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Letter to the Editor

Geometry of Pulsar Beams: Relative Orientations of Rotation Axis, Magnetic Axis, and Line of Sight

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Summary: The observed polarisation angle variation in pulsars is analysed on the basis of the magnetic pole model. The angles α between the rotation and magnetic axes and β between magnetic axis and line of sight are estimated using (a) the shape of the polarisation angle (θ) vs pulse longitude (ϕ) curve, (b) the magnitude of the central gradient $d\theta/d\phi$ in the main pulse, and (c) the magnitude and sign of the gradient in the interpulse. We conclude that β can have either sign, with outer lines of sight (i.e., away from the rotation axis) probably predominating. Our result is contrary to certain theories which predict a complete absence of outer lines of sight. We also conclude that α is not randomly distributed; small angles seem to be preferred.

Key words: Pulsar, polarisation, interpulse.

1. Introduction

It is now widely though not universally accepted that the radiation from pulsars originates over the magnetic poles of rotating neutron stars. In the context of this model (Radhakrishnan and Cooke, RC, 1969), two angles α and β (Fig. 1) describe the pulsar geometry. There have been few attempts to estimate these angles observationally. RC, in their pioneering work, used polarisation information to place upper bounds on α and β for pulsar 0833-45 ($\alpha<80^\circ$, $\beta<10^\circ$) and also suggested that $\beta<0^\circ$ (and $\alpha \sim 90^\circ$ because of the interpulse) for pulsar 0531+21. Manchester and Taylor (1977) made least squares fits of the observed polarisation angle variation in four pulsars and gave estimates of β (since the least squares could not fix α, they assumed $\alpha = 60^\circ$). More recently, Narayan and Vivekanand (NV, 1982) proposed a model for the geometry of pulsar 0950+08, including estimates of α ($\simeq 10^\circ$) and β ($\simeq -5^\circ$), using the polarisation data of Backer and Rankin (BR, 1980). Apart from these, the only other pulsar where some information is available on the beam geometry angles is the binary pulsar 1913+16, where, if one assumes that the rotation axis is normal to the plane of the orbit, then one deduces that $\alpha + \beta = i = 47^\circ$ (Taylor, 1980).

In the absence of more detailed information, it is conventional to assume that α is distributed uniformly on a sphere and that β lies within $\pm \sim 6^\circ$ (assuming circular beams and a duty cycle of 4%). Recently, NV have used statistical arguments to show that pulsar beams are probably highly elongated with $|\beta|$ going as high as $\sim 22^\circ$, suggesting that the conventional views on α and β could be significantly in error. In this paper we estimate values of these angles for a number of individual pulsars by analysing the available polarisation data (mostly from BR). We then investigate, among other questions, (a) the sign of β, which Arons and co-workers [see e.g., Arons, 1979] suggest will be always negative (on theoretical grounds), (b) the range of β which NV claim to be rather large and (c) the distribution of α.

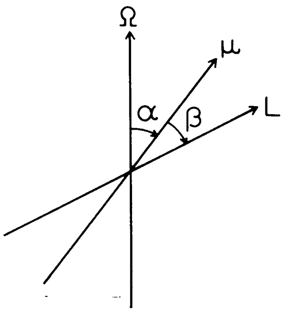

Fig. 1: Ω, μ and L are the directions of the rotation axis, magnetic axis and line of sight at an instant when all three are in the same plane. The angles α and β are considered positive in the direction away from the rotation pole. Positive and negative β are called outer and inner lines of sight respectively.

Following RC we assume that the polarisation of the (radio) radiation reflects the orientation of the magnetic field in the vicinity of the pulsar and that the projected magnetic field has a purely radial structure when viewed down the magnetic axis. BR have established the validity of these assumptions by means of careful polarisation observations on a number of pulsars.

We use three essentially independent arguments, all based on the RC model, and these are presented separately (methods A, B and C below). Our discussion makes use of the following: (a) In the RC model, when the magnetic field lines are projected on the star surface, they form great circles passing through the magnetic poles. (b) The line of sight traces a small circle of constant 'latitude' $\pi/2-\alpha-\beta$ (referred to the rotation axis) and the polarisation angle at any pulse longitude ϕ is the angle at the corresponding point on the surface of the star between the local magnetic great circle and the line of sight small circle. (c) From (a) and (b), it can be shown (Manchester and Taylor, 1977) that the gradient of the polarisation angle θ with respect to ϕ at the centre of the pulse (i.e., the point of closest approach to the magnetic pole) is given by

$$| d\theta/d\phi |_{\mathrm{central}} = |\sin \alpha/\sin \beta| \qquad (1)$$

The actual sign of $d\theta/d\phi$ has no useful information for our present purposes.

