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THE EVOLUTION OF CORRELATION FUNCTIONS IN THE ZELDOVICH APPROXIMATION AND ITS IMPLICATIONS FOR THE VALIDITY OF PERTURBATION THEORY

SOMNATH BHARADWAJ
Raman Research Institute, Bangalore 560 080, India,[1] and Joint Astronomy Program, Indian Institute of Science, Bangalore 560 012, India; somnath@rri.ernet.in
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ABSTRACT

We investigate whether it is possible to study perturbatively the transition in cosmological clustering from a single-streamed flow to a multistreamed flow. We do this by considering a system whose dynamics is governed by the Zeldovich approximation (ZA) and calculating the evolution of the two-point correlation function using two methods, (1) distribution functions and (2) hydrodynamic equations without pressure and vorticity. The latter method breaks down once multistreaming occurs whereas the former does not. We find that the two methods yield the same results to all orders in a perturbative expansion of the two-point correlation function. We thus conclude that we cannot study the transition from a single-streamed flow to a multistreamed flow in a perturbative expansion. We expect this conclusion to hold even if full gravitational dynamics (GD) is used instead of ZA.

We use ZA to look at the evolution of the two-point correlation function at large spatial separations, and we find that, until the onset of multistreaming, the evolution can be described by a diffusion process in which the linear evolution at large scales is modified by the rearrangement of matter on small scales. We compare these results with the lowest order nonlinear results from GD. We find that the difference is only in the numerical value of the diffusion coefficient, and we interpret this physically.

We also use ZA to study the induced three-point correlation function. At the lowest order of nonlinearity, we find that, as in the case of GD, the three-point correlation does not necessarily have the hierarchical form. We also find that at large separations the effect of the higher order terms for the three-point correlation function is very similar to that for the two-point correlation, and in this case too the evolution can be described in terms of a diffusion process.

Subject headings: cosmology: theory — galaxies: clusters: general — large-scale structure of universe — methods: analytical

1. INTRODUCTION

The inviscid hydrodynamic equations without pressure and vorticity (herafter the HD equations) are often used to describe the evolution of disturbances in an expanding universe filled with collisionless particles that interact only through Newtonian gravity. The disturbances that are usually considered are such that, initially, all the particles at any point have the same velocity, i.e., it is a single-streamed flow. Such a situation is correctly described by the HD equations. As the disturbances evolve, the particle trajectories intersect and there are particles with different velocities at the same point, i.e., the flow becomes multistreamed. When this occurs, the HD equations are no longer valid. This is because the HD equations neglect the local stress tensor associated with the moments of the velocity about the mean velocity at a point.

The BBGKY hierarchy of equations obeyed by the distribution functions can be used instead of the HD equations. The distribution functions keep track of the position and velocity of the particles, and these equations are valid even if multistreaming occurs. The question we would like to address in this paper is whether we can study the effects of multistreaming by using distribution functions perturbatively to follow the evolution of the disturbances.

We examine the perturbative evolution of the density-density two-point correlation function for Gaussian initial conditions in a universe with $\Omega = 1$. The perturbative expansion of this function using the HD equations has been studied by many authors (Juszkiewicz 1981; Vishniac 1983; Fry 1994). In a recent paper (Bharadwaj 1996, hereafter Paper II), we have calculated the lowest order nonlinear term for the two-point correlation function using the moments of the BBGKY hierarchy. These equations are based on the distribution functions and are valid even in the multistreamed regime. The two different methods of calculation (HD and BBGKY) are found to yield the same result at the lowest order of nonlinearity, and hence, to this order, distribution functions have not been able to capture any effect of multistreaming. In this paper, we investigate whether, by going to higher orders of perturbation theory, we shall be able to study any effects of multistreaming or if it is a limitation of perturbation theory that it cannot follow the transition from a single-streamed flow to a multistreamed flow.

Because of the difficulty in calculating the higher order terms in a perturbative treatment of gravitational dynamics (GD), we look at a simpler system in which we use the Zeldovich approximation (ZA) (Zeldovich 1970) to determine the motion of the particles. In this situation too the transition from a single-streamed flow to a multistreamed flow occurs, and we can analyze it to see whether, in a perturbative calculation using distribution functions, we can include any effects of multistreaming that would be missed if the HD equations were used instead.

[1] Postal address.

In § 2, we discuss the evolution equations. In § 3, we use distribution functions to calculate the evolution of the two-point correlation function. In § 4, we perform the same calculation using the HD equations and compare the result with that obtained in § 3.

Bond & Couchman (1988) have studied the evolution of the two-point correlation function using ZA, and the calculation presented in § 3 is on similar lines. In a more recent paper, Schneider & Bartlemann (1995) studied the evolution of the power spectrum in ZA. For a comprehensive article on various aspects of ZA, the reader is referred to the review by Shandarin & Zeldovich (1989).

In Paper II, we investigated the lowest order nonlinear correction (using GD) to the two-point correlation for initial power spectra of the form $P(k) \propto k^n$ at small k and an exponential or Gaussian cutoff at large k. We found that, for $0 < n \leq 3$, the numerical results for the nonlinear correction to the two-point correlation function at large x could be fitted by a simple formula. We also interpreted this formula in terms of a simple diffusion process. In § 5 of this paper, we investigate the evolution of the two-point correlation function at large separations using ZA and compare it with the results from GD.

In § 6, we look at the evolution of the induced three-point correlation function using ZA. This was first calculated for GD by Fry (1984), who concluded that for power-law initial conditions, at large separations, the three-point correlation function could be described by the hierarchical form, in which it can be written in terms of products of the two-point correlation function evaluated at the separations involved. In an earlier paper (Bharadwaj 1994, hereafter Paper I), we calculated the same quantity and found that these conclusions were not fully correct. We showed that the three-point correlation function at some length scale depends not only on the two-point correlation at the same length scales but on all smaller scales as well. As a result we found that the hierarchical form is true for only a class of initial conditions and that there is a class for which it does not hold. In this paper, we first calculate the three-point correlation function at the lowest order of nonlinearity for ZA and compare it to the results from GD. We then go on to study the effect of the higher order nonlinear terms at large separations.

The calculations using ZA are valid for any value of Ω_0, but whenever we make comparisons with GD, it is for the specific value $\Omega = 1$.

A similar calculation has been performed by Grinstein & Wise (1987), who studied the evolution of skewness of the density field averaged over a Gaussian ball. In addition, Munshi & Starobinsky (1994) have considered the evolution of the skewness of the density field for ZA and various other approximations, and Bernardeau et al. (1994) have calculated the evolution of the skewness of the density field averaged over top-hat filters. All these calculations have been done at the lowest order of nonlinearity.

In § 7, we present a discussion of the results obtained and the conclusions.

2. EVOLUTION OF THE DISTRIBUTION FUNCTION

The Zeldovich approximation defines a mapping from the initial position of a particle to its position at any later instant. If $x_\mu(t)$ is the comoving coordinate of a particle at any time t, the initial instant being t_0 and $b(t)$ the growing mode in the linear analysis of density perturbations, this mapping is

$$x_\mu(t) = x_\mu(t_0) + b(t)u_\mu \, . \tag{1}$$

The quantity u_μ is related to the peculiar velocity $v_\mu(t)$ at any instant by

$$v_\mu(t) = a(t) \frac{d}{dt} x_\mu(t) = a(t)\dot{b}(t)u_\mu \, , \tag{2}$$

where $a(t)$ is the scale factor.

We consider a system of particles whose motion is governed by this mapping. This can be described by a distribution function $f(x, u, t)$, where $f(x, u, t)d^3x\, d^3u$ is the number of particles in the volume d^3x around the point x and having a value of u in an interval d^3u around u.

We can see that Liouville's theorem is true for the mapping defined in equation (1). Using this, we can obtain the equation for the time evolution of the distribution function f,

$$f(x, u, t) = f[x - b(t)u, u, t_0] \, . \tag{3}$$

We can also use equation (1) to obtain a differential equation for the evolution of the distribution function:

$$\frac{\partial}{\partial b} f(x, u, b) + u_\mu \frac{\partial}{\partial x_\mu} f(x, u, b) = 0 \, , \tag{4}$$

where we use the growing mode b instead of time as the evolution parameter.

