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Neutrinos in compact objects

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The trapping of neutrinos in strong gravitational fields is considered. The zero-mass Dirac equation for curved space-time is specialized to the case of a static spherically symmetric object of constant density. The WKB method is applied to obtain the energy spectrum of the semibound states of the neutrinos. The probability of penetration through the barrier presented by the gravitational field is calculated for each level and the lifetime of a neutrino in the well obtained. It is shown that in the energy domains of interest the classical approximation is applicable and the total fraction of neutrinos trapped after emission is obtained. It is shown that neutrino trapping may take place in neutron stars with stiff equations of state.

I. INTRODUCTION

About a decade ago one of the present authors, while studying the scattering of waves of different spins by black holes, noticed that the effective potential in the case of neutrinos comprised both a well and a barrier. The well depth was found to increase with the angular parameter of the neutrino. However, a WKB analysis revealed the total absence of bound states and hence of the trapping of neutrinos in the potential well. It was conjectured at that time that such bound states should exist if the black hole were to be replaced by a sufficiently collapsed material object. We now return to this question and what follows is a natural sequel to the considerations pertaining to neutrinos in the field of a Schwarzschild black hole.[1]

We consider in this work[2] the possible trapping of neutrinos in the strong gravitational fields of compact objects. In compact dense objects, and in explosive situations, neutrinos are considered to be of importance in the transport of energy from the region of energy production to the outside. This is because neutrinos interact weakly with matter. However, in the situations where the gravitational field is sufficiently strong, the neutrinos may be gravitationally trapped in the region of production. If this happens, the neutrinos will be less efficient as vehicles of energy transport than they are normally expected to be. Trapped neutrinos may also affect the rates of nuclear processes by forming a degenerate sea. We explore in this paper the formalism necessary for the treatment of gravitationally trapped neutrinos.

Since the trapping will occur in strong gravitational fields, it is necessary to use the general theory of relativity to describe the field. The neutrinos themselves are described using the zero-mass Dirac equation generalized to curved spacetimes. Our aim will be to see if the gravitational field can act as a potential well and produce quasi-bound-state solutions of the Dirac equation. We develop the formalism, using for the background gravity the field due to a static spherically symmetric distribution of matter, and later specialize to the case of an object of constant density. Application of the WKB method shows that neutrinos are gravitationally trapped for durations of time which depend upon the quantum numbers of the neutrino and the mass and radius of the gravitating object. We neglect the back reaction of the neutrinos on the metric.

Neutrinos in curved spacetime were first considered by Brill and Wheeler,[3] who were looking into the possibility of building neutrino geons. They considered neutrinos in the field of a spherically symmetric thin-shell geon, but they did not treat in detail the application of the WKB method to the problem of trapping. The Dirac equation is a differential equation of the first order, whereas the WKB method is applicable to a second-order equation of the type of the Schrödinger equation. If the Dirac equation is reduced to the second order by taking derivatives, and solutions of these equations obtained in the WKB approximation, it is important to notice that a transformation of these solutions is necessary before solutions of the original first-order equations are obtained. We show that only if this is done can the WKB method be applied in the usual way.

A drawback of the Brill-Wheeler formalism is that in it neutrino and antineutrino states are always mixed. It is possible to have pure neutrino and pure antineutrino states by using curved-spacetime Dirac matrices which are related to those used by Unruh[4] in connection with the second quantization of the neutrino field in the Kerr metric. We use this representation of the Dirac matrices and separate out in the usual manner the angular part so that a pair of coupled first-order differential equations involving only the radial coordinate is obtained. This is then reduced to a pair of uncoupled second-order equations to which the WKB method is applied.

We present numerical results for various values of the parameter $2M/R$ and show how, from the set of curves presented, information regarding the trapping and escape of neutrinos may be obtained. It follows from the calculations that in the energy regime of interest, the spectrum of energy levels is nearly continuous. It is possible in this regime to treat neutrinos classically and assume that they move along null geodesics. We obtain in the classical approximation the distribution function for the neutrino impact parameter and from this calculate the fraction of neutrinos trapped after emission in the entire volume of the star. This fraction is independent of the energy of the neutrinos.

II. THE ZERO-MASS DIRAC EQUATION IN THE GENERAL SPHERICALLY SYMMETRIC METRIC

The generalization of the flat-spacetime Dirac equation to curved spacetime has been described in some detail by Brill and Wheeler[3] and in the references given by them. For curved spacetime the Dirac matrices γ^k $(k=0,1,2,3)$, which transform together as a contravariant vector under arbitrary transformation of the coordinates, satisfy the relations

$$\gamma^i\gamma^k+\gamma^k\gamma^i=2g^{ik}, \tag{1}$$

where g^{ik} is the metric tensor. Equation (1) is clearly a generalization of the flat-spacetime relations satisfied by the Dirac matrices. The neutrino, which is taken to be a zero-rest-mass particle of spin $\frac{1}{2}$, is described by a four-component spinor ψ. A covariant derivative ∇_k of the spinor is defined by

$$\nabla_k\psi=\left(\frac{\partial}{\partial x^k}-\Gamma_k\right)\psi, \tag{2}$$

where the spinor affinity Γ_k is determined uniquely up to an additive multiple of the unit matrix by

$$\gamma^i{}_{;k}-\Gamma_k\gamma^i+\gamma^i\Gamma_k=0, \tag{3}$$

where a semicolon indicates the usual covariant derivative for the metric g_{ik}. By an appropriate spinor transformation $\psi\to s\psi$, $\gamma^k\to s\gamma^k s^{-1}$, the Γ_k can be chosen such that

$$\mathrm{Tr}\,\Gamma_k=0. \tag{4}$$

For every spinor ψ an adjoint spinor $\bar\psi$ is defined by

$$\bar\psi=\psi^\dagger\alpha, \tag{5}$$

where a dagger denotes Hermitian conjugation (complex conjugate of the transposed matrix) and the matrix α satisfies the equations

$$\alpha\gamma^k-\gamma^{k\dagger}\alpha=0 \tag{6}$$

and

$$\alpha_{,k}+\Gamma_k^\dagger\alpha+\alpha\Gamma_k=0, \tag{7}$$

where a comma indicates ordinary differentiation. The covariant derivative of the adjoint spinor $\bar\psi$ is given by

$$\nabla_k\bar\psi=\frac{\partial\bar\psi}{\partial x^k}+\bar\psi\Gamma_k. \tag{8}$$

The massless Dirac equation for curved spacetime is

$$\gamma^k\nabla_k\psi=0 \tag{9}$$

and the equation satisfied by the adjoint spinor $\bar\psi$ is

$$(\nabla_k\bar\psi)\gamma^k=0. \tag{10}$$

It is known that the neutrino is purely left-handed, i.e., the spin of the neutrino is always polarized antiparallel to its momentum. The neutrino is therefore really a two-component object. The extra degrees of freedom from the four-component spinor ψ are removed by imposing the constraint

$$(1+i\gamma^5)\psi=0, \tag{11}$$

where

$$\gamma^5=-\frac{\epsilon^{ijkl}\gamma_i\gamma_j\gamma_k\gamma_l}{4!\sqrt{-g}} \tag{12}$$

and ϵ^{ijkl} is the totally antisymmetric tensor density with $\epsilon^{0123}=1$. The constraint equation for the antineutrino, which is purely right handed, is

$$(1-i\gamma^5)\psi=0. \tag{13}$$

For any two solutions ψ_1 and ψ_2 of Eq. (9), a current four-vector j^k is defined by

$$j^k=\bar\psi_1\gamma^k\psi_2. \tag{14}$$

The current is conserved, i.e., $\nabla_k j^k=0$, and it follows that the integral

$$\int_\sigma\sqrt{-g}\,j^k d\sigma_k$$

is independent of the spacelike hypersurface σ on which it is evaluated. An inner product is now defined by (Unruh[4])

$$\langle\psi_1,\psi_2\rangle=\int_\sigma\sqrt{-g}\,\bar\psi_1\gamma^0\psi_2 d^3x. \tag{15}$$

