Fractional Spin from Gravity

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Friedman and Sorkin have observed that the total angular momentum of an asymptotically flat, vacuum, quantum gravitational field in 3+1 dimensions need not be integral. We pursue this idea in the context of asymptotically flat 2+1 gravity, which is an exactly solvable model. We find that, for nontrivial spatial topologies, the quantized pure gravitational field has states of fractional spin. These states are dynamically allowed, in the sense that they solve *all* the constraints of the theory.

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The possibility of half-integer spin states in pure quantum gravity was first suggested by Finkelstein and Misner [1]. Their work was partly motivated by the "geometrization of physics" program, whose success depends on the existence of such states. This idea was followed up by Friedman and Sorkin [2] (hereafter referred to as FS), who showed that the topology of space was central to the discussion. They gave *necessary* conditions on the spatial topology for the existence of spinorial states in pure quantum gravity and listed some three-manifolds that fulfill these conditions. One might have naively expected the quantum gravitational field to have integer spin, since the graviton has spin 2 in linearized theory. The work of FS suggests that this naive expectation fails, due to nonperturbative effects. The basic idea of "spin 1/2 from gravity" is exciting, since it opens up the possibility of constructing fermions from pure spacetime.

We emphasize that this question concerns *pure* gravity, with no matter present, either in the form of fields or "punctures." Producing spinorial states from spinorial matter is hardly remarkable. The novelty of FS is precisely that there is *no* matter present. Their construction is similar in spirit to those of [3], which use (integer spin) Higgs and gauge fields to produce a fermionic soliton. In both cases, the theory has a "gauge" symmetry, which is exploited to permit a 2π rotation to act nontrivially on the system. The construction of FS is *different* in that their spin 1/2 states are due to nontrivial spatial topology rather than matter.

We briefly review FS before going on to the subject of the present paper. Let Σ denote the spatial manifold and suppose the spacetime manifold $\mathcal{M}$ to be of the form $\mathcal{M} = \Sigma \times \mathbb{R}$. We suppose that Σ has a single asymptotic region and that all topological complications are contained within a compact region $\mathcal{B}$, whose boundary is a topological sphere. We work within the canonical approach to quantum gravity. As is well known, the basic canonical variables are subject to constraints. The constraints generate "gauge" transformations and, in the quantum theory, are imposed on the allowed states as operator equations. The traditional canonical variables (the metric on a spatial slice and its conjugate momentum) are subject to the diffeomorphism constraints and the Hamiltonian constraint. The diffeomorphism constraint plays a kinematical role and expresses the invariance of the quantum state under reparametrizations of the spatial slice. The dynamical role in quantum gravity is played by the Hamiltonian constraint, which generates transformations that move the spatial slice Σ in $\mathcal{M}$. Setting aside the Hamiltonian constraint for the moment, we focus on the role of the diffeomorphism constraint. Let Diff be the group of asymptotically trivial diffeomorphisms of Σ and Diff_0 its identity component. The diffeomorphism constraints require that all quantum states are invariant under the action of Diff_0, which is thus singled out as the gauge group of the theory. Let $\mathcal{B}$ be properly contained in a region $\mathcal{B}'$, whose boundary is also spherical. Let $\mathcal{N} = \Sigma - \mathcal{B}'$ be a neighborhood of infinity. Consider now the following element $\mathcal{R}$ of Diff: $\mathcal{R}$ reduces to the identity when restricted to $\mathcal{B}$ and $\mathcal{N}$. On the thick spherical shell $\mathcal{U}=\mathcal{B}' - \mathcal{B}$, $\mathcal{R}$ is a differential rotation of the nested spheres, through an angle which varies continuously from zero on the innermost sphere to 2π on the outermost one. $\mathcal{R}$ has the physical meaning of a 2π rotation of the system relative to its environment. If $\mathcal{R}$ is in Diff_0, then all physical states are invariant under a 2π rotation and so have integral angular momentum. If, on the other hand, $\mathcal{R}$ is *not* in Diff_0, this argument cannot rule out spinorial states. It is elementary to verify that a 4π rotation of the system ($\mathcal{R} \circ \mathcal{R}$) *is* in Diff_0 [4]. The much harder question of whether $\mathcal{R}$ is in Diff_0 has been answered in the mathematical literature [5]. The answer depends on the topology of Σ. For some spatial topologies, including $\mathbb{R}^3$, $\mathcal{R}$ *is* in Diff_0 and these topologies only support integer angular momentum states of quantum gravity. For other spatial topologies $\mathcal{R}$ is not in Diff_0 and these could support spinorial states. The main observation of FS is that there exist "spinorial" three-manifolds and so spin 1/2 from gravity is a possibility.

