A quasi-one-dimensional micrometer-length quantum wire containing repulsively interacting electrons and connected to wide electron reservoirs at the two ends is a nontrivial mesoscopic transport system of considerable current interest, made realizable experimentally by the recent advances in nanoheterostructure technology. A minimum electronic model for such a system is a homogeneous one-dimensional Luttinger liquid (1DLL) for the quantum wire, terminating into two 1D leads containing noninteracting electrons. The latter is to simulate the higher-dimensional electronic model for such a system is a homogeneous one-dimensional Luttinger liquid (1DLL) for the quantum wire, terminating into two 1D leads containing noninteracting electrons. The derived expression for the conductance shows broad, roughly equidistant resonances as a function of the bias voltage, this being a manifestation of the spin-charge separation. In the limit of zero bias and zero barrier, the tunneling conductance reduces to the ballistic contact value e^2/h per channel. [S0163-1829(96)51020-9]
amplitude at \( x = L \) will be \( t(e)g_R(L,E) \) with \( g_R(x,E)=G_R(x,E)/G_R(0,E) \). The barriers at \( x = L \) transmits \( t(E)g_R(L,E)t(E) \) and reflects \( t(E)g_R(L,E)r(E) \), where \( r(E) \) is the amplitude reflection coefficient for the barrier, with \( |r(E)|^2 + |t(E)|^2 = 1 \). This reflected amplitude is now propagated to the left barrier at \( x = 0 \) with an amplitude \( t(E)g_R(L,E)r(E)g_L(-L,E) \), and so on for the multiple scatterings. The total transmitted amplitude \( T(E) \) is then given by the series

\[
T(E) = t(E)g_R(L,E)t(E) + t(E)g_R(L,E)r(E)g_L(-L,E) + \cdots
\]

\[
= \frac{r^2(E)g_R(L,E)}{1 - r^2(E)g_R(L,E)g_L(-L,E)}.
\]

Here we have assumed the two barriers to be identical and symmetrical. The tunneling conductance \( G(V) \) with \( E \) set equal to \( eV \) is now given by

\[
G(V) = \left( \frac{dI}{dV} \right) = e^2 \left( \frac{\partial n}{\partial E} \right) |T(E)|^2
\]

\[
= \left( e^2/\pi \hbar \right) |r(E)|^4 f_0^2(e\Delta v LV/2h\nu_s^2)
\]

\[
\times \frac{1 - \exp(iQr^2(E)f_0^2(e\Delta v LV/2h\nu_s^2))}{1 - \exp(iQr^2(E)f_0^2(e\Delta v LV/2h\nu_s^2))},
\]

where \( Q = 2L[k_F+E/\hbar v_F] \). This is our main result.

It is readily seen that in the limit \( V \to 0 \) and \( t(E) \to 1 \), \( r(E) \to 0 \), i.e., in the zero-bias zero-barrier limit, the conductance reduces to the well-known expression \( e^2/\pi \hbar \) (as for the spin case). As function of the bias voltage \( V \), the differential tunneling conductance \( G(V) \) shows oscillations, with equispaced peaks separated by \( \Delta V \approx (h\nu_s/eL)(\nu_s/\Delta V) \). For \( \nu_s \approx 10^5 \text{ m}^2 \text{s}^{-1}, \Delta V \approx 0.1 \text{ mV} \), we get \( 0.5 \text{ mV} \). Of course, the bias voltage must be kept low enough to keep the number of active transverse channels fixed (=1 in the present case). These oscillations, of course, disappear with the velocity difference \( \Delta V \to 0 \), suggesting their origin in the spin-charge separation.

Above, we have taken the leads to contain noninteracting electrons. If the leads are modeled by yet another 1D Luttinger liquid, all we have to do is to recalculate the prefactor \( e^2/\pi \hbar \) in our Eq. (3) for a 1DL. This can be done very generally through an identity stating that for a system in a stationary state the expectation value of the time derivative of the current operator \( j(x) \) must vanish.\(^{12}\) Applying this to a 1DL in the presence of a local potential \( \delta U(x) \), and using the well-known expression\(^{14} \) for \( j(x) \), we get for the change in the local electron density \( \delta n(x) \) due to \( \delta U(x) \) as

\[
- \Delta U(x)(K/2\pi \hbar v_F^2)
\]

for \( (\partial_n/\partial E)v_F^2 = K/2\pi \hbar \), where \( K \) is the standard interaction parameter as defined in Ref. 11. This gives \( G(V) \) (with interacting leads) \( = KG(V) \) (with noninteracting leads).

