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THE DIFFUSE GAMMA-RAY BACKGROUND AND THE PULSAR MAGNETIC WINDOW

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ABSTRACT

We have investigated in detail the hypothesis advanced earlier that the spark discharges in the Ruderman and Sutherland model are triggered by diffuse background gamma rays. We find that such triggering can be effective, but only within a narrow range around 2.5×10^{12} gauss of the surface magnetic field. The position of this magnetic window is quite insensitive both to the radius of the neutron star and to the scaling with radius of the polar gap size. A long period cutoff at 4 s for pulsars also follows as a result.

Assuming that gamma-ray triggering plays a role and that the fields are dipolar, those neutron stars with surface magnetic fields within the window will have a higher probability of functioning as pulsars. This offers a natural explanation for the peaking (around this value) of derived pulsar magnetic fields which has been a puzzle for many years. The observed spread in magnetic fields, derived from measurements of pulsar slowdown rates on the assumption of dipole braking and a constant radius, could be attributed to the presence of multipole components and to the spread in the neutron star radii.

Subject headings: gamma rays: general — pulsars — stars: neutron

I. INTRODUCTION

Several models have been proposed so far to explain the radio emission from pulsars (see Manchester and Taylor 1977 for a discussion of these models). Although it has not yet been possible for any of them to account for all of the details of pulse radiation, the Ruderman-Sutherland (RS) model (Ruderman and Sutherland 1975) possesses a number of features that agree with many of the observed characteristics.

In the RS model (applicable only when the rotation and magnetic axes of the neutron star are nearly antiparallel), the inhibition of ion emission from the stellar surface is crucial and leads to the formation of a vacuum region above the pole with a strong electric field component parallel to the magnetic field. In this region, the gap, any charged particle or a photon capable of producing electron-positron pairs can, by a combination of curvature radiation and pair production processes, create sufficient charges to short the gap electric field. The spark discharge then injects a high-energy beam of positrons into the magnetosphere, the energy of the beam being supplied by the parallel electric field. Again, by a combination of curvature radiation and pair production, these injected particles multiply further in the magnetosphere (Sturrock 1971). Bunching in this plasma has then been invoked to account for the observed coherent radio radiation from pulsars.

Once the gap has been discharged, the sparking can recur only if fresh charges are introduced into the gap where the parallel electric field has built up again. Ruderman (1980) states, "the discharge must continually be restarted on a time scale of 10–10^2 μs, the time it [the discharge] would take to reach either the boundary or an inner field line with no curvature." One process to achieve this involves emission of ions from patches of the polar cap heated by the previous discharge (Cheng and Ruderman 1980). We wish, however, to explore another process (Radhakrishnan 1980), involving only the diffuse gamma-ray background which must always be present and which can be discounted only when some other, more powerful source is operative.

The diffuse background gamma rays with energies above 1.02 MeV can supply charged particles to the gap on the 10–10^2 μs time scale, if first, they manage to reach the gap, and second, they produce pairs within it. Let the optical depth be $\tau_{\rm mag}$ for the first stage and $\tau_{\rm gap}$ for the second. Then, of the diffuse gamma-ray flux n_0 (cm^{-2} s^{-1} sr^{-1} MeV^{-1}), which is incident on the star, the part which reaches the gap is

$$n_1 = n_0 e^{-\tau_{\rm mag}} . \tag{1}$$

Of this, that part which produces pairs within the gap is

$$n_2 = n_1(1 - e^{-\tau_{\rm gap}}) . \tag{2}$$

Henceforth, we shall work with the efficiency of this process, η, where

$$\eta = e^{-\tau_{\rm maf}}(1 - e^{-\tau_{\rm gap}}) , \tag{3}$$

$$= e^{-\tau}(e^{\tau_{\rm gap}} - 1) , \tag{4}$$

and $\tau = \tau_{\rm mag} + \tau_{\rm gap}$. The optical depth τ receives contributions from various possible processes. Among these we shall concentrate only on pair production by gamma rays in an external magnetic field, but first we touch briefly on other processes.

Scattering of the diffuse gamma rays by electrons and positrons in the magnetosphere, e.g., Compton scattering by particles in Landau orbitals, is expected to be negligible and hence will not introduce any asymmetry in the originally isotropic background. The scattering cross sections are not available for the most general case, but particular cases are discussed by Milton *et al.* (1974) and Herold (1979); in any event, substantial degradation of gamma rays to lower energies is not expected. However, the gamma rays would produce electron-positron pairs in such collisions, as well as by the process which we later consider in detail (i.e., pair production in an external magnetic field). We ignore the particles produced outside the gap by these processes as unimportant in this discussion, but their role may need to be investigated further. Thus, as mentioned before, only pair production in the pulsar magnetic field is considered in determining η, which directly bears on the rate of sparking.