The discussion above makes no reference to the Hamiltonian constraint and is, in this respect, incomplete. States of quantum gravity must satisfy *all* the constraints of the theory. It does seem arbitrary [6] to impose some constraints while ignoring others. A practical reason for following this arbitrary procedure is that the Hamiltonian constraint, which is also known as the Wheeler-Dewitt equation, is notoriously hard to solve. While fractional spin states are allowed kinematically, it is not entirely clear whether these states survive the imposition of the Hamiltonian constraint. Semiclassical arguments are advanced in FS to suggest that they do. Given the intractable nature of the Hamiltonian constraint, it seems hard to do any better than this in 3+1 dimensions.

Since FS, there has been considerable work (see [7–10], and references therein) in this area. There is some interest in spin-statistics relations (or rather, the lack of them) for topological geons. Reference [8] is devoted to the interplay between spin, statistics, and diffeomorphism invariance in generally covariant theories. Of particular relevance to us is the remark [8] that fractional spin states are allowed in 2+1 quantum gravity. This is the 2+1 analog of the FS result. The principal difference is that in 2+1 dimensions, $\mathcal{R} \circ \mathcal{R}$ need not be in Diff_0. Consequently, the spin need not be half integral. Just as in FS, this result is kinematic, in the sense that Ref. [8] does not impose the Hamiltonian constraint (see also [10], p. 275). In short, Ref. [8] shows that fractional spin states *can* exist in 2+1 quantum gravity, but leaves open the question of whether they *do* exist. The present paper is devoted to filling this gap. We first show, following [8], that fractional spin states are kinematically allowed in 2+1 gravity.

The diffeomorphism $\mathcal{R}$ is defined exactly as in FS save for the obvious difference that the regions $\mathcal{B}$ and $\mathcal{B}'$ have *circular* boundaries and the region $\mathcal{U} = \mathcal{B}' - \mathcal{B}$ is annular. It is easily checked that for the trivial topology $\Sigma = \mathbb{R}^2$, $\mathcal{R}$ *is* in Diff_0: simply rotate the disk $\mathcal{B}$ by 2π and untwist $\mathcal{U}$. Do there exist spatial topologies for which $\mathcal{R}$ is not in Diff_0? To answer this question, fix a base point σ at infinity. Let $\pi_1(\Sigma, \sigma)$ be the fundamental group of Σ relative to the base point σ. Any element $\mathcal{T}$ of Diff maps points of Σ to points of Σ and reduces to the identity at infinity. Curves starting and ending at σ are also mapped to other such curves. Further, this mapping also preserves homotopy equivalence relations between curves. Thus, each element $\mathcal{T}$ of Diff defines a natural action $\tilde{\mathcal{T}}$: $\pi_1(\Sigma, \sigma) \to \pi_1(\Sigma, \sigma)$ on the fundamental group. Clearly, if $\mathcal{T}$ belongs to Diff_0, $\tilde{\mathcal{T}}$ acts trivially on $\pi_1(\Sigma, \sigma)$. (We say that a map "acts trivially" if it is the identity map.) So if $\tilde{\mathcal{R}}$ has a nontrivial action on $\pi_1(\Sigma, \sigma)$, it follows that $\mathcal{R}$ is not in Diff_0. Let us denote by c the element of $\pi_1(\Sigma, \sigma)$ represented by the curve that starts from σ, loops once around $\mathcal{B}'$ anticlockwise, and returns to σ. Since $\mathcal{R}$ is a "2π twist of infinity," $\tilde{\mathcal{R}}$ acts on $\pi_1(\Sigma, \sigma)$ by conjugation (see Fig. 1) with respect to c:

$$\tilde{\mathcal{R}}: \pi_1(\Sigma, \sigma) \to c\pi_1(\Sigma, \sigma)c^{-1}. \quad (1)$$

