

***c*-axis resistivity and high- T_c superconductivity**

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Recently we had proposed a mechanism for the normal-state *c*-axis resistivity of the high- T_c layered cuprates that involved *blocking* of the single-particle tunneling between the weakly coupled planes by strong intraplanar electron-electron scattering. This gave a *c*-axis resistivity that tracks the *ab*-plane *T*-linear resistivity, as observed in the high-temperature limit. In this work this mechanism is examined further for its implication for the ground-state energy and superconductivity of the layered cuprates. It is now argued that, unlike the single-particle tunneling, the tunneling of a bosonlike pair between the planes prepared in the BCS-type coherent trial state remains *unblocked* inasmuch as the latter is by construction an eigenstate of the pair-annihilation operator. The resulting pair delocalization along the *c* axis offers energetically a comparative advantage to the paired-up trial state, and thus stabilizes superconductivity. In this scheme the strongly correlated nature of the layered system enters only through the *blocking* effect, namely, that a given electron is effectively repeatedly *monitored* (intraplanarly scattered) by the other electrons acting as an environment, on a time scale shorter than the interplanar tunneling time. [S0163-1829(98)04722-5]

The two electronic-structural features, now become central to a proper understanding of the normal-state resistivity as well as the high-temperature superconductivity of the highly anisotropic marginal metals, namely, the layered cuprates, are the strong electron-electron correlation and their effectively low (two) dimensionality.^{1,2} We have, thus, the oxygen-hole-doped CuO_2 planes representing the strongly correlated electronic system, while the weak interplanar tunneling through the thick spacer layers of the reservoir oxides, e.g., SrO , Bi_2O_3 , etc., gives the near two dimensionality. In our recent work,^{3,4} it was shown that these two features, namely, the strong intraplanar electron-electron scattering and the weak interplanar tunneling, can give rise to a *c*-axis resistivity that tracks the *T*-linear metal-like *ab*-plane resistivity in the high-temperature limit, with an insulatorlike upturn at low enough temperatures. The observed resistivity upturn can, however, be a precursor phenomenon close to the transition at T_c , and has indeed been attributed to superconducting fluctuations giving a decrease (a virtual gap) in the single-particle tunneling density of states.^{5,6} Also, the metal-like *c*-axis resistivity [$\rho_c(T)$] can have a magnitude not bounded by Mott's maximum metallic resistivity.⁷ These results are in qualitative agreement with the measured $\rho_c(T)$ on high-quality single-crystal samples that reflect, presumably, their intrinsic transport behavior.⁸ Furthermore, the *c*-axis transport is found to be necessarily incoherent as indeed supported by observations.^{2,9} This mechanism for incoherent *c*-axis transport was also proposed independently by Leggett,⁹ and has now been followed up by a number of workers in the field.¹⁰ It has been invoked to explain the anomalous *c*-axis magnetoresistance in the normal state of high- T_c cuprates.¹¹ The physics underlying our proposed mechanism is that of the *blocking* of the weak interplanar tunneling by the relatively strong intraplanar inelastic scattering.

This is, of course, a particular case of the celebrated quantum Zeno effect, namely, the suppression of transition between two weakly coupled Hilbert subspaces due to strong intrasubspace coupling to environment.¹² Thus, in the present case the two neighboring CuO_2 planes (e.g., of the bilayer), coupled weakly through a small interplanar tunneling matrix element, constitute the two electronic subspaces, and the strong intraplanar scattering of a given electron by the other electrons represents the intrasubspace environmental coupling. We will now examine this *blocking* effect further for its implication for the ground-state energy of and for the superconductive electron pairing in these layered strongly correlated systems. Our main finding is that the strong intraplanar electron-electron scattering does indeed, at zero temperature, *block* the single-electron interlayer tunneling but *not* the tunneling of (the time-reversed) electron pairs. The resulting interplanar pair delocalization energetically favors the pairing globally and hence stabilizes superconductivity. The calculation is done for a simple bilayer model. The present work is much in the spirit of, and complements the work of, Chakravarty, Subdo, Anderson, and Strong¹³ and that of Kumar,¹⁴ all based on the idea of confinement by orthogonality catastrophe.¹⁵ We also discuss in this context how the present mechanism differs essentially from the several other pairing mechanisms that involve interlayer tunneling.