We shall show that, when pulsar magnetic fields are weak, most of the diffuse gamma rays reach the gap but cannot be stopped within it. On the other hand, when fields are strong, very few gamma rays reach the gap. Thus, sparks can be reinitiated easily by gamma rays only when the neutron star magnetic fields are neither too weak nor too strong. Furthermore, the time scale associated with reinitiation turns out to be 10–10^2 μs. Other consequences are discussed later.

In § II we enumerate our assumptions and derive an expression for η. Section II contains details of approximations used to evaluate η numerically. Results and concluding remarks appear in § IV.

II. THE EFFICIENCY OF SPARK INITIATION

In the RS model, the regions above the magnetic poles of a neutron star whose spin and magnetic axes are nearly antiparallel develop a potential drop of the order of 10^{12} V parallel to the magnetic field $\boldsymbol{B}$. The gaps are devoid of plasma; their formation depends crucially on the crustal ionic binding energies being high enough to prevent the parallel electric field from pulling ions out of the crust. The extent of the gap is about 10^4 cm. Although in the RS model, multipole fields are invoked near the star, we take the neutron star magnetic field to be dipolar for the sake of simplicity. In § IV, we remark on how this could affect the final result.

As stated earlier, we consider only the diffuse gamma-ray background to be effective in restarting sparks. Further, the attenuation of the gamma rays is attributed solely to electron-positron pair creation in the dipolar pulsar magnetic field.

The gap size is taken to be 10^4 cm as in the RS model. It may vary with the stellar radius (see Appendix for a discussion of this possibility).

The mean free path l for pair creation by a gamma ray with an energy E ($E > 1.02$ MeV $= 2m_e c^2$) depends sensitively on the component of magnetic field perpendicular to the gamma-ray path. It is (Erber 1966)

$$l = \frac{1}{\alpha}\frac{\hbar}{m_e c}\frac{B_q}{B_\perp} f(\chi) , \tag{5}$$

$$= 1.016 \times 10^6 \frac{1}{B_\perp} \exp\left(\frac{60.15 \times 10^{12}}{B_\perp E}\right) , \tag{6}$$

where α is the fine-structure constant, $B_\perp$ the perpendicular component of the magnetic field, $B_q = 4.41 \times 10^{13}$ gauss, and $\chi = (E/2m_e c^2)(B_\perp/B_q)$ with

$$f(\chi) = 4.35 e^{4/3\chi} \quad (\chi \ll 1) . \tag{7}$$

In equation (7), $f(\chi)$ is written only for the case when $\chi \ll 1$. It will be seen that we never need to overstep this condition.

The photon trajectory is given by

$$\boldsymbol{u} = \hat{u}_1 + \upsilon \hat{a} , \tag{8}$$

where all lengths are in units of the stellar radius $R_* = R_6 \times 10^6$ cm. The trajectory is parameterized by υ, which equals zero at the stellar surface (i.e., when $u = 1$). Unit vectors $\hat{u}_1$ and $\hat{a}$ define, respectively, the point of incidence on the stellar surface and the direction of incidence of the gamma ray. In spherical polar coordinates, they are given by (θ_1, ϕ_1) and (θ_0, ϕ_0). Now

$$\boldsymbol{B}(\boldsymbol{r}) = \left(\frac{3(\boldsymbol{B}_0 \cdot \boldsymbol{r})\boldsymbol{r} - r^2 \boldsymbol{B}_0}{r^5}\right) R_*^3 , \tag{9}$$

where $2\boldsymbol{B}_0$ is the polar surface magnetic field taken to be along the z-axis. ($\boldsymbol{B}_0$ is the field usually inferred from pulsar slowdown as in eq. [35]). Using $\boldsymbol{u} = \boldsymbol{r}/R_*$, we get

$$B_\perp^2(\upsilon) = \frac{B_0^2}{4u^{10}} \{[u^2 \sin\theta_0 - 3 \sin\gamma(\upsilon \cos\theta_0 + \cos\theta_1)]^2 + 6u^2(\upsilon \cos\theta_0 + \cos\theta_1)[\cos(\theta_0 - \gamma) - \cos\theta_0]\} , \quad (10)$$

where

$$u^2 = 1 + 2\upsilon \cos\gamma + \upsilon^2 \quad (11)$$

and $\cos\gamma = \hat{u}_1 \cdot \hat{a}$.

