Applicability of the van der Pauw–Hall measurement technique to implanted samples

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The application of the van der Pauw–Hall measurement technique to implanted samples in which the mobility varies with depth has still not been fully justified. A proof that the technique is in fact applicable in this situation is given.

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The general validity of the van der Pauw–Hall measurement technique[1] for arbitrarily shaped samples was recently questioned.[2] Beck[3] showed that arbitrary shapes could be used for homogenous samples. However, the technique has been widely used for implanted samples in which the properties vary with depth. Petritz[4] has considered the case of inhomogenous samples but only in the standard Hall geometry. Further, his analysis is based on a microscopic transport model, the Boltzmann equation. However, the final result for the measured Hall coefficient depends only on the macroscopic transport coefficients suitably averaged over thickness [see eq. (8)]. In this paper we treat the problem in the van der Pauw geometry using just the macroscopic relation between the current density $\mathbf{J}$ and the electric field $\mathbf{E}$.

We first note that van der Pauw's original treatment[1] as well as the arguments given by Beck[3] require that the current density $\mathbf{J}$ equals its value in zero magnetic field $\mathbf{J}_0$. In what follows, the suffix 0 is used for quantities in zero magnetic field. If $\mathbf{J} = \mathbf{J}_0$ we have

$$\mathbf{E} = \mathbf{E}_0 - [\mu(z)/c]\mathbf{E}_0 \times \mathbf{B},$$

where the mobility μ depends on the depth z in general. This implies $\nabla \times \mathbf{E} = -(B_z/c)(\partial\mu/\partial z)\mathbf{E}_0 \neq 0$ which is unacceptable in a steady-state situation. The current lines remain unaltered by the application of a magnetic field only when the sample is homogenous or has variations in carrier concentration alone with mobility remaining constant. We now show that in the general case, the current density integrated over thickness does remain unchanged with magnetic field. Assuming isotropy in the xy plane, in a notation similar to Refs. 1 and 2, we have

$$J_x(x,y,z) = \sigma_{xx}(z)E_x(x,y) + \sigma_{xy}(z)E_y(x,y) \tag{1a}$$

and

$$J_y(x,y,z) = -\sigma_{xy}(z)E_x(x,y) + \sigma_{xx}(z)E_y(x,y). \tag{1b}$$

We take E_x and E_y to be independent of z and comment on this later. Define the components I_x and I_y of the current per unit width as follows:

$$I_x = \int_0^w J_x(x,y,z)dz, I_y = \int_0^w J_y(x,y,z)dz. \tag{2}$$

The integration is over the thickness w of the sample. I_x and I_y are functions of x and y. Substituting Eq. (2) in Eq. (1) and solving for the electric field, we find

$$E_x = P_{xx} I_x + P_{xy} I_y, \tag{3a}$$

$$E_y = -P_{xy} I_x + P_{xx} I_y, \tag{3b}$$

where

$$P_{xx} = \sum_{xx} \Big/ \sum^2, \quad P_{xy} = -\sum_{xy} \Big/ \sum^2, \tag{4a}$$

$$\sum_{xx} = \int_0^w \sigma_{xx} dz, \quad \sum_{xy} = \int_0^w \sigma_{xy} dz, \quad \sum^2 = \sum_{xx}^2 + \sum_{xy}^2. \tag{4b}$$

The charge conservation condition for steady current reads

$$\nabla \cdot \mathbf{I} = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} = 0. \tag{5}$$

Substituting Eq. (3) into $\nabla \times \mathbf{E} = 0$ and using Eq. (5) gives

$$\nabla \times \mathbf{I} = \frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y} = 0. \tag{6}$$

Equations (5) and (6), together with the sample geometry and the boundary conditions at the current leads, are enough to determine $I_x(x,y)$ and $I_y(x,y)$ which are therefore independent of the magnetic field.

We can now use Eq. (4) and the field independence of I_x and I_y to calculate the Hall voltage V_H, defined as half the change in voltage between contacts c and d at the edge of the sample when the magnetic field is reversed.

$$\begin{aligned} V_H &= (1/2)\int_c^d [E_x(B) - E_x(-B)]dx \\ &\quad + [E_y(B) - E_y(-B)]dy \\ &= -P_{xy}(B)\int_c^d (I_x\, dy - I_y dx). \end{aligned} \tag{7}$$

In writing down Eq. (7), the symmetry properties of P_{xy} and P_{xx}, viz., that they are odd and even under reversal of the field, have been used. The line integral from c to d is nothing but the total current I through the sample. The measured Hall coefficient is given by

$$R = -V_H \frac{w}{BI} = \frac{wP_{xy}}{B} = \frac{w\int_0^w \sigma_{xy}\, dz}{B\left(\int_0^w \sigma_{xx}\, dz\right)^2} \quad (8)$$

dropping corrections of order B^2 in the denominator. Equation (8) is just the result found by Petritz[4] which we now see depends neither on the assumption of the standard Hall geometry nor on the microscopic transport model.

The solution both here [Eq. (8)] and in the treatment given by Petritz has been found on the assumption that E_x and E_y are independent of z. Strictly speaking, this condition cannot apply to the boundary region along the vertical edge of the sample if mobility varies with depth. In that case, with magnetic field on, the Hall angle $\tan^{-1}(\sigma_{xy}/\sigma_{xx})$ being different in various x-y planes, the normal component of the current density **J** cannot vanish at this boundary if J_z is zero. Note that the normal component of the integrated current density I can still be taken to vanish on the boundary. For example, one might have

$$\mathbf{J}\cdot\mathbf{n} > 0 \text{ for } 0 < z < w/2$$

and

$$\mathbf{J}\cdot\mathbf{n} < 0 \text{ for } w/2 < z < w \text{ with } \mathbf{I}\cdot\mathbf{n} = 0.$$

To satisfy the true boundary condition $\mathbf{J}\cdot\mathbf{n} = 0$, we have to inject some current into the lower half of the thickness and withdraw an equal current from the upper half. The extra superposed currents and electric fields due to this dipolar distribution die off rapidly as we move from the edge to the interior of the sample, giving corrections of order w/L where L is a typical transverse dimension. If $w/L < < 1$, as is usually the case, the voltage contribution from the boundary region with nonzero E_z can be neglected as compared to that from the rest of the sample with zero E_z. In short, Eq. (8) applies to a sample if $w/L < < 1$.

In conclusion, the validity of the van der Pauw–Hall measurement technique for implanted samples has been confirmed, and the measured Hall coefficient can be expressed in terms of the depth dependent transport coefficients in the same manner as found by Petritiz[4] for a particular geometry and transport model.

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