The following comment on our Eq. (2) for the transmission amplitude seems in order. As mentioned earlier, this equation is exact in the case of a strictly one-electron problem, or, equivalently, in the absence of mutual interaction in the wire, where this can be obtained directly by wave-function matching. In this case we can propagate the injected wave amplitude \( \psi_m(x=0,E) \) at the incident energy \( E \) through the composition rule\(^{13} \)

\[
\psi(x,E) = G_0(x-x',E)\left( \frac{i\hbar}{2m} \frac{\partial}{\partial x'} \right) \psi_m(x',E),
\]

where \( (i\hbar/2m) \frac{\partial}{\partial x'} \) gives the mean of the velocities on the two sides of \( x'=0 \), and \( G_0 \) is the Green function for a one-electron problem for \( x>0 \). This ensures that the \( \psi(x,E) \) for \( x>0 \) is matched to the incident amplitude \( \psi_m(0,E) \) at \( x=0 \). In the present case we have assumed such a matching for the case of our 1DLL, made plausible in the absence of backscattering.

A number of remarks are in order now to clarify the main approximation involved in the idealized Luttinger-liquid model adopted here, and the extent of its robustness relevant to our treatment of tunneling. First, we have considered here a minimum model of spin Luttinger liquid that retains the spin-charge separation, but without the additional complication of the anomalous power-law correlation function. The latter involves scattering between the left- and the right-going branches caused by the unscreened short-ranged two-body repulsion. Our expression [Eq. (1)] for the electron propagator is valid only in the corresponding limit of the coupling constant analysis.\(^{10,11} \) Second, the question naturally arises as to whether our tunneling results are robust against this neglect of interbranch backscattering. This question becomes particularly relevant in view of the recent published work\(^{14} \) where the short-ranged repulsive interaction is shown to lead to vanishing linear conductance even for an arbitrarily weak one-body scattering—a subtle manifestation of the (soft) Coulomb gap. Inasmuch as in our case the tunnel barrier does cause backscattering, one could conclude that there would be a vanishing tunneling conductance—a vanishing bias voltage. The latter, of course, is the whole point. We have considered here the differential tunneling conductance as a function of the bias voltage. The latter involves carrier injection at energies \( V \) away from the Fermi level. The vanishing of the tunneling density of states (and hence of the linear conductance) due to the repulsive interaction and the one-body scattering mentioned above refer strictly to the zero bias-voltage limit, i.e., the condition at the Fermi level. Thus, our tunneling conductance and its resonant qualitative features should persist at nonzero bias voltage. It is only in the limit of zero bias voltage that they will be washed out by the soft Coulomb gap due to the relevant repulsive interaction. Of course, we do expect qualitative modifications at all bias voltages due to the soft Coulomb gap.

Finally, we would like to point out that the earlier study of spin-charge separation\(^{15} \) involved the effect of an Aharonov-Bohm (AB) magnetic flux through a Luttinger-liquid loop on the transmission through the loop. This is qualitatively quite different from the present study in that our tunneling at finite bias voltage involves carrier injection away from Fermi level while their AB-flux effects are related to the conditions at Fermi level. The latter could, therefore, be relatively more subject to the blockade effects discussed above.
In conclusion, we have shown that the tunneling conductance of a Luttinger-liquid quantum wire connected to the leads through tunnel barriers shows resonances as function of the bias voltage. This is due to the spin-charge separation and should be observable. Physical arguments based on forward-only scattering for the 1DLL are given to justify our Landauer-type one-electron-scattering approach. Our result agrees with the known result in the appropriate limit of zero bias and no barrier.

One of us (V.A.) would like to thank Professor C. N. R. Rao, President of the Jawaharlal Nehru Centre for Advanced Scientific Research, for financial support during the course of this work.

11 For a review, see V.J. Emery, in Highly Conducting One-Dimensional Conductors, edited by J.T. Devreese (Plenum, New York, 1979).
12 This closed form for $\delta n/\delta U$ obtains only for the special case of the 1DLL model considered here. For a general case of interacting electrons, there is no such closed form. Surprisingly, the same identity also gives a closed form density-response to $\delta U(x)$ for a quantum Hall liquid consistent with its known incompressibility.