To avoid complications which do not affect the result, we shall approximate the gap by a point, such that $\theta_1 = 0$. Then, since $\gamma = \theta_0$,

$$b_\perp(\upsilon) = \frac{B_\perp}{B_0} = \frac{\sin\theta_0 |\upsilon - \upsilon_1|(\upsilon + \upsilon_2)}{2(1 + 2\upsilon\cos\theta_0 + \upsilon^2)^{5/2}} , \quad (12)$$

where

$$\upsilon_{1,2} = \frac{\cos\theta_0}{2} \pm \left(\frac{\cos^2\theta_0}{4} + 2\right)^{1/2} . \quad (13)$$

With this expression for $b_\perp$, we get, with $B_0 = B_{12} \times 10^{12}$ gauss,

$$l(\upsilon) = 1.01 \times 10^{-6} \frac{\exp[60.15/B_{12} E b_\perp(\upsilon)]}{B_{12} b_\perp(\upsilon)} . \quad (14)$$

Now the optical depths are

$$\tau_{a,b} = R_6 \cdot 10^6 \int_a^b \frac{d\upsilon}{l(\upsilon)} = 0.98 \times 10^6 R_6 B_{12} \int_a^b d\upsilon\, b_\perp(\upsilon) \exp\left[-\frac{60.15}{B_{12} E b_\perp(\upsilon)}\right] , \quad (15)$$

and $\tau = \tau_{0,\infty}$, $\tau_{\rm gap} = \tau_{0,\upsilon g}$, where $\upsilon_g = r_g/(R_6 \cdot 10^6)$ and r_g is the gap size in centimeters. Now given the diffuse gamma-ray flux, $n_0(E)$, the number of sparks that can be started in the gap per second (assuming each pair starts one) is (taking $\theta_1 = 0$)

$$N = 2\pi A_{\rm gap} \int_0^{\pi/2} \sin\theta_0 \, d\theta_0 \int_{2m_e c^2}^{\infty} dE n_0(E) \eta(R_6, B_{12}, E, \theta_0) , \quad (16)$$

where $A_{\rm gap}$ is the gap area and η follows from equations (4) and (15).

III. THE EVALUATION OF THE EFFICIENCY

Obviously it is not expedient to attempt to compute N in the form given above. For this reason, we make a series of approximations, using freely the insights gained from sample numerical evaluations.

As is well known, the pair production processes depend on physical parameters predominantly through the combination χ (see eq. [7]), and therefore the combination which plays the determining role in our calculations is

$$A = \frac{60.15}{B_{12} E \sin\theta_0} . \quad (17)$$

It will be seen that the relevant value of A is ~ 20.

It is now seen in equation (15) that the integral for τ is dominated by the small υ region. This is to be expected physically as most of the gamma rays break down near the star where the magnetic field is stronger. Therefore, we can use the Taylor expansion for $1/b_\perp(v)$ near $v = 0$:

$$\frac{1}{b_\perp(\upsilon)} = \frac{1}{\sin\theta_0}\left[1 + \tfrac{9}{2}\upsilon\cos\theta_0 + \frac{\upsilon^2}{2}(3 + \tfrac{21}{4}\cos^2\theta_0) + \cdots\right] . \quad (18)$$

Retaining only the first two terms in equation (18), we get

$$\tau_{ab} = \tau_0 \int_a^b d\upsilon (1 - B\upsilon) e^{-AB\upsilon} , \quad (19)$$

where

$$\tau_0 = 59.22 \times 10^{12} \frac{R_6 \cdot e^{-A}}{E \cdot A} \quad (20)$$

and

$$B = \tfrac{9}{2} \cos \theta_0 . \tag{21}$$

Retention of quadratic or higher order terms in equation (18) will not improve the accuracy in the values of η by more than a few percent. However, the approximation which we have used (eq. [19]) is clearly invalid when θ_0 approaches 90°, although numerically it was seen that θ_0 can safely be as much as 80°. We shall not have occasion to exceed this value. Further, while the approximation is truly valid only for $A \gg 1$, it suffices if $A > 1$.