We are interested in the evolution of the statistical properties of an ensemble of such systems. Every member of the ensemble initially has the particles uniformly distributed. Initially each particle can be labeled by its coordinate x_μ. The particles are given velocities $u_\mu(x)$. The velocity field is the gradient of a function $\psi(x)$, which for each system is a different realization of a Gaussian random field. It is assumed that ψ is statistically homogenous and isotropic. The statistical properties of the ensemble are initially fully specified by the two-point correlation of ψ, which is defined as $\phi(x) = \langle \psi(0)\psi(x) \rangle$, where the angle brackets denote ensemble averaging.

The statistical quantity whose evolution we shall focus on in this paper is the density two-point correlation function $\xi(x, t)$. This is defined by the relation

$$\langle \rho \rangle^2[1 + \xi(x)] = \langle \rho(0)\rho(x) \rangle \, , \tag{5}$$

where $\rho(x)$ is the mass density. This is just the number density of particles multiplied by the mass of each particle, which is assumed to be the same for all the particles.

3. THE TWO-POINT CORRELATION USING DISTRIBUTION FUNCTIONS

In this section, we look at the evolution of the ensemble-averaged two-point distribution function ρ_2. This is defined as

$$\rho_2(x^1, x^2, u^1, u^2, t) = \langle f(x^1, u^1, t) f(x^2, u^2, t) \rangle . \tag{6}$$

From homogeneity and isotropy, we can also say that

$$\rho_2(x^1, x^2, u^1, u^2, t) = \rho(x, u^1, u^2, t) , \tag{7}$$

where

$$x_\mu = x_\mu^2 - x_\mu^1 . \tag{8}$$

The density two-point correlation function is related to the zeroth moment of the two-point distribution function with respect to u:

$$\langle \rho \rangle^2 [1 + \xi(x, t)] = \int \rho_2(x, u^1, u^2, t) d^3u^1 \, d^3u^2 . \tag{9}$$

In this paper, we normalize $\langle \rho \rangle = 1$.

The initial two-point distribution is a Gaussian in the velocities and hence specified by the covariance matrix

$$T_{\mu\nu}^{ab}(x) = \langle u_\mu^a u_\nu^b \rangle (x) = \int u_\mu^a u_\nu^b \rho_2(x, u^1, u^2, t_0) d^3u^1 \, d^3u^2 , \tag{10}$$

where a, b take values 1, 2. The initial two-point distribution function then is the Gaussian distribution

$$\rho_2(x, u^1, u^2, t_0) = \frac{1}{(2\pi)^3 \sqrt{\Delta T(x)}} \exp\left[-\frac{1}{2} u_\mu^a u_\mu^a (T^{-1})_{\mu\nu}^{ab}(x) \right] , \tag{11}$$

where $\Delta T(x)$ is the determinant of the covariance matrix. In terms of the potential ϕ, we have

$$\langle u_\mu^1 u_\nu^2 \rangle = -\partial_\mu \partial_\mu \phi(x) , \qquad \langle u_\mu^1 u_\nu^1 \rangle = -\tfrac{1}{3} \nabla^2 \phi(0) \delta_{\mu\nu} . \tag{12}$$

We use equation (3) to obtain the time evolution of ρ_2,

$$\rho_2(x, u^1, u^2, t) = \rho_2[x - (u^2 - u^1) b(t), u^1, u^2, t_0] . \tag{13}$$

This may also be written

$$\rho(x, u^1, u^2, t) = \int \delta^3\{x' - [x - (u^2 - u^1) b(t)]\} \rho_2(x', u^1, u^2, t_0) d^3x' . \tag{14}$$

Using the Fourier expansion of the Dirac delta function and equation (11), we have

$$\rho(x, u^1, u^2, t) = \int \left(\frac{1}{2\pi}\right)^3 \exp[ik_\mu(x'_\mu - x_\mu)] \exp[ik_\mu(u_\mu^2 - u_\mu^1) b(t)] \frac{1}{(2\pi)^3 \sqrt{\Delta T(x')}} \exp\left[-\frac{1}{2} u_\mu^a u_\mu^a (T^{-1})_{\mu\nu}^{ab}(x') \right] d^3k \, d^3x' . \tag{15}$$

Using this in equation (9) and evaluating the u-integrals, we obtain

$$1 + \xi(x, t) = \left(\frac{1}{2\pi}\right)^3 \int \exp[ik_\mu(x'_\mu - x_\mu)] \exp\left[-\frac{b^2(t)}{2} k_\mu k_\nu F_{\mu\nu}(x') \right] d^3x' \, d^3k , \tag{16}$$

where

$$F(x)_{\mu\nu} = -\tfrac{2}{3} \nabla^2 \phi(0) \delta_{\mu\nu} + 2 \partial_\mu \partial_\mu \phi(x) . \tag{17}$$

Computing the k-integral, we obtain the two-point correlation as

$$1 + \xi(x, t) = \frac{1}{(2\pi)^{3/2} b^3(t)} \int \frac{1}{\sqrt{\Delta F(x')}} \exp\left[-\frac{1}{2b^2(t)} (x'_\mu - x_\mu)(x'_\nu - x_\nu) F_{\mu\nu}^{-1}(x') \right] d^3x' . \tag{18}$$

Instead of integrating equation (16), if we do a Taylor expansion of

$$\exp[-\tfrac{1}{2} b^2(t) k_\mu k_\nu F_{\mu\nu}(x')]$$

and then evaluate the k- and the x'-integrals, we obtain

$$1 + \xi(x, t) = \sum_{n=0}^{\infty} \frac{b^{2n}}{n!} \partial_{\mu_1} \partial_{\nu_1} \dots \partial_{\mu_n} \partial_{\mu_n} \left\{ \left[\partial_{\mu_1} \partial_{\nu_1} \phi(x) - \delta_{\mu_1 \nu_1} \frac{\nabla^2 \phi(0)}{3} \right] \cdots \left[\partial_{\mu_n} \partial_{\nu_n} \phi(x) - \delta_{\mu_n \nu_n} \frac{\nabla^2 \phi(0)}{3} \right] \right\} . \tag{19}$$

Nowhere above has any assumption been made about the number of streams in the flow. Equation (18) obviously has the effects of multistreaming built into it. Equation (19) is what one would obtain if one did a perturbative expansion of the distribution function and calculated the two-point correlation function. Whether by performing the perturbative analysis in this way (i.e., using distribution functions) we are able to include the effects of multistreaming is what has to be checked.

4. THE TWO-POINT CORRELATION USING THE HYDRODYNAMIC EQUATIONS

In this section, we shall work in the single-stream approximation. We consider any one member of the ensemble described previously. Its evolution is described by equation (4). If we take the zeroth moment of this equation with respect to u, using the definitions

$$\rho(x, b) = m \int f(x, u, b) d^3u \ , \qquad \rho(x, b) v_\mu(x, b) = m \int u_\mu f(x, u, b) d^3u \ , \tag{20}$$

we have the continuity equation

$$\frac{\partial}{\partial b} \rho(x, b) + \partial_\mu[\rho(x, b) v_\mu(x, b)] = 0 \ . \tag{21}$$

Next, taking the first moment of equation (4) and using equation (21), we have

$$\rho(x, b)\left[\frac{\partial}{\partial b} v_\mu(x, b) + v_\nu(x, b)\partial_\nu v_\mu(x, b)\right] + m \int [v_\nu(x, b) - u_\nu][v_\mu(x, b) - u_\mu] f(x, u, b) d^3u = 0 \ . \tag{22}$$

In the single-stream approximation the last term in the above equation is dropped, and we have

$$\frac{\partial}{\partial b} v_\mu(x, b) + v_\nu(x, b)\partial_\nu v_\mu(x, b) = 0 \ . \tag{23}$$

We shall use equations (21) and (23) to perturbatively evolve the density and velocity fields of the system. We then take ensemble averages and use these equations to calculate the two-point correlation function.

