Comment on "Evolving Geometric Phase and Its Dynamical Manifestation as a Frequency Shift: An Optical Experiment"

A frequency shift of a laser beam resulting from periodic changes in its state of polarization has recently been reported by Simon, Kimble, and Sudarshan.[1] This has been interpreted as an evolving geometric phase resulting from a changing circuit spanning the state space given by a single Poincaré sphere (PS). I wish to point out a difficulty with this interpretation and suggest an alternative.

In this experiment, the combination of the quarter-wave plate QWP2 and the mirror returns left-circular light as left circular, not as right circular as assumed by the authors. At first sight then it appears that the circuit on the PS is retraced during the return journey of the beam, yielding a net zero geometric phase. The correct explanation of the observed phase lies in another fact that needs to be taken into account: The true state space of evolution of the beam is a direct product of the PS and the classical **k** space, where **k** is the wave vector. While in the single-pass experiments of the type reported by us,[2] one can ignore the **k** space, in double-pass experiments one cannot.

How does one visualize circuits in this product space? Fortunately, the relevant **k** space under discussion is one-dimensional (in fact only two points, $-k$ and $+k$). For a coherent beam of fixed intensity, that PS uses up only two out of the three visualizable dimensions. One could therefore "tag-on" a 1D k space to the radial coordinate of the PS. Let each k have its own PS, and we suggest that a sphere of radius R represent $k = \ln R$. All spheres with $R > 1$ then correspond to light propagating in the positive direction and those with $R < 1$, the negative direction. Here we need only two spheres, $-k$ and $+k$. The polarization states on each sphere are represented according to the well-established PS representation, as shown in Fig. 1. It follows that a linear vibration in space that is represented by an azimuth $+\alpha$ on one sphere is represented by $-\alpha$ on the other (return journey). For the case when the output of QWP1 is left circular, the track of the state point in the product space is shown in Fig. 1 as *APBLQM*. The dotted line represents the jump caused by the mirror reflection. The net geometric phase is half the total solid angle subtended by the track at the center. This is obtained simply by projecting the entire track on any one of the spheres.[3] As long as the jump can be represented by a line in the equatorial plane, i.e., light remains linearly polarized, the net phase equals the solid angle of one of the spherical triangles. It should be noted that in the product space, the evolution of the beam is noncyclic. It is thus an example of the partial-cycle Berry phases predicted recently.[4]

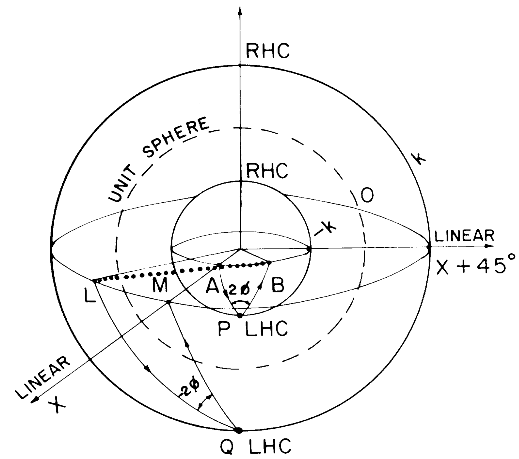

FIG. 1. Circuits in the product space for the case when the output of QWP1 is left-circular light.

We show elsewhere that in a full treatment of the Berry phase in the direct product of the 3D **k** space and the PS, ideas like the "modified momentum space,"[5] the "space of spin directions,"[6] and the "generalized PS"[7] have a natural explanation.[8]

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[3]A different alignment of the two PS's can make the projected circuit look like that of Ref. 1. The essential point is that one needs two PS's.

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