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Method A

From eq. (1) it is seen that the value of $|d\theta/d\phi|$ gives $|\beta|$ as a function of α, but does not determine the actual values of α and β, nor even the sign of β (for exceptions, see B below). However, the detailed shape of the curve (Fig.2 shows typical examples) has much more information than the central gradient. A striking characteristic of the θ-ϕ curve for positive β (we call this an "outer" line of sight since L lies outside the angle formed by Ω and μ, Fig. 1)is its flattening as one moves away from the pulse centre, with a maximum polarisation swing less than 180°. On the other hand, the θ-ϕ curve for negative β ("inner" line of sight) is monotonic. Fig. 2 further shows that the difference between the curves for $+\beta$ and $-\beta$ at a given pulse longitude increases as α decreases. Because of these effects, given accurate observations of θ as a function of ϕ, it is possible to estimate α and β by means of curve fitting procedures. Table 1 shows the results we obtain by least squares fits on the excellent data of BR.(It is because of the improved data that we obtain tighter estimates of α and β than Manchester and Taylor, 1977). For each pulsar, we have computed the mean observed θ as a function of pulse longitude from the published histograms, applying the relevant offset (estimated from the data) in the case of orthogonal polarisation modes (Manchester et al., 1975; Backer et al., 1976). In order to include only the most reliable data, in our least squares fits, we have retained only those longitudes where BR could estimate the polarisation angle in at least 15% of the pulses (symbol '2' or better in the second last column marked 'A' in their histograms). Table 1 gives our estimates of α and β for seven pulsars as well as the percentage probabilities of occurrence of outer and inner lines of sight (computed by standard statistical techniques in terms of the residuals). In the rest of BR's pulsars, the parameters are too poorly determined to be of interest. The last column in Table 1 gives the rms difference between the observed and fitted polarisation angles. We estimate that our procedure of computing values of θ from BR's histograms has contributed an rms error of about 0.5° to 1.0°.

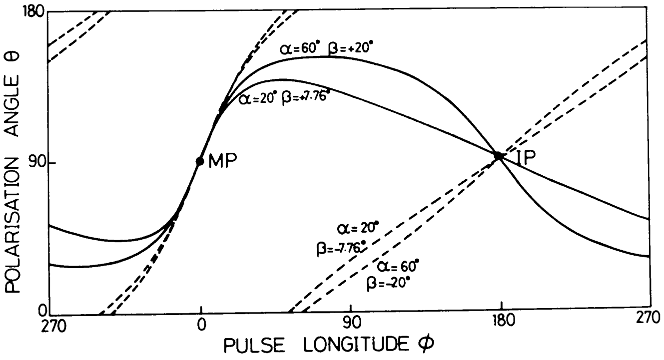

Fig.2: Variation of polarisation angle θ across 360° of pulse longitude ϕ. The solid lines correspond to positive β and the dashed lines to negative β. The points MP and IP are the centres of the main and inter pulses. All the four curves have the same value of $d\theta/d\phi$ at MP, but deviate from one another away from this point. Note (a) the topological distinction between positive and negative β curves, (b) the increased difference between positive and negative β curves at small values of α for the same value of $(d\theta/d\phi)_{MP}$, and (c) the opposite slopes of solid and dashed lines at the interpulse.

Table 1: Estimates of α and β of seven pulsars, obtained by least squares fits of the data of BR (method A). Values in parentheses are doubtful because of the relatively larger rms residuals. Where limits are given on α, the value of β corresponds to the extreme value of α.

Pulsar	α	β	Probability (%) of Positive β	Probability (%) of Negative β	Rms residual in polarisation angle
0301+19	$70^\circ \pm 11^\circ$	$+3.0^\circ$	97%	3%	1.0°
0525+21	$\leqslant 20^\circ$	$+0.60^\circ$	100%	0%	1.4°
0950+08[a]	$\leqslant 25^\circ$	-12°	0%	100%	3.8°
1133+16	$(90^\circ \pm 60^\circ)$	$(+6.0^\circ)$	(50%)	(50%)	1.7°
1929+10	$50^\circ \pm 61^\circ$	$+32^\circ$	69%	31%	0.7°
2016+28	$\leqslant 30^\circ$	$+5.7^\circ$	100%	0%	1.4°
2020+28	$(\leqslant 25^\circ)$	$(+2.9^\circ)$	(100%)	(0%)	2.1°

[a] Values taken from NV. The rms error in θ is larger because the whole data, including the interpulse, have been fitted, and not just the most accurate data as in the other cases. Similar values of α and β are obtained even if the main pulse data alone are considered.