Let us first consider the possible *blocking* of the single-electron interplanar tunneling due to intraplanar scattering at zero temperature. Now, in the high-temperature limit the in-plane inelastic scattering can be viewed as a stochastic field acting on a given electron attempting to tunnel out of plane. This general picture is well known and well supported, experimentally as well theoretically, in the context of decoherence.^{15,16} The problem becomes rather subtle at low (zero) temperature, and is best probed in the present context

by calculating the change ΔE_0 of the ground-state energy E_0 of a weakly coupled bilayer as a function of the strength (λ) of the in-plane electron-electron scattering, maintaining, of course, the system in the normal state, i.e., without breaking spontaneously any global symmetry, such as the one responsible for superconductivity. *Blocking* effect is expected to reduce the change ΔE_0 as λ is increased. This is readily concluded by using the Hellmann-Feynman¹⁷ *charging* technique involving in the present case an integration with respect to the interplanar tunneling matrix element ($-t_\perp$) as the variable coupling parameter.

The Hamiltonian for a bilayer of the weakly coupled planes, labeled A and B , can be written as

$$H = H_A + H_B + H_{AB}(\text{single particle}), \quad (1)$$

where the intraplanar Hamiltonians H_A and H_B describe the two interacting electron subsystems of the isolated planes A and B , and

$$H_{AB}(\text{single particle}) = -\eta t_\perp \sum_{\mathbf{k}\sigma} (a_{\mathbf{k}\sigma}^\dagger b_{\mathbf{k}\sigma} + b_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma}), \quad (2)$$

with $t_\perp = t_\perp^* > 0$, the tunneling Hamiltonian with the creation/annihilation fermionic operators $a_{\mathbf{k}\sigma}^\dagger/a_{\mathbf{k}\sigma}$ ($b_{\mathbf{k}\sigma}^\dagger/b_{\mathbf{k}\sigma}$) referring to the planes A (B). Here tunneling is taken to conserve the in-plane wave vector \mathbf{k} and the spin projection σ . The tunneling matrix element $-t_\perp$ is taken to be small in a sense to be made precise later. The dimensionless parameter η is to be set equal to 1 at the end. The exact ground-state energy $E_0(\eta)$ of the bilayer varies with η parametrically according to the Hellmann-Feynman¹⁷ theorem as

$$\partial E_0(\eta)/\partial \eta = \langle \eta | H_{AB} | \eta \rangle \quad (3)$$

giving

$$\Delta E_0 \equiv E_0(1) - E_0(0) = -2t_\perp \int_0^1 d\eta \sum_{\mathbf{k}} \langle \eta | a_{\mathbf{k}}^\dagger b_{\mathbf{k}} | \eta \rangle, \quad (4)$$

where $|\eta\rangle$ denotes the exact bilayer ground state for a given value of the parameter η . Here we have dropped the spin projection label σ , and the wave-vector summation includes summation over σ .

Expressing the equal-time correlation $\langle \eta | a_{\mathbf{k}}^\dagger b_{\mathbf{k}} | \eta \rangle$ in terms of the imaginary part of the retarded Green function $G_{AB}^R(\mathbf{k}, w)$ [$\equiv G_\perp^R(\mathbf{k}, w)$], we get

$$\Delta E_0 = \frac{2t_\perp}{\pi} \sum_{\mathbf{k}} \int_0^\infty d\omega \int_0^1 d\eta \text{Im} G_\perp^R(\mathbf{k}, w). \quad (5)$$

Now, the exact retarded Green function G_\perp^R for the correlated metallic planes A and B coupled by the weak tunneling $-\eta t_\perp$ is clearly not known. We can, however, adopt the following viewpoint. In the absence of the interplanar tunneling, the correlated electron planes A and B can be well modeled by the semiphenomenological marginal Fermi liquid,¹⁸ which is known to be consistent with the T -linear ab -plane resistivity. The corresponding retarded in-plane (\parallel) Green function $G_{AA}^R(k, w) = G_{BB}^R(\mathbf{k}, w) \equiv G_\parallel^R(\mathbf{k}, w)$ is then given by

$$G_\parallel^R(\mathbf{k}, w) = 1/[w - \epsilon_{\mathbf{k}} - \Sigma^R(\mathbf{k}, w)] \quad (6)$$

with

$$\text{Re} \Sigma^R(\mathbf{k}, w) = \lambda w \ln(w/w_c), \quad \text{Im} \Sigma^R(\mathbf{k}, w) = -\lambda \frac{\pi}{2} w,$$

with $w_c > w > 0$. Henceforth we will drop the superscript R .