Finally,

$$\tau = \frac{\tau_0}{AB}\left(1 - \frac{1}{A}\right), \tag{22}$$

and

$$\tau_{\rm mag} = \tau - \tau_{\rm gap} = \frac{\tau_0}{AB} e^{-ABv_g}\left(1 - \frac{1}{A} - Bv_g\right). \tag{23}$$

Substitution of these optical depths in equation (4) gives us η. But even with these simplified expressions for τ, further calculations demand further approximations. The efficiency η is a very highly peaked function of both E and θ_0. The values E_m and θ_m, where η assumes its maximum, are interrelated. The ranges around E_m and θ_m are very narrow, outside of which η rapidly falls by orders of magnitude. With this numerical hindsight, we can approximately locate the maximum of η as a function of τ with $\tau_{\rm mag} = \beta\tau$, $0 < \beta < 1$. Then it is seen that $d\eta/d\tau = 0$ is solved with β being a slowly varying function of τ when

$$(1 - \beta)\tau = -\ln \beta . \tag{24}$$

When both $1/A$ and $C = \cos \theta_0/22.22R_6$ are $\ll 1$, the solution of equation (24) is

$$A_m = \frac{60.15}{B_{12} E \sin \theta_m} = 23.19\left[1 + \frac{0.5 - 0.36B_{12}^2 E^2 \log (E/R_6)}{1 + 4.06B_{12}^2 E^2}\right], \tag{25}$$

the maximum value of η is

$$\eta_m = 0.017 \frac{(A_m + 1) \cos \theta_m}{R_6}, \tag{26}$$

and correspondingly

$$\tau_m = 1 - \frac{1}{A_m},$$
$$\beta_m = 1 - C_m(A_m + 1), \tag{27}$$

with

$$C_m = \frac{0.045 \cos \theta_m}{R_6} . \tag{28}$$

Using the behavior of η away from the maximum,

$$\int_0^{\pi/2} \eta \sin \theta_0 d\theta_0 = \frac{(1 - \beta_m)\tau_m \sin \theta_m \tan \theta_m}{A_m + 1 + \sec^2 \theta_m} \tag{29}$$

to a very good approximation, and thus

$$N = 2\pi A_{\rm gap} \int_{2m_e c^2}^{\infty} dE n_0(E) \frac{(1 - \beta_m)\tau_m \sin \theta_m \tan \theta_m}{A_m + 1 + \sec^2 \theta_m} . \tag{30}$$

Taking the flux of gamma rays, $n_0(E) = 1.1 \times 10^{-2} E^{-2.3}$ (from Trombka *et al.* 1978) and $A_{\rm gap} = \pi r_g^2$, we have

$$N = \frac{3.54 \times 10^9}{R_6 B_{12}^2} \int_{1.02}^{\infty} dE \frac{\theta(E - E_0) E^{-4.3}}{[A_m + (2A_m^2 - A_{\rm min}^2)/(A_m^2 - A_{\rm min}^2)](A_m - 1)}, \tag{31}$$

where

$$A_{\rm min} = 60.15/B_{12} E , \tag{32}$$

and θ_m is the solution of equation (25) for given values of B_{12} and E.

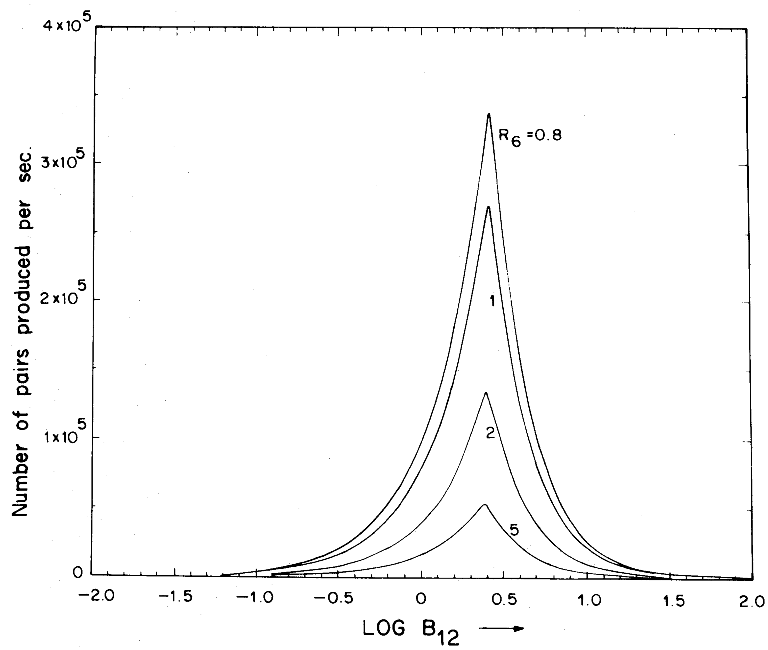

FIG. 1.—The number of pairs produced per second within the gap by the diffuse gamma-ray background as a function of the pulsar magnetic field. R_6 labels the stellar radius in units of 10^6 cm.

