbation on an incident plane wave

where $\eta = -\frac{\lambda}{2\pi}\frac{\partial\phi}{\partial r}$ is the extra refractive bending angle of a ray at transverse location $(x+r)$ on the screen, $\frac{\partial\phi}{\partial r}$ is the phase gradient in the r direction at the screen and λ the wavelength of observation. The phase perturbation at the screen is related to the power spectrum of the density irregularities as

$$\phi(\bar{S}) \propto \iint d^2\bar{q}\,|Q(\bar{q})|^{1/2} \exp[\mathrm{i}\bar{q}\bar{S} + \mathrm{i}\psi(\bar{q})], \quad (3)$$

where $\bar{S} = (x, y)$ is the spatial lag; $Q(\bar{q})$ the power spectrum of the density perturbations; $\bar{q} = (q_z, q_y)$ the wave vector and the phases $\psi(\bar{q})$ are random for a Gaussian process. Thus, the extra refractive bending angle $\bar{\eta} \equiv (\eta_x, \eta_y)$ is given by:

$$\begin{aligned}\bar{\eta} &\propto \frac{\partial\phi}{\partial\bar{S}} = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right)\\ &\propto [\iint d^2\bar{q}\,\mathrm{i}q_x|Q(\bar{q})|^{1/2}\exp(\mathrm{i}\bar{q}\bar{S}+\mathrm{i}\psi),\\ &\quad \iint d^2\bar{q}\,\mathrm{i}q_y|Q(\bar{q})|^{1/2}\exp(\mathrm{i}\bar{q}\bar{S}+\mathrm{i}\psi)]\\ &= [\iint d^2\bar{q}\exp(\mathrm{i}\bar{q}\bar{S})\,\Phi'_x,\ \iint d^2\bar{q}\exp(\mathrm{i}\bar{q}\bar{S})\,\Phi'_y], \end{aligned} \quad (4)$$

where $\Phi'_x = \mathrm{i}q_x|Q(\bar{q})|^{1/2}\exp(\mathrm{i}\psi)$ and $\Phi'_y = \mathrm{i}q_y|Q(\bar{q})|^{1/2}\exp(\mathrm{i}\psi)$.

A power-law spectrum is generally assumed and defined by $Q(q) = Q\,q^{-\beta}$. However, the value of β is still at issue. Theoretical studies for a range of values of β have been reported (see Goodman and Narayan, 1985; Roberts and Ables, 1982). These show that values of $\beta = 11/3$ and 4.3 are both consistent with most observations, although $\beta = 4.3$ is favoured by the large observed modulation index. However, some recent observational evidence (Gwinn et al., 1988) seems to favour $\beta = 11/3$. In our computation we will consider both $\beta = 11/3$ and 4.3.

2. Simulations and results

Our simulations involve the following steps:

(i) Hermitian symmetric spectra corresponding to Φ'_x, Φ'_y are generated in the range $0 < |\bar{q}| \leqq q_{\rm ref}$ for an assumed value of β, where the phases $\psi(\bar{q})$ are randomized. The cutoff at $q_{\rm ref}$ excludes the short-wavelength scales which are already accounted for in diffraction broadening of the image.

(ii) The spectra are then Fourier transformed to obtain the two components of the extra refractive bending angles $(\bar{\eta})$ at the screen.

(iii) The magnitude of $\bar{\eta}$ (w.r.t. σ) is adjusted, so as to satisfy the relation derived by RNB between the structure function of $\bar{\eta}$ and the parameter σ. This is given by:

$$\begin{aligned}&\frac{L^2}{\sigma^2}\langle(\bar{\eta}(\bar{S})-\bar{\eta}(0))^2\rangle\\ &= 2K\left(\frac{S}{\sigma}\right)^{\beta-4}\int_0^\infty x^{3-\beta}(1-J_0(x))\exp\left[-\frac{x^2}{2(S/\sigma)^2}\right]\mathrm{d}x, \end{aligned} \quad (5)$$

