Comment on "Spinning Cosmic Strings and Quantization of Energy"

Mazur[1] has recently derived the intriguing result that in the background space-time of a spinning string, energy must *necessarily* be quantized. It is well known[2] that such a result obtains in the background space-time of a gravitational monopole: Time must be a periodic coordinate and energy must quantized. By use of an exact analogy between rotation in general relativity and magnetic fields, both of these results can be translated to the more familiar context of electromagnetism. The latter corresponds to Dirac's celebrated result that electric charge is quantized in the field of a magnetic monopole. However, the electromagnetic analog of the former is clearly not true. The spinning string corresponds, in electromagnetism, to a charged infinite solenoid or flux tube. The presence of a solenoid does not imply charge quantization. We therefore thought it worthwhile to examine more closely the arguments that led to the main conclusion of Ref. 1, and we find that energy *need not* be quantized in the background of a spinning string. This restores the complete parallel between the electromagnetic and gravitational cases.

The fallacy in the argument is elementary and becomes apparent if one correctly keeps track of the identification of points under multivalued coordinate transformations. Following Ref. 1, the metric of a spinning string is

$$ds^2 = -(dt - A\,d\phi)^2 + \rho^2\alpha^2 d\phi^2 + d\rho^2 + dz^2, \qquad (1)$$

where $\alpha = 1 - 4GM$, $A = -4GJ$, and M and J are two parameters representing the mass and angular momentum per unit length of the string. The ranges of the coordinates (t,ϕ,ρ,z) are $-\infty < t < \infty$, $0 \le \phi \le 2\pi$, $0 < \rho < \infty$, $-\infty < z < \infty$, and the points

$$(t,\phi,\rho,z) \text{ and } (t,\phi+2\pi,\rho,z) \qquad (2)$$

are identified. The transformation $T = t - A\phi$, $\phi' = \alpha\phi$ reduces the line element (1) to locally Minkowskian form. In terms of the new coordinates (T,ϕ',ρ,z), (2) translates into the identification of (T,ϕ',ρ,z) and $(T-2\pi A,\phi+2\pi\alpha,\rho,z)$. Implementing correctly the single valuedness of the wave function, we find

$$\Psi(T-2\pi A,\phi'+2\pi\alpha,\rho,z) = \Psi(T,\phi',\rho,z), \qquad (3)$$

and not

$$\Psi(T-2\pi A,\phi',\rho,z) = \Psi(T,\phi',\rho,z), \qquad (4)$$

as implied in Eq. (7) of Ref. 1. We now consider eigenstates of "energy" and "angular momentum,"

$$\Psi(T,\phi',\rho,z) = \exp\left(-\frac{i}{\hbar}(E'T - L'\phi'\right) f(\rho,z),$$

and find using (3) that E' and L' obey the condition

$$AE' + \alpha L' = \text{integer}, \qquad (5)$$

which does *not* imply the quantization of E'. If in the original coordinate system, the variables conjugate to t and ϕ are denoted E and L, respectively, then $E' = E$, $\alpha L' = L - AE$, and we see that Eq. (5) simply represents the quantization condition for the angular momentum L.

One is, of course, at liberty to identify points

$$(t,\phi,\rho,z) \text{ and } (t+P,\phi,\rho,z) \qquad (6a)$$

in the spinning-string space-time and arrive at a new space-time in which energy *is* quantized in units h/P. But this is not forced on us (unlike in the case of the monopole). If one does make the identification (6a) with the value

$$P = 2\pi A \qquad (6b)$$

suggested by Mazur, one finds that a *globally* well-defined coordinate transformation $T = t - A\phi$ reduces the (Kerr-type) spinning-string metric to

$$ds^2 = -dT^2 + \alpha^2\rho^2 d\phi^2 + d\rho^2 + dz^2,$$

a (Schwarzschild-type) nonspinning metric. Thus the condition (6) renders the angular momentum of the string unobservable. Conversely, requiring that the angular momentum of the string be unobservable leads to (6) and energy quantization. But this is an *extra* assumption that one need not feel obliged to make.

The question of causality-violating regions is peripheral to the whole discussion. One cannot avoid such regions by declaring time to be a periodic coordinate. In fact, this leads to causality violation over all space-time and not just for $\rho < \rho_0$.

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J. Samuel and B. R. Iyer
Raman Research Institute
Bangalore 560080, India

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[1] P. O. Mazur, Phys. Rev. Lett. **57**, 929 (1986).
[2] See references in J. Samuel and B. R. Iyer, Curr. Sci. **55**, 818 (1986).