Now, for sufficiently small t_\perp , one can assume the electron-electron scattering to take place on a time scale much less than the tunneling time \hbar/t_\perp , and, therefore, ignore the vertex corrections to the interplanar tunneling. We can then at once write down from the Dyson equation for the retarded interplanar (\perp) Green function G_\perp :

$$G_\perp(\mathbf{k}, w) = \frac{\eta t_\perp [G_\parallel(\mathbf{k}, w)]^2}{1 - \eta^2 t_\perp^2 [G_\parallel(\mathbf{k}, w)]^2}. \quad (7)$$

Now, substituting from Eqs. (6) and (7) into Eq. (5) and performing the \mathbf{k} integration with a constant two-dimensional density of states n_0 , we get

$$\delta e = \int_0^\infty dw [I(W, t_\perp, w) - I(-W, t_\perp, w) + I(W, -t_\perp, w) - I(-W, -t_\perp, w)], \quad (8)$$

where dimensionless energy change

$$\delta e = \Delta E_0 / (4n_0 t_\perp^2 / \pi), \quad (9)$$

with

$$\begin{aligned} I(W, t_\perp, w) = & -t_\perp \arctan \left[\frac{W + t_\perp - w + \lambda w \ln(w/w_c)}{(\pi/2)\lambda w} \right] - [W - w + \lambda w \ln(w/w_c)] \\ & \times \arctan \left[\frac{t_\perp w \lambda \pi/2}{\{[W + t_\perp - w + \lambda w \ln(w/w_c)][W - w + \lambda w \ln(w/w_c)] + (\lambda w \pi/2)^2\}} \right] \\ & + \frac{\pi}{4} \lambda w \ln \frac{\{[W + t_\perp - w + \lambda w \ln(w/w_c)]^2 + (\lambda w \pi/2)^2\}}{\{[W - w + \lambda w \ln(w/w_c)]^2 + (\lambda w \pi/2)^2\}}, \end{aligned} \quad (10)$$

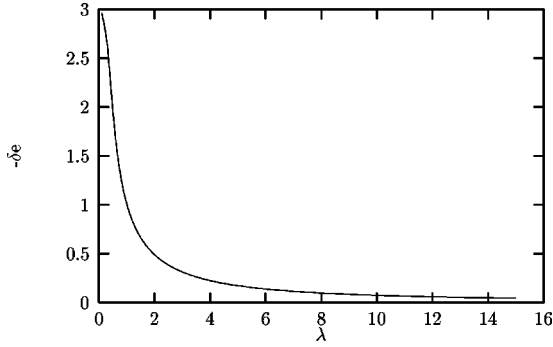


FIG. 1. Dimensionless energy gain due to interplanar single-particle tunneling as function of intraplanar interaction parameter λ . The decreasing energy gain with increasing λ demonstrates the blocking effect. The plot is for the choice of parameters $(t_{\parallel}/t_{\perp})=20.0$ and $(\omega_c/t_{\perp})=100$.

where W is the two-dimensional bandwidth.

In Fig. 1, we have plotted the energy gain (reduction $-\delta e$ in the ground-state energy), due to delocalization by interplanar single-particle tunneling ($-t_{\perp}$), against the intraplanar electron interaction strength λ . It is readily seen that the energy gain is a sharply decreasing function of the intraplane interaction strength λ . This clearly demonstrates the effective *blocking* of the single-particle interplanar tunneling by intraplanar scattering—the quantum Zeno effect—as anticipated on physical grounds.

Thusly encouraged, we now address the rather subtle question as to how this blocking of the single-particle tunneling becomes ineffective against the tunneling of (time-reversed) bosonic pairs. In order to clearly appreciate this point, let us consider the two electronic subsystems forming the bilayer to be prepared in a BCS-like trial many-body state ψ (in the absence of tunneling). Thus we have for the decoupled bilayer,

$$|\psi\rangle = \prod_{\mathbf{k},q} (u_{\mathbf{k}} + v_{\mathbf{k}} a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger}) (u_{\mathbf{q}} + v_{\mathbf{q}} b_{\mathbf{q}\uparrow}^{\dagger} b_{-\mathbf{q}\downarrow}^{\dagger}) |0\rangle$$