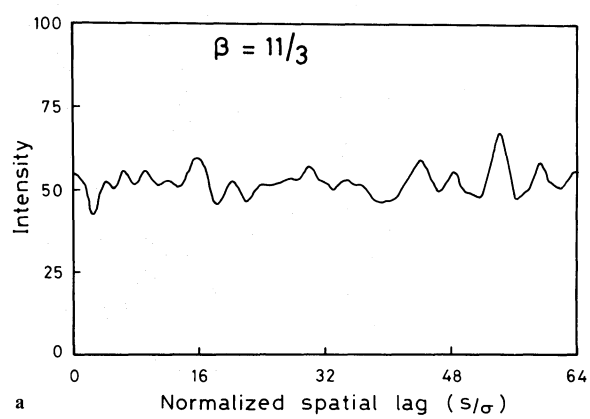
where J_0 is the Bessel function.

(iv) Instead of computing the $F(x, y)$ using Eq. (2), we do the following to reduce the number of computations. We first find the intensity that would have been observed if the diffraction broadening of the image were neglected. This intensity distribution is then convolved with a Gaussian corresponding to the diffraction-broadened image. This procedure gives the same result as would be obtained from Eq. (2), following integration over r.

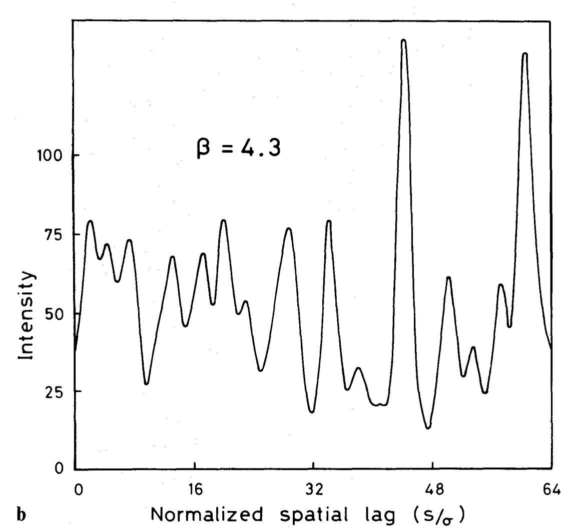
(v) This simulated intensity pattern in the plane of the observer can now be used to find m over all possible cuts in the plane as a function of $(S_{\rm obs}/\sigma^*)$, where $S_{\rm obs}$ is the spatial extent of the observed intensity pattern and σ^* is defined as the lag at which the normalized structure function has a value of 0.5. The σ^* is related to σ by $(\sigma^*/\sigma) = 1.16$ and $(\sigma^*/\sigma) = 1.46$ for $\beta = 11/3$ and 4.3 respectively. It is easy to see that

$$(S_{\rm obs}/\sigma^*) = (t_{\rm obs}/t_{\rm ref}) \quad (6)$$

as $t_{\rm ref} = \sigma^*/v$, where v is the speed of the observer relative to the intensity pattern.



a



b

Fig. 2a and b. An arbitrary cut through the simulated F distribution: **a** for $\beta = 11/3$; **b** for $\beta = 4.3$