We therefore consider the results of Table 1 to be quite reliable for the four pulsars with a final rms of less than 1.5°. The results on pulsar 1133+16 are probably sound while those on 2020+28 are much less certain. Pulsar 0950+08 has been earlier studied in detail by NV and their results quoted here are considered quite reliable.

Method B

Since $|\beta|$ cannot be less than α (see Fig. 1), eq. (1) shows that $|d\theta/d\phi|_{central} > 1.0$ for inner lines of sight. Thus any pulsar with a gradient less than 1.0 must have positive β. Moreover, if we assume equal emission from both magnetic poles, then we have the obvious inequality $\alpha + \beta < 90^\circ$ for the main pulse. Combining these two results we can place rigorous upper limits on α for all pulsars having $|d\theta/d\phi|_{central} < 1.0$. Table 2 shows four such pulsars among those observed by BR.

Table 2: Limits on α for four pulsars estimated by means of method B. These pulsars have positive values of β (equal to $90^\circ - \alpha$ for the extreme value of α). Values in parentheses are doubtful because of the relatively larger rms residuals.

Pulsar	$|d\theta/d\phi|$ [a]	α	Rms residual in polarisation angle
0540+23	0.97 ± 0.05	$\leqslant 44^\circ$	1.1°
1237+25*	(0.64 ± 0.12)	$(\leqslant 33^\circ)$	4.3°
1541+09	(0.46 ± 0.04)	$(\leqslant 25^\circ)$	4.8°
1944+17	0.68 ± 0.03	$\leqslant 35^\circ$	1.7°

[a] Obtained by least squares straight line fits on the data of BR.

The values of the gradient were computed by least squares fits on the histograms of BR using all data corresponding to symbol '.' and above. Two of the four pulsars viz., 1237+25* and 1541+09, display very unusual θ-ϕ curves, not consistent with the RC model; this is reflected in the large rms residuals. However, the results on pulsars 0540+23 and 1944+17 are quite unambiguous and imply positive values of β (outer lines of sight) and the upper limits on α shown in Table 2.

Method C

Fig. 2 shows that the polarisation angle gradient in the interpulse has the same sign as that of the main pulse for negative β and the opposite sign for positive β. This is therefore a very straightforward technique of determining the sign of β (RC, 1969; see also Hankins and Cordes, 1980). Thus, in Table 3, of the five pulsars with polarisation observations on both main pulse and interpulse, three have inner lines of sight (negative β) and two have outer (positive β).

Table 3: Values of α and β for five pulsars estimated using method C. Values in parentheses are doubtful because the interpulse gradient is ambiguous.

Pulsar	$(d\theta/d\phi)_{\mathrm{central}}$		α	β		$\left\lvert\frac{\beta_{IP}}{\beta_{MP}}\right\rvert$	Flux ratio^{a} S_{IP}/S_{MP}
	Main Pulse	Inter Pulse		Main Pulse	Inter Pulse		
0531+21	6.0^{b}	$+3.1^{b}$	86°	-9.6°	$+18^{\circ}$	1.8	0.36
0823+26	14.3^{c}	-4.6^{d}	82°	$+4.0^{\circ}$	$+12^{\circ}$	3.0	0.005
0950+08	2.0^{e}	$+0.67^{e}$	10°	-5.0°	-15°	3.0	0.018
1055-52	1.6^{f}	$(+2.1^{f})$	(95°)	$(+38^{\circ})$	(-28°)	(0.74)	(0.85)
		$(+0.66^{g})$	(20°)	(-12°)	(-28°)	(2.3)	(0.85)
1929+10	1.48^{c}	-0.58^{d}	35°	$+23^{\circ}$	$+87^{\circ}$	3.8	0.02

a Taken from Manchester and Taylor (1977).
b From the fit of Kristian et al., (1971) on optical polarisation data.
c Least squares fit on the data of BR.
d From Rankin and Benson (1981).
e From NV.
f From Fig. 12 of McCulloch et al., (1978).
g From Fig. 3 of McCulloch et al., (1976)