The neutrino equation may be written in the form

$$i\frac{\partial\psi}{\partial x^0}=H\psi,\quad H=i[\Gamma_0-(\gamma^0)^{-1}\gamma^\mu\nabla_\mu],\quad \mu=1,2,3. \tag{16}$$

The operator H is Hermitian in the inner product defined in Eq. (15) and it plays the role of the Hamiltonian. If a normal-mode solution is defined by

$$i\frac{\partial\psi}{\partial x^0}=H\psi=\omega\psi, \tag{17}$$

the eigenvalues ω are real and may be identified with the energy eigenvalues.

We now specialize the above formalism to the case of the spherically symmetric metric in the usual Schwarzschild coordinates,

$$ds^2=e^\nu dt^2-e^\lambda dr^2-r^2(d\theta^2+\sin^2\theta\, d\varphi^2). \tag{18}$$

Before we proceed, we have to choose a set of matrices satisfying Eq. (1). A set of matrices could be obtained by using the vierbien formalism outlined by Brill and Wheeler,[3] but then mixed neutrino and antineutrino states are obtained. We find it convenient to use γ matrices related to Unruh's representation,[4] for these lead to pure neutrino and pure antineutrino states. Our γ matrices are defined by

$$\gamma^t=e^{-\nu/2}\tilde{\gamma}^0,\ \gamma^r=e^{-\lambda/2}\tilde{\gamma}^3,\quad \gamma^\theta=\frac{1}{r}\tilde{\gamma}^1,\quad \gamma^\varphi=\frac{1}{r\sin\theta}\tilde{\gamma}^2, \tag{19}$$

where the $\tilde{\gamma}^k$ are flat-spacetime Dirac matrices in the Bjorken-Drell[5] representation. We choose

$$\alpha=\tilde{\gamma}^0. \tag{20}$$

The matrix γ^5 is now given by

$$\gamma^5=\tilde{\gamma}^0\tilde{\gamma}^1\tilde{\gamma}^2\tilde{\gamma}^3. \tag{21}$$

For this choice of the γ matrices, the spinor affinities are

$$\Gamma_t=\frac{\nu'}{4}e^\nu\gamma^1\gamma^0,\quad \Gamma_2=0,$$
$$\Gamma_\theta=\frac{r}{2}\gamma^2\gamma^1,\quad \Gamma_\varphi=\frac{r\sin^2\theta}{2}(\gamma^3\gamma^1+r\cot\theta\gamma^3\gamma^2). \tag{22}$$

The wave equation (9) now reduces to

$$\gamma^t\frac{\partial\psi}{\partial t}+\gamma^r\left(\frac{\partial}{\partial r}+\frac{\nu'}{4}+\frac{1}{r}\right)\psi+\gamma^\theta\left(\frac{\partial}{\partial\theta}+\tfrac{1}{2}\cot\theta\right)\psi+\gamma^\varphi\frac{\partial\psi}{\partial\varphi}=0. \tag{23}$$

We have defined energy normal modes in Eq. (17). The angular normal modes are similarly defined by

$$i\frac{\partial\psi}{\partial\varphi}=m\psi. \tag{24}$$

If the spinor ψ is written as

$$\psi=\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}, \tag{25}$$

where α_1, α_2 are two-component objects, in the Bjorken-Drell representation the constraint equation (11) implies

$$\alpha_1=\alpha_2. \tag{26}$$

The radial and angular parts of Eq. (23) may now be separated by writing ψ as

$$\psi=\frac{e^{-i\omega t}e^{-im\varphi}}{e^{\nu/4}(r^2\sin\theta)^{1/2}}\begin{pmatrix}\eta\\ \eta\end{pmatrix},\quad \eta=\begin{pmatrix}R_1(r)S_1(\theta)\\ R_2(r)S_2(\theta)\end{pmatrix}, \tag{27}$$

and the radial and angular functions satisfy the coupled first-order differential equations

$$e^{\nu/2-\lambda/2}\frac{dR_1}{dr}-i\omega R_1=\frac{ke^{\nu/2}}{r}R_2,$$
$$e^{\nu/2-\lambda/2}\frac{dR_2}{dr}+i\omega R_2=\frac{ke^{\nu/2}}{r}R_1, \tag{28}$$

and

$$\frac{dS_1}{d\theta}+\frac{mS_1}{\sin\theta}=kS_2,$$
$$\frac{dS_2}{d\theta}-\frac{mS_2}{\sin\theta}=-kS_1, \tag{29}$$

respectively, where k is a constant. It has been shown by Schrödinger[6] that k has a spectrum of positive and negative integral eigenvalues. The angular functions S_1 and S_2 are normalized so that

$$\int_0^\pi S_1^2(\theta)d\theta=\int_0^\pi S_2^2(\theta)d\theta=\frac{1}{4\pi}. \tag{30}$$

If antineutrino states are obtained using the constraint (13), ω in Eq. (27) is replaced by $-\omega$.

It is convenient at this stage to introduce a new variable r^* defined by

$$dr^*=e^{\lambda/2-\nu/2}dr. \tag{31}$$

Introducing

$$D\equiv\frac{d}{dr^*},\quad A=\frac{ke^{\nu/2}}{r}, \tag{32}$$

the radial equations reduce to the simple form

$$DR_1-i\omega R_1=AR_2,$$
$$DR_2+i\omega R_2=AR_1. \tag{33}$$

Defining functions F_1 and G_1 by

$$F_1=R_1+R_2,\quad G_1=\frac{1}{i}(R_1-R_2), \tag{34}$$

the radial equations may be written as

$$(D-A)F_1 = -\omega G_1\,,$$
$$(D+A)G_1 = \omega F_1\,. \tag{35}$$

These are just the first-order radial equations obtained by Brill and Wheeler[3] using a different set of γ matrices.

Operating on the two equations in (35) by $D+A$ and $D-A$, respectively, and adding and subtracting the resulting equations leads to two uncoupled second-order equations:

$$(D^2+\omega^2)F_1 - A^2F_1 - DAF_1 = 0\,,$$
$$(D^2+\omega^2)G_1 - A^2G_1 + DAG_1 = 0\,. \tag{36}$$

It is convenient to use these equations in analyzing the neutrino problem. However, not all solutions of the second-order equations (36) are solutions of the first-order equations (35). Given, therefore, solutions F and G of Eqs. (36), it is necessary to modify them so that solutions of the first-order equations are obtained. For any given solutions F and G of (36), this is achieved by defining

$$F_1 = \frac{1}{2\omega}[(D+A)G + \omega F]\,,$$
$$G_1 = \frac{1}{2\omega}[-(D-A)F + \omega G]\,. \tag{37}$$

If F and G are solutions of (36), F_1 and G_1 are always solutions of (35). If F, G are already solutions of (35), the transformation (37) gives $F_1 = F$, $G_1 = G$. The corresponding equations for antineutrinos are obtained by replacing ω with $-\omega$.