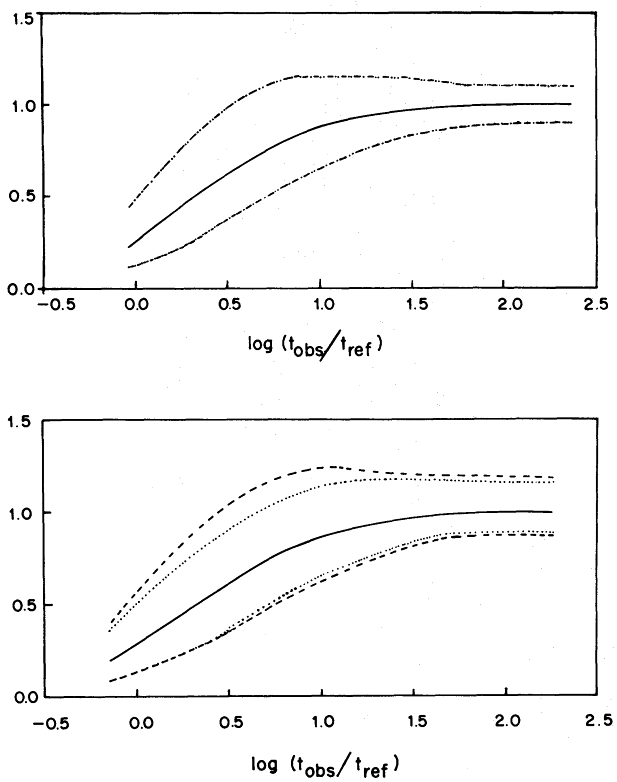


Fig. 3a and b. The dependence of (m_a/m) on (t_{obs}/t_{ref}) inferred from the simulations. The continuous curve corresponds to the mean value of (m_a/m) [also (m_a^{SF}/m)]. The dotted and dashed curves indicate the rms deviations on either side for (m_a/m) and (m_a^{SF}/m) respectively: **a** for $\beta = 11/3$; **b** for $\beta = 4.3$

Following the above procedure, simulations were performed using a 256×256-point grid for the distribution. Figure 2 shows an arbitrary cut of the F distribution for the two values of β. Here, we assume $L = \lambda = C_{-4} = 1$, where C_{-4} is the turbulence level as defined by RNB and L and λ are in kpc and meters respectively.

The dependence of (m_a/m) on (t_{obs}/t_{ref}) is shown in Fig. 3. For comparison, we also computed the dependence of (m_a^{SF}/m) on (t_{obs}/t_{ref}) (see Fig. 3) where $m_a^{SF} = m_a \bar{F}_a/\bar{F}$ is the modulation index as implied by the structure function of F. The number of independent cuts of the F distribution used in these computations ranged from 16 for the longest data stretch to 4000 for the shortest data stretch.

3. Discussion

Our results clearly demonstrate the effect of short observation periods on the apparent modulation index. It is worth noting that the variations of (m_a/m) with (t_{obs}/t_{ref}) obtained for the values $\beta = 11/3$ and 4.3 are rather similar. The values of the modulation indices are however different in the two cases (see Fig. 2). The relation between (m_a/m) and (t_{obs}/t_{ref}) can be used to obtain estimates of the true modulation index from an apparent modulation index derived from observations over short periods. Such a correction is essential, particularly in those cases where the apparent modulation indices are to be used for further analysis. For example, the variability data obtained by Slee et al. (1986) may be affected by the effect discussed here. This may change the conclusions to be drawn from their data. It is significant that the comparison of the results for m_a and m_a^{SF} show that their mean values depend on (t_{obs}/t_{ref}) in the same way. The distribution of m_a around its mean value is generally asymmetric. To bring out such an asymmetry clearly, we compute the rms deviations for the modulation index values above and below the average value separately. Such rms deviations in the two cases (m_a and m_a^{SF}) are similar when $\beta = 11/3$ (see Fig. 3a). This is not surprising as $\beta = 11/3$ corresponds to a weak modulation case, where $\bar{F}_a$ is not too different from $\bar{F}$. However, the "upper-side" rms deviation of m_a^{SF} differs noticeably, though not considerably, from that of m_a, when $\beta = 4.3$ (see Fig. 3b). This can be explained if the event that $m_a^{SF} > \overline{m_a^{SF}}$ for a data stretch has some correlation with $\bar{F}_a > \bar{F}$. Such a correlation is quite natural for a variation of $F(t)$, as seen in Fig. 2b, with large positive spikes (large values of F tend to be accompanied by steep variations).

Thus, we conclude that the structure function computations can be used confidently to obtain the required correction depending on (t_{obs}/t_{ref}), even in a case with strong modulation (such as for $\beta = 4.3$) where the sample mean is not the true mean. The results presented here also make it possible to assign error bars to the corrected modulation index derived in this way.

In most theoretical studies, the estimates of the refractive fluctuations in other observables e.g. source size, pulse width, decorrelation bandwidth, etc. also correspond to an infinite observing period. Therefore, it is also desirable to estimate the effect of short observing periods on the mean and variance of fluctuations in these quantities. It should be possible to study this effect for other observables using direct simulations similar to those employed here.

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