$$\propto e^{\phi(\alpha^{\dagger} + \beta^{\dagger})} |0\rangle \equiv |A\rangle |B\rangle, \quad (11)$$

where $\alpha^{\dagger}(\alpha), \beta^{\dagger}(\beta)$ are the pair-creation (-annihilation) operators for the two planes A and B of the bilayer. Here

$$\phi\alpha^{\dagger} = \sum_{\mathbf{k}} \left(\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} \right) a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger}, \quad (12)$$

$$\phi\beta^{\dagger} = \sum_{\mathbf{k}} \left(\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} \right) b_{\mathbf{k}\uparrow}^{\dagger} b_{-\mathbf{k}\downarrow}^{\dagger}, \quad (13)$$

where we can take as usual the operators α 's and β 's to be bosonic to a good approximation. Thus, the unnormalized trial function $|\psi\rangle$ is a coherent state, i.e., a phased superposition of states with different number of pairs, with $|\phi|^2$ representing eventually the mean bosonic pair-occupation number for each of the planes A and B .

However, these trial coherent states $|A\rangle$ and $|B\rangle$ are certainly not the ground states for the isolated two-dimensional electronic subsystems A and B , with (repulsive) electron-electron interaction in general. The crucial observation, however, is that the coherent states $|A\rangle$ and $|B\rangle$ are, respectively,

eigenstates of the pair-annihilation operators α and β . If, therefore, we now introduce a pair-tunneling ($-t'_{\perp}$) term in the Hamiltonian, $H_{AB}(\text{pair}) = -t'_{\perp}(\alpha^{\dagger}\beta + \beta^{\dagger}\alpha)$ (with $t'_{\perp} \neq t_{\perp}$ in general), we at once verify that

$$\langle \psi | H_{AB}(\text{pair}) | \psi \rangle = -2t'_{\perp} |\phi|^2. \quad (14)$$

This implies adiabatic transfer of the bosonic pairs between the two planes A and B of the now coupled bilayer prepared in the coherent state $|\psi\rangle$. This interplanar pair delocalization in turn stabilizes the trial state $|\psi\rangle$ energetically. Thus, given that the single-particle tunneling ($-t_{\perp}$) is blocked effectively while the pair tunneling ($-t'_{\perp}$) is not as suggested by the above, we can expect $t'_{\perp} \gg t_{\perp}$ and the coherent state $|\psi\rangle$ to be stabilized relative to the normal state. This is the central point of the pairing mechanism proposed in this work.

Once this is accepted, the energetic stabilization of such a BCS-like paired-up state due to the dominance of the pair tunneling over the single-particle tunneling can be readily treated within a mean-field approximation. For this, consider the reduced Hamiltonian that should suffice for describing the low-energy phenomena:

$$H_{red}(\text{bilayer}) = H_A + H_B + H_{AB}(\text{single particle}) + H_{AB}(\text{pair})$$

with

$$H_A = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} a_{\mathbf{k}\sigma}^{\dagger} a_{\mathbf{k}\sigma} + U_{\text{eff}} \sum_{\mathbf{k}\mathbf{k}'} a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger} a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}'\uparrow},$$

$$H_B = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} b_{\mathbf{k}\sigma}^{\dagger} b_{\mathbf{k}\sigma} + U_{\text{eff}} \sum_{\mathbf{k}\mathbf{k}'} b_{\mathbf{k}\uparrow}^{\dagger} b_{-\mathbf{k}\downarrow}^{\dagger} b_{-\mathbf{k}'\downarrow} b_{\mathbf{k}'\uparrow},$$

$$H_{AB}(\text{single}) = -t_{\perp} \sum_{\mathbf{k}} (b_{\mathbf{k}\sigma}^{\dagger} a_{\mathbf{k}\sigma} + \text{H.c.}),$$

$$H_{AB}(\text{pair}) = -t'_{\perp} \sum_{\mathbf{k}\mathbf{k}'} (b_{\mathbf{k}\uparrow}^{\dagger} b_{-\mathbf{k}\downarrow}^{\dagger} a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}'\uparrow} + \text{H.c.}), \quad (15)$$

where $t'_{\perp} \gg t_{\perp} > 0$, and U_{eff} can even be moderately repulsive ($U_{\text{eff}} > 0$) as considered by Zecchina,¹⁹ except for the retention of the single-particle tunneling here. The latter enables us to treat the effect of the degree of blocking of single-particle tunneling explicitly. Note that the reduced Hamiltonian maintains the condition of pairing involving electrons in time-reversed states.