The appearance of the step function $\theta(E - E_0)$ comes in as follows. For all values of R_6 and B_{12} where the efficiency η is not negligible, $A_m \sim 21$ determines the range of E. Thus, for high values of B_{12}, the relevant energy value is well below $2m_e c^2$ and thus $E_0 = 1.02$; but, for low values of B, this energy is very high. In this case, we take E_0 as the lowest energy where η can be appreciable for any value of θ_0 at all. In our approximation, E_0 follows from the condition $\sin \theta_m = 1$ applied to equation (25):

$$E_0 = \left\{\frac{2.61}{B_{12}[1 + 0.10 \log (B_{12} R_6)]}\right\} \quad (B_{12} < 2.46) . \tag{33}$$

The B_{12} dependence of N can be seen from equation (31) without any calculation. By virtue of the slow variation of A_m, the integral depends on B_{12} predominantly through $E^{-4.3}$. Let the value of B_{12} when E_0 in equation (33) is 1.02 be B_{peak}. Then for $B_{12} \geq B_{\text{peak}}$ the integral does not contribute any B_{12} dependence and

$$N \propto B_{12}^{-2} ; \qquad B_{12} \geq B_{\text{peak}} . \tag{34a}$$

On the other hand, when $B_{12} < B_{\text{peak}}$, E_0, the lower limit of the integral, depends roughly inversely on B_{12}. This results in

$$N \propto B_{12}^{1.3} ; \qquad B_{12} < B_{\text{peak}} . \tag{34b}$$

The integral in equation (31) was evaluated numerically and the results are shown in Figure 1. The turnover is always at the value of B_{12} equal to B_{peak}.

IV. RESULTS AND CONCLUSIONS

From Figure 1 we see that sparking in the polar gap can be triggered by the gamma-ray background only when the value of the neutron star magnetic field lies within a narrow window. Although the connection between the total and the radio luminosities of pulsars is still obscure, it is clear that pulsars would be observable only when the sparking takes place. Thus, our results would imply that the magnetic fields derived from observations should peak around 2.5×10^{12} gauss.

In passing we would like to note that Ruderman and Sutherland (1975) remarked parenthetically that the maximum number of effective gamma rays would be $\sim 10^5$ s^{-1}. Our number is somewhat higher, permitting reinitiation in a time scale of ~ 10 μs which is of the order of the observed microstructure.

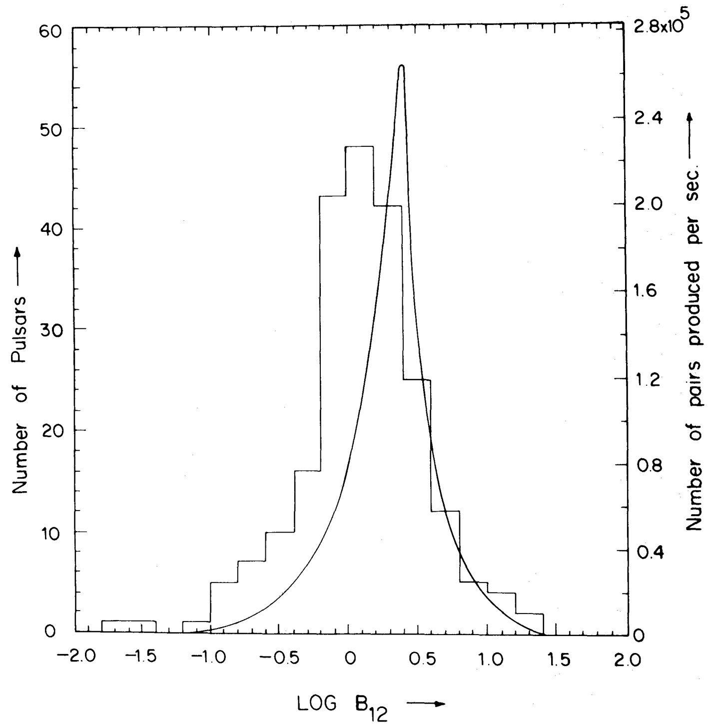

FIG. 2.—Histogram of the number of pulsars with derived fields lying in equal logarithmic intervals. The curve is the magnetic window of Fig. 1 for $R_6 = 1$. Magnetic fields were either taken from, or derived using periods and slowdown rates from, Taylor and Manchester (1975), Manchester *et al.* (1976), Gullahorn and Rankin (1978), Taylor, Fowler, and McCulloch (1979), and Newton, Manchester, and Cooke (1981).