The second-order equations (36) are of the type

$$\frac{d^2\chi}{dr^{*2}} + [\omega^2 - \xi(r^*)]\chi = 0\,, \tag{38}$$

with the "potential term" $\xi(r^*)$ suitably defined. Such an equation can be solved in the WKB approximation,[7] and the solution is given by

$$\chi(r^*) = \frac{C_1}{[p(r^*)]^{1/2}}\exp\left(i\int^{r^*} p\,dr^*\right) + \frac{C_2}{[p(r^*)]^{1/2}}\exp\left(-i\int^{r^*} p\,dr^*\right), \tag{39}$$

where

$$p(r^*) = [\omega^2 - \xi(r^*)]^{1/2}\,. \tag{40}$$

For any solution ψ of Eq. (9), a conserved particle number density current is given by

$$j^k = \bar{\psi}\gamma^k\psi\,. \tag{41}$$

The net number of particles flowing *out* of a surface $r=$ constant per unit Schwarzschild time t is then given by

$$\frac{\partial N}{\partial t} = \int_{r=\text{constant}} \sqrt{-g}\,j^r d\theta\,d\varphi = |R_1|^2 - |R_2|^2 = \tfrac{1}{2}i(F_1^*G_1 - F_1G_1^*)\,, \tag{42}$$

where the asterisk denotes complex conjugation. In what follows we will find it convenient to express $\partial N/\partial t$ directly in terms of solutions F and G of the second-order equations. We will not write down such an expression as it is rather unwiedly. We will need only an approximate expression which will be introduced in the next section after conditions for its validity are discussed.

III. COMPACT SPHERICALLY SYMMETRIC UNIFORM DENSITY OBJECTS

In the previous section we reduced the Dirac equation to a form suitable for any static, spherically symmetric metric. We will now consider a particular solution of Einstein's equations belonging to this class—the spacetime metric in the interior and exterior of a static, spherically symmetric object with uniform density σ,

$$\sigma = \text{constant}\,. \tag{43}$$

If the radial coordinate at the boundary of the object (star) is taken to be $r=R$, the metric in the interior and exterior is given by (18) with

$$\left.\begin{aligned} e^\nu &= \frac{1}{4}\left[3\left(1-\frac{2M}{R}\right)^{1/2} - \left(1-\frac{2Mr^2}{R^3}\right)^{1/2}\right]^2 \\ e^\lambda &= \left(1-\frac{2Mr^2}{R^3}\right)^{-1}\end{aligned}\right\}(r<R) \tag{44}$$

and

$$e^\nu = e^{-\lambda} = \left(1-\frac{2M}{r}\right), \quad r>R \tag{45}$$

if a system of units with $c=G=1$ is used. Introducing

$$\rho = \frac{r}{M}, \quad a = \frac{1}{4}\left(\frac{2M}{R}\right)^3, \quad b = 3\left(1-\frac{2M}{R}\right)^{1/2}, \tag{46}$$

Eqs. (44) and (45) may be written as

$$e^\nu = \tfrac{1}{4}[b-(1-a\rho^2)^{1/2}]^2\,, \quad e^\lambda = (1-a\rho^2)^{-1}\,, \quad \rho < R/M \tag{47}$$

and

$$e^\nu = e^{-\lambda} = \left(1-\frac{2}{\rho}\right)^{-1}, \quad \rho > R/M\,. \tag{48}$$

To prevent a pressure singularity at the center it is required that

$$\frac{2M}{R} < \frac{8}{9}, \quad \text{i.e.,} \quad R > \frac{9M}{4}\,. \tag{49}$$

Using (47) and (48), Eqs. (36) may be reduced to the form (to avoid confusion we use F and G when writing second-order equations and F_1 and G_1 in the first-order equations)

$$\frac{d^2F}{dr^{*2}}+[\omega^2-\xi(r^*)]F=0,\quad \frac{d^2G}{dr^{*2}}+[\omega^2-\eta(r^*)]G=0, \tag{50}$$

with

$$\left.\begin{matrix}\xi(r^*)\\ \eta(r^*)\end{matrix}\right\}=\frac{k^2}{4M^2\rho^2}[b-(1-a\rho^2)^{1/2}]^2$$
$$\mp\frac{k}{4M^2\rho^2}[b-(1-a\rho^2)^{1/2}]+(1-a\rho^2)^{1/2}$$
$$\pm\frac{ak}{4M^2}[b-(1-a\rho^2)^{1/2}],\quad \rho<R/M \tag{51}$$

and

$$\left.\begin{matrix}\xi(r^*)\\ \eta(r^*)\end{matrix}\right\}=\frac{k^2}{M^2\rho^2}\left(1-\frac{2}{\rho}\right)$$
$$\mp\frac{k}{M^2\rho^2}\left(1-\frac{2}{\rho}\right)^{1/2}\left(1-\frac{3}{\rho}\right),\quad \rho>R/M \tag{52}$$

with the upper signs used in $\xi(r^*)$ and the lower signs in $\eta(r^*)$ in Eqs. (51) and (52).

In the exterior potentials ($\rho>R/M$), it is obvious that for large k the term $\sim k^2$ dominates the term $\sim k$ for all values of $\rho>R/M$. The dominance of terms $\sim k^2$ over terms $\sim k$ for large k occurs also in the interior potential, i.e., for $\rho<R/M$ provided

$$\frac{2M}{R}<\frac{119}{144}. \tag{53}$$

In the following we will assume that this condition is always satisfied. The potentials now take the simple form

$$\xi_{\rm int}(r^*)=\eta_{\rm int}(r^*)$$
$$=\frac{k^2}{4M^2\rho^2}[b-(1-a\rho^2)^{1/2}],\quad \rho<R/M \tag{54}$$

and

$$\xi_{\rm ext}(r^*)=\eta_{\rm ext}(r^*)=\frac{k^2}{M^2\rho^2}\left(1-\frac{2}{\rho}\right),\quad \rho>R/M. \tag{55}$$

Since the potentials in the two equations (50) are now identical, we can take $F=G$. In the present approximation Eq. (42) reduces to

$$\left(\frac{\partial N}{\partial t}\right)\simeq\frac{i}{4\omega}(FDF^*-F^*DF). \tag{56}$$

The interior potential $\xi_{\rm int}(r^*)\to\infty$ as $\rho\to 0$. Further, $\xi_{\rm int}(r^*)$ has a minimum occurring at

$$\rho=2\left(\frac{2M}{R}\right)^{-3/2}\left(\frac{8-9(2M/R)}{9-9(2M/R)}\right)^{1/2}, \tag{57}$$

with

$$\xi_{\rm int\,min}=\frac{k^2}{8M^2}\left(\frac{2M}{R}\right)^3\left[4-\frac{9}{2}\left(\frac{2M}{R}\right)\right]. \tag{58}$$

The minimum exists when $2M/R>\frac{2}{3}$, i.e., if $R<3M$ (if this condition is not satisfied, the minimum should occur outside the surface of the star, which is not allowed). The exterior potential $\xi_{\rm ext}(r^*)\to 0$, as $\rho\to\infty$, and it has a maximum of

$$\xi_{\rm ext\,max}=\frac{k^2}{27M^2} \tag{59}$$

occurring at $\rho=3$, i.e., $r=3M$. Clearly the maximum will exist only if $R<3M$. We will hereafter consider values of $2M/R$ such that

$$\frac{2}{3}<\frac{2M}{R}<\frac{119}{144},\ \text{i.e.,}\ \frac{288M}{119}<R<3M. \tag{60}$$

In this range the minimum in $\xi_{\rm int}(r^*)$ and the maximum in $\xi_{\rm ext}(r^*)$ exist. The potential $\xi(r^*)$ and its derivative are continuous across the surface $\rho=R/M$ because of the continuity of the metric and it first derivatives across the surface. For the range indicated in (60), a potential well of the type shown in Fig. 1 is therefore obtained.