Consider first the case of U_{eff} negative (attractive). Introducing the anomalous averages in the spirit of the mean-field approximation (MFA), we get

$$H_{MFA} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}^{\dagger} a_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}\sigma} b_{\mathbf{k}\sigma}^{\dagger} b_{\mathbf{k}\sigma}$$

$$+ \Delta V \sum_{\mathbf{k}} (a_{-\mathbf{k}\downarrow} a_{\mathbf{k}\uparrow} + a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger})$$

$$+ \Delta V \sum_{\mathbf{k}} (b_{-\mathbf{k}\downarrow} b_{\mathbf{k}\uparrow} + b_{\mathbf{k}\uparrow}^{\dagger} b_{-\mathbf{k}\downarrow}^{\dagger})$$

$$- t_{\perp} \sum_{\mathbf{k}} (b_{\mathbf{k}\sigma}^{\dagger} a_{\mathbf{k}\sigma} + a_{\mathbf{k}\sigma}^{\dagger} b_{\mathbf{k}\sigma}), \quad (16)$$

where the s -wave gap parameter $\Delta = \sum_{\mathbf{k}'} \langle a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}'\uparrow} \rangle = \sum_{\mathbf{k}'} \langle b_{-\mathbf{k}'\downarrow} b_{\mathbf{k}'\uparrow} \rangle$ and $V = (U_{\text{eff}} - t_{\perp}'/2)$. After straightforward diagonalization, the self-consistent gap equation for Δ turns out to be

$$\Delta = -\frac{1}{2} \sum_{\mathbf{k}} \frac{\Delta V [1 - 2f(\epsilon_{\mathbf{k}} - t_{\perp})]}{2\sqrt{\Delta^2 V^2 + (\epsilon_{\mathbf{k}} - t_{\perp})^2}} - \frac{1}{2} \sum_{\mathbf{k}} \frac{\Delta V [1 - 2f(\epsilon_{\mathbf{k}} + t_{\perp})]}{2\sqrt{\Delta^2 V^2 + (\epsilon_{\mathbf{k}} + t_{\perp})^2}}, \quad (17)$$

where as usual f is the Fermi function, $f(\epsilon) = 1/(e^{\epsilon/k_B T} + 1)$. The corresponding equation for the critical temperature T_c is then (for $U_{\text{eff}} - t_{\perp}' < 0$)

$$1 = -\left(\frac{U_{\text{eff}} - t_{\perp}'}{2}\right) \frac{1}{2} \sum_{\mathbf{k}} \left[\frac{1}{2(\epsilon_{\mathbf{k}} - t_{\perp})} \tanh\left(\frac{\epsilon_{\mathbf{k}} - t_{\perp}}{k_B T_c}\right) + \frac{1}{2(\epsilon_{\mathbf{k}} + t_{\perp})} \tanh\left(\frac{\epsilon_{\mathbf{k}} + t_{\perp}}{k_B T_c}\right) \right]. \quad (18)$$

This reduces to the usual expression in the limit $t_{\perp} \rightarrow 0$ (i.e., total blocking of the single electron tunneling). From the above equation one can get the following expression for T_c :

$$k_B T_c = (4\gamma/\pi) \sqrt{W^2 - t_{\perp}'^2} \exp\{-2/[N(t_{\perp}' - U_{\text{eff}})]\}, \quad (19)$$

where W , N being the half-bandwidth and the constant density of states of the two-dimensional plane, respectively, and γ is Euler's constant. It is also readily seen from Eq. (19) that incomplete suppression of the single-particle blocking ($t_{\perp} \neq 0$) leads to a reduction in T_c . Thus we recover our claim that the blocking of the single-particle tunneling relative to the pair-tunneling stabilizes the paired-up superconducting state.