The magnetic field of a pulsar is derived from the observed values of the period P and the slowdown rate $\dot{P}$ through the mechanism of energy loss. The proposed mechanisms such as loss due to very low frequency magnetic dipole radiation (Gunn and Ostriker 1969) or due to relativistic particles (Goldreich and Julian 1969) lead to almost identical expressions for the derived magnetic field B_d. We shall therefore adopt the relation given by the dipole radiation mechanism although it may not be the main mechanism for energy loss. The derived magnetic field B_d is given by (Manchester and Taylor 1977)

$$B_d^2 = \frac{3c^3 I P \dot{P}}{8\pi^2 R_*^6} . \tag{35}$$

The unknowns in equation (35) are the stellar moment of inertia I and the stellar radius R_*. Since massive neutron stars tend to be more compact than less massive ones, the moments of inertia predicted by a large variety of equations of state for the neutron star interior lie in a very narrow range around 10^{45} g cm^2. (Arnett and Bowers 1977). We have assumed this value for I and a constant value of $R_* = 10$ km for deriving the values of B_d.

In the histogram in Figure 2 we have shown the number of pulsars with derived magnetic fields lying in equal logarithmic intervals; for exponential decay of the magnetic field, these intervals represent equal intervals of time. The curve in Figure 1 for $R_6 = 1$ is also shown in Figure 2. The statistical errors in the histogram are not shown, but it is seen that the peaks in the histogram and the curve are very close. This seems to us to support strongly the interpretation that the probability of observing a pulsar is very high when its magnetic field lies in or close to the very narrow range required for incoming gamma rays to initiate sparking. We note, however, that in Figure 2, the width of the distribution of B_d is much larger than the curve of Figure 1, particularly for lower fields.

The existence of a concentration of the derived fields has been known for a long time (e.g., Greenstein 1972). It can now be interpreted as a selection effect if the diffuse background gamma rays play an important part in the triggering of polar cap discharges.

On this basis, we can predict that neutron stars with very strong fields are not likely to function as pulsars. In the RS model, pulsar activity can continue at most until the period P reaches the critical value $P_{\rm crit}$ (eq. [39] in Ruderman and Sutherland 1975) given by

$$P_{\rm crit} \propto 1.7 \times B_{12}^{8/13} R_6^{21/13} \text{ s} . \tag{36}$$

Thus, if B_{12} is large, e.g., ~ 10, P_{crit} can be correspondingly large, ~ 7 s. But, according to our hypothesis, neutron stars with periods longer than ~ 4 s are unlikely to be pulsars, since from equation (36) the field would be above the upper limit specified by the window. The longest period observed for a pulsar is 4.3 s.

We would like to comment on the fact that in Figure 2 the width of the distribution of B_d is much larger than the curve of Figure 1. This would imply that spark initiation can in fact take place at values of derived magnetic field which differ by over an order of magnitude from the value of the peak in Figure 1. Since for such magnetic fields the number of paris produced per second falls to a few percent of its peak value, we consider this unlikely and look for other causes to explain the discrepancy.

Assuming that the present hypothesis is correct, there are still a number of variables which will affect the expected agreement of the histogram and curve in Figure 2. We have assumed a constant gap of 10^4 cm in all of the above. We show in the Appendix that the location of the magnetic window is insensitive to the dependence on radius assumed for the gap size although the number of pairs per second does vary (Fig. 3). In our calculation of the window we also ignored the progressive gravitational blueshift of the gamma-ray photons as they approached the star. This can be taken into account by substituting the appropriate υ-dependence of E in equation (15) for τ, when part of the diffuse gamma-ray spectrum below 1.02 MeV will also become relevant. Because of the sensitive dependence on photon energy, this blueshift—of all general relativistic effects—may be expected to have the greatest effect on the magnetic window. For this reason we have also evaluated it in detail but present here only a summary of the results (Fig. 4). It will be seen there that our earlier conclusions remain essentially unaltered by taking the blueshift into account.

Very little is known about the spread in masses and, consequently, about the spread in radii of neutron stars; it is therefore difficult to judge whether the contribution which must be there from this cause will alter the picture. On the other hand, the presence of multipole fields, such as are invoked in the RS model and which have been discussed by Flowers and Ruderman (1977), may be expected to affect the results significantly. Such higher multipoles would modify the curves and shift the peaks in Figure 1 where the x-axis is the dipole component, which alone determines B_d. A proper calculation of curves similar to those in Figure 1 taking higher order moments into account will be more formidable than for the dipole case considered earlier; but the sense in which the curves will be modified can be seen easily without recourse to calculations.