Consider a neutrino with quantum numbers ω and k in the interior of the star. The energy of the neutrino is $E=\hbar\omega$. Comparison with the flat-spacetime Dirac equation (i.e., using an equivalence principle argument) shows that k is related to the total angular quantum number j by

$$|k|=j+\tfrac{1}{2},\quad j=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\ldots. \tag{61}$$

For large k the total angular momentum J is

$$J=[j(j+1)]^{1/2}\hbar\simeq|k|\hbar. \tag{62}$$

The neutrino finds itself in a potential well with the barrier height $k^2/27M^2$. If the neutrino energy

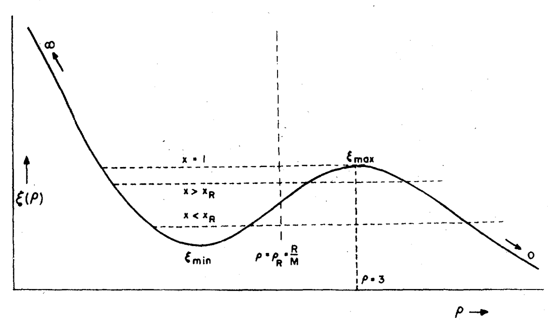

FIG. 1. A schematic representation of the potential $\xi(\rho)$. The line $x<x_R$ represents a neutrino with energy $E<[\xi(R/M)]^{1/2}$, where $x=E^2/\xi_{\rm ext,max}$ [see Eq. (67)]. Both the turning points are within the surface of the star. In the notation of Eqs. (72) and (73), $I=I_1$, and $I'=I_1+I_2$. For $x>x_R$, the energy $E>[\xi(R/M)]^{1/2}$, the outer turning point is beyond the surface of the star, and $I=I_1+I_2$, $I'=I_2$. The line $x=1$ is used to determine the number of bound states for a given k.

$E > (k^2/27M^2)^{1/2}$, the radial wave function is oscillatory for all values of $\rho > \rho_1$, where ρ_1 is the turning point obtained as the lesser root of the quadratic equation $E^2 = \xi_{int}(\rho)$. (We sometimes find it convenient to express the potentials ξ and η as functions of r or ρ.) For $E < (k^2/27M^2)^{1/2}$, let ρ_1 and ρ_2 be the inner and outer turning points, respectively, which are obtained as solutions of $E^2 = \xi(\rho)$ (see Fig. 1). The wave function in this case is (i) real and decaying with decreasing ρ for $\rho < \rho_1$, (ii) oscillatory for $\rho_1 < \rho < \rho_2$, (iii) real and decaying with increasing ρ for $\rho_2 < \rho < \rho_3$, where ρ_3 is the greater real root of the equation $E^2 = \xi_{ext}(\rho)$, and (iv) oscillatory for $\rho > \rho_3$. The form of the wave function suggests that the neutrino is trapped in a semibound state in the gravitational potential well, and that it eventually penetrates through the potential barrier and escapes to infinity. The average lifetime of the neutrino against barrier penetration can be obtained by following a procedure similar to the one used in the virtual level theory of α decay.[7] However, it is sufficient in the present case, as in the case of α decay, to use the WKB method to obtain the probability of penetration.

In applying the WKB approximation, it is first assumed that there is no penetration through the barrier. The energy eigenvalues are then discrete, and are those values which satisfy the Bohr-Sommerfeld quantization condition[8]

$$(n_k + \tfrac{1}{2}) = \int_{\rho=\rho_2}^{\rho_3} p\, dr^*, \quad p = [\omega^2 - \xi(r^*)]^{1/2}, \tag{63}$$

where n_k is a positive integer and ρ_1 and ρ_2 are the turning points. For a given k, the total number of eigenvalues N_k is obtained as the largest positive integer less than or equal to the right-hand side for $E^2 = \omega^2 = k^2/27M^2$ ($\hbar = 1$). Having obtained the energy eigenvalues, the penetration can be "switched on" and the penetration factor obtained by comparing the number of particles flowing out of a r = constant surface in the regions $\rho_1 < \rho < \rho_2$ and $\rho > \rho_3$, respectively. Using Eq. (56) and WKB functions F, which are appropriately joined in passing from one region to another (see Ref. 7), the penetration factor is found to be

$$P = \exp\left(-2\int_{\rho_2}^{\rho_3} p\, dr^*\right) = \exp\left[-2\int_{\rho_2}^{\rho_3}\left(\frac{k^2 e^\nu}{M^2\rho^2} - \omega^2\right)^{1/2} e^{\lambda/2-\nu/2} d\rho\right], \tag{64}$$

where ρ_2 and ρ_3 are defined above. It must be noted that for consistency of the calculation, P should be small compared to unity.

In the usual WKB calculations for massive particles, the probability of penetration Π is taken to be $\Pi = P \times \nu$, where ν is the frequency of approach to the turning point ρ_2 by the particle assumed to be traveling along a classical trajectory in the same potential. In the present case we take ν to be the frequency with which a neutrino traveling along a null geodesic, with angular momentum $J = \hbar k$ and energy $E = \hbar\omega$ in the potential $\xi(\rho)$ approaches r_2. ν is obtained from

$$\nu^{-1} = 2M \int_{\rho_1}^{\rho_2} e^{(\lambda-\nu)/2}\left(1 - \frac{q^2 e^\nu}{M^2\rho^2}\right)^{-1/2} d\rho, \tag{65}$$

where $q = J/E$ is the "impact parameter" (see Sec. V for the details). The probability of penetration and the average lifetime τ of the neutrino in the well are then

$$\Pi = \nu P, \quad \tau = \frac{1}{\Pi}, \tag{66}$$

respectively.

IV. NUMERICAL RESULTS

We will consider in this section the energy eigenvalues, penetration factors, and lifetimes of the semi-bound states which result for various values of M and R, with $2M/R$ in the range indicated in (60).