Using equation (21), we can obtain an equation for the first derivative of the two-point correlation function:

$$\frac{\partial}{\partial b} \{\langle\rho\rangle^2[1 + \xi(x, b)]\} = -\langle \partial^1_\mu[\rho(x^1) v_\mu(x^1)] \rho(x^2)\rangle - \langle \rho(x^1) \partial^2_\mu[\rho(x^2) v_\mu(x^2)]\rangle \ . \tag{24}$$

Using the normalization $\langle\rho\rangle = 1$, the above equation may be written

$$\frac{\partial}{\partial b} \xi(x, b) = -\partial^{a_1}_{\mu 1}\langle \rho(1) v^{a_1}_{\mu 1} \rho(2)\rangle \ . \tag{25}$$

We can use equations (21) and (23) to obtain equations for the higher derivatives of the two-point correlation,

$$\frac{\partial^n}{\partial b^n} \xi(x, b) = (-1)^n \partial^{a_1}_{\mu 1} \partial^{a_2}_{\mu 2} \ldots \partial^{a_n}_{\mu n} \langle \rho(1) v^{a_1}_{\mu 1} v^{a_2}_{\mu 2} \ldots v^{a_n}_{\mu n} \rho(2)\rangle \ . \tag{26}$$

Next we write the two-point correlation function as a Taylor series in powers of the growing mode b:

$$\xi(x, b) = \sum_{n=1}^{\infty} \frac{b^n}{n!} \frac{\partial^n}{\partial b^n} \xi(x, b)_{b=0} \ . \tag{27}$$

It should be noted that this allows us to express the two-point correlation function at any instant in terms of the derivatives of the two-point correlation function at the initial instant. Next, using equation (26), we obtain

$$\xi(x, b) = \sum_{n=1}^{\infty} \frac{b^n(-1)^n}{n!} \partial^{a_1}_{\mu 1} \partial^{a_2}_{\mu 2} \ldots \partial^{a_n}_{\mu n} \langle \rho(1) v^{a_1}_{\mu 1} v^{a_2}_{\mu 2} \ldots v^{a_n}_{\mu n} \rho(2)\rangle_{b=0} \ . \tag{28}$$

Then, using the fact that the initial density is uniform, we can write the two-point correlation function at an arbitrary time in terms of the initial velocities only, i.e.,

$$\xi(x, b) = \sum_{n=1}^{\infty} \frac{b^n}{n!} (-1)^n \partial^{a_1}_{\mu 1} \partial^{a_2}_{\mu 2} \ldots \partial^{a_n}_{\mu n} \langle v^{a_1}_{\mu 1} v^{a_2}_{\mu 2} \ldots v^{a_n}_{\mu n}\rangle_{b=0} \ . \tag{29}$$

In addition, the initial velocity field is Gaussian, and hence all the odd terms in equation (29) are zero. We can then write this equation as

$$\xi(x, b) = \sum_{n=1}^{\infty} \frac{b^{2n}}{(2n)!} \partial^{a_1}_{\mu 1} \partial^{b_1}_{\nu 1} \ldots \partial^{a_n}_{\mu n} \partial^{b_n}_{\nu n} \langle v^{a_1}_{\mu 1} v^{b_1}_{\nu 1} \ldots v^{a_n}_{\mu n} v^{b_n}_{\nu n}\rangle_{b=0} \ . \tag{30}$$

For the Gaussian initial velocity field, we have

$$\langle v^{a_1}_{\mu 1} v^{b_1}_{\nu 1} v^{a_2}_{\mu 2} v^{b_2}_{\nu 2} \ldots v^{a_n}_{\mu_n} v^{b_n}_{\nu_n} \rangle = \sum_P \langle v^{a_1}_{\mu 1} v^{b_1}_{\nu 1} \rangle \langle v^{a_2}_{\mu 2} v^{b_2}_{\nu 2} \rangle \cdots \langle v^{a_n}_{\mu_n} v^{b_n}_{\nu_n} \rangle \, , \tag{31}$$

where the sum is over all possible ways of pairing the u's.

Using this and the fact that the derivatives are symmetric in all the indices involved, we have for the initial velocity field

$$\partial^{a_1}_{\mu 1} \partial^{b_1}_{\nu 1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} \langle v^{a_1}_{\mu 1} v^{b_1}_{\nu 1} \ldots v^{a_n}_{\mu_n} v^{b_n}_{\nu_n} \rangle = \frac{(2n)!}{n!\,2^n} \partial^{a_1}_{\mu 1} \partial^{b_1}_{\nu 1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} (\langle v^{a_1}_{\mu 1} v^{b_1}_{\nu 1} \rangle \cdots \langle v^{a_n}_{\mu_n} v^{b_n}_{\nu_n} \rangle) \, . \tag{32}$$

This, when used in equation (30), yields

$$\xi(x, b) = \sum_{n=1}^{\infty} \frac{b^{2n}}{n!\,2^n} \partial^{a_1}_{\mu 1} \partial^{b_1}_{\nu 1} \partial^{a_2}_{\mu 2} \partial^{b_2}_{\nu 2} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} (\langle v^{a_1}_{\mu 1} v^{b_1}_{\nu 1} \rangle \langle v^{a_2}_{\mu 2} v^{b_2}_{\nu 2} \rangle \cdots \langle v^{a_n}_{\mu_n} v^{b_n}_{\nu_n} \rangle)_{b=0} \, . \tag{33}$$

Summing the superscripts $a_1, b_1, \ldots, a_n, b_n$ over the values 1 and 2 and using the fact that, for the initial velocity field,

$$\langle v^a_\mu v^b_\nu \rangle = \begin{cases} -\frac{1}{3}\nabla^2\phi(0)\delta_{\mu\nu} \, , & \text{if } a = b \, , \\ \partial^a_\mu \partial^b_\nu \phi(x) \, , & \text{if } a \neq b \, , \end{cases} \tag{34}$$

we have

$$1 + \xi(x, t) = \sum_{n=0}^{\infty} \frac{b^{2n}}{n!} \partial_{\mu 1}\partial_{\nu 1} \ldots \partial_{\mu_n}\partial_{\mu_n} \left\{ \left[\partial_{\mu 1}\partial_{\nu 1}\phi(x) - \delta_{\mu_1\nu_1} \frac{\nabla^2\phi(0)}{3} \right] \cdots \left[\partial_{\mu_n}\partial_{\nu_n}\phi(x) - \delta_{\mu_n\nu_n} \frac{\nabla^2\phi(0)}{3} \right] \right\} . \tag{35}$$

This is the same as equation (19), which was obtained by using distribution functions. Thus we see that the perturbative calculation of the two-point correlation function by use of distribution functions includes no effects of multistreaming, and hence we reach the conclusion that it is not possible to perturbatively follow the transition from a single-streamed flow to a multistreamed flow.

5. THE TWO-POINT CORRELATION AT LARGE SEPARATIONS

In this section, we investigate the evolution of the two-point correlation function in the regime in which it can be studied perturbatively, and we examine the behavior at large separations. The initial conditions for the evolution of the cosmological correlations may be expressed in terms of the potential $\phi(x)$ or, equivalently, in terms of the matter two-point correlation in the linear epoch, $\xi^{(1)}(x, t)$. The two are related by the equation

$$\xi^{(1)}(x, t) = b^2(t)\nabla^4\phi(x) \, . \tag{36}$$

Usually, the initial conditions are given in terms of the matter two-point correlation $\xi^{(1)}(x, t)$ or its Fourier transform $b^2(t)P_1(k)$, which is the power spectrum. One then has to invert equation (36) to obtain the potential $\phi(x)$ and its derivatives. In doing so, one has freedom in choosing boundary conditions, and the effect of changing the boundary condition is

$$\nabla^2\phi(x) \to \nabla^2\phi(x) + C_1 \, , \qquad \phi(x) \to \phi(x) + \tfrac{1}{6}C_1 x^2 + C_2 \, . \tag{37}$$

Equation (19) for the two-point correlation function is invariant under these transformations, and we are free to choose any boundary condition. For initial conditions under which the integral $\int_0^\infty \xi^{(1)}(x, t)x\,dx$ [or $\int_0^\infty P_1(k)dk$] is finite, we can choose the boundary condition $\lim_{x\to\infty} \nabla^2\phi(x) = 0$. We then have

$$\langle u^2 \rangle = -\nabla^2\phi(0) = \int_0^\infty \xi^{(1)}(x)x\,dx \, . \tag{38}$$