As multipole fields will fall off faster than the dipole field, their contribution to τ_{gap} will be much greater than to τ_{mag}. The number of gamma rays reaching the gap will be roughly the same but the number that produce pairs within it will be much greater than before. Thus, pulsars with values of dipole field lower than that of the window, but with higher multipole contributions, will also have a reasonable number of pairs produced within the gap to reinitiate sparks. It would therefore be consistent with our hypothesis to find pulsars with B_d much lower than the window value, as seen in the histogram of Figure 2, if their surface field strengths had significant multipole contributions. The statistical errors in Figure 2 are too large to warrant attributing the slight discrepancy between the peaks to this

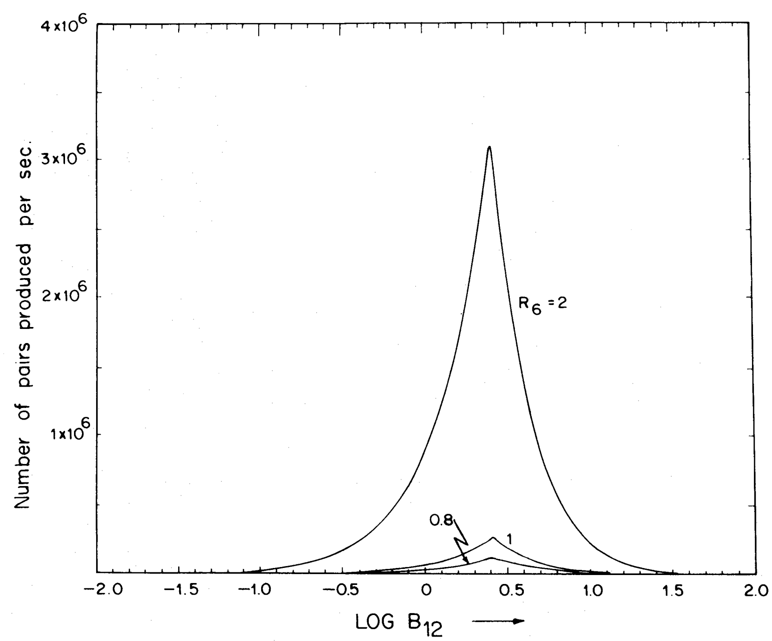

FIG. 3.—The number of pairs produced per second as in Fig. 1 but with the gap size varying as $R_6^{3/2}$ (see Appendix)

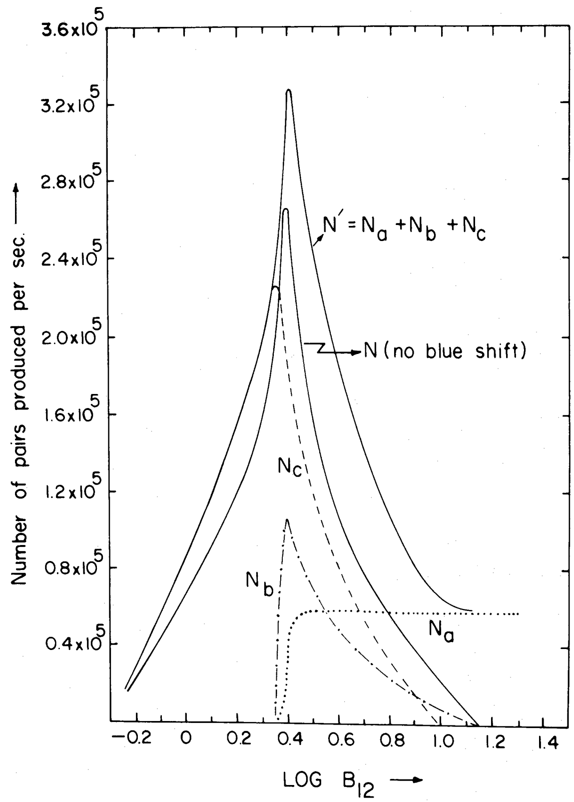

FIG. 4.—The effect of including the gravitational blueshift. The number of pairs produced per second by photons in energy ranges *a*, *b*, and *c*, and their total N', all for $R_6 = 1$. The energy ranges are (*a*) $E_0 < E < E_1$; (*b*) $E_1 < E < 1.02 = 2m_e c^2$; and (*c*) $1.02 < E$, where E_0 and E_1 are the energies that are blueshifted to $2m_e c^2$ at the stellar surface and on top of the gap respectively. Thus, $\tau_{mag} = 0$ for photons of case (*a*), and they can produce pairs only after entering the gap; for high enough magnetic fields, they always contribute, and N' does not go to zero. For case (*b*), τ_{mag} gets a contribution only when v is less than v_0, the value of v where the blueshifted energy crosses the threshold of $2m_e c^2$. Case (*c*) is very similar to the case dealt with in detail in § III. Also shown for comparison is the curve of Fig. 2.

cause. But in any case, we consider the gross characteristics of the histogram to be consistent with the hypothesis of spark initiation by the gamma-ray background.