For this purpose, it is convenient to express the energy of a given (semi) bound state as a fraction of $\xi_{ext\,max}$. For a given value of k we write

$$E^2 = x\,\xi_{ext\,max} = \frac{xk^2}{27M^2}. \tag{67}$$

The energy values $E = (\xi_{ext\,max})^{1/2}$ and $E = (\xi_{int\,min})^{1/2}$ correspond to

$$x = x_{max} = 1, \quad x = x_{min} = \left(\frac{27}{2}\right)\left(\frac{2M}{R}\right)^3\left[1 - \frac{9}{8}\left(\frac{2M}{R}\right)\right], \tag{68}$$

respectively. The value of x which corresponds to $E = [\xi(\rho = R/M)]^{1/2}$ is

$$x = x_R = \frac{27}{4}\left(\frac{2M}{R}\right)^2\left(1 - \frac{2M}{R}\right). \tag{69}$$

We now define two integrals I_1 and I_2 by

$$I_1 = 2\int_{\rho_1}^{\rho_2}\left\{\frac{x}{27} - \frac{1}{4\rho^2}[b - (1 - a\rho^2)^{1/2}]^2\right\}^{1/2} \times [b - (1 - a\rho^2)^{1/2}]^{-1}(1 - a\rho^2)^{-1} d\rho, \tag{70}$$

$$I_2 = \int_{\rho_3}^{\rho_4}\left[\frac{x}{27} - \frac{1}{\rho^2}\left(1 - \frac{2}{\rho}\right)\right]^{1/2}\left(1 - \frac{2}{\rho}\right)^{-1} d\rho, \tag{71}$$

with the limits of integration to be specified in the following. In terms of these integrals the Bohr-Sommerfeld quantization condition reduces to

$$n_k = \frac{k}{\Pi} I - 0.5, \tag{72}$$

with I defined as follows: (i) For $E^2 \leq \xi(\rho = R/M)$, i.e., $x < x_R$, $I = I_1$ with ρ_1 and ρ_2 the lesser and the greater root, respectively of the quadratic equation $E^2 = \xi_{int}(\rho)$; (ii) For $E^2 > \xi(\rho = R/M)$, i.e., $x > x_R$, $I = I_1 + I_2$ with ρ_1 the lesser root of $E^2 = \xi_{int}(\rho)$, $\rho_2 = \rho_3 = \rho_R = R/M$, and ρ_4 the lesser positive root of the cubic equation $E^2 = \xi_{ext}(\rho)$. With our definition of x, the limits of integration are all independent of k. The integral and the limits of integration depend only on x and the ratio $2M/R$.

For a given value of $2M/R$, in order to determine the value of k for which the first bound state occurs, the integral is evaluated for $x = 1$, which corresponds to the highest value of energy for which a bound state may occur. The value of k is then increased through positive integral values until n_k exceeds unity by a fraction (we consider only positive integral values of k for convenience, but all our results are true also for negative values of k if k is replaced everywhere by $|k|$). The exact energy of the bound state is determined by finding the value of x for which $n_k = 1$. No bound state is possible for lesser values of k. In general, the number of bound states for a given value of k is equal to the greatest integer less than or equal to the right-hand side of Eq. (72).

For a given energy level, Eq. (64) for the penetration factor may be expressed as

$$P = \exp(-2kI'), \tag{73}$$

where the integral I' is defined as: (i) $I' = I_1 + I_2$ for $x < x_R$, with ρ the greater root of the equation $E^2 = \xi_{int}(\rho)$, $\rho_2 = \rho_3 = \rho_R = R/M$, and ρ_3 the greater positive root of the equation $E^2 = \xi_{ext}(\rho)$; (ii) $I' = I_2$ for $x > x_R$, with ρ_3 and ρ_4 the lesser and greater positive roots respectively of the equation $E^2 = \xi_{int}(\rho)$. The penetration probability per second Π and the average lifetime τ are then obtained using Eq. (66). ν is obtained from

$$\nu = \left(\frac{54GMI''}{c^3}\sqrt{x}\right)^{-1} \text{sec}^{-1}, \tag{74}$$

with (i) for $x \leq x_R$, $I'' = I_3$, where I_3 is defined as in (70) except that the expression within the parentheses is now raised to the power $-\frac{1}{2}$, and (ii) for $x > x_R$, $I'' = I_3 + I_4$ with I_4 defined as in (71) with the expression within the parentheses in the integrand raised to the power $-\frac{1}{2}$.

We present in Table I some values of k and the corresponding number of bound states for $2M/R = 0.7465$, which is the mean of the upper and lower limits on the allowed values of $2M/R$. In Table II we have listed the x values for each of the nine energy levels for $k = 166$. The energy E corresponding to each x is $E = 5.17 \times 10^8\, M^{-1}(g)k\sqrt{x}$ (keV). Against each level are shown the penetration factor P and the lifetime τ. It is found that in the range of $2M/R$ being considered, the value of $\sqrt{x}I''$, which occurs in Eq. (74), is a slowly varying function of x and $2M/R$, with $\sqrt{x}I'' = 0.011$. The error in assuming that $\sqrt{x}I''$ has this constant value is small (~5–10%). For $M \simeq 2M_\odot$, this value of $\sqrt{x}I''$ gives $\nu = 2.11 \times 10^5$ sec^{-1}.

We have made similar calculations for various values of $2M/R$ in the allowed range. To avoid the tedium of long tables, the results are presented graphically. It is possible to use the curves to get an approximate idea of the distribution of energy levels and the corresponding penetration factors for any value of $2M/R$.

In Fig. 2 we have plotted as a function of $2M/R$ the value of the integral I of Eq. (72) for $x = 1$. Given a value of $2M/R$, the corresponding value of I is determined from this curve. Equation (72) can then be used to determine the number of bound-state energy levels for any value of $2M/R$.

In Fig. 3 we have plotted, for various values of $2M/R$, the average spacing between the energy levels for a given value of k, as a function of k. Given $2M/R$ and having determined from Fig. 2 the values of k for which bound states occur, Fig. 3 may be used to determine the average energy separation $\overline{\Delta x}$ between the levels for each k.

In Fig. 4 we have plotted x maximum, averaged over various k as a function of $2M/R$. It is possible from this plot to determine approximately the highest energy level for any k for a given value of $2M/R$. Progressively subtracting from this level the average level separation $\overline{\Delta x}$ for a given k, the energy spectrum for the required value can be generated. For every $2M/R$ there are some

TABLE I. N_k is the number of bound states allowed for various values of the angular momentum quantum number k. $2M/R$ is taken to be 0.7465.

k	27	44	62	79	96	114	131	149	166	184
N_k	1	2	3	4	5	6	7	8	9	10

TABLE II. The energy spectrum for $2M/R = 0.7465$, $k = 166$, in terms of the parameter $x = E^2/\xi_{max}$.

n_k	x	P	τ (sec)
1	0.9161	$\sim 10^{-13}$	$\sim 5 \times 10^7$
2	0.9270	$\sim 10^{-13}$	$\sim 5 \times 10^7$
3	0.9381	$\sim 10^{-13}$	$\sim 5 \times 10^7$
4	0.9492	$\sim 10^{-13}$	$\sim 5 \times 10^7$
5	0.9603	$\sim 1 \times 10^{-9}$	$\sim 5 \times 10^3$
6	0.9713	2.76×10^{-7}	1.7×10^1
7	0.9818	7.16×10^{-5}	6.6×10^{-2}
8	0.9916	1.24×10^{-4}	4×10^{-2}
9	0.9999	9.48×10^{-1}	5×10^{-6}