If Σ is the connected sum of $\mathbb{R}^2$ and a Riemann surface $\mathcal{S}_g$ of genus g, $\pi_1(\Sigma, \sigma)$ is the free group on $2g$ letters $(a_\alpha, b_\alpha), \alpha = 1 \cdots g$, representing the $2g$ basic cycles of $\mathcal{S}_g$. The product $c = c_1 c_2 \cdots c_g$ of their commutators $c_\alpha = [a_\alpha b_\alpha] = a_\alpha b_\alpha a_\alpha^{-1} b_\alpha^{-1}$ loops around $\mathcal{B}'$. Clearly, conjugation by c is *not* the identity homomorphism on $\pi_1(\Sigma, \sigma)$ for $g \neq 0$. It follows that $\mathcal{R}$ is *not* in Diff_0 and

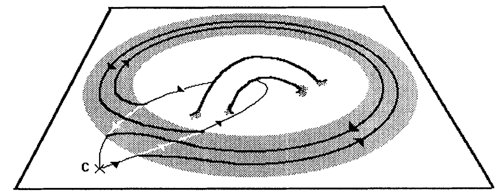

FIG. 1. Space containing a topological geon and the effect of $\mathcal{R}$, a clockwise 2π twist of infinity relative to the geon. $\mathcal{R}$ is the identity diffeomorphism everywhere except in the shaded annular region, denoted $\mathcal{U}$ in the text. The effect of $\mathcal{R}$ is best shown by its action on (equivalence classes of) homotopically nontrivial curves. A representative curve is shown above, which starts from the cross C, goes under the handle, and returns to C. Under the action of $\mathcal{R}$, the curve is changed only in the shaded grey region $\mathcal{U}$. The original (white on grey) and final (black on grey) locations of the curve in $\mathcal{U}$ after the action of $\mathcal{R}$ are shown. This illustrates Eq. (1) of the text.

so fractional spin states are kinematically allowed in 2+1 quantum gravity. We now go on to realize this possibility in the framework of a dynamical theory. For clarity, we give details just for the simplest nontrivial case, $g = 1$ and drop the index α. Restoring α yields the general case.

In the last few years, it has been realized that pure quantum gravity in 2+1 dimensions is an exactly solvable model. Achucarro and Townsend [11] noticed that this theory can be derived from a Chern-Simons action. Witten [12] (following a suggestion of Ashtekar's [13] in the context of 3+1 gravity) gave a covariant quantization of the theory in the connection representation, for compact spatial topologies. For the canonical formulation see, for example, [14]. This work extends easily also to the asymptotically flat case, which is of primary interest here (see below). Do there exist quantum states in asymptotically flat 2+1 gravity which have fractional spin? The answer is yes, and this is the main point of this paper. Below, we construct states of quantum gravity, which solve *all* the constraints and have fractional spin.

The basic variables we use to describe the gravitational field are [12, 13] a triad [15] of 1-forms $e^i = e^i_a dx^a$ and an SO(2,1) valued connection 1-form $A^i = A^i_a dx^a$ on $\mathcal{M}$. ($i, j = 0, 1, 2$ are internal, frame indices and $a, b = 0, 1, 2$ are World indices.) In the usual manner, one can construct the metric tensor g_{ab} from the triad e^i_a and the field strength F^i from the connection A^i. The basic fields (e^i, A^i) must, of course, be subject to asymptotic conditions. The asymptotic structure of the gravitational field in 2+1 dimensions is quite different [16] from the 3+1 case: the metric does not become Minkowskian at infinity, but only "conical." In standard polar coordinates t, r, ϕ in a neighborhood of infinity $ds^2 = -(dt - \zeta\, d\phi)^2 + dr^2 + \alpha^2 r^2 d\phi^2$. These conical metrics, which can be constructed [17] by identifying points in Minkowski space, are parametrized by α

and ζ and describe systems with total energy $2\pi(1-\alpha)$ and angular momentum $2\pi\alpha\zeta$. Conical metrics are stationary and axisymmetric, but do not admit any other global Killing vector fields. Since the asymptotic symmetry group is only $\mathbb{R}\times SO(2)$ (and not the Poincarè group), the only well defined global charges are the total energy and angular momentum of the system [18, 19]. We fix a fiducial two-parameter family of fields $(e_\infty(\alpha,\zeta), A_\infty(\alpha,\zeta))$ describing a conical metric at infinity: $e_\infty = (dt - \zeta\, d\phi, dr, \alpha r\, d\phi), A_\infty = (\alpha\, d\phi, 0, 0)$. The asymptotic conditions we impose on (e, A) are that they are *equal* to (e_∞, A_∞), for some α, ζ, in a neighborhood $\mathcal{N}$ of infinity [20]. This choice of asymptotic conditions may seem rather drastic compared to prescribing falloff rates [21] for the fields as one is used to in 3+1 gravity. In fact, in 2+1 dimensions no physics is lost by this choice, since the space of classical solutions is not reduced by it. Henceforth, all triads and connections are supposed to obey these conditions. We use the standard Einstein action supplemented by a surface term

$$I = \int_{\mathcal{M}} e^i \wedge F_i + \int_{\partial\mathcal{M}} e^i \wedge A_i. \quad (2)$$