If we now further assume that the interpulse corresponds to the point of closest approach to the second magnetic pole, then we have

$$(d\theta/d\phi)_{\mathrm{interpulse}} = -\sin\alpha/\sin(2\alpha+\beta) \quad (2)$$

where the gradient of the main pulse has been taken to be positive and β is positive or negative as the case may be. Eq. (2) is correct even if the radiation in the interpulse comes from the same pole as the main pulse as in NV's model for pulsar 0950+08. Using Eqs. (1) and (2) and the observed polarisation angle gradients at the main pulse and interpulse, it is possible to estimate both α and β. The results are given in Table 3. For pulsar 0531+21 we confirm the results of RC who estimated that $\alpha \sim 90^{\circ}$, $\beta < 0$. In the case of pulsar 1055-52, the magnitude of the interpulse gradient is not clear and hence our estimates of α and β are in considerable doubt. The negative sign of β is however quite unambiguous.

Conclusions

From the summarized results of Tables 1,2 and 3, we conclude the following:

(a) Pulsars 0950 + 08 and 1929 + 10 have been analysed by two methods, viz., A and C. Tables 1 and 3 show that in both cases the results are consistent, confirming the validity of the arguments used.

(b) There is clear evidence for the occurrence of both inner and outer lines of sight. We note in particular that positive β values have been deduced using all three independent arguments A, B and C. Therefore, given the RC model, outer lines of sight are quite inescapable. The theory of pulsar electrodynamics developed by Arons and others [e.g., Arons, 1979], which makes a clear prediction that only negative values of β are allowed, needs to be reconsidered.

(c) Table 1 shows a preponderence of pulsars with positive β. A partial explanation could be that the least squares works best in those pulsars having small values of α; a positive β is then more likely than negative β on solid angle considerations. Alternatively, it is possible that there could be some systematic distortion of the (assumed) radial field lines leading to a θ-ϕ curve similar to that expected (in the RC model) for small values of α and positive β (Fig. 2). If so, the results of Table 1 may be in doubt.

(d) The various reliable indications in Tables 1, 2 and 3 give the magnitude of β (including interpulses) to be $\sim 22^{\circ}$ (we have used the maximum value of β where only upper limits are available; this will tend to increase our estimate of $<|\beta|>$). This gives a beam size of $|\beta|_{\max} \sim 44^{\circ}$ which is consistent with 22° estimated by NV using an independent statistical approach. However, there appears to be a preponderence of small values of β, whereas one expects a uniform distribution out to $\beta_{\max}$. This could mean that the apparent luminosity falls off with increasing $|\beta|$ (see (f) below). Alternatively, it might be a consequence of many of the $|\beta|$ values being upper bounds.

(e) The values of α that we have estimated appear to be generally rather small. This can be partly accounted for by the fact that arguments A and B both work best at small α (argument C, on the other hand, prefers large α). However, even after allowing for this, there seems to be a residual preference for small α. For example, of the total of 18 pulsars observed by BR, 5 pulsars (viz., 0525+21, 0950+08, 1929+10, 1944+17, 2016+28) have $\alpha \leqslant 35^{\circ}$. Even if none of the other 13 has $\alpha \leqslant 35^{\circ}$, this is still more than the 1.9 pulsars expected for a random distribution of α and β (with $|\beta|_{\max} = 22^{\circ}$). Our result might mean that pulsars work better at small values of α. Alternatively, one could propose that the pulsar magnetic axis aligns with the rotation axis with age (as suggested by Goldreich, 1971). However, we feel that this is unlikely since the estimated ages $(P/2\dot{P})$ of the pulsars with small α are not significantly different from the others in BR's sample. Moreover, four of the five pulsars with $\alpha < 35^{\circ}$ have periods shorter than average.

(f) The three pulsars in Table 3 with very weak radio interpulses have $|\beta(\mathrm{interpulse})/\beta(\mathrm{main\ pulse})| \simeq 3.0$. On the other hand, pulsar 0531+21 with a β ratio of 1.8 has a fairly strong interpulse. Thus there seems to be some evidence for a monotonic fall off of radio flux with the latitude offset, $|\beta|$, between the magnetic pole and the line of sight. The limited data in Table 3 suggest that the integrated pulse strength varies approximately as $|\beta|^{-3}$.

(g) Of the 18 pulsars observed by BR, the analyses of this paper have led to unambiguous estimates of α and β in 8 cases. The value of high quality polarisation data is obvious.

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* In the case of PSR 1237+25 there is good reason to believe that it has actually ~ 180° polarisation swing across the pulse with the central rapid variation region having pronounced distortions.