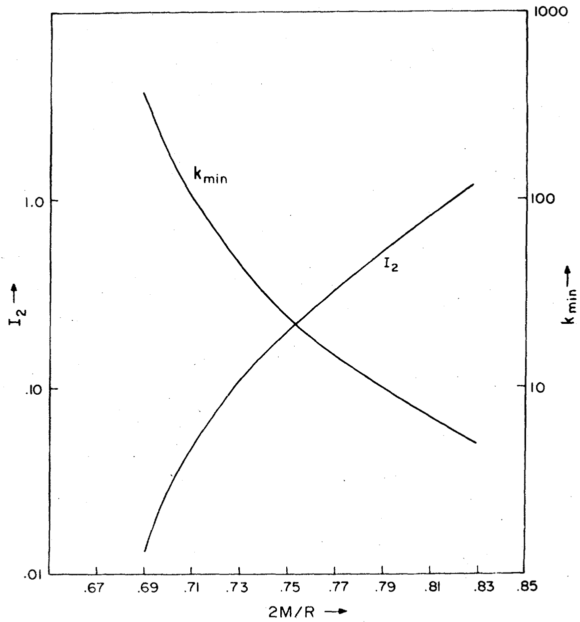

FIG. 2. The integral I_2 is shown as a function of $2M/R$. The value of k at which the first bound state occurs is also shown as a function of $2M/R$.

k values for which only one energy level exists. Such levels are not included in the above scheme and a separate plot is made in Fig. 5 of the x value for this level as a function of $2M/R$.

It is found that the integral I' of Eq. (73) can be very well approximated as a linear function of the parameter x. We show in Table III the correlation coefficient, and the parameters α and β, for a straight-line fit of the type $I' = \alpha + \beta x$. Having obtained the approximate energy spectrum as described above, the penetration factors can be determined from a straight-line fit of this type. Using $\sqrt{n}\, I'' \simeq 0.011$ in Eq. (74) then leads to the lifetime $\tau = 1/P\nu$ in each case. It should be noticed that an error $\Delta I'$ in determining I' leads to the error $\Delta P = -kP\Delta I'$ in determining P. Since $k \sim 10^2$, a small error in determining I' can lead to an error of magnitude unity in determining P.

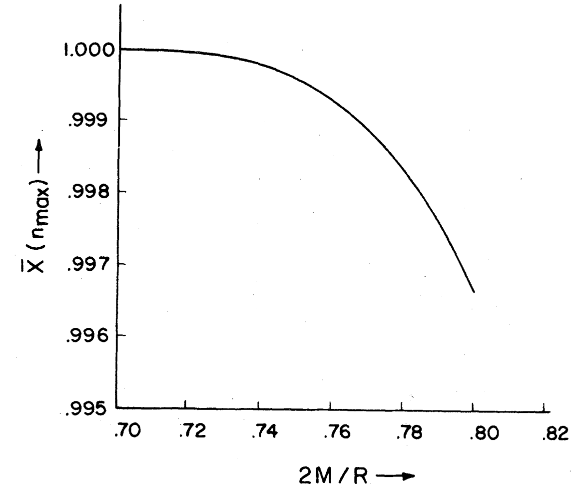

FIG. 4. The value of x for the highest energy level averaged over k values shown as a function of $2M/R$, where $x = E^2/\xi_{ext,max}$ [see Eq. (67)].

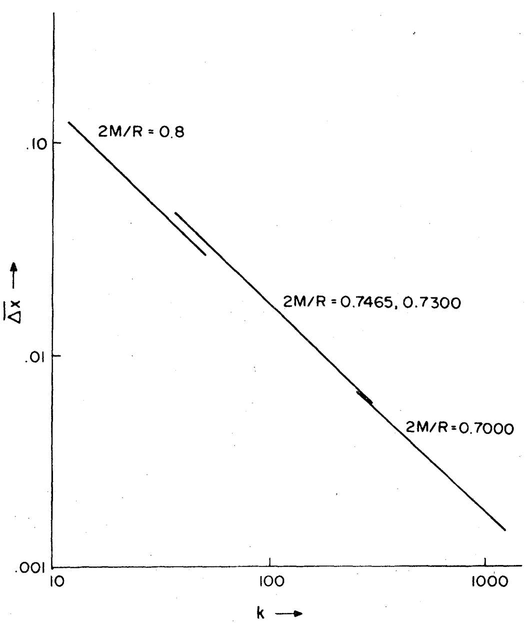

FIG. 3. Average spacing between levels plotted as a function of x for various values of $2M/R$.

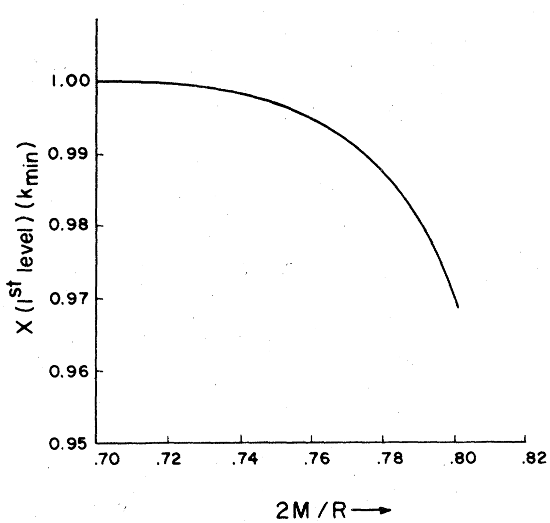

FIG. 5. The value of x for the first energy level (corresponding to k_{min}, one bound state) shown as a function of $2M/R$, where $x = E^2/\xi_{ext,max}$ [see Eq. (67)].

TABLE III. The parameters α and β and the correlation coefficient for the straight-line fit $I' = \alpha + \beta x$.

$2M/R$	α	β	r
0.7000	1.6660	−1.6663	0.9976
0.7300	1.6367	−1.6376	0.9997
0.7465	1.6585	−1.6546	0.9996
0.8000	1.7196	−1.7263	0.9992

V. CLASSICAL TREATMENT OF NEUTRINO TRAPPING

We have seen in the previous section that it is possible to trap neutrinos in a gravitational potential well for periods of ~10^7 seconds. For neutrinos of energy E and angular momentum $\hbar k$, trapping occurs for $E \leq (\xi_{\text{ext max}})^{1/2} = \hbar c^3 k / 3\sqrt{3} GM$ with k sufficiently high. For any energy the fraction of neutrinos trapped will depend upon how the k values are distributed amongst the neutrinos. To determine the distribution function in k, we will take advantage of the fact that in the regime of interest the problem may be treated in the classical approximation.

For a spherically symmetric object of mass $M = 2M_\odot \simeq 4\times 10^{33}$ g, the energy up to which trapping is in principle possible is $E = (\hbar c^3 k/3\sqrt{3}GM)$ erg $= 1.3\times 10^{-15} k$ (keV). The values of k that we considered in the previous section were ~10^2; for these the energy for which trapping may occur is very low. For trapping to occur in (say) the keV range, it is necessary to have $k \sim 10^{15}$. For values of k as large as this, there is a nearly continuous distribution of energy eigenvalues. Moreover, the de Broglie wavelength of such neutrinos is ~10^{-9} cm, which is very small compared to the radius $R \simeq 10^6$ cm of an object with $M = 1.5 M_\odot$, $2GM/c^2 R \simeq 1$. One can therefore safely apply the classical approximation and assume that neutrinos move along definite trajectories which are null geodesics (by virtue of the assumed zero mass of the neutrino).