It can be checked that varying this action within the allowed space of field configurations does yield the Einstein field equations. The action and asymptotic conditions completely define the classical theory. Recasting the theory in Hamiltonian form, one finds that the canonically conjugate pair A_a and $\tilde{\eta}^{ab} e_b$ ($\tilde{\eta}^{ab}$ is the antisymmetric Levi-Civita tensor density; from now on, $a, b = 1, 2$ only) are subject to the following constraints:

$$\mathcal{G}(\Lambda) = \int_\Sigma \Lambda_k (\mathcal{D}\wedge e)^k = 0, \quad (3)$$

$$\mathcal{F}(\xi) = \int_\Sigma \xi_k F^k = 0. \quad (4)$$

The smearing functions Λ and ξ must vanish at infinity for $\mathcal{G}(\Lambda)$ and $\mathcal{F}(\xi)$ to be differentiable functions on the phase space. (3) is the Gauss' law constraint, which reflects the local SO(2,1) gauge invariance of the action. The constraints (4) reflect the invariance of the action under asymptotically trivial *spacetime* diffeomorphisms. They imply both the Hamiltonian constraint *and* the two diffeomorphism constraints (which generate *spatial* diffeomorphisms) of standard canonical gravity.

We now apply Dirac quantization in the connection representation. Quantum states are represented by wave functionals $\psi[A]$ on the space of connections on Σ. We impose the classical constraints (3),(4) as operator equations on the allowed quantum states. (4) implies that $\psi[A]$ has support only on flat connections. (3) then requires that $\psi[A]$ be a gauge invariant functional on the space of flat connections. $\psi[A]$ is therefore a function on Q, the moduli space of flat connections [22]. Elements of Q are denoted q. The space of physical states [23] (those that satisfy all the constraints) is the linear space $\mathcal{H} = \{\psi : Q \to \mathbb{C}\}$ of all complex functions $\psi(q)$ on Q.

Let us now describe the space Q more precisely. A connection is completely characterized (modulo gauge) by its holonomy $H_\gamma(A) = \mathrm{P}\,\exp[\int_\gamma A] \in SO(2,1)$ around all loops γ based at σ. For flat connections, much less information is needed. The holonomy only depends on the homotopy class of the loop: it is enough to know $H_{[\gamma]}(A)$ for all elements $[\gamma]$ of $\pi_1(\Sigma,\sigma)$. Since $\pi_1(\Sigma,\sigma)$ is generated by a and b, this information is completely contained in the two SO(2,1) elements $H_a(A)$ and $H_b(A)$. These elements are unchanged by gauge transformations (since all gauge transformations die out at infinity) and thus exactly capture the gauge invariant information in the flat connection A. The asymptotic conditions on the fields imply that $H_a(A)H_b(A)H_b^{-1}(A)H_a^{-1}(A) = H_{aba^{-1}b^{-1}}(A) = H_c(A) = H_c(A_\infty) \in SO(2)$. We thus identify q with a pair (H_a, H_b) of SO(2,1) elements subject only to the condition that their commutator must lie in the SO(2) subgroup of SO(2,1).

Are there states in $\mathcal{H}$ which have fractional spin? In order to answer this, we need to know the behavior of the elements of $\mathcal{H}$ under $\mathcal{R}$, a 2π rotation of the system relative to its environment. Let $\hat{\mathcal{R}}$ be the action of $\mathcal{R}$ on $\mathcal{H}$: $\hat{\mathcal{R}}\psi(q) = \psi(\mathcal{R}_* q)$, where $\mathcal{R}_* q$ is the pullback of the flat connection 1-form by the diffeomorphism $\mathcal{R}$. The effect of $\mathcal{R}$ on a flat connection A is easily computed from the action (1) of $\tilde{\mathcal{R}}$ on $\pi_1(\Sigma,\sigma)$:$H_a(\mathcal{R}_* A) = H_{\tilde{\mathcal{R}}a}(A)$. Thus $\mathcal{R}_* q = q'$, where q' represents the pair $(H_c H_a H_c^{-1}, H_c H_b H_c^{-1})$. Thus $\mathcal{R}_*$ acts on Q by conjugation with respect to H_c. This action is clearly not the identity on Q. It follows then that $\hat{\mathcal{R}}$ acts nontrivially on $\mathcal{H}$. This is the central technical result of this letter.