In addition, if at large x the function $\partial_\mu\partial_\nu\phi(x)$ is monotonically decreasing and $\partial_\mu\partial_\nu\phi(x) \ll (\delta_{\mu\nu}/3)\nabla^2\phi(0)$, we can then neglect all but one of the $\partial_\mu\partial_\nu\phi(x)$ terms that appear in equation (19). For initial conditions in which the power spectrum has the form $P(k) \propto k^n$ at small k and a cutoff at large k, the conditions discussed above are satisfied for $n > -1$. For these cases, we obtain, for the two-point correlation function at large x,

$$\xi(x, t) = \sum_{n=0}^{\infty} \frac{b^{2(n+1)}}{n!} \left[\frac{-\nabla^2\phi(0)}{3} \right]^n (\nabla^2)^n \nabla^4\phi(x) \, . \tag{39}$$

Using this, we obtain the lowest order nonlinear correction to the two-point correlation function at large x,

$$\xi^{(2)}(x, t) = \tfrac{1}{3}b^2\langle u^2 \rangle \nabla^2\xi^{(1)}(x, t) \, . \tag{40}$$

In Paper II, we calculated the same quantity using GD, and we found that, for $0 < n \leq 3$, at large x the results can be fitted by the formula

$$\xi^{(2)}(x, t) = 0.194b^2\langle u^2 \rangle \nabla^2\xi^{(1)}(x, t) \, . \tag{41}$$

We find that the two equations are very similar, differing only in the numerical coefficient. In Paper II we also interpreted equation (41) in terms of a simple heuristic model based on a diffusion process. We consider a particular member of the ensemble and look at the evolution of the density in volume elements located at separation x from each other. We assume that the density in each volume element grows according to linear theory and that the volume elements are rearranged randomly on small scales because of their peculiar velocities. Based on this model, we obtained an equation identical to equation (40). Thus we see that this model yields an exact description of what happens in ZA at large scales in the regime in which the perturbative treatment is valid. In ZA, as in our heuristic model, the velocity of the particles is fixed whereas, in GD, the particle velocity changes as evolution proceeds. We believe that this is responsible for the smaller diffusion coefficient for GD as compared to ZA.

Going back to equation (39) and writing it in Fourier space, we obtain for the power spectrum

$$P(k, t) = \left[\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{-b^2k^2\langle u^2\rangle}{3}\right)^n\right] b^2 P_1(k) . \tag{42}$$

Summing the terms in square brackets, we have

$$P(k, t) = \exp\left(\frac{-b^2k^2\langle u^2\rangle}{3}\right) b^2 P_1(k) , \tag{43}$$

which in real space gives us

$$\xi(x, t) = \frac{1}{[\sqrt{\pi}2L(t)]^3}\int_0^{\infty} \exp\left\{-\frac{(x-x')^2}{[4L(t)]^2}\right\}\xi^1(x', t)d^3x' , \tag{44}$$

where

$$L^2(t) = \tfrac{1}{3}b^2(t)\langle u^2\rangle . \tag{45}$$

The length scale $L(t)$ is the rms deviation of the particles from their Lagrangian (or initial) positions at any time t. We see that the nonlinear evolution of the two-point correlation function at large x corresponds to a convolution of the linear two-point correlation with a Gaussian whose width is proportional to $L(t)$. This is consistent with our interpretation of the evolution in terms of a diffusion process.

For the case in which the initial power spectrum has the form

$$P_1(k) = Ae^{-\alpha^2k^2}k^n , \tag{46}$$

using equation (39) at small k, we have, for the nonlinear power spectrum at small k,

$$P_1(k) = Ae^{-[\alpha^2+L^2(t)]k^2}k^n . \tag{47}$$

Using equations (46) and (47) and the fact that

$$\int e^{ikx}e^{-\beta^2\alpha^2k^2}P_1(k)d^3k = \frac{1}{\beta^{3+n}}\int_{-\infty}^{\infty} e^{ikx/\beta}e^{-\alpha^2k^2}P_1(k)d^3k , \tag{48}$$

we obtain, for the nonlinear two-point correlation function at large x,

$$\xi_1(x, t) = \{1 + [L(t)/\alpha]^2\}^{-(3+n)/2}\xi_2^{(1)}\{x/\sqrt{1+[L(t)/\alpha]^2}, t\} . \tag{49}$$

This formula relates the nonlinear two-point correlation at some separation x at time t to the linear two-point correlation at a smaller separation at the same time. Thus, at large x, for small values of the two-point correlation, we have information being transferred out from the smaller scales to the larger scales.

We next numerically investigate the evolution of the two-point correlation function at large separations for the initial power spectrum $P_1(k) = 0.5e^{-k^2}k$. Figure 1 shows the function $\xi^{(1)}(x)$ versus x. This function, multiplied by the square of the scale factor, yields the linear two-point correlation $\xi^{(1)}(x, t)$. At large x, the function $\xi^{(1)}(x)$ has a negative sign and a power-law behavior as x^{-4}. We investigate the evolution of the two-point correlation function at the large separation $x = 10$. We do this using four different approximations:

1. Linear perturbation theory;
2. Linear theory plus the lowest order nonlinear correction using GD (Paper II);
3. The result obtained from summing the whole perturbation series for the ZA with the extra assumptions about the evolution at large separations made in this section, i.e., equation (49);
4. The nonperturbative two-point correlation calculated using ZA (eq. [18]).

This exercise allows us to investigate two different issues. The first thing that we can check is how well ZA approximates GD. This can be done by comparing (2) with (3) and (4). In this section, we have made some assumptions about the large-x behavior of the two-point correlation function and arrived at the diffusion picture for the evolution. We can put these assumptions to the test by comparing (3) with (4). The results are shown in Figure 2. We find that all four approximations match in the early stages of the evolution. The two-point correlation at this separation is initially negative, and this value evolves according to linear theory, where it gets multiplied by b^2. The different approximations start to differ as the evolution