For a null geodesic in the spherically symmetric, static Schwarzschild spacetime, we have the following first integrals of motion,[9]

$$\dot{\varphi} = \frac{J}{r^2}, \quad \dot{t} = e^{-\nu} E, \quad \dot{r}^2 = e^{\nu-\lambda}\left(1 - \frac{q^2 e^\nu}{r^2}\right), \tag{75}$$

where the overdot indicates differentiation with regard to some affine path parameter λ. J and E are constants of motion which may be taken to be the angular momentum and energy, respectively. $q = J/E$ is the impact parameter. From these equations it follows that

$$\left(\frac{dr}{d\varphi}\right)^2 = \frac{r^4 e^{-(\nu+\lambda)}}{q^2}\left(1 - \frac{q^2 e^\nu}{r^2}\right). \tag{76}$$

Consider a neutrino emitted at an arbitrary point $\mathcal{P}(r, \theta, \varphi)$ making an angle ζ in the locally orthonormal frame of a static observer, with the radial direction (see Fig. 6). Then

$$\tan\zeta = \frac{q}{(r^2 e^{-\nu} - q^2)^{1/2}}. \tag{77}$$

Consider a three-dimensional volume element with proper volume dv at $\mathcal{P}$. Let n_0 be the number of neutrinos emitted per unit proper time, per unit proper volume at $\mathcal{P}$. The number of neutrinos emitted in dv in Schwarzschild time dt is $n_0 dv e^{\nu/2} dt$. Since there is no preferred direction for the emitters to be polarized in (the coupling of the emitters to the gravitational field being neglected), the emission is isotropic in the locally orthonormal frame. The number of neutrinos emitted with angles between ζ and $\zeta + d\zeta$ with the radial direction is therefore $(n_0/2)\sin\zeta\, d\zeta\, dv e^{\nu/2} dt$. Let $n(\zeta, r)$ be the number of neutrinos emitter per unit Schwarzschild time, per unit interval in ζ, per unit interval in r at the radial coordinate r and with any θ and φ. $n(\zeta, r)$ is related to n_0 through

$$n(\zeta, r) = 2\pi n_0 e^{\lambda/2+\nu/2} r^2 \sin\zeta. \tag{78}$$

Let $n(q, r)$ be the number of neutrinos emitted per unit r interval, unit t interval and unit q interval. Since $\sin\zeta = q/re^{-\nu/2}$, for a given q the angle of emission can be either ζ or $\pi - \zeta$. Since $n(\zeta; r) \sim \sin\zeta$, $n(\zeta, r) = n(\pi - \zeta, r)$ and we have

$$n(q, r)\, dq\, dr\, dt = 2n(\zeta, r)\, d\zeta\, dr\, dt, \tag{79}$$

i.e.,

$$\begin{aligned} n(q, r)\, dq &= 2n(\zeta, r)\frac{d\zeta}{dq}\, dq \\ &= 4\pi n_0 e^{\nu+\lambda/2} r q (r^2 e^{-\nu} - q^2)^{-1/2} dq. \end{aligned} \tag{80}$$

We have used the locally measured isotropy of emission to obtain the distribution function $n(q, r)$.

For a neutrino emitted at a given r, since the right-hand side of Eq. (76) should be greater than zero, we have

$$1 - \frac{q^2 e^\nu}{r^2} > 0, \text{ i.e., } q < re^{-\nu/2}. \tag{81}$$

At any given point, therefore, $q_{\max} = re^{-\nu/2}$. For this value of q, $\zeta = \zeta_{q_{\max}} = \pi/2$. As ζ increases beyond this, q decreases from $q_{\max}$ to its minimum value $q = 0$ at $\zeta = \pi$.

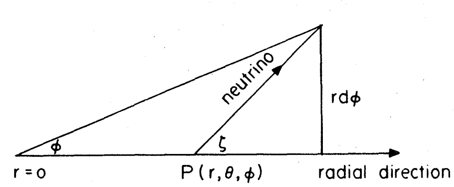

FIG. 6. Relation between the angles ζ and φ.

Writing Eq. (75) as

$$\left(\frac{dr}{dt}\right)^2=\frac{e^{\nu-\lambda}}{E^2}[E^2-\xi(r)], \tag{82}$$

it is clear that the form of the effective potential $\xi(r)=J^2e^\nu/r^2$, which determines the turning points and the motion of the null geodesic, is the same as the quantum mechanical potential with large k. Consider a neutrino emitted with a fixed energy E and impact parameter $q=J/E$. The neutrino sees a potential for which the maximum is $J^2/27M^2=q^2E^2/27M^2$. The neutrino is trapped when $E^2<q^2E^2/27M^2$, i.e., when $3\sqrt{3}M<q$. Of all the neutrinos emitted at radial coordinate r with the distribution function $n(q,r)$, those with q in the range $3\sqrt{3}M<q<q_{max}$ are trapped and those with q in the range $0<q<3\sqrt{3}M$ escape. The fraction of neutrinos trapped is

$$F(r)=\frac{\int_{q_e}^{q_{max}}n(q,r)dq}{\int_0^{q_{max}}n(q,r)dq}, \quad q_e=3\sqrt{3}M. \tag{83}$$

Using Eq. (80), $F(r)$ may be reduced to the simple form

$$F(r)=\left(1-\frac{\xi(r)}{\xi_{max}}\right)^{1/2}, \quad \xi(r)=\frac{J^2e^\nu}{r^2}. \tag{84}$$

The trapping can occur only for those values of r at which $q_e<q_{max}$, i.e., $3\sqrt{3}M<re^{-\nu/2}$. Let r_{min} be the lesser root of the quadratic equation $e^\nu/r^2=1/27M^2$. For $r<r_{min}$, $3\sqrt{3}M>re^{-\nu/2}$, i.e., $\xi_{int}(r)>\xi_{ext\,max}$, i.e., $E^2>\xi_{ext\,max}$, since always $E^2>\xi_{int}(r)$. It follows that neutrinos which can reach the region with $r<r_{min}$ will escape to infinity. In order to calculate the total number of neutrinos trapped on emission in the object, $dr\,dt\int_{q_e}^{q_{max}}\times n(q,r)dq$ has to be integrated from r_{min} to R. The fraction of all neutrinos trapped is therefore

$$F_T=\frac{N_T}{N}=\frac{\int_{r_{min}}^{R}e^{(\nu+\lambda)/2}r^2\left(1-\frac{\xi(r)}{\xi_{max}}\right)^{1/2}dr}{\int_0^R e^{(\nu+\lambda)/2}r^2dr}, \tag{85}$$

where N is the total number of neutrinos emitted by the object in Schwarzschild time Δt (say) and N_T is the number of the neutrinos trapped. Using the notation of the previous sections, this result may be expressed in terms of $\rho=R/M$:

$$F_T=\frac{\int_{\rho_{min}}^{\rho_R}\rho^2(1-a\rho^2)^{-1/2}[b-(1-a\rho^2)^{1/2}]\left(1-\frac{27}{4\rho^2}[b-(1-a\rho^2)^{1/2}]^2\right)^{1/2}d\rho}{\int_0^{\rho_R}\rho^2(1-a\rho^2)^{-1/2}[b-(1-a\rho^2)^{1/2}]d\rho}. \tag{86}$$

In the above expressions it has been assumed that n_0, which is the number of neutrinos emitted per unit proper time, per unit proper volume, is a constant throughout the volume of the star. The fraction of neutrinos trapped is independent of E, because the barrier height available is proportional to the energy.