Having established that the action of $\hat{\mathcal{R}}$ on $\mathcal{H}$ is nontrivial, it is easy to construct states belonging to $\mathcal{H}$ which have fractional spin. The "Bloch" wave function

$$\psi_\theta(q) = \sum_{n=-\infty}^{\infty} e^{-i2\pi n\theta} \hat{\mathcal{R}}^n \psi(q) \quad (5)$$

satisfies

$$\hat{\mathcal{R}}\psi_\theta(q) = e^{i2\pi\theta}\psi_\theta(q). \quad (6)$$

Comparing this with the phase $e^{i2\pi s}$ picked up by a spin s state on 2π rotation, we conclude that s has fractional part θ.

While the action of $\mathcal{R}_*$ on the space Q is nontrivial, there do exist individual flat connections, which are fixed points of this action. An example is the zero connection $A = 0$. It follows from (5) that ψ_θ vanishes at these fixed points for nonzero θ. Had *all* points of Q been fixed points of $\mathcal{R}_*$, the Bloch state (5) would have vanished identically. The main work of this paper consisted in showing that not all points of Q are fixed points of $\mathcal{R}_*$. We emphasize that this result is not implied by general kinematical analyses such as [8].

From (5) we see that $\mathcal{H}$, the space of solutions to the constraints, splits up into sectors labeled by θ: $\mathcal{H} = \bigcup_\theta \mathcal{H}_\theta$. A superselection rule applies [6] to these sectors. All physical observables are invariant under 2π rotations, and do not connect different θ sectors. It suffices to restrict our attention to one sector $\mathcal{H}_\theta$ at a time. θ can thus be viewed as a quantization ambiguity: different choices of θ (more correctly, $e^{i2\pi\theta}$) lead to different quantum theories. Since $\hat{\mathcal{R}}^m \neq \mathbf{1}$ for any nonzero m, it follows [8] that θ can have any real value.

Because of our interest in spin, we have focused entirely on $\mathcal{R}$, a 2π rotation of the system relative to infinity. There are also other elements of Diff which are not in Diff_0. $\text{Diff}/\text{Diff}_0$ is called the mapping class group (see [8, 24] for a discussion of this group in a related context) of Σ. This group is generated by Dehn twists about the cycles of the Riemann surface. A little reflection shows that $\mathcal{R}$ commutes with the other Dehn twists (see [8]). It follows that the action of these other Dehn twists on the physical states lies entirely within a fixed θ sector $\mathcal{H}_\theta$. One can construct phase representations of these other Dehn twists within $\mathcal{H}_\theta$. The presence of these other elements of the mapping class group does not affect the construction (5) of fractional spin states.

In conclusion, we have *dynamically* realized the idea of FS in 2+1 quantum gravity. We find that there exist states (5) which satisfy *all* the constraints and have fractional spin. It is nice to know that the idea of fractional spin from pure gravity does work in this simple context. There is much to be understood about topological geons. The work described above may provide a simple toy model for discussing some of the questions raised in [6].

It is of relevance to compare the work of this paper with [25], which considers 2+1 gravity on a plane with punctures representing the location of particles. Reference [25] uses holonomies to descrbe the gravitational field and relates these to physically important questions like the geometric interpretation of the theory and particle scattering [26]. The work of [24, 25] does illuminate our analysis by providing geometric and physical intuition for the holonomies. There are, however, several important differences in motivation and conclusions between the present work and [25]. Since our interest is in producing fractional spin from *vacuum* 2+1 quantum gravity, we explicitly exclude punctures on the grounds that they constitute matter. Carlip [25] suggests that the mapping class group (which in his case allows for braiding of punctures) acts trivially [Eq. (5.3) of Ref. [25]) on the state space. The present paper explores alternative quantizations in which the mapping class group acts trivially on the *ray* space, but nontrivially on the state space. These alternatives do lead to physically distinct predictions, such as the possibility of the total angular momentum of an isolated system being fractional.

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