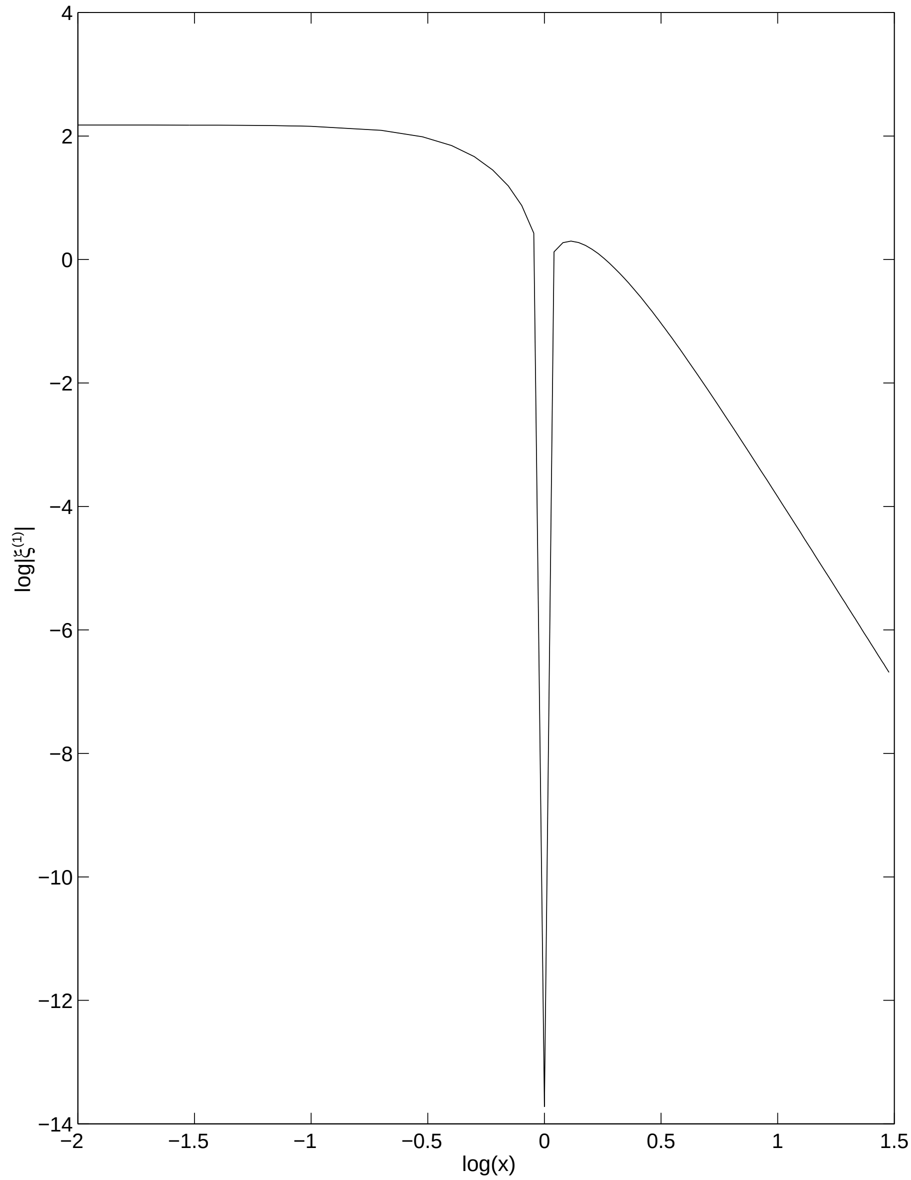

FIG. 1.—Initial two-point correlation as a function of separation for the power spectrum $P(k) = 0.5e^{-k^2}k$

proceeds. The first thing to note is that they begin to differ much before $\xi(x, t) \sim 1$, where one would naively expect the perturbation series to break down. This is a consequence of the nonlocal nature of the nonlinear terms for the two-point correlation. As discussed in Paper II, this can be understood from equation (38),

$$\langle u^2 \rangle = \int_0^\infty \xi^{(1)}(x)x\,dx \;,$$

which shows that the nonlinear correction depends on the linear two-point correlation condition at all scales, and the major contribution to this integral comes from the small scales. The small scales become strongly nonlinear very early in the evolution, and it is because of this that the nonlinear term starts contributing at large x even when $\xi(x, t) \ll 1$. In all the approximations (i.e., [2], [3], and [4]), the effect of the initial deviation from the linear theory is to make the growth rate faster than $b^2(t)$. In the initial stages, approximations 2, 3, and 4 exhibit qualitatively similar behavior, but as the evolution proceeds we find that (4) starts showing a behavior completely different from (2) and (3). We find that the rapidly decreasing function (4) slows its decrease and then starts to increase. This is quite different from the behavior of (2) and (3), which continue to decrease. This difference is because of the effects of multistreaming. In ZA, the correlations are washed out after multistreaming occurs. Until the onset of multistreaming, the diffusion picture (3) matches quite well with the full ZA, i.e., (4). A comparison of (2), (3), and (4) shows that ZA qualitatively predicts the same behavior as GD, and the quantitative difference may be attributed to the difference in the diffusion coefficients. In the case of the actual gravitational dynamics (nonperturbative), we expect that the results may be different because there the particles will get "stuck" in bound objects once multistreaming occurs (e.g., the adhesion model; Gurbatov, Saichev, & Shandarin 1989). As a result of this, the mean

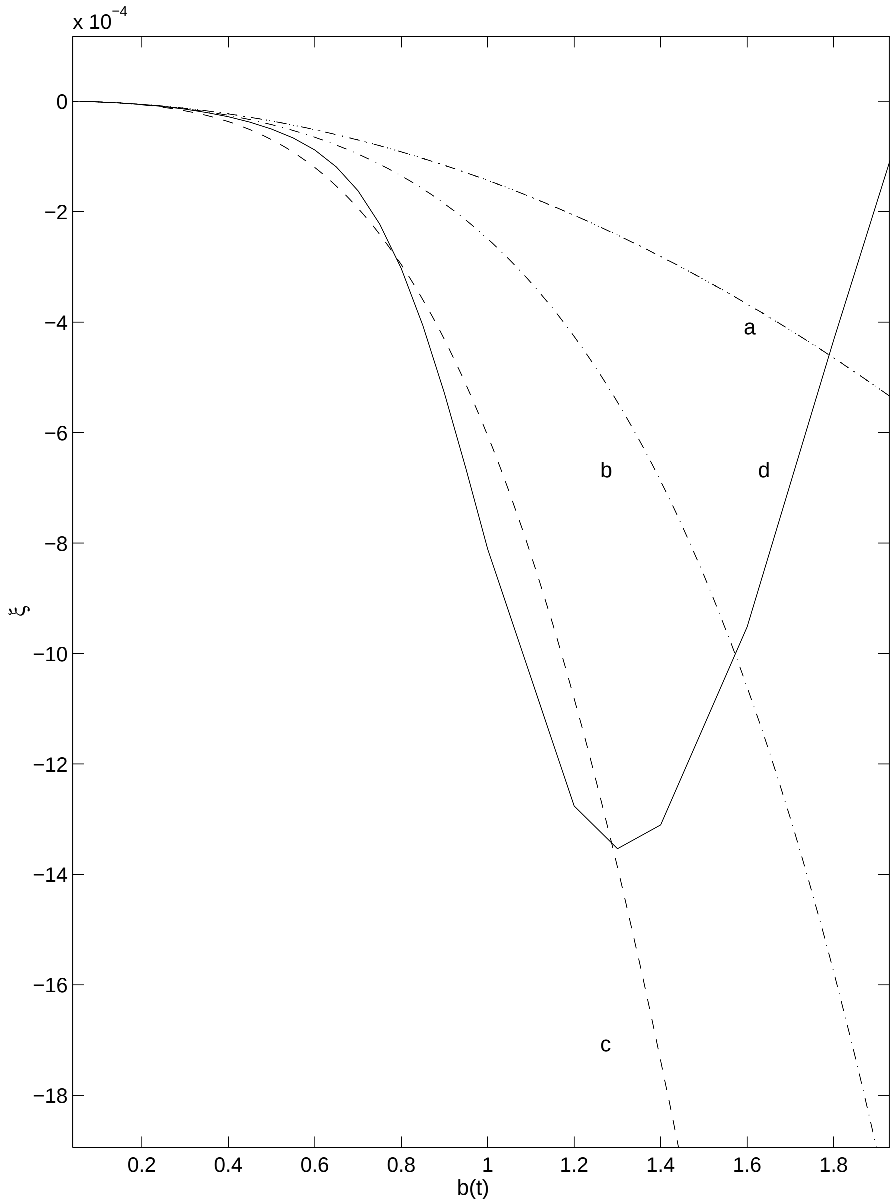

FIG. 2.—Two-point correlation at a fixed separation $x = 10$ as a function of the growing mode $b(t)$ for linear theory (a), linear theory plus lowest order nonlinear correction using GD (b), nonlinear evolution using ZA and the assumptions made in § 5 about the large-x behavior (c), and nonperturbative ZA (d).

square displacement of the particles will be much less than in ZA or in perturbative GD. Although we expect this diffusion picture to hold for the actual evolution of the two-point correlation function at large x, the perturbative treatment of GD and also calculations using ZA may overestimate what would be obtained in N-body simulations. Incidentally, the regime treated here would be difficult to study by use of such simulations since it involves the low-amplitude tail of the two-point correlation function, which would be limited by the size of the box, and it would require a large dynamical range.

6. THE THREE-POINT CORRELATION FUNCTION

We use ZA to follow the evolution of the N-point correlation function. It is possible to do this nonperturbatively by following a line of reasoning very similar to that in § 3. However, since ZA is a good substitute for gravitational dynamics only in the weakly nonlinear regime, we prefer to carry out the investigation perturbatively.

We first consider the evolution of the ensemble-averaged N-point distribution function $\rho_N(x^a, u^a, t)$. This is a generalization of the ensemble-averaged two-point distribution function introduced in § 3, and the superscript a refers to the various points, i.e., 1, 2, ..., N, in phase space that are arguments of this function. Using equation (3), we obtain for the time evolution of this function

$$\rho_N(x^a, u^a, t) = \rho_N[x^a - b(t)u^a, u^a, t_0] . \tag{50}$$

Expanding this in a perturbative series and using $a_1, a_2, \ldots, a_n$ for n indices that independently take values between 1 and N, and using $\mu_1, \mu_2, \ldots, \mu_n$ for n corresponding Cartesian components, we have

$$\rho_N(x^a, u^a, t) = \sum_{n=0}^{\infty} \frac{(-b)^n}{n!} u^{a_1}_{\mu_1} u^{a_2}_{\mu_2} \ldots u^{a_n}_{\mu_n} \partial^{a_1}_{\mu_1} \partial^{a_2}_{\mu_2} \ldots \partial^{a_n}_{\mu_n} \rho_N(x^a, u^a, t_0) . \tag{51}$$

For both kinds of indices, the Einstein summation convention holds: all the a^i have to be summed over the range 1 to N whenever they appear twice, and the μ_i have to be summed over the three Cartesian components whenever the indices are repeated.

To calculate the N-point correlation function, we take velocity moments of the N-point distribution function:

$$\int_0^{\infty} \rho_N(x^a, u^a, t) d^{3N}u = \sum_{n=0}^{\infty} \frac{(-b)^n}{n!} \partial^{a_1}_{\mu_1} \partial^{a_2}_{\mu_2} \ldots \partial^{a_n}_{\mu_n} \langle u^{a_1}_{\mu_1} u^{a_2}_{\mu_2} \ldots u^{a_n}_{\mu_n} \rangle . \tag{52}$$

All the terms where n is odd are zero, and only the terms with even n contribute. We also have

$$\langle u^{a_1}_{\mu_1} u^{a_2}_{\mu_2} \ldots u^{a_n}_{\mu_n} u^{b_1}_{\nu_1} u^{b_2}_{\nu_2} \ldots u^{b_n}_{\nu_n} \rangle = \langle u^{a_1}_{\mu_1} u^{b_1}_{\nu_1} \rangle \cdots \langle u^{a_n}_{\mu_n} u^{b_n}_{\nu_n} \rangle + \text{permutations} . \tag{53}$$

Using the fact that $\partial^{a_1}_{\mu_1} \partial^{a_2}_{\mu_2} \ldots \partial^{a_{2n}}_{\mu_{2n}}$ is symmetric in all the indices, we can add all the permutations to obtain, for the terms with even n,

$$\partial^{a_1}_{\mu_1} \partial^{b_1}_{\nu_1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} \langle u^{a_1}_{\mu_1} u^{b_1}_{\nu_1} \ldots u^{a_n}_{\mu_n} u^{b_n}_{\nu_n} \rangle = \frac{(2n)!}{2^n n!} \partial^{a_1}_{\mu_1} \partial^{b_1}_{\nu_1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} (T^{a_1 b_1}_{\mu_1 \nu_1} \ldots T^{a_n b_n}_{\mu_n \nu_n}) . \tag{54}$$

where $T^{ab}_{\mu\nu} = \langle u^a_\mu u^b_\nu \rangle$ is the covariance matrix introduced in § 3, generalized for the N-point distribution function.

Using this in equation (51), we have

$$\int_0^{\infty} \rho_N(x^a, u^a, t) d^{3N}u = \sum_{n=0}^{\infty} \frac{(b^2)^n}{2^n n!} \partial^{a_1}_{\mu_1} \partial^{b_1}_{\nu_1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} (T^{a_1 b_1}_{\mu_1 \nu_1} \ldots T^{a_n b_n}_{\mu_n \nu_n}) . \tag{55}$$

In the above equation, for a fixed value of n, there will be a term with n pairs $(a_1 b_1), (a_1 b_2), \ldots, (a_n b_n)$, where each index is independently summed over values 1 to N. Thus, for a fixed value of n, the total contribution is a sum of N^{2n} terms, each corresponding to a different set of values for the position indices. In any one of these N^{2n} terms, there can be two kinds of pairs:

A. If $a_i = b_i$, then $T^{a_i b_i}_{\mu_i \nu_i} = -\frac{1}{3}\delta_{\mu_i \nu_i} \nabla^2 \phi(0)$ is a constant.
B. If $a_i \neq b_i$, then $T^{a_i b_i}_{\mu_i \nu_i} = \partial^{a_i}_{\mu_i} \partial^{a_i}_{\nu_i} \phi(a_i, b_i)$ is a function of the separation between these two points.

Any of the terms can be represented by a directed graph with N vertices and n edges. The pairs of kind A correspond to an edge connecting a vertex to itself, and a pair of kind B corresponds to an edge connecting two different vertices (Fig. 3). The integral $\int_0^\infty \rho_N(x^a, u^a, t) d^{3N}u$ then corresponds to a sum of graphs with N vertices and the number of edges going from 0 to infinity.

The quantity $\int_0^\infty \rho_N(x^a, u^a, t) d^{3N}u \, d^3x^1 \, d^3x^2 \ldots d^3x^N$ is the mean number of particles we expect to find in the volume d^3x^1 at x^1, in d^3x^2 at $x^2, \ldots,$ and in d^3x^N around x^N. This has contribution from the lower (i.e., $N-1, \ldots, 1$ point) correlation functions also. The residue when the contributions from the lower correlation functions have been removed is called the *reduced N-point correlation function.* Hereafter we shall refer to this as the N-point correlation function. The graphs that do not connect all N points correspond to functions that do not refer to all N points, and these are the contributions from the

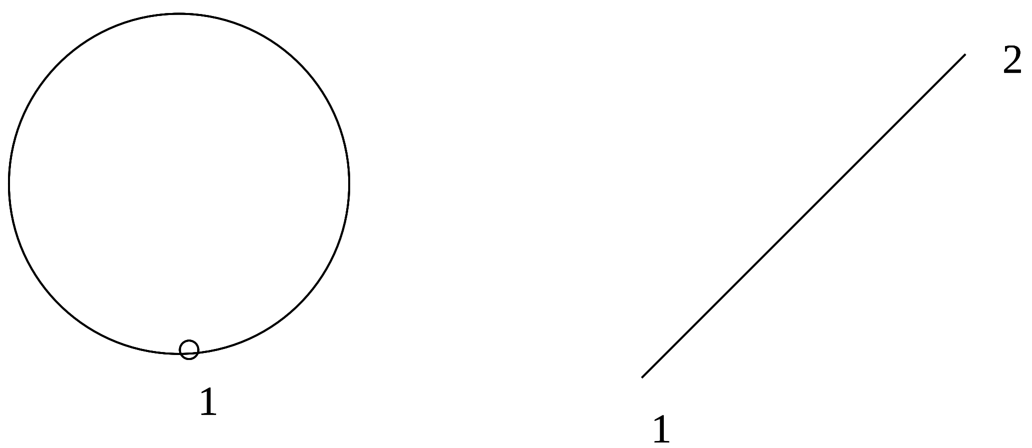

FIG. 3.—The two possible kinds of edges. Type A (*left*) connects a vertex to itself, and type B (*right*) connects two different vertices.

lower correlations. The reduced N-point correlation can be calculated by considering only the connected graphs with N vertices. The lowest order contribution to the N-point correlation corresponds to the connected graphs with the least number of edges. These graphs are the tree graphs, and they have $N - 1$ edges. The other terms that contribute to the N-point correlation can be generated by adding more edges to the tree graphs.

We use equation (55) to calculate the three-point correlation function. The lowest order at which the three-point correlation develops is $n = 2$, and this can be written

$$\zeta^{(1)}(1, 2, 3, t) = \tfrac{1}{2} b^4 \partial_{\mu_1}^{a'_{11}} \partial_{\mu_2}^{a'_{12}} \partial_{\mu_3}^{a'_{12}} \partial_{\mu_4}^{a'_{13}} (T_{\mu_1 \mu_2}^{a'_{11} a'_{12}} T_{\mu_3 \mu_4}^{a'_{12} a'_{13}}) , \tag{56}$$

where a'_{11}, a'_{12}, and a'_{13} are to be summed over all possible permutations of 1, 2, and 3. Equation (56) corresponds to the only possible tree graph with three vertices a'_{11}, a'_{12}, and a'_{13} and two edges (a'_{11}, a'_{12}) and (a'_{12}, a'_{13}) (Fig. 4).