Some remarks are in order at this point. For an attractive U_{eff} , the present pair-tunneling mechanism may well be viewed as an amplification of the (s -wave) superconductive pairing preexisting in the isolated planes. (This, of course,

remains true also for the case when the isolated planes support d -wave pairing, arising from the spin-fluctuation mechanism, say.) We would, however, like to emphasize here that our present mechanism provides for a global stabilization of the condensed state even when the pairing potential for the individual pairs (U_{eff}) is repulsive, but, of course, sufficiently small and the isolated planes are not superconducting on their own. Thus, for a short-ranged repulsive potential our interlayer mechanism based on the Zeno effect can stabilize a coherent condensate, albeit of d -wave pairs. (The case of a strongly negative U_{eff} supporting a repulsive bound state lying above the top of the band is quite different. It can give high-lying d -wave pairs that may get stabilized coherently through interlayer pair tunneling mechanism.) Indeed, the present mechanism involves global stabilization, and cannot be reduced to a pairing potential arising, say, from virtual exchange of some excitations.

Our work is closest in spirit to that of Chakravarty *et al.*,¹³ which it complements. It is, however, different from the earlier interlayer pair mechanism of Wheatley, Hsu, and Anderson²⁰ that involves spin charge separation and exchange of spinons mediating the interlayer tunneling of a pair of the otherwise confined physical electrons of opposite spins. It is, however, quite likely that the non-Fermi liquid feature involved in this exotic model in the ultimate analysis is yet another route to realizing the quantum Zeno effect in the extreme total confinement by the orthogonality catastrophe.

In conclusion, we have extended the mechanism proposed by us earlier for the c -axis resistivity, involving the blocking of the interplanar single-particle tunneling by the intraplanar scattering, to low temperatures to possibly explain the high- T_c superconductivity of the layered cuprates. We have given an argument based on coherence and supported by simple analysis that, in contrast to the single-particle tunneling, the tunneling of the bosonic pairs remains unblocked and thus stabilizes the superconducting state. In this scheme, the strongly correlated nature of the two-dimensional layers enters only through this single-particle blocking effect.

¹See P. W. Anderson, *The Theory of Superconductivity in the High T_c Cuprates* (Princeton Univ. Press, Princeton, 1997).
²See D. G. Clarke and S. P. Strong, *Adv. Phys.* **46**, 545 (1997).
³N. Kumar and A. M. Jayannavar, *Phys. Rev. B* **45**, 5001 (1992).
⁴N. Kumar, T. P. Pareek, and A. M. Jayannavar, *Mod. Phys. Lett. B* **11**, 347 (1997).
⁵L. B. Ioffe *et al.*, *Phys. Rev. B* **47**, 8936 (1993).
⁶G. Balestrino *et al.*, *Phys. Rev. B* **47**, 6037 (1993).
⁷N. F. Mott, *Philos. Mag.* **22**, 7 (1970).
⁸T. A. Friedmann *et al.*, *Phys. Rev. B* **42**, 6217 (1990).
⁹A. J. Leggett, *Braz. J. Phys.* **22**, 129 (1992).
¹⁰See Y. Zha, *Philos. Mag. B* **74**, 497 (1996).
¹¹D. Yan, Ya-Sha Yi, Zhao-bin Su, and Lu Yu (unpublished).
¹²For a direct and relevant demonstration of the quantum Zeno effect, see B. Nagels, L. J. F. Hermans, and P. I. Chapovsky,

Phys. Rev. Lett. **79**, 3097 (1997). Also references cited therein.
¹³S. Chakravarty *et al.*, *Science* **261**, 337 (1993).
¹⁴N. Kumar, *Curr. Sci.* **64**, 754 (1993).
¹⁵A. J. Leggett *et al.*, *Rev. Mod. Phys.* **59**, 1 (1987).
¹⁶A. Stern, Y. Aharonov, and Y. Imry, *Phys. Rev. A* **41**, 3436 (1990); also W. M. Zurek, *Phys. Today* **44** (10), 36 (1991).
¹⁷R. P. Feynman, *Phys. Rev.* **56**, 340 (1939); H. Hellmann, *Acta Physicochim. URSS* **6**, 913 (1935); D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Addison-Wesley, Reading, 1966), Vol. I.
¹⁸C. M. Varma *et al.*, *Phys. Rev. Lett.* **63**, 1996 (1989).
¹⁹R. Zecchina, *Mod. Phys. Lett. B* **4**, 1817 (1993); see also Z. Tešanović, *Phys. Rev. B* **36**, R2364 (1987).
²⁰J. M. Wheatley, T. C. Hsu, and P. W. Anderson, *Phys. Rev. B* **37**, R5897 (1988).