It would be of interest to see how far the picture we have presented above depends for its validity on other details of the model. If, for example, outer gaps play a crucial role in pulsar radiation (Holloway 1973; Cheng and Ruderman 1977), the diffuse background would play a very minor role, if any, since these gaps occur in regions where the magnetic fields would be much weaker and also modified due to magnetospheric contributions.

We see from equation (25) that the pulsar magnetic field has a collimating effect on the incident gamma rays depending on their energies. In models involving an outer gap (e.g., Cheng and Ruderman 1977), the supply of gamma rays from the outer gap may well exceed that from the diffuse background; but, if the spectrum is similar, these gamma rays would be most effective for reinitiating the discharge at the polar gap if the outer gap has a preferred location. Gamma rays produced from both gaps and observed directly would then show a double-peaked profile with a separation dictated by this geometry. One of the remarkable features of the observed gamma-ray pulses from the Crab and Vela pulsars is that they both have a double-peaked profile with a separation of 0.4 times the period. We plan to investigate the question whether in fact this feature can be explained as we have suggested above.

In any event, if the essential features of the RS model are correct, then, in the absence of other satisfactory mechanisms for spark reinitiation, the inevitable supply of more than 10^5 pairs per second from the diffuse background will lead one to the conclusions mentioned earlier in this section.

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APPENDIX

EFFECT OF VARIATION OF GAP SIZE WITH STELLAR RADIUS

In the calculations above, the size of the polar gap r_g was taken as a constant, 10^4 cm. Though this is always the correct order of magnitude for r_g, the height of the gap in the RS model is given by (eq. [22] in Ruderman and Sutherland 1975)

$$h \sim 4.05 \times 10^5 R_6^{1/7} B_{12}^{-4/7} P^{5/7} \text{ cm} ,$$

where P is the pulsar period in seconds.

As an extreme third case, we can take the size of the gap to be the same as the size of the region through which positive current can leave the star (eq. [4] in Ruderman and Sutherland 1975)

$$r_g \equiv 10^4 R_6^{3/2} P^{1/2} \text{ cm} .$$

We summarize these three cases by writing

$$r_g = 10^4 \cdot R_6^n \qquad (n = 0, 1/7, 3/2) .$$

Since the second case does not differ by much from the first, we consider only $n = 0$ and 1.5. Then, the condition valid for both values of n is obtained when equation (25) is replaced by

$$A_m = A_1\left(1 - \frac{\ln [\![(B_{12} \cdot E)\{1 + [\ln (B_{12} \cdot R_6^n)/23.19]\}]\!] + y_1 - \ln (\frac{1}{2} + 1/y_1)}{A_1 + 1 + \mu_1 y_1 + [2\mu_1/(y_1 + 2)]}\right),$$

where

$$A_1 = 23.19 + n(B_{12} R_6^n) ,$$
$$A_{\min} = 60.15/(B_{12} \cdot E) ,$$
$$y_1 = (0.045) R_6^{n-1} (A_1^2 - A_{\min}^2)^{1/2} ,$$

and

$$\mu_1 = A_1^2/(A_1^2 - A_{\min}^2) .$$

Now this value of A_m as in the previous calculations leads to

$$N = 3.54 \times 10^9 \frac{R_6^{3n-1}}{B_{12}^2} \int_{E_0}^{\infty} \frac{dE\, E^{-4.3}}{[A_m + (2A_m^2 - A_{\min}^2)/(A_m^2 - A_{\min}^2)](A_m - 1)} .$$

However with $B_{\text{peak}} = 2.46/(1 + 0.09 \log R_6)$,

$$E_0 = 1.02 \qquad (B_{12} \geq B_{\text{peak}}) ,$$

and

$$E_0 = 2.61/[1 + 0.10 \log (B_{12} R_6)] B_{12} \qquad (B_{12} < B_{\text{peak}}) ,$$

as before. We notice that when $n = 1.5$, as R_6 increases, values of N actually increase as $R_6^{7/2}$ instead of falling inversely with R_6 as in the previous case. The location of the peak in the N versus B_{12} plot is, however, again determined by B_{peak}, and since B_{peak} is independent of n, the magnetic window does not shift as seen in Figure 3. This can be seen qualitatively also from equation (4) where the n-dependence enters only through τ_{gap}.

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