Using

$$\partial_\mu \nabla^2 \phi(x) = \frac{x_\mu}{x^3} \int_0^\infty \xi^{(1)}(y) y^2 \, dy = \frac{1}{3} x_\mu \overline{\xi^{(1)}}(x) , \tag{57}$$

we have

$$\zeta^{(1)}(1, 2, 3, t) = \frac{b^4}{2} \Bigg[(1 + \cos^2 \theta_{xy}) \xi^{(1)}(x, t) \xi^{(1)}(y, t) + \cos \theta_{xy} \frac{2}{3} \frac{d}{dx} \xi^{(1)}(x, t) y \overline{\xi^{(1)}}(y, t) + \frac{2}{3} (1 - 3 \cos^2 \theta_{xy}) \xi^{(1)}(x, t) \overline{\xi^{(1)}}(y, t) - \frac{1}{3} (1 - 3 \cos^2 \theta_{xy}) \overline{\xi^{(1)}}(x, t) \overline{\xi^{(1)}}(y, t) \Bigg] , \tag{58}$$

where

$$x = x^{a'_{12}} - x^{a'_{13}} , \qquad y = x^{a'_{12}} - x^{a'_{13}} , \qquad \theta_{xy} = x_\mu y_\mu / xy . \tag{59}$$

This explicitly exhibits the dependence of the lowest order induced three-point correlation function on the initial two-point correlation function. We see that the three-point correlation depends on both $\xi^{(1)}(x, t)$ and $\overline{\xi^{(1)}}(x, t)$. Thus we see that the small scales can influence the three-point correlation at large scales through the quantity $\overline{\xi^{(1)}}(x, t)$. The lowest order induced three-point correlation function calculated using ZA is very similar to that calculated by studying gravitational dynamics perturbatively at the lowest order beyond the linear theory (Paper I), the difference being only in the numerical factors.

We next calculate the higher order terms that contribute to the three-point correlation function. These are generated by adding more edges to the tree graphs. Figures 5 and 6 illustrate the simplest cases, where we add only one edge to the tree graph. Next consider any of the graphs with $n > 2$ edges. In these graphs, the tree graph can be embedded in $\binom{n}{2}$ ways. Using

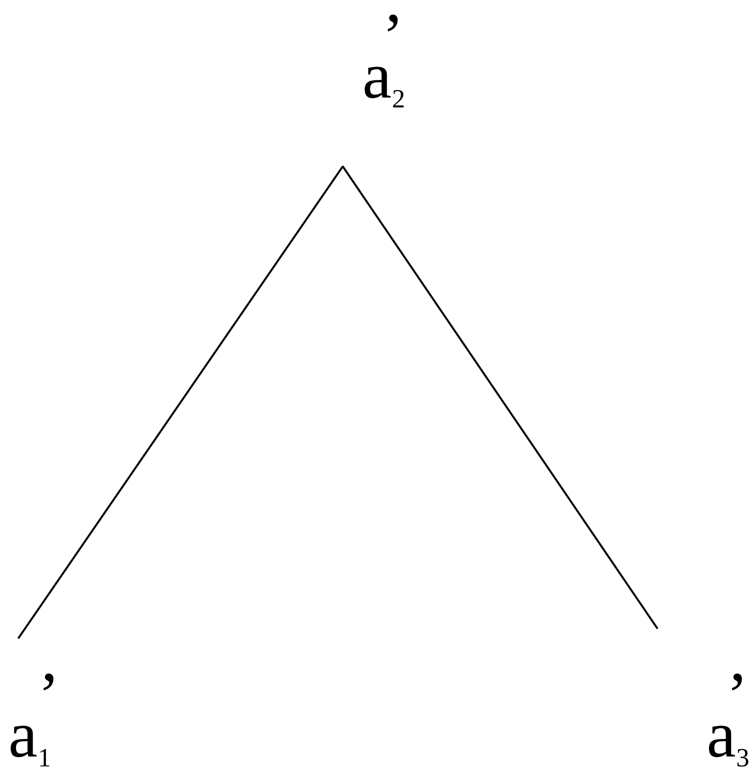

FIG. 4.—Tree graph corresponding to the lowest order induced three-point correlation function

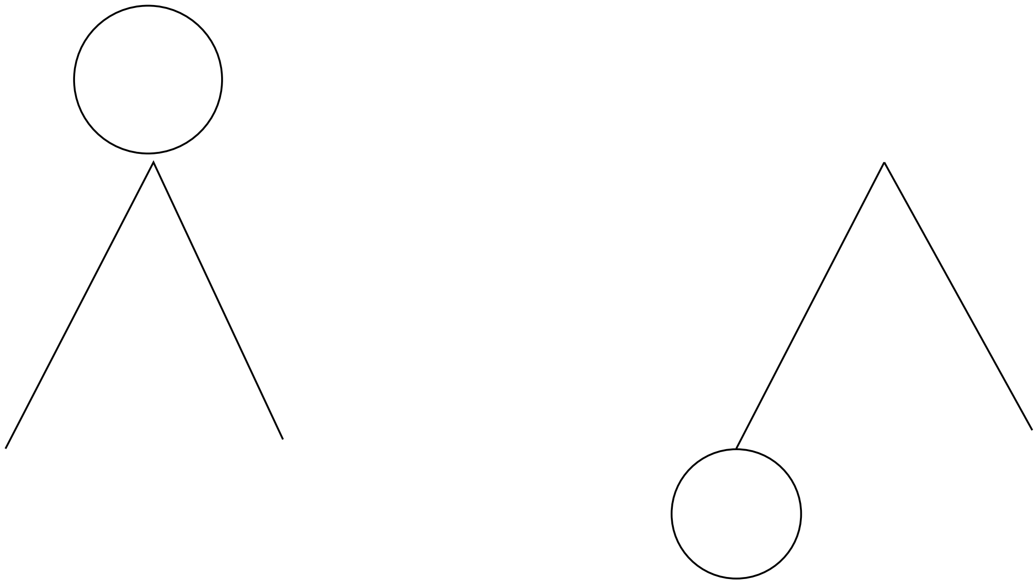

FIG. 5.—Some of the graphs corresponding to the contribution to the three-point correlation function at one order beyond the lowest. These graphs are all obtained by adding edges to the tree graph. This figure shows those cases in which the extra edge connects a vertex to itself.

this in equation (55), we have

$$\zeta(1, 2, 3, t) = \sum_{n=0}^{\infty} \frac{b^{2(n+2)}}{2^{n+1} n!} \partial^{a_1}_{\mu_1} \partial^{b_1}_{\nu_1} \dots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} \partial^{a'_1}_{\alpha_1} \partial^{a'_2}_{\alpha_2} \partial^{a'_2}_{\alpha_3} \partial^{a'_4}_{\alpha_4} (T^{a_1 b_1}_{\mu_1 \nu_1} \dots T^{a_n b_n}_{\mu_n \nu_n} T^{a'_1 a'_2}_{\alpha_1 \alpha_2} T^{a'_2 a'_3}_{\alpha_3 \alpha_4}) \,. \tag{60}$$

As discussed in the previous section, at large x the contributions from the terms with $a_i = b_i$ will dominate, i.e., at the lowest order, graphs of the kind shown in Figure 5. Thus, at large x, the three-point correlation function may be written

$$\zeta(1, 2, 3, t) = \sum_{n=0}^{\infty} \frac{b^{2n}}{2^n n!} \left[-\frac{1}{3} \nabla^2 \phi(0) \right]^n (\nabla^{a_1})^2 (\nabla^{a_1})^2 \dots (\nabla^{a_n})^2 \xi_3^{(1)}(1, 2, 3, t) \,, \tag{61}$$

where the index a_i indicates at which point the Laplacian acts, and it is to be summed over the values 1, 2, and 3. In Fourier space, we have

$$F_3(k^1, k^2, k^3, t) = \sum_{n=0}^{\infty} \frac{b^{2n}}{2^n n!} \left[\frac{1}{3} \nabla^2 \phi(0) \right]^n [(k^{a_1})^2 + (k^{a_2})^2 + \dots + (k^{a_n})^2] F_3^{(1)}(k^1, k^2, k^3, t) \,, \tag{62}$$

where F_3 is the Fourier transform of the three-point correlation and $F_3^{(1)}$ is the Fourier transform of the lowest order three-point correlation. The terms can be summed to obtain

$$F_3(k^1, k^2, k^3, t) = \exp\left\{ -\frac{1}{2} \frac{b^2 \langle u^2 \rangle}{3} [(k^1)^2 + (k^2)^2 + (k^3)^2] \right\} F_3^{(1)}(k^1, k^2, k^3, t) \,, \tag{63}$$

which gives us, in real space,

$$\zeta(x^1, x^2, x^3, t) = \frac{1}{[\sqrt{2\pi} L(t)]^9} \int \exp\left[-\frac{(x^a - y^a)^2}{2L^2(t)} \right] \zeta^{(1)}(y^1, y^2, y^3, t) d^9 y \,. \tag{64}$$

Thus, at large separations, the effect of including the higher order terms for the three-point correlation function is to convolve the lowest order induced three-point correlation with a Gaussian of width $L(t)$. As with the two-point correlation function, this too can be interpreted in terms of a diffusion process.