Vilhu[10] has independently performed a similar classical calculation. He has, however, taken the fraction of escaping neutrinos at r to be $2\bar{\zeta}/\pi$, where $\bar{\zeta}$ is the half-angle of the escape cone at r. This amounts to omitting the factor $\sin\zeta$ which appears in the distribution function (78). Omitting this factor corresponds to replacing $[1-\xi(r)/\xi_{max}]^{1/2}$ in Eqs. (84) and (85) with $\cos^{-1}[\xi(r)/\xi_{max}]^{1/2}$. In Table IV we have listed the values of F_T and F_T'/F_T for several values of $2M/R$, where F_T' is the fraction of neutrinos trapped when the $\sin\zeta$ factor is omitted from the distribution function, as is done by Vilhu. It is seen that the absence of the $\sin\zeta$ factor matters only for high values of $2M/R$. In the last column of Table IV we have shown the values F_V given by Vilhu for the total fraction of neutrinos trapped. It is seen that our results are between 15 and 4 times in excess of Vilhu's. It is difficult to see where the discrepancy arises, as the manner in which Vilhu averages over the volume of the star is not clear. The numerical results are shown graphically in Fig. 7. Included in this figure are some points not shown in Table IV. We have shown for comparison the values for the fraction of neutrinos trapped as obtained by Vilhu.

In the classical approximation, the trapping of

TABLE IV. F_T is the fraction of neutrinos trapped in the potential well. F_T'/F_T = [fraction trapped with $\sin\zeta$ omitted from distribution function (70)]/(fraction trapped with $\sin\zeta$ included in the distribution function). F_V is the fraction trapped in Vilhu's calculation.

$2M/R$	F_T	F_T'/F_T	F_V
0.6700	0.5579×10^{-3}	1.0000	
0.6800	0.8523×10^{-2}	1.0003	
0.7000	0.0485	1.0021	
0.7100	0.0781	1.0037	
0.7300	0.1513	1.0085	
0.7330	0.1635	1.0093	0.01
0.7465	0.2213	1.0141	
0.7600	0.2825	1.0201	
0.7890	0.4172	1.0380	0.07
0.8000	0.4673	1.0468	
0.8100	0.5115	1.0560	
0.8264	0.5805	1.0735	
0.8440	0.6479	1.0962	0.16
0.8600	0.7028	1.1203	
0.8890	0.7847	1.1700	0.25

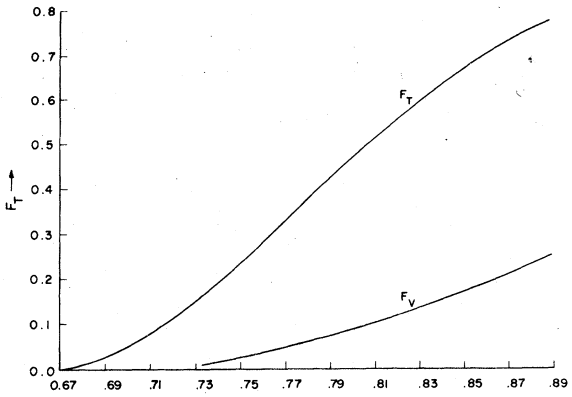

FIG. 7. F_T is the total number of neutrinos trapped in the gravitational field. F_V is the total number trapped as calculated by Vilhu.

neutrinos is permanent for there is no penetration through the barrier. In the quantum-mechanical formalism too the penetration would turn out to be negligibly small, as the values of k which occur in the penetration factor $P=e^{-2kl'}$ are very large in the classical region.

VI. ASTROPHYSICAL APPLICATIONS AND CONCLUSION

It is not our intention here to consider in detail the effects which gravitationally trapped neutrinos can have an astrophysical phenomena. We will only point out some circumstances in which a gravitational well may occur and leave the detailed calculations to a future report.

The maximum and the minimum in the potential ξ appear only when $R < 3M$, i.e., when $R < 1.5R_S$, where R_S is the Schwarzschild radius of a spherically symmetric object of mass M. Of the known equilibrium configurations, only a neutron star can be compact enough to satisfy this condition. Even in this case the radius can be made sufficiently small only for stiff equations of state. It is seen from the M-R curves given by Canuto[11] that at the peaks of the curves corresponding to stiff equations of state, $R < 1.5R_S$ with $2M/R = R_S/R = 0.67$. For a constant density object with this value of $2M/R$, the calculations of this paper show that ~0.05% of neutrinos produced are trapped, irrespective of their energy. The analysis is of course not directly applicable to a neutron star, as the matter density varies in its interior. Nevertheless, our numbers may be used to get an approximate idea of how much time it would take to produce neutrino degeneracy which would affect further neutrino production.

For a neutron star of mass $M \simeq 2.2M$, the radius $R = 9.7$ km if $2GM/c^2R$ is to be 0.67. If it is assumed that the density is uniform, this corresponds to a density of $\sigma = 1.14\times 10^{15}$ g cm^{-3} $= 2.13\sigma_{\rm nucl}$, where $\sigma_{\rm nucl} = 3.7\times 10^{14}$ g cm^{-3} is the nuclear density. Bahcall and Wolf have considered the neutrino luminosities of neutron stars due to various reactions. As an example, we will use their results for the reaction $n+n \to n+p+e+\bar{\nu}$ to estimate the luminosity of our constant-density object. The expression for the (anti) neutrino luminosity for the above reaction is[12]

$$L_\nu = (\sigma/\sigma_{\rm nucl})^{2/3} T_9{}^8 (1+F)\times 10^{20}\ {\rm erg\,cm^{-3}\,sec^{-1}}, \quad (87)$$

where $F = [1 - 2.25(\sigma_{\rm nucl}/\sigma)^{4/3}]^{1/2}$, and T is the temperature in billions of degrees. Using $T_9 = 0.4$ and $\sigma = 2.13\sigma_{\rm nucl}$, $L_\nu = 1.94\times 10^{17}$ erg cm^{-3} sec^{-1}. Taking the typical energies of the neutrinos to be $E = kT/3$, the neutrino emission due to the above reaction is $N = 3.64\times 10^{25}$ cm^{-3} sec^{-1}. For $2GM/c^2R \simeq 0.67$, 0.05% of these neutrinos are gravitationally trapped. This means that in about 1.3×10^5 sec., the Fermi level of the degenerate sea rises to 100 keV, inhibiting the further emission of neutrinos with energies less than this.

The presence of a degenerate sea of neutrinos can affect the rates of nuclear reactions taking place inside the star. Moreover, if neutrinos are trapped, they will not be able to transport energy to the outside, and this can have serious consequences on the thermal evolution of the star. These considerations might become especially interesting in the case of a collapsing phase which leads to the formation of a compact, dense object. However, during the collapse the interior metric is time-dependent, and it will be necessary to extend the formalism to this case before the possibility of trapping can be investigated. Preliminary results by Dhurandhar and Vishveshwara[13] show that in the case of collapse, neutrino pulses and bursts are obtained. Detailed calculations regarding the possibility and the nature of trapping are in progress.

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