FIG. 6.—Same as Fig. 5, but for those cases in which the extra edge connects two different vertices

7. DISCUSSION AND CONCLUSIONS

We find that when we calculate the two-point correlation function as a series in powers of the growing mode, we obtain the same answer as when we perform the calculation by using distribution functions or in the single-stream approximation. Since the first method is valid even after multistreaming occurs and the second method breaks down once multistreaming occurs, once multistreaming has occurred we would expect to obtain different answers from the two methods. But the two results match to all orders in the expansion parameter. We therefore conclude that, even though these equations are valid in the multistreaming epoch, if we start from single-streamed initial conditions we cannot perturbatively calculate any effect due to multistreaming, e.g., vorticity or pressure. This limitation arises from the fact that the full two-point correlation function for ZA, which includes the effects of multistreaming, is an exponential in $1/b^2$. All the derivatives of the function $(1/b)e^{-A/b^2}$ vanish at $b = 0$. As a result, if we try to expand this function in a series in powers of b around $b = 0$, we find that coefficients of all the powers of b are zero. If one considers the power spectrum instead, it is of the form $e^{-\alpha k^2 b^2}$. This function can be expressed as a power series in b^2, and one might think that it is possible to perturbatively study the effects of multistreaming by working in Fourier space instead of real space. Such a conclusion would be erroneous, as none of the terms in this expansion would have the effects of multistreaming. It would be possible to study the effects of multistreaming only if it were possible to sum the whole series. This point is further illustrated in the Appendix, where we consider a simpler example in which a similar situation occurs.

Shandarin & Zeldovich (1989) presented a formula for N, the mean number of streams at any point, in a situation in which the particles are moving in one dimension under ZA. At small b, this formula is of the form $N = 1 + e^{-A/b^2}$, where A is a constant characterizing the initial conditions. If we expand this in powers of b, the coefficients for all the terms are zero, and we find that the mean number of streams is 1. This confirms that the effects of multistreaming cannot be studied perturbatively. Although in this analysis we used ZA, we expect this to hold for full gravitational dynamics too, as derived at the lowest order of nonlinearity in Paper II.

In our comparison of the two-point correlation function at large separations, we find that the results obtained using ZA are quite similar to the lowest order nonlinear results obtained using GD and that both can be interpreted in terms of a diffusion process in which the rearrangement of matter on small scales affects the two-point correlation at large scales. In ZA, for an initial power spectrum with $n > -1$, the mean square displacement of the particles from their original positions is $L^2(t) = b^2(t)\langle u^2 \rangle$, and this makes its appearance in the formula for the nonlinear corrections to the two-point correlation function obtained using ZA. Interpreting the results from GD in a similar fashion, for an initial power spectrum with $n > 0$, we have $L^2(t) \sim 0.58 b^2(t)\langle u^2 \rangle$. In Paper II, we also considered the case with $n = 0$, and for this case we found $L^2(t) \sim 1.49 b^2(t)\langle u^2 \rangle$. The differences can be understood in terms of the fact that, in ZA, the particles move along trajectories calculated using linear GD, whereas, when we take into account nonlinear corrections, the trajectories are modified by tidal forces. In the equations for the evolution of the two-point correlation function, the tidal force acts through the three-point correlation function. The tidal force of the third particle (in the three-point correlation) will cause the other two particles to move toward or away from one another. This effect will be strongly dependent on the spatial behavior of the three-point correlation function. For the cases with $n > 0$, the induced three-point correlation has the hierarchical form at large x, whereas, for the case with $n = 0$, the induced three-point correlation does not have this form. We propose that it is because of this that the effect of the tidal forces is different in these two cases and that, in the former, the effect of the tidal forces is to reduce the mean square displacements relative to ZA whereas in the latter case it increases it. Thus, indirectly, it is a diagnostic of the effect of the back-reaction of the three-point correlation function on the pair velocity, which in turn affects the two-point correlation.

We find that for ZA, at large x, we can sum all terms in the perturbative series, and the nonlinear two-point correlation function is related to the linear two-point correlation by a convolution with a Gaussian of width $\propto L(t)$. We also find that for special initial conditions in which the power spectrum has a Gaussian cutoff at large k, the evolution at large x can be described by a simple scaling relation according to which the information propagates outward.

We also find that this picture based on diffusion provides a good description of the evolution under ZA until the onset of multistreaming. Based on this, we suggest that the evolution of the two-point correlation function in GD can also be described by a diffusion process until the onset of multistreaming.

We have calculated the lowest order induced three-point correlation function using ZA, and we find that it is very similar to the result obtained using GD; the two differ only in the numerical factors. We also investigated the effect of the higher order nonlinear terms, and we find that at large x we can sum the whole perturbation series. We find that the expression obtained after taking into account the nonlinear corrections is related to the lowest order three-point correlation function by a convolution with a Gaussian of width $\propto L(t)$. This is very similar to the evolution of the two-point correlation function at large separations. It can be shown that a similar relation holds for the higher correlation functions also, but we do not pursue this matter in this paper.

The author would like to thank Rajaram Nityananda for his advice, encouragement, and many very useful suggestions and discussions.

APPENDIX

Consider a Gaussian function of the variable x with standard deviation σ. We are interested in the power series expansion of this function in σ around $\sigma = 0$. We can do this expansion by taking the Fourier transform of the Gaussian, i.e.,

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} = \frac{1}{2\pi} \int e^{ikx} e^{-k^2\sigma^2/2} dk \,, \tag{A1}$$

and then doing a Taylor expansion (convergent) of $e^{-k^2\sigma^2/2}$. We then obtain

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} = \frac{1}{2\pi} \int e^{ikx} \sum_{n=0}^{\infty} \left(-\frac{1}{2} k^2\sigma^2\right)^n \frac{1}{n!} dk \,, \tag{A2}$$

which gives us

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{1}{2} \sigma^2 \frac{d^2}{dx^2}\right)^n \delta(x) \,. \tag{A3}$$

Equation (A3) can also be derived if we take the Gaussian function and directly do a Taylor expansion in σ, i.e., without going to Fourier space.

We see that the series expansion is entirely made up of Dirac delta functions and their derivatives, and hence it has nonzero value only when $x = 0$. This should be compared with the original Gaussian function, which has nonzero value even if $x \neq 0$. We see that in this case the Taylor expansion fails to capture an important aspect of the original function, and we can attribute this to the fact that we are performing the Taylor expansion of a function that is an exponential in $1/\sigma^2$.

If instead of working in real space we work in Fourier space, we find that we have to deal with the Taylor expansion of a function that is an exponential in σ^2 instead of $1/\sigma^2$. There is no problem expanding this function in a Taylor series, and one might be led to think that the limitation of the Taylor expansion in real space can be overcome by going to Fourier space. But this turns out to be wrong. On comparing equations (A2) and (A3), we see that each term in the expansion in Fourier space corresponds to some derivatives of a Dirac delta function, and hence it cannot capture any of the effects missed if the analysis is done in real space. These effects can be included only if one is able to sum the series in Fourier space and then do the